Mathematisches Forschungsinstitut Oberwolfach

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Geometrie

1. Oktober - 7. Oktober 2000

The present conference was organized by V. Bangert (Freiburg), Yu. D. Burago (St. Petersburg) and U. Pinkall (Berlin). The 47 participants came from 9 countries, about half of them from Germany and larger groups from Switzerland, the U.S.A., Russia and Belgium. The official program consisted of 20 lectures among them two mini-series (of 2 talks each) given by S. Buyalo (St. Petersburg) on "Spaces of curvature bounded above" and by S. Tabachnikov (University Park) on "Billiards".

The lectures covered a wide range of new developments in geometry with emphasis on the areas "submanifolds, Dirac operators and integrable systems" and "Riemannian geometry and its generalizations". Thursday evening was devoted to geometric videos and to a computer demonstration by K. Polthier.

Every participant had prepared a poster on his/her recent research. These were posted in the lecture building and stimulated scientific exchange among the participants that ranged from private discussions to informally organized additional talks.

Abstracts

A Modern Introduction to Finsler geometry

Juan Carlos Alvarez Paiva

A Finsler manifold is a manifold together with the choice of a norm on each tangent space. The length of a curve is defined as the integral of the norms of its velocity vectors and the distance between two points is defined as the infimum of the lengths of all curves joining them. In this talk I stress the interactions between metric geometry, calculus of variations, and convex geometry as the way to approach Finsler geometry.

After defining the volume of an n-dimensional Finsler manifold as the symplectic volume of its unit codisc bundle divided by the volume of the n-dimensional euclidean unit ball, I discuss the recent results of Burago, Alvarez and Fernandes on the minimality of k-planes in finite-dimensional normed spaces and Finsler metrics on $\mathbb{R}^n$ for which geodesics are straight lines (Hilbert's fourth problem).

We shall also study the geometry of unit spheres on normed spaces and sketch the recent proof by Alvarez of the following conjecture of J.J. Schäffer.

The girth of a normed space - the infimum of the lengths of all curves on the unit sphere joining a pair of antipodes - equals the girth of its dual.

Properties of the stable norm in codimension one

Franz Auer (joint work with Victor Bangert)

Let (M, g) be an n-dimensional compact oriented Riemannian manifold. The *stable norm* $\|h\|$ of $h \in H_{n-1}(M,\mathbb{R})$ is defined as the infimum of the volumes $\text{vol}_g(c) = \Sigma|r_i|\text{vol}_g(\sigma_i)$ of all Lipschitz cycles $c = \Sigma r_i\sigma_i$ representing h. It can be equivalently characterized as follows: $\|h\|$ is the infimum of the L^1 norm $|\omega| = \int |\omega(x)|_g \, d\,\text{vol}_g$ over all closed 1-forms ω representing the Poincaré dual $\alpha^h \in H^1(M,\mathbb{R})$ of h; or: $\|h\|$ is the *minimum* of the masses $\mathbf{M}_g(T)$ of all closed $(n-1)$-currents T representing h.

Properties of the stable norm are described by properties of its unit norm ball $B = \{h \in H_{n-1}(M,\mathbb{R}) \mid \|h\| = 1\}$.

In the following theorem $h \cdot h' \in H_{n-2}(M,\mathbb{R})$ denotes the intersection class of h and $h' \in H_{n-1}(M,\mathbb{R})$.

Theorem 1. *If ∂B contains the segment $\overline{hh'}$ then $h \cdot h' = 0$.*

To formulate the next theorem we need to introduce the *Abelian covering* $p : \bar{M} \to M$, where $\bar{M}$ is the quotient $\tilde{M}/\ker H$ of the universal covering $\tilde{M}$ by the kernel of the natural map $H : \pi_1(M) \to H_1(M,\mathbb{R})$. Note that the image $H_1(M,\mathbb{Z})_{\mathbb{R}} := H(\pi_1(M)) \subseteq H_1(M,\mathbb{R})$ acts on $\bar{M}$ as the group of decktransformations and is isomorphic to $\mathbb{Z}^{b_1(M)}$, where $b_1(M)$ is the first Betti number of M.

Theorem 2. *If $H_1(\bar{M},\mathbb{R}) = 0$ then ∂B is strictly convex.*

Theorem 3. *On every compact manifold of dimension $n \geq 3$ there exists a metric g such that the unit norm ball $B = B_g$ of the associated stable norm on $H_{n-1}(M,\mathbb{R})$ is strictly convex.*

On the other hand, there are examples of Riemannian manifolds where B is a polytope. We say that the direction of $h \in H_{n-1}(M,\mathbb{R})$ is *irrational* if h is not a multiple of an integer class.

