MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 41/2000

Arbeitsgemeinschaft: Operaden und ihre Anwendungen

08.10. - 14.10.2000

Die Arbeitsgemeinschaft zum aktuellen Thema "Operaden und ihre Anwendungen" stand unter der Leitung von C.-F. Bödigheimer (Bonn), J.-L. Loday (Strasbourg) und B. Richter (Bonn). Die Leiter hatten ein Programm mit 17 Vorträgen zusammengestellt und nach dem Eingang der Anmeldungen an die Teilnehmer verteilt. Die Vorträge wurden während der Tagung noch durch rege Diskussionen, Anmerkungen und Ausblicke sowie durch weitere kurze Vorträge und Diskussionsrunden ergänzt. Die von den Sprechern verfassten Kurzfassungen der Vorträge sind in diesem Tagungsbericht zusammengestellt.
Es war das Ziel der Arbeitstagung, möglichst viele (inbesondere junge) Wissenschaftler mit der "Operadensprache" vertraut zu machen, die Anwendungsergebnisse in den verschiedenen Zweigen der Mathematik und Physik vorzustellen und einen Rahmen für die Kommunikation zu schaffen, um die offenen Probleme anzugehen.
Folgende inhaltliche Schwerpunkte wurden diskutiert:

- Algebraische Operaden und die Koszul-Eigenschaft
- Die Struktur von Schleifenräumen (die Ursprünge der Operadentheorie)
- Operaden in der Theorie der Modulräume und der Abbildungsklassengruppen
- Anwendungen auf Vertexalgebren und die Grothendieck-Teichmüller Gruppe

Während der Tagung wurde durch Abstimmung das Thema der nächsten Arbeitsgemeinschaft festgelegt. Es wird die "Stringtheorie" sein.

Vortragsauszüge

Michael Brinkmeier

Operads and Monads

Let $(\mathcal{C}, \otimes, k)$ be an arbitrary complete and cocomplete symmetric monoidal category, such that the left and right distributivity laws

$$(A \sqcup B) \otimes C \simeq (A \otimes C) \sqcup (B \otimes C) \text{ and } A \otimes (B \sqcup C) \simeq (A \otimes B) \sqcup (A \otimes C)$$

hold. Furthermore let Σ the category of finite sets $\underline{n} = \{1, \ldots, n\}$, including the empty set $\underline{0}$, and bijections. A *symmetric object* A in $\mathcal{C}$ is a functor $A : \Sigma^{op} \to \mathcal{C}$. A morphism of two symmetric objects is natural transformation of the two functors.
On the category $\Sigma\mathcal{C}$ of symmetric objects and morphisms between them, a monoidal (but not symmetric) structure can be defined, whose product is given by

$$(A \square B)[n] := \bigsqcup_{m \geq 0} A[m] \otimes_{\Sigma_m} B[m, n]$$

where Σ_m denotes the symmetric group and

$$B[m, n] := \bigsqcup_{n_1 + \cdots + n_m = n} \left(\bigotimes_{i=1}^{m} B[n_i] \otimes_{\Sigma_{n_1} \times \cdots \times \Sigma_{n_m}} \Sigma_n \right),$$

or, in more algebraic terms

$$B[m, n] := \operatorname{Ind}_{\Sigma_{n_1} \times \cdots \times \Sigma_{n_m}}^{\Sigma_n} \left(\bigsqcup_{n_1 + \cdots + n_m = n} \bigotimes_{i=1}^{m} B[n_i] \right).$$

The unit object of this monoidal structure on $\Sigma\mathcal{C}$ is the symmetric object I given by

$$I[n] = \begin{cases} k & \text{if } n = 1 \\ \emptyset & \text{otherwise} \end{cases}$$

where $\emptyset$ is the initial object of $\mathcal{C}$.

Definition: *An* operad (A, μ, η) *is a monoid in the monoidal category* $(\Sigma\mathcal{C}, \square, I)$, *i.e. it consists of a symmetric object* A, *a* multiplication $\mu : A \square A \to A$ *and a* unit $\eta : I \to A$, *such that the following diagrams commute.*

$$\begin{tikzcd} A\square A\square A \arrow[r, "\mu(\mu\otimes A)"] \arrow[d, "\mu(A\otimes\mu)"'] & A\square A \arrow[d, "\mu"] \\ A\square A \arrow[r, "\mu"'] & A \end{tikzcd} \qquad \begin{tikzcd} I\square A \arrow[r, "\eta\otimes A"] \arrow[dr, "\simeq"'] & A\square A \arrow[d, "\mu"] & A\square I \arrow[l, "A\otimes\eta"'] \arrow[dl, "\simeq"] \\ & A & \end{tikzcd}$$

To each symmetric object A an endofunctor $A(-) : \mathcal{C} \to \mathcal{C}$, the *Schur functor*, is assigned, which is given on objects by

$$A(X) := \bigsqcup_{n \geq 0} A[n] \otimes_{\Sigma_n} X^n.$$

For this functor we have natural isomorphisms

$$(A\Box B)(X) \simeq A\left(B(X)\right) \text{ and } I(X) \simeq X.$$

If A is an operad this functor is a *monad* in $\mathcal{C}$, i.e. a monoid in the monoidal category of endofunctors of $\mathcal{C}$ with the composition as product.