Theorem 4. *Assume* $b := b_1(M) \geq 2$. *Then every face* F *of* B *is a polytope with a most* $2(b-1)$ *vertices. If* e *is the number of vertices of* F *with irrational direction then* $e \leq b-1$ *and* F *has at most* $2(b-1)-e$ *vertices.*

Normal biquotients with completely integrable geodesic flows

Yaroslav V. Bazaikin

Let M be a Riemannian manifold of dimension n. The geodesic flow on M is integrable, if there exist first integrals $f_1, \ldots, f_n : T^*M \to R$ which are independent almost everywhere and in involution (i.e. all Poisson brackets $\{f_i, f_j\}$ vanish).

Thimm proposed a method which allows one to construct first integrals using isometries of M, and integrability of the geodesic flow under Thimm's method takes place if the isometry group of M is large. He used this method for real and complex Grassmanian manifolds, which are homogeneous spaces. The next natural step was done by Paternain and Spatzier: they used Thimm's method for the Gromoll-Meyer sphere and Eschenburg spaces, which are biquotients of Lie group. The biquotient construction is the following. Let G be a Lie group with some (usually homogeneous) metric and $U \subset G \times G$ be a Lie subgroup which acts on $G : (g_1, g_2) \in U : g \mapsto g_1 g g_2^{-1}$. If this action is free and isometric, the factor-space of G by U is a Riemannian manifold G/U and is called biquotient of G.

Biquotients were introduced by Gromoll and Meyer for describing the first nonnegatively curved metric on one exotic Milnor sphere. Then biquotients were used for constructions of positively curved metrics (first such examples were found by Eschenburg).

In order to study properties of known examples of positively curved metrics, the author used Thimm's method for normal biquotients of general form and obtained the following estimate for the numbers of independent first integrals:

Theorem. *Let* $M = H\backslash G/K$ *be a biquotient of a compact Lie group* G *with biinvariant metric. Let* $v = (h+k)^{\perp} \subset g$, *where* g, h, k *are the Lie algebras of the groups* G, H, K. *Consider some chains of Lie algebras*

$$h = h_0 \subset \ldots \subset h_l = g, k = k_0 \subset \ldots \subset k_m = g,$$

and $r_1 = rank(\{h_i\}_i, v), r_2 = rank(\{k_i\}_i, v), r_3 = rank(G)$. *Then the geodesic flow on* M *has at least* $r_1 + r_2 - r_3$ *independent integrals in involution.*

The definition of rank of a chain is the following. If $h \subset g$ is a short 2-chain and $v \subset g$, then let $rank((g,h),v) = \max_{X \in v} \dim pr_{h^{\perp}}(Z(Ker(ad(X))))$. For an arbitrary chain $\{h_i\}_i, v \subset h_l$ let $rank(\{h_i\}_i, v) = \Sigma_{i=0}^{l-1} rank((h_{i+1}, h_i), pr_{i+1}(v))$, where $pr_i : h_l \to h_i$ is the orthogonal projection. Using this theorem the author established the following fact:

Proposition. *All known manifolds with nonhomogeneous positively curved metrics have ones with completely integrable geodesic flows.*

Harmonic maps in unfashionable geometries

Francis E. Burstall

Many special surfaces in classical differential geometry are characterised by the property that an appropriate Gauss map is harmonic and then the integrable features of their geometry (spectral deformation, Bäcklund transformation, algebro-geometric solutions and so on) can be inferred from those of harmonic maps.

Thus a special case of the Ruh-Vilms theorem asserts that a surface has constant mean curvature if and only if its Gauss map is a harmonic map into the 2-sphere and, similarly, a surface has constant negative Gauss curvature if and only if its Gauss map is Lorentz harmonic with respect to the metric induced on the surface by the second fundamental form.