Definition: *An algebra (X, ϵ) over an operad A consists of an object X in $\mathcal{C}$ and a morphism $\epsilon : A(X) \to X$, such that the following diagrams commute.*

$$\begin{tikzcd} A\left(A(X)\right) \simeq (A\Box A)(X) \arrow[r, "\mu(X)"] \arrow[d, "A(\epsilon)"'] & A(X) \arrow[d, "\epsilon"] \\ A(X) \arrow[r, "\epsilon"'] & X \end{tikzcd} \qquad \begin{tikzcd} I(X) \arrow[r, "\eta(X)"] \arrow[dr, "\simeq"'] & A(X) \arrow[d, "\epsilon"] \\ & X \end{tikzcd}$$

A morphism $f : (A, \epsilon) \to (A', \epsilon')$ is a morphism $x : A \to A'$ in $\mathcal{C}$, which is compatible with the strucutres, i.e.

$$f \circ \epsilon = \epsilon' \circ A(f).$$

Stephan Mohrdieck

Important Examples of Operads

Operads are thought to encode certain structure. If an object A of the underlying category $\mathcal{C}$ carries this structure the corresponding operad should act on it. This is the notion of an algebra over an operad.
To be more precise let $C(k)_{k\in\mathbb{N}}$ be an operad in $\mathcal{C}$. An object $A \in \mathrm{Ob}(\mathcal{C})$ is a $C(k)_{k\in\mathbb{N}}$-algebra, if for all natural numbers k there are morphisms

$$\Theta : \quad C(k) \otimes A^{\otimes k} \to A, \tag{1}$$

fulfilling the conditions of associativity, equivariance with respect to the symmetric groups Σ_k, $k \in \mathbb{N}$ and unitality.
The basic algebraic examples are in the category $\mathcal{C} = R-$mod of modules over a commutative ring R with unit 1. They are called Ass, Com and Lie corresponding to the associative, commutative and Lie-algebras.
In particular we have:
$Ass(k) = R[\Sigma_k]$, the group ring of the symmetric group, which acts on $Ass(k)$ by the naural right action.
$Com(k) = R$ furnished with the trivial Σ_k-representation.
Now, $A \in R-$ mod is an associative resp. commutative Lie-algebra, iff A is an $Ass(k)_{k\in\mathbb{N}}$, $Com(k)_{k\in\mathbb{N}}$ resp $Lie(k)_{k\in\mathbb{N}}$-algebra.
An important topological example is the operad of little n-cubes $C_n(k)_{k\in\mathbb{N}}$.
Here $C_n(k)$ is given by the embedding of k copies of the unit cube I^n in $\mathbb{R}^n$ into I^n such that the images of the different copies of the unit cube are disjoint. Furthermore we require each embedding to be affine linear and such that the axes of the small cubes are parallel to axes of the big cube.

This operad acts in the category of pointed topological spaces. One easily sees that the n-fold loop space $\Omega^n X := Hom_{\mathcal{C}}(S^n, s_0; X, x_0)$ is a $C_n(k)_{k\in\mathbb{N}}$-algebra. (Here, S^n is the n-dimensional sphere).
A theorem, the so called recognition principle for n-fold loop spaces states that in a way the inverse is also true:
Recognition Principle: If a pointed connected space (Y, y_0) is an algebra over the operad of little n-cubes, then it is weakly homotopy equivalent to an n-fold loop space $\Omega^n X$ for a suitable (X, x_0).

Jörg Sixt, Fabian Theis

Koszul Duality I

After introducing Hochschild (co)homology for graded k-algebras over a field k, we start studying quadratic algebras. A quadratic algebra is of the form $A := A(V, R) := T(V)/(R)$ with V a finite dimensional k-vectorspace, $T(V)$ its tensor algebra and R a subspace of $V^{\otimes 2}$. The advantage of graded algebras lies in the fact that they have quadratic duals $A^! := A(V^*, R^\perp)$. Here, $R^\perp$ denotes the perpendicular subspace. By analyzing the classical Cobar complex, one sees that the diagonal cohomology of A is $A^!$, i.e.

$$H_p^p(A) = A_p^!.$$

A is called Koszul if the cohomology vanishes everywhere else. This is equivalent to saying that $H_1^1(A)$ already generates H^* as a bigraded algebra.
By using what Loday calls a twisting cochain we equip the tensor product of a d.g. coalgebra and a d.g. algebra with a differential. This is then applied to define the Koszul complex $K(A) := A^{!*} \otimes A$ and to show that $K(A)$ is a subcomplex of the Bar Hochschild complex $B(k, A, A)$.
In the main theorem of our talk, we prove that the following are equivalent:

(i) K(A) is acyclic, i.e. a free resolution of k and hence a smaller resolution than the Bar resolution.

(ii) A is Koszul

(iii) $A^!$ is Koszul

(iv) The canonical inclusion of $A^{!*}$ in the (cohomologically written) $B(k, A, k)$ is a quasi-isomorphism.