It is interesting that harmonic maps into pseudo-Riemannian symmetric spaces also arise in this context: a surface in S^n is Willmore if and only if its *conformal Gauss map* is harmonic. This is a map into the indefinite Grassmannian that parametrises 2-spheres in S^n and geometrically represents a congruence of 2-spheres having partial second order contact with the Willmore surface. In this talk I shall report on work in progress with Udo Hertrich-Jeromin and show that similar constructions are available in Lie sphere and projective differential geometry. Moreover, both geometries can be treated at the same time in a practical manifestation of the celebrated line-sphere correspondence of Lie. In both cases, surfaces in 3-space are studied via their lifts into a contact manifold which can be viewed as the space of lines in a quadric. From this lift we construct a "Gauss map" taking values in a certain Grassmannian which can be viewed as a congruence having partial third order contact with the underlying surface. In Lie sphere geometry our Gauss map is the Lie congruence of cyclides while in projective differential geometry it is the Lie congruence of quadrics. The Gauss map we construct is conformal in an appropriate sense and its energy integral defines a functional on the underlying surface whose extremals are the minimal surfaces of Lie sphere and projective geometry. I shall give a simple, conceptual and uniform argument to show that a surface is minimal in Lie sphere or projective geometry precisely when its Gauss map is harmonic.

Spaces of Curvature Bounded Above, Parts 1 and 2

Sergei V. Buyalo

A short survey von Curvature Bounded Above (CBA) spaces. Local properties, constructions and results are discussed.

Contents

1. Motivation
2. Defining CBA
3. Propagation from local to global
4. Recognizing CBA-spaces
5. Different types of convergence
6. Infinitesimal properties of CBA-spaces
7. Gluing CBA-spaces
8. 2-dimensional polyhedra

For details see www.pdmi.ras.ru/preprint/2000/index.html
Preprint 15

Circle Patterns and Integrable Systems

Tim Hoffmann

Circle patterns and packings have been widely studied in the past, as they can be viewed as discretizations of conformal maps. This basically started with the Theorem of Koebe ('34) which says that any triangulation of the Riemann sphere may be realized as the tangency graph of a circle packing and the resulting circle configuration is unique (up to inversions and automorphisms). Later Thurston conjectured ('85) that one can approximate the Riemann map with this. This was proven '87 by Rodin and Sullivan. But until recently there was no relation to integrable systems. This relation first appeared when Bobenko and Pinkall noticed that Schramm's SG-circle patterns can be viewed as special case of their integrable discrete conformal maps. With this on hand Bobenko and Agafonov were able to construct a Schramm pattern analogue of z^α using methods from integrable theory ('00). The only explicitly known examples of circle patterns with hexagonal combinatorics have been the Dayle spirals. This could be extended by Bobenko and H. ('00) to conformally symmetric packings. They are characterized by the condition that every elementary flower of 7 circles possesses a Möbius involution fixing the centercircle and sending the i-th neighbor to the $(i+3)$rd. This gives a 3-parameter family of packings that can be described explicitly and that contains the Dayle spirals as a special case. Another example of an integrable circle pattern was found by Bobenko, H. and Suris in '00. It is a hexagonal pattern where in each point three circles intersect and the 6 intersection points on each circle have multi-ratio $=-1$ $\left(\text{i.e. } \frac{z_1-z_2}{z_2-z_3}\frac{z_3-z_4}{z_4-z_5}\frac{z_5-z_6}{z_6-z_1}=-1\right)$. Again using integrable methods one can construct a pattern analogue of the function z^α.

Real surfaces in complex surfaces

Wilhelm Klingenberg

We study real surfaces $\Sigma^2 \looparrowright (M^4, J)$ in almost-complex four-manifolds, and in particular, the following phenomena:

a) complex points: $\sigma \in \Sigma : T_\sigma\Sigma = JT_\sigma\Sigma$ of embedded surfaces.

b) double points of immersed totally real surfaces.

We give a smoothing construction that transforms a) to b). Secondly, we study the moduli space of J-holomorphic curves of disc-type in M^4

$$\mathcal{M}\left(M^4,\Sigma^2\right)=\left\{f:(D,\partial D)\to\left(M^4,\Sigma^2\right):\overline{\partial}_J f=0\right\}$$

bounded by Σ^2. We prove for $\sigma =$ elliptic complex point of Σ:

Theorem α) $\partial\mathcal{M} \sim$ complex points of Σ.
β) $\Sigma^2 \in C^{k,\alpha}$ - smooth $\Rightarrow \overline{\mathcal{M}}$ is a $C^{\frac{k}{2},\frac{\alpha}{2}}$ smooth 1-manifold.

The proof involves an asymptotic analysis of a Riemann - Hilbert problem via a blow-up of the CR-singularity of Σ that transforms a) to b). The regularity result is optimal.