From (iv), one gets immediately that $B(k, A, k)^*$ is a minimal model of the algebra $A^!$. A corollary then is the Koszul duality for Koszul algebras:

$$H^* H^* A = A$$

Ulrich Bunke, Thomas Lehmkul

Koszul Duality II

We introduced linear operads and collections. There is a forgetful functor mapping operads to collections, which admits an adjoint, the free operad on a collection. Its existence was shown using a construction, which involved labeled trees. Working in the differential graded case we defined the dual operad (cobar construction) as the free operad on a suitable collection derived from the original operad. Applying the cobar construction twice we proved that the operad obtained in this way is quasi isomorphic to the original one.
We introduce the concept of a quadratic operad and its quadratic dual. The latter is the image of the cobar construction. If this projection is a quasi-isomorphism, the quadratic operad is called *Koszul*. For any algebra over a quadratic operad we construct a complex, which is the usual Hochschild complex in case of the associative operad. The main theorem states that a quadratic operad $\mathcal{P}$ is Koszul if and only if for any free $\mathcal{P}$-algebra this complex is exact in degree < -1.

Lutz Hille

Classical Homology Theories

Quadratic operads. All operads are k–linear, where k is a ground field of characteristic zero. We consider the free operad $F(E)$ generated by elements in degree two, that is the collection E consists only of $E(2)$, all other spaces $E(i)$ for $i > 2$ are zero. Moreover, let R be a Σ_3 invariant subspace in $\mathrm{Ind}_{\Sigma_2}^{\Sigma_3}(E)$. We form the quotient of the free operad $F(E)$ by the ideal generated by R and denote the resulting operad by $\mathcal{P}(E, R)$. Such an operad is called *quadratic*. For any quadratic operad we can form the *quadratic dual operad* $\mathcal{P}(E^{\vee}, R^{\perp})$, where $E^{\vee}$ denotes the dual space twisted with the sign representation and $R^{\perp}$ denotes the perpendicular subspace.

The cobar complex associated with a quadratic operad. To any quadratic operad and any $n \geq 2$ we can ssociate a complex, called the *cobar complex*,

$$\mathcal{P} \otimes \det(k^n) \longrightarrow \bigoplus_{\substack{n\text{–trees}T\\ |T|=1}} \mathcal{P}(T)^* \otimes \det(T) \longrightarrow \cdots \bigoplus_{\substack{n\text{–trees}T\\ |T|=n-2}} \mathcal{P}(T)^* \otimes \det(T).$$

Moreover, we define a dual operad $\mathbf{D}(\mathcal{P})$ by twisting the complexes C_n with the determinant operad $\Lambda(n)$. The collection of complexes $\mathbf{D}(\mathcal{P})$ has a natural structure of a dg-operad and there exists a natural isomorphism

$$\mathbf{D}(\mathbf{D}(\mathcal{P})) \longrightarrow \mathcal{P}.$$

Homology of algebras over a quadratic operad. Let $\mathcal{P} = \mathcal{P}(E, R)$ be a quadratic operad and A an algebra over $\mathcal{P}$. We define a complex

$$C_n(A) := \left(A^{\otimes n} \otimes \mathcal{P}^!(n)^{\vee}\right)_{\Sigma_n}.$$

The differential is defined similar to the differential in the cobar complex, where first we replace $\mathcal{P}$ by its quadratic dual and second we replace the trees T by its induced action on

A. This way C_* becomes a complex and we can speak about the homology of the algebra A over the operad $\mathcal{P}$.
An operad is called *Koszul* if the canonical map $\mathbf{D}(\mathcal{P}) \longrightarrow \mathcal{P}^!$ induced by the isomorphism $H^0(\mathbf{D}(\mathcal{P})) \longrightarrow \mathcal{P}^!$ is an isomorphism of dg-operads.
This is equivalent to the vanishing of the homology of the complex $C_n(A)$ for any free algebra A over the operad $\mathcal{P}$.

Examples of quadratic operads. The main examples of k–linear operads we are interested in are the operads $\mathcal{L}$ie, $\mathcal{A}$ss, and $\mathcal{C}$om. Algebras over these operads are Lie algebras, associative algebras and commutative algebras, respectively, in the usual sense, however possibly without unit element. There are further examples of k–linear operads like the operad of Gerstenhaber algebras and the operad of Poisson algebras.

Proposition: *The operads $\mathcal{L}ie$, $\mathcal{A}ss$, and $\mathcal{C}om$ are quadratic and its quadratic dual is $\mathcal{C}om$, $\mathcal{A}ss$, and $\mathcal{L}ie$, respectively. Moreover, these three operads are Koszul.*

Classical homology theories. *Lie algebra homology.* Let $\mathfrak{g}$ be a finite dimensional Liealgebra of dimension n. The Chevalley-Eilenberg complex for the trivial $\mathfrak{g}$–module k is

$$0 \longrightarrow \Lambda^n \mathfrak{g} \longrightarrow \cdots \longrightarrow \Lambda^2 \mathfrak{g} \longrightarrow \mathfrak{g} \longrightarrow k.$$

The homology of this complex coincides with the groups $\mathrm{Tor}^{\mathfrak{U}(\mathfrak{g})}(k, k)$. Moreover, this complex coincides with the complex defined above for the operad $\mathcal{L}$ie. Thus its homology is the usual Lie algebra homology for the Lie algebra $\mathfrak{g}$.

Hochschild homology. Let A be an associative algebra viewed as an algebra over the operad $\mathcal{A}$ss. The Hochschild complex is

$$\cdots \longrightarrow A^{\otimes n} \longrightarrow \cdots \longrightarrow A \otimes A \longrightarrow A \longrightarrow 0.$$

This can be seen as a resolution of the algebra A as a $A \otimes A^{op}$–algebra (that is as an A–bimodule. For any A-bimodule M one gets a resolution of M as an A–bimodule just by tensoring the complex with M (from the left). For an augmented algebra A we consider the trivial bimodule k via the augmentation map $A \longrightarrow k$. This way we obtain a resolution of k

$$\cdots \longrightarrow A^{\otimes n-1} \longrightarrow \cdots \longrightarrow A \longrightarrow k \longrightarrow 0.$$

Again this complex coincides with the complex defining the homology of A over the operad $\mathcal{A}$ss.

André-Quillen homology and Harrison homology. Let A be a commutative algebra, that is an algebra over the operad $\mathcal{C}$om. Let Q_* be a free (commutative) resolution of A and let

$$\mathbb{L}_n(A) := \Omega^1_{Q_n} \otimes_{Q_n} A$$

be the cotangent complex of A. For any A–module M, André-Quillen homology is defined as

$$D_n(A; M) := H_n(\mathbb{L}_n(A) \otimes_A M).$$

It is known that $D_n(A; M) = 0$ for any smooth algebra A. In particular, this holds for A a polynomial ring. This implies that the André-Quillen homology coincides with the homology defined above for the operad $\mathcal{C}$om, however to get an isomorphism on the level of complexes one must use so-called Harrison homology, which is isomorphic to André-Quillen homology:

$$\mathrm{Harr}_n(A; M) = D_{n-1}(A; M).$$

Peter Schneider

Homotopy Algebras

The central task of this lecture was to explain the notion of minimal models and to sketch the proof of their existence. We first reviewed the classical theory for graded-commutative differential graded algebras over a field of characteristic zero. The minimality requirement says that the image of the differential should consist of decomposable elements. A minimal model for a d.g.a. A is a map of d.g.a's $M \longrightarrow A$ which is a quasi-isomorphism and where M is minimal and, as a graded algebra, is free. Such a minimal model exists if A is connected and simply connected. Moreover, if $M \longrightarrow A$ and $M' \longrightarrow A$ are two minimal models then $M \cong M'$ by an isomorphism which is compatible with the maps up to homotopy.
It then was explained how his notion extends to differential operads over a field of characteristic zero k. The simply connectedness assumption here becomes the requirement that the first space of the operad is equal to k. The proof of the existence of minimal models was given. In the case of Koszul operads a functorial construction of minimal models is possible through the cobar complex of the Koszul dual.
By definition, a homotopy algebra over an operad $\mathcal{S}$ is an algebra over a minimal model of $\mathcal{S}$. Two examples of this were discussed. Homotopy associative algebra are the same as A_∞-algebras. Secondly, (homotopy) Gerstenhaber algebras are (homotopy) algebras over the homology operad of the little squares operad.

Sigrid Wortmann

Loop spaces

In this talk we explained how operads may be used to identify $H-$spaces as loop spaces. $H-$spaces were introduced as a generalization of topological groups, they are spaces X endowed with a continuous multiplication and a unit. In 1956 Milnor showed that the loop space ΩX of a space X (which is here and in the sequel assumed to be of the homotopy type of a (connected) countable $CW-$complex) has the homotopy type of a topological group. Extending his methods Dold and Lashof proved in 1959 a delooping result for certain associative $H-$spaces. These construction led Stasheff to the definition of an A_n-*structure* on a space $(X, *)$, i.e. an $n-$tuple of maps

$$\begin{array}{ccccccccc} X & = & E_1 & \subset & E_2 & \subset & \dots & \subset & E_n \\ & & \downarrow^{p_1} & & \downarrow^{p_2} & & & & \downarrow^{p_n} \\ * & = & B_1 & \subset & B_2 & \subset & \dots & \subset & B_n \end{array}$$

such that the maps $(p_i)_* : \pi_q(E_i, X) \to \pi_q(B_i)$ are isomorphisms, together with a contracting homotopy $h : CE_{n-1} \to E_n$ satisfying $h(CE_{i-1}) \subset E_i$ for all i. Here, CE_{n-1} is the cone over E_{n-1}.
On the other hand the concept of homotopy associativity is generalized by the concept of A_n-*spaces*. To define them we introduced (the underlying cell complex of) Stasheff's non-symmetric operad $\{K_i\}_{i\geq 2}$:

$$K_2 = * \text{ and } K_i = CL_i,\ L_i = \bigcup_{r+s=i+1} \bigcup_{1\leq k\leq r} (K_r \times K_s)_k \text{ for } i > 2.$$

The $(K_r \times K_s)_k$ are copies of $K_r \times K_s$ corresponding to the insertion of one pair of parentheses ( ) in i symbols: $1\,2 \dots (k\,k{+}1 \dots k{+}s{-}1) \dots i$. The intersection $(K_r \times K_s)_k \cap (K_{r'} \times K_{s'})_{k'}$ corresponds to inserting two pairs of parentheses : $\dots(\dots(\dots)\dots)\dots$ or $\dots(\dots)\dots(\dots)\dots$.

(The relation with the operad C_1 of little intervals is easily seen.) The complex $K_\bullet$ is in fact isomorphic to $I^{\bullet-2}$. An A_n-space is now defined to be a space $(X,*)$ together with forms $(M_i)_{2\le i\le n}$, $M_i : K_i \times X^i \to X$ such that

- M_2 is multiplication with unit,
- M_3 defines homotopy associativity for M_2,
- M_i, $i > 4$ define higher homotopy associativity involving $(M_j)_{j\le i}$.