Total curvature of complete submanifolds of $\mathbb{E}^n$

Wolfgang Kühnel (joint work with Franki Dillen)

The classical Cohn-Vossen inequality states that for any Riemannian 2-manifold the difference between $2\pi\chi(M)$ and the total curvature $\int_M K\, dA$ is always nonnegative. For complete open surfaces in E^3 this curvature defect can be interpreted in terms of the length of the curve "at infinity". The goal of this paper is to investigate higher dimensional analogues for open submanifolds of euclidean space with cone-like ends. This is based on the extrinsic Gauss-Bonnet formula for compact submanifolds with boundary and its extension "to infinity". It turns out that the curvature defect can be positive, zero, or negative, depending on the shapespac of the ends "at infinity". We give an explicit example of a 4-dimensional hypersurface in E^5 where the curvature defect is negative, so that the direct analogue of the Cohn-Vossen inequality does not hold. Furthermore we study the variational problem for the total curvature of hypersurfaces. It turns out that for open hypersurfaces with cone-like ends the total curvature is stationary if and only if each end has vanishing Gauss-Kronecker curvature in the sphere "at infinity". For this case of stationary total curvature we prove a result on the quantization of the total curvature.

Can one tie a knot with 1 foot of 1 inch (or 12 cm of 1 cm) rope?

Robert Kusner

The geometric complexity of a knot might be measured by the shortest length of rope with given diameter needed to tie the knot; a combinatorial measure of knot complexity is the minimum number of crossings in a planar representation of the knot. Biologists studying knotted DNA are now interested in ropelength, while mathematicians have used the minimum crossing number for more than a century to help classify knots. Both of these knot invariants are simple to define, yet difficult to compute. This talk explores the relationship between them, using some simple notions from the theory of minimal surfaces. Two new geometric ideas - the overcrossing number of, and the cone angle subtended by, a knot or link - are introduced; these bring us very close to answering the title question!

Bilipschitz embeddings of metric spaces into space forms

Urs Lang (joint work with Conrad Plaut)

The question of which metric spaces admit a bilipschitz embedding into some (finite-dimensional) euclidean space has received a lot of attention in recent work. The results obtained so far indicate that there is no simple answer to this question. An obvious fact is that a metric space X which is bilipschitz embeddable into a euclidean space is necessarily *doubling*, i.e., there is a constant L such that for all $x \in X$ and $r > 0$, the closed ball $B(x, 2r)$ can be covered by L closed balls of radius r. On the other hand, the Heisenberg group with its Carnot-Carathéodory metric is an example of a doubling metric space that is not bilipschitz embeddable in any euclidean space. We develop some basic geometric tools for studying this problem, such as decomposition and gluing and embedding via distance functions. As it comes to global issues we admit also real hyperbolic spaces as targets. The theorem stated below subsumes some of the obtained results. A *geodesic bicombing* of a geodesic metric space (X, d) is an assignment of a (minimizing) geodesic $c_{xz} : [0,1] \to X$ from x to z to every pair $(x, z) \in X \times X$. We call a geodesic bicombing $\{c_{xz}\}$ *weakly convex* if there is a constant $\gamma \geq 1$ such that

$$d(c_{xz}(t), c_{xz'}(t)) \leq \gamma t d(z, z')$$

for all $x, z, z' \in X$ and $t \in [0,1]$.

Theorem. *Let (X, d) be a metric space with a weakly convex geodesic bicombing $\{c_{xz}\}$. Suppose that for all $x, y \in X$ and $t \in (0,1)$ there exists a $z \in X$ with $y = c_{xz}(t)$. Then the following holds: X admits a bilipschitz embedding into some euclidean space if and only if X is doubling. X admits a bilipschitz embedding into some real hyperbolic space if and only if X is Gromov hyperbolic and doubling up to some scale. Moreover, if X is complete, then in either case the image of such an embedding is a lipschitz retract in the target space.*

The proof of the "hyperbolic" part of this result uses a quasi-isometric embedding theorem for Gromov hyperbolic spaces proved recently by Bonk and Schramm. We discuss some applications to isoperimetric filling inequalities for cycles in all dimensions. The new theory of currents in metric spaces due to Ambrosio and Kirchheim provides a suitable framework to formulate isoperimetric inequalities in metric spaces. Our embedding results can be used to extend Gromov's construction of isoperimetric fillings for cycles in Hadamard manifolds to a class of weakly convex metric spaces, as well as to prove in a simple way (higher-dimensional) linear isoperimetric inequalities for certain Gromov hyperbolic spaces.