The main result of our talk was

Theorem: (Stasheff) *A space $(X,*)$ admits an A_n-structure if and only if it is an A_n-space. This implies in particular that $(X,*)$ is a loop space if and only if it admits an $A_\infty-$form $(M_i)_{i\ge 2}$.*

The proof was sketched. The first step is to construct an A_n-structure from $(X,(M_i)_{2\le i\le n})$. The spaces are just $\mathcal{E}_i := K_{i+1} \times X^i$ and $\mathcal{B}_i := K_{i+1} \times X^{i-1}$ and the inclusions are defined using the forms $(M_j)_{2\le j\le i}$. This construction is used in the second step again. Starting with an A_n-structure one defines inductively forms M_j together with commutative diagrams

$$\begin{array}{ccc} \mathcal{E}_j & \to & E_j \\ \downarrow & & \downarrow \\ \mathcal{B}_j & \to & B_j \end{array} \quad \text{for } j < i.$$

Alexander Schmidt

Iterated loop spaces I

The talk introduced the *little n-cube operad $\mathcal{C}_n$* and gave an idea how to prove the *Recognition Principle*: Every n-fold loop space is a $\mathcal{C}_n$-space and every connected $\mathcal{C}_n$-space has the weak homotopy type of an n-fold loop space

In the remainder of the talk the Dyer-Lashof operations on the homology $H_*(X,\mathbb{F}_p)$ of a $\mathcal{C}_n$-space X where introduced, where we restricted to the case $p = 2$. Finally, a sketch of their construction was given.

Stefan Schwede

Iterated Loop Spaces II

The operad $\mathcal{C}_n$ of little n-cubes can be embedded into the operad of $(n+1)$-cubes by taking product with the additional coordinate direction. This way every $\mathcal{C}_{n+1}$-space becomes a $\mathcal{C}_n$-space. The Browder operation is an obstruction for a given $\mathcal{C}_n$-action to extend to a $\mathcal{C}_{n+1}$-action. For $n \ge 2$ the Browder operation and the multiplication make the homology of a $\mathcal{C}_n$-space into a Gerstenhaber-algebra.

The operad $\mathcal{C}_\infty$ is defined as the colimit of the operads $\mathcal{C}_n$. The recognition principle says that every grouplike $\mathcal{C}_\infty$-space is an infinite loop space. The operad $\mathcal{C}_\infty$ is an " E_∞-operad", i.e., an operad whose k-th space is Σ_k-free and non-equivariantly contractible for every $k \ge 1$. We give two further example of E_∞-operads, the translation operad of the symmetric groups and the linear isometries operad.

Thomas Wenger

Formality and the Deligne conjecture

The subject of the talk was the problem of defining the structure of a homotopy Gerstenhaber algebra on the Hochschild cochains $C^*(A)$ of an associative algebra over a field of characteristic zero. Its existence can be interpreted as an answer to a question posed by P. Deligne in a letter, which has since then been called the Deligne conjecture. He asked if the action of homology operad of the little discs operad on $HH^*(A)$ coming from the isomorphism of this operad with the operad describing Gerstenhaber algebras (denoted by G) can be lifted to the cochain level.
The proof of existence was sketched along the lines of the preprint QA /0003052 by V. Hinich, which presents the existence proof of D. Tamarkin including some simplifications. To begin with, it uses an operad B_∞ describing B_∞ algebras in the sense of H. Baues, i.e. graded vector spaces X such that the cofree coalgebra cogenerated by $X[1]$ is equipped with the structure of a differential graded bialgebra. The analogous notion for dg Lie bialgebras is called a $\tilde{B}$ algebra, again described by an operad bearing the same name. The first main step in the proof is the action of B_∞ on $C^*(A)$ by the so-called braces, which goes back to M. Gerstenhaber and A. Voronov. Then a morphism of operads is constructed from the homotopy Gerstenhaber operad (defined to be or identified as the minimal model of the (Koszul) operad G) into B_∞. For this, the key idea is to interpret the result of P. Etingof and D. Kazhdan on (de-)quantization of Lie bialgebras as giving an isomorphism between the operads B_∞ and $\tilde{B}$ and then to observe that almost by definition, G_∞ maps to $\tilde{B}$.
It was pointed out that alternative answers to the question raised by P. Deligne have been given by J. McClure and J. Smith and by M. Kontsevich and Y. Soibelmann.

Markus Szymik

Batalin-Vilkovisky algebras

Let $(A_\bullet, \Delta)$ be a graded commutative algebra $A_\bullet$ together with an operator Δ which satisfies $\Delta^2 = 0$ and raises degree by one. One can define a bracket by

$$[a,b] = (-)^a \Delta(ab) - (-)^a \Delta(a)b - a\Delta(b).$$

If $(A_\bullet, [\ ])$ is a Gerstenhaber algebra, $(A_\bullet, \Delta)$ is called a Batalin-Vilkovisky algebra. This is equivalent to a certain relation for $\Delta(abc)$ corresponding to the Poisson relation.
On the rational homology of a space, an operator like Δ arises from an action of the circle group $\mathbb{T}$. For example, the rational homology of a double loop space is a Batalin-Vilkovisky algebra due to the action of $\mathbb{T}$ on the sphere by rotations.
In general, and this was presented in the talk as the main result, a Batalin-Vilkovisky algebra is the same as an algebra over the homology of the operad of framed discs, which is build from the operad of ordinary discs by taking the action of $\mathbb{T}$ on the disc into account. That result can be proven by reduction to the corresponding result relating Gerstenhaber algebras and the operad of ordinary discs in the same way.