Quaternionic Plücker Formula and Dirac Eigenvalue Estimates

Franz Pedit (joint work with D. Ferus, K. Leschke, U. Pinkall)

Let L be a quaternionic holomorphic line bundle of degree d over a compact Riemann surface M of genus g, and let $H \subset H^0(L)$ be an $n+1$ dimensional linear system. Then the Willmore energy of L satisfies the quaternionic Plücker formula:

$$\frac{\mathcal{W}}{4\pi} \geq (n+1)(n(1-g)-d) + \text{ord } H,$$

where ord H is the total singularity order of the linear system computed similarly to the complex holomorphic case. We give three applications of this formula:

1. Classical Plücker formula for holomorphic curves in $\mathbb{CP}^n$
2. Lower bounds for an eigenvalue λ of multiplicity m of the Dirac operator over a surface of genus g:

$$\text{area}_M \lambda^2 \geq \begin{cases} 4\pi m^2 & g = 0 \\ \frac{\pi}{g}(m^2 - g^2) & g \geq 1 \end{cases}$$

3. Area estimates for constant mean curvature tori in 3-space in terms of their spectral genus g:

$$\text{area} \geq \begin{cases} \frac{\pi}{4}(g+2)^2 & g \text{ even} \\ \frac{\pi}{4}((g+2)^2 - 1) & g \text{ odd} \end{cases}$$

Isospectral Metrics on Spheres

Dorothee Schüth

The Laplace operator acting on functions on a closed Riemannian manifold has a discrete spectrum of eigenvalues. Two manifolds are *isospectral* if they have the same spectrum. The "classical" way of constructing such manifolds is by the so-called Sunada method. With this method, families of isospectral manifolds arose as quotients of a common Riemannian covering by different discrete subgroups of isometries. In particular, those manifolds were always locally isometric to each other and (at least in the case of closed manifolds) never simply connected.

Various examples of locally nonisometric and even simply connected isospectral manifolds were discovered in the 1990's. All of these manifolds arose as principal torus bundles endowed with certain invariant Riemannian metrics. The dimension of the torus must be at least two in order to allow nontrivial examples. Since a *sphere* is never realizable as a principal $T^{k\geq 2}$-bundle, it seemed at first hopeless to obtain in this way any examples of isospectral metrics on spheres (a longstanding dream of the isospectral community). Recently, however, Carolyn Gordon developed a generalized version in which the torus actions used for the construction are no longer assumed to be free, but just effective. Using her new method, she obtained continuous families of isospectral metrics on the spheres $S^{n\geq 8}$ of dimension at least eight.

We reformulate Gordon's method in a slightly streamlined way and apply it to construct pairs of isospectral metrics on S^6 and even on S^5. We also obtain continuous isospectral families of metrics on S^7. Moreover, we show that in each of these examples the metrics can be chosen equal to the standard metric outside subsets of arbitrarily small volume.

Connections with Proper Symplectic Holonomy Groups

Lorenz Schwachhöfer

Let (M, ω) be a symplectic manifold. We call a torsion free connection ∇ on M *symplectic* if $\nabla\omega \equiv 0$. This is equivalent to saying that the holonomy of ∇ is contained in the symplectic group $\mathrm{Sp}(V)$. We say that such a connection has *proper symplectic holonomy* if the holonomy of ∇ is irreducible and properly contained in $\mathrm{Sp}(V)$.

The classification of possible irreducible holonomy groups of torsion free connections has been completed some time ago. In the process of this classification, some holonomies were discovered which were not contained in the classical list of Berger. As it turns out, the missing entries are precisely the proper symplectic holonomy groups and there is a one-to-one correspondence between proper symplectic holonomy groups and quaternionic symmetric spaces.

Recently, the following has been achieved.

Theorem 1 *For each connection with proper symplectic holonomy, the dimension of the local symmetry group is at least the rank of the holonomy group (and thus, in particular, positive). Moreover, equality holds iff the local symmetry group is abelian.*

Since there is always a large amount of symmetries, it is a reasonable question to consider *homogeneous* manifolds with proper symplectic holonomy. Here, our results are the following.