Natalie Wahl

Moduli spaces I

$\mathcal{M}^s_{g,r}$, the moduli space of Riemann surfaces of genus g, with r boundary components and s punctures, is (in most cases) a classifying space for the mapping class group $\Gamma^s_{g,r} = \pi_0 Diff^+(F^s_{g,r}, \partial)$ of the corresponding surface ($F^s_{g,r}$ is a surface of genus g, with r boundary components and s punctures). This is a consequence of the contractibility of the Teichmüller space, the moduli space of marked Riemann surfaces.
Attaching surfaces on punctures defines an operad structure on $\overline{\mathcal{M}}^s_{g,r}$, the compactification of $\mathcal{M}^s_{g,r}$. Those attaching maps produce singularities. Gluing along boundaries allows to work without singularities. However, gluing complex structures is not an easy operation. We present an operad induced by gluing along the boundaries, but using $B\Gamma^s_{g,r}$, which is homotopic to the moduli space. In this case, we "glue" diffeomorphisms which are the identity on the boundary. The operad is shown to be a double loop space operad, result which will be improved to an infinite loop space structure in the talk moduli space II.
This operad is build using a general construction of "operads with families of groups". The same construction provides a braid groups operad, which is equivalent to the little disks operad, and a ribbon braid groups operad equivalent to the framed disks operad.

Manfred Lehn

The vanishing of the BV-structure on the BRST-cohomology of a TCFT

Let $\tilde{\mathcal{M}}_{g,n+1}$ denote the space of isomorphism classes of Riemann surfaces C of genus g with $n+1$ disjoint biholomorphic embeddings $\varphi_0, \ldots, \varphi_n : \mathbb{D} \to C$, and let $\tilde{\mathcal{M}}_0(n) := \tilde{\mathcal{M}}_{0,n+1} \subset \tilde{\mathcal{M}}(n) := \coprod_{g\geq 0} \tilde{\mathcal{M}}_{g,n+1}$. Sewing curves $C \setminus \varphi_i(\mathbb{D}^\circ)$ along their boundaries defines an operad $\tilde{\mathcal{M}}$ that contains $\tilde{\mathcal{M}}_0$ as a suboperad. The natural inclusion of the framed little discs operad $\mathcal{P} \to \mathcal{M}_0$ is a homotopy equivalence. In particular, any algebra over $H_*\tilde{\mathcal{M}}$ inherits a Batalin-Vilkovisky structure. Let $t \in H_0\tilde{\mathcal{M}}$ correspond to the point class of a genus 1 surface with two disc embeddings. Then $H_*\tilde{\mathcal{M}} \cong \mathbb{Q}[t]$. We deduce the following theorem of Tillmann from a theorem of Harer-Ivanov on the stabilisation of the mapping class group homology $H_*(\Gamma_{g,n})$ for increasing genus:

Theorem: *The element t is locally nilpotent on $H_n(\tilde{\mathcal{M}})$ for $n > 0$. In particular, if A is an algebra over $H_*(\tilde{\mathcal{M}})$, then the induced BV-structure $(\Delta, [-,-])$ on A becomes trivial after localization of t.*

David J. Green

Moduli spaces II

In the first half of the talk, the construction of Ulrike Tillmann's surface operad $\mathcal{M}$ was treated in considerable detail.
Then Tillmann's theorem that every $\mathcal{M}$-space is an infinite loop space after group completion was stated, and the most important steps in the proof were presented. Harer's theorem that the homology of the mapping class groups stabilises as the genus increases plays a key role. The main corollary of Tillmann's theorem is that $\mathbb{Z} \times B\Gamma^+_\infty$ is an infinite loop space.

Stefan Kühnlein

Torsors and the Grothendieck-Teichmüller group

After an introduction to quantization deformation we stated the fact that Kontsevich's proof of the existence of a quantization deformation (for a given Poisson manifold) bases heavilly on the construction of an isomorphism ϕ between two homotopy Lie-algebras, the one controlling deformations of associative, the other that of Poisson-algebras. (This is where operads and the formality conjecture for $\mathcal{C}_2$ come in!) Let T be the set of all these isomorphisms. It turns out that T is a pro-algebraic bitorsor. The isomorphism ϕ gives a point on $T(\mathbf{C})$ and therefore corresponds to a ringmorphism $\alpha : \mathcal{O}(T) \longrightarrow \mathbf{C}$. In the following we will describe the construction of a ring $\mathcal{P}_{Tate}$ together with a morphism to $\mathbf{C}$ such that α factors through $\mathcal{P}_{Tate}$ by a map β. The affine scheme $\mathrm{Spec}(\mathcal{P}_{Tate})$ seems to be a proalgebraic bitorsor of great importance in arithmetic geometry.