Theorem 2 *A manifold with a connection with proper symplectic holonomy is locally homogeneous iff its scalar curvature is constant. This happens iff the first and second order derivatives of the scalar curvature vanish at a single point of M.*

Theorem 3 *For each proper symplectic holonomy group H, there are - up to coverings - finitely many homogeneous spaces $M = G/K$ with a G-invariant torsion free connection with holonomy H.*

1. *The possible pairs (G, K) can be given explicitly in each case.*
2. *Only one of these homogeneous spaces is reductive. Its holonomy is the 4-dimensional representation of $H \cong SL(2, \mathbb{F})$ ($\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$).*

The Second Hull of a Knotted Curve

John M. Sullivan (joint work with J. Cantarella, R. Kusner)

The convex hull of a set K in space consists of points "enclosed" (in a certain sense) by K. When K is a closed curve, we define its higher hulls, consisting of points which are "multiply enclosed" by the curve. Our main theorem shows that if a curve is knotted then it has a nonempty second hull.

Billiards, Parts 1 and 2.

Serge Tabachnikov

This is a survey of selected topics on mathematical billiards. The topics are:

1). definitions and principal constructions: conventional and nonconventional billiards (Finsler billiards, dual billiards, projective billiards)

2). periodic orbits in multi-dimensional billiards (recent result by M. Farber and myself)

3). integrable billiards, around Birkhoff's conjecture (Bialy's theorem; generalized Birkhoff's conjecture for outer billiards)

4). geometry of projective billiards and exact transverse line fields (including complete integrability of geodesically equivalent metrics).

Dirac operators and conformal invariants of tori in 3-space

Iskander A. Taimanov

We show how to assign to any immersed torus in $\mathbb{R}^3$ or S^3 a Riemann surface such that the immersion is described by functions defined on this surface. We call this surface the spectrum or the spectral curve of the torus. The spectrum contains important information about conformally invariant properties of the torus and, in particular, relates to the Willmore functional. We propose a simple proof that for isothermic tori in $\mathbb{R}^3$ (this class includes constant mean curvature tori and tori of revolution) the spectrum is invariant with respect to conformal transformations of $\mathbb{R}^3$. We show that the spectral curves of minimal tori in

S^3 introduced by Hitchin and of constant mean curvature tori in $\mathbb{R}^3$ introduced by Pinkall and Sterling are particular cases of this general spectrum. The construction is based on the Weierstrass representation of closed surfaces in $\mathbb{R}^3$ and the construction of the Floquet-Bloch varieties of periodic differential operators.

A-integrability of geodesic flows

PETER J. TOPALOV

We consider a class of (pseudo) Riemannian metrics, called A-integrable metrics, that admit integrals in involution of a special form. One of the main properties of these metrics is the existence of hierarchies. Any A-integrable metric defines a big family (hierarchy) of A-integrable metrics and any A-integrable metric from a given hierarchy determines uniquely the whole hierarchy. Many classical integrable Hamiltonian systems that appear in geometry and mechanics lie in such hierarchies. For example, the hierarchy that corresponds to the standard sphere includes also the Poisson sphere and the ellipsoid. The hierarchy of the Hyperbolic plane includes the analog of the Poisson sphere that corresponds to the free motion of the rigid body in Minkowski space. The Clebsch case of motion of the rigid body corresponds to the hierarchy of the Euclidean space. The main properties of the A-integrable metrics are investigated.

Exotic spheres and positive Ricci curvature

WILDERICH TUSCHMANN

The relations between curvature and differentiable structures on spheres are still quite obscure. Whereas many exotic spheres admit metrics with positive Ricci curvature, not a single such sphere is known to carry a positive sectional curvature metric, and in certain dimensions half of the exotic spheres do not even support positive scalar curvature.
In view of Cheng's maximal diameter theorem - though this result is known to be not even topologically rigid - one may ask whether it is possible to isolate the standard sphere among all other topological spheres or, more generally, among all other complete Riemannian manifolds with positive Ricci curvature by using merely curvature and diameter assumptions.
The following differentiable diameter sphere theorem which I discussed in my talk gives a positive answer to this question and shows that any violation of smooth rigidity, in particular any which is modelled on an exotic sphere, must be accompanied by a blow-up of sectional curvatures:

For any given m and C there exists a positive constant $\epsilon = \epsilon(m, C) > 0$ such that any m-dimensional complete Riemannian manifold with $Ricc \geq m-1$, $K \leq C$ and diameter $\geq \pi - \epsilon$ is diffeomorphic to the standard m-sphere.

Edited by Ursula Wöske

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