Conjecture (Kontsevich): *Spec(β) is an isomorphism of proalgebraic bitorsors.*

To construct $\mathcal{P}_{Tate}$, one first defines $\mathcal{P}^+$ to be the $\mathbf{Q}$-vector space generated by symbols $[X, D, \gamma, \omega]$, where X runs through all smooth equidimensional varieties over $\mathbf{Q}$ (say $d = \dim X$), D is a divisor with normal crossings on X, $\gamma \in H_d(X(\mathbf{C}), D(\mathbf{C}), \mathbf{Q})$, and $\omega \in \Omega^d(X)$. These symbols obey the relations coming from $\mathbf{Q}$-linearity in ω and γ, change of variables and Stokes' Theorem. In particular, $\int_\gamma \omega$ is well-defined on $\mathcal{P}^+$, inducing a map $\mathrm{ev} : \mathcal{P}^+ \longrightarrow \mathbf{C}$. It is conjectured that ev is injective (which we assume now). $\mathcal{P}^+$ is a ring, as the product of two integrals again is an integral: the ring of effective periods. Let $\mathcal{P} := \mathcal{P}^+[1/2\pi \mathrm{i}]$.

Theorem: (Nori) *$\mathcal{P}$ is the ring of functions of the proalgebraic torsor of isomorphisms between Betti- and deRham-cohomology.*

Now $\mathcal{P}_{Tate}$ is the ring generated by $(2\pi\mathrm{i})^{\pm 1}$ and entries of period matrices which are rationally triangulisable having only powers of $2\pi\mathrm{i}$ on the diagonal (and some conditions on discriminants must be satisfied.) It is implied by theorems of Goncharov, that the integrals defining α all take there values in $\mathcal{P}_{Tate}$. On the other hand, one conjectures that the vector bundles on (the bitorsor) $\mathrm{Spec}(\mathcal{P}_{Tate})$ with commuting actions of the groups acting on $\mathrm{Spec}(\mathcal{P}_{Tate})$ form *the* category of mixed, unramified Tate-motives (over $\mathbf{Q}$ with values in $\mathbf{Q}$). Then, again, some conjectures of Beilinson imply that the quotient of the motivic Galois group acting simply transitively on $\mathrm{Spec}(\mathcal{P}_{Tate})$ is isomorphic to the Grothendieck-Teichmüller group GT.
NB: GT is the automorphism group of the tower of pro-nilpotent completions of the braid groups $\mathcal{B}_n = \pi_1(\mathcal{C}_2(n))$, and it certainly should play some role in studying $\mathrm{Chains}(\mathcal{C}_2)$. To understand this is part of Kontsevich's conjecture.

Yorck Sommerhäuser

Vertex operator algebras

We review the work of Y. Z. Huang on the geometric description of vertex operator algebras. Vertex operator algebras are defined via a graded vector space, called the state space, and a map, called the state-field correspondence, from the state space to the space of fields, which are formal distributions with coefficients in the endomorphism algebra of the state space. On the other hand, geometric vertex operator algebras are mappings from the partial operad of moduli spaces of punctured spheres to the partial endomorphism operad. Huang's work gives a one-to-one correspondence between these two objects by looking at correlation functions of vertex operator algebras.

Dan Fulea

Planar algebras and C^*–algebras

The task of the talk was to give a compact description of VAUGHAN JONES' formalism [Jo] (Jones, V.F.R: Planar algebras I, see also the conference program) of planar operads related to the construction of subfactors. For the purposes of the conference, taking advantage on the common knowledges of the participants, I decided to rephrase the formalism of the planar algebras [Jo], expressing it in a categorial fashion.

• In a (symmetric) tensor category $\mathcal{A}$ there is a formalism of making diagram calculus for morphisms (Kassel, Christian: Quantum groups, XIV.1,2(,3)). A morphism $f : U \to V$ in $\mathcal{A}$ is figured as a box $\boxed{f}$ with two vertical strands into and out of the box: $\boxed{f}$. The composition of morphisms is done by vertical concatenation $\begin{array}{c}\boxed{f_1}\\ \boxed{f_2}\end{array} \doteq \boxed{f_1 f_2}$. The tensor product is given by horizontal concatenation: $f_1 \otimes f_2$ is represented by $\boxed{f_1}\boxed{f_2}$, completed with vertical strands. The identity of an object is a vertical strand $|$. (We don't specify the source and the target of a morphism, for they are determined by the morphisms involved. Exception: The identity morphism. In this case it should be clear from the context.)

• An iterated composition of iterated tensor products of morphisms in $\mathcal{A}$ gives rise to a diagram of not overcrossing strands in the plane with boxed marks out of $Hom_{\mathcal{A}}$. Isolating this structure one can define labelled tangles (without overcrossings). The labels are elements of a language $(L = \sqcup L_k, \cdot, \otimes)$. The *set* L_k of symbols of size k of L is given by $\cdot$ an *associative* "vertical" composition rule. The "horizontal" composition rule is given by $\otimes : L_k \times L_l \to L_{k+l}$. Let $\mathcal{T}\langle L\rangle$ denote the category of tangles with marks in L.

• Analogously to the operad of little squares one can now define after linearization with the functor $R[\cdot]$ from sets to R–modules the folowing *planar operad* $\mathcal{P}(\cdot, L)$ of R–modules:

$$\mathcal{P}(n, L) := R\left[\bigsqcup_{k\geq 0} Hom_{\mathcal{T}\langle\mathcal{L}^{\square}\rangle}(k \to k)_n\right] ,$$

with the following meanings:

(i) $L^{\square}$ is the language obtained from L by adjoining an empty box $\square = \square_k$ in each size k to L_k and considering the universal language $(L^{\square}, \cdot, \otimes)$ with a canonical embedding $(L, \cdot, \otimes) \to (L^{\square}, \cdot, \otimes)$.

(ii) Form the category $\mathcal{T}\langle\mathcal{L}^{\square}\rangle$ which is generated by tangles, symbols of the language L and the empty boxes. The number n of empty boxes of a morphism in $Hom_{\mathcal{T}\langle\mathcal{L}^{\square}\rangle}(k \to k)$ is called the valence, and $Hom_{\mathcal{T}\langle\mathcal{L}^{\square}\rangle}(k \to k)_n$ denotes the set of morphisms of valence n with a chosen order of the n empty boxes. Let $f \in Hom_{\mathcal{T}\langle\mathcal{L}^{\square}\rangle}(k \to k)_n$ be a morphism. If k_i is the size of the i^{th} box of f, we define the color of f to be $K := (k_1, \ldots, k_n)$.

(iii) The set of morphisms of color K generate in $\mathcal{P}(n, L)$ the R–module $\mathcal{P}(n, L)_K$.

The operadic multiplication

$$\gamma : \mathcal{P}(n, L)_K \ \otimes \ \mathcal{P}(k_1, L) \otimes \cdots \otimes \mathcal{P}(k_n, L) \to \mathcal{P}(k_1 + \cdots + k_n)$$

is given on generators by formal substitution into the empty boxes of $\mathcal{P}(n, L)_K$ of generators from the other tensor factors. This substitution is possible, only when "the colors match". The unit morphism is $1 \to \square$. (Substituting an empty box into an empty box gives an empty box.)

• The main naive example of a $\mathcal{P}$–algebra is given by the language of tensors $T = \left(T^{j_1 \ldots j_k}_{i_1 \ldots i_k}\right) = T^J_I$ with all indices i_s, j_s: $s = 1, .., k$ in a fixed set.

The multiplication of tensors $T = T^J_I$ and $S = S^K_J$ is $ST := \sum_J T^J_I S^K_J$ and the tensor product of $T = T^J_I$, $S = S^N_M$ is $TS = (T^J_I S^N_M)^{J \sqcup N}_{I \sqcup M}$.

The operadic action on tensors is given by (coloured) substitution of the tensors into the empty boxes of an operadic operation and evaluating the result in tensors using $(\cdot, \otimes)$.

• The main example [Jo] pf planar algebras ($\mathcal{P}$–algebras) is connected to the construction of pairs $A \subset B$ of a subfactor of a factor of (C^*)–algebras.

– A conditional expectation $E : B \to A$ is an A–bimodule morphism giving the identity, when restricted to A. Let E be also non–degenerate. We also write $E : B \to B$ for the composition of the conditional expectation $B \to A$ and the inclusion $A \to B$.

– One has the so called *fundamental construction*: Associate to $A \subset B$ the pair $B \subset End_A(B)$. Iterating this procedure one becomes a tower $A \subset B \subset End_A(B) \subset \ldots$, that we homogeneously denote by $M_0 \subset M_1 \subset M_2 \subset \ldots$. Then $E =: E_2$ is an element of M_2. Analogously one has elements $E_3 \in M_3 \ldots$

– Assume the algebra $B \otimes_A B$ has a unit $\sum u_i \otimes v_i$. Define $\tau = Index E := \sum u_i v_i$. It belongs to the center of B, which is in our case $\mathbb{C}$. Consider also $\delta \in \mathbb{C}$, $\delta \tau^2 = 1$.

– The endomorphism group $End_A(B)$ of B is generated by B and E, and seen as a right A–module it is isomorphic to $B \otimes_A B$. An endomorphism f corresponds to $\sum f(u_i) \otimes v_i$. One has in general $M_k \cong B \otimes_A B \otimes_A \cdots \otimes_A B$ (k tensor factors).

– Consider the intersection of the sequence of inclusions $M_0 \subset M_1 \subset M_2 \ldots$ with $M'_0 \supset M'_1 \supset M'_2 \supset \ldots$.

$$\begin{array}{ccccccccccc}
 & M_0 & \subset & M_1 & \subset & M_2 & \subset & \cdots & \subset & M_k & \subset \cdots \\
 & & & & & & & & & & \\
M'_0 & M'_0 \cap M_0 & \subset & M'_0 \cap M_1 & \subset & M'_0 \cap M_2 \subset & & \cdots & & \subset M'_0 \cap M_k \subset & \cdots \\
\cup & & & \cup & & \cup & & & & \cup & \\
M'_1 & & & M'_1 \cap M_1 & \subset & M'_1 \cap M_2 \subset & & \cdots & & \subset M'_1 \cap M_k \subset & \cdots \\
 & & & & & & & & & & \\
\vdots & & & & & \ddots & & \ddots & & \ddots &
\end{array}.$$

Then the operad $\mathcal{P}(\cdot, M'_0 \cap M.)/\sim$ generated by tangles, symbols in B end empty boxes organizes the (R–linearized) quotient $\mathcal{T}\langle L\rangle/\sim$ of tangles $\mathcal{T}\langle L\rangle$ of diagrams in the language L generated by B as an *algebra*. The relation $\sim$ indentifies each internal circuit of tangles with $\delta \in \mathbb{C}$. In this operadic algebra structure, E_k is mapped to the tangle $k \to k$, which joins the last two upper and the last two lower strands of k, and each $x \in B$ is mapped to $\boxed{x}$. One can in the above picture (and more general in the framework of Popa–systems) "diagonally extend" the operadic algebra structure.

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