Mathematisches Forschungsinstitut Oberwolfach

Report No. 46 / 2000

The Mathematics of Discrete Tomography

December 3rd – December 9th, 2000

The basic problem of discrete tomography is to reconstruct finite point sets that are accessible only through some of their discrete X-rays. Questions of discrete tomography are relevant in image processing, data compression, and data security; the dominating motivation for the conference, however, was the need for practical reconstruction techniques, particularly in material sciences.

The conference was organized by Richard Gardner (Western Washington University, Bellingham) and Peter Gritzmann (Technische Universität München). It brought together 41 scientists from mathematics, computer science, physics, material science, chemistry, biology, and radiology. There were 28 talks focussing on mathematical models and algorithms – both from a theoretical and a practical point of view – and on various kinds of real world challenges. Additionally, there was an informal discussion (moderated by the organizers) on the interface between the mathematical core problems in discrete tomography and the various applications presented.

Abstracts

Discrete tomography: Designs, codes, and quantum physics

Thomas Beth

We survey the following examples and applications of discrete tomography:
- SNR-improved measurements by weighing designs
- Radon transforms based on affine planes AG(2,p)
- Wigner transforms in digital optics
- Phase space tomography for spinning tops
- Discrete Wigner transforms

As a recent research result we describe an application of inverse discrete tomography in the area of quantum state stabilization by error-correcting quantum codes. The construction of the error-free-subspace decomposition is presented as an example of the inverse slice theorem of tomography.

Reconstructing convex lattice sets

Sara Brunetti

A convex lattice set F in the integer lattice is a finite subset of $\mathbb{Z}^2$ that is equal to the intersection of its convex hull with $\mathbb{Z}^2$. A lattice direction is a vector $v \in \mathbb{Z}^2 \setminus \{0\}$ and the 1-dimensional X-ray of F parallel to v provides line sums giving the number of elements of F on each line parallel to v. The focus of this talk is on the problem proposed by Gritzmann in 1997 who asked whether given functions $f_1, \ldots, f_m$ encode the X-rays of some convex lattice set in $\mathbb{Z}^2$. We propose a polynomial time algorithm solving the problem for X-rays taken in seven mutually non parallel lattice directions or in four suitable directions. This algorithm exploits a result of Alain Daurat establishing uniqueness for a subclass of that of convex lattice sets.

Radiography and tomography with fast neutrons at the FRM-II

Thomas Bücherl

At the Technical University of Munich (TUM) a new research reactor is under construction. For its instrumentation a tomography system using fast neutrons for the examination of dense and large volume objects is actually set-up. The status of this project, some examples measured at the old research reactor FRM and the related mathematical problems (challenges) are reported.

Remarks on multidimensional assignment problems

Rainer E. Burkard

We survey results on the classical assignment polytope, the axial 3-index assignment polytope and the planar 3-index assignment polytope. For reconstructing fixed, but unknown, feasible solutions it might be useful to introduce probabilities p_{ij} and p_{ijk} for the events $x_{ij} = 1$ and $x_{ijk} = 1$, respectively. Maximizing the product of probabilities of feasible solutions is equivalent to minimizing the sums $\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$ and

$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} c_{ijk} x_{ijk}$ subject to the given constraints, where $c_{ij} = -\log p_{ij}$ and $c_{ijk} = -\log p_{ijk}$. Some methods for assigning such probabilities are outlined. This leads to the classical linear assignment problem (which may also be used for alignments) and to 3-index assignment problems.

Solution methods for 3-index assignment problems are sketched which resemble the Hungarian method for the classical assignment problem. Moreover, a Lagrangean relaxation method for the axial 3-index assignment problem is explained. Further comments concern cases when 3-index problems are polynomially solvable. In particular the role of (permuted) Monge arrays is stressed.

Determination of Q-convex sets by X-rays

Alain Daurat

Q-convexity is a new notion of convexity which depends on a set of directions. It is linked with the convexities which have been studied in discrete tomography: every HV-convex polyomino is Q-convex along the directions H and V and every (usually) convex set is Q-convex for any set of directions. In this talk we prove that the result of Gardner and Gritzmann about uniqueness of the totally convex sets which have prescribed projections along some sets of directions can be extended to Q-convex sets.

Reconstructing domino tilings under tomographic constraints

Christoph Dürr

(joint work with E.Goles, I.Rapaport, E.Rémila, and M.Chrobak, G.Woeginger)

Fix a set of colored tiles. Given an integer n and two vectors r and c we want to construct a partial tiling of the finite grid $[n] \times [n]$ such that its horizontal and vertical projections are respectively r and c. This reconstruction problem can be NP-complete or have polynomial complexity depending on the fixed set of tiles. In this talk we will list different sets and the associated complexity. Then we describe how a general construction can be applied to prove NP-hardness for most reconstruction problems.

Application of the Radon transform in image analysis

Ulrich Eckhardt

In document processing one deals with binary digital images or subsets of the digital plane $\mathbb{Z}^2$. Any set in $\mathbb{Z}^2$ can be represented by its boundary, and a closed curve in the plane can be represented as a set of tangent lines thus yielding a curve in the domain $\mathcal{S}_1 \times [0, \infty)$ of the Radon transform. This mapping is investigated. It turns out that in this interpretation the Radon transform has surprising properties. Specifically, it can be shown that under certain conditions the backtransformation is a continuous mapping.

In 1929 Heinrich Tietze published a paper on local characterization of convex sets. For image processing applications a discrete version of Tietze's Theorem is desired. It is shown, however, that in general a local characterization of convex digital sets cannot be found. Some consequences of this negative result are presented and its relation to the Radon transform is given.

Complexity of the 3-dimensional reconstruction of a lattice set from its one-dimensional X-rays parallel to the axis

Yan Gerard

It is known since 1957 (Ryser-Gale) that a matrix of 0 and 1 with given numbers of 1 in each row and column can be computed in polynomial time. This 2-dimensional problem can be extended in 3D by considering subsets of $\{1, \dots, m\} \times \{1, \dots, n\} \times \{1, \dots, l\}$ and their number of elements in each line parallel to the axis. This 3D generalization is known to be NP-complete since 1994 (Irving-Jerrum), and we are interested in what happens when the size l is a small integer:

When $l \geq 4$, the NP-completeness of the problem is a corollary of the NP-completeness of the 4-colors-problem (Chrobak-Dürr, 1998).

When $l = 3$, we prove that the problem is still NP-complete with an original reduction of the 3D-matching problem.

When $l = 2$, the problem can be reduced to a maximum flow problem with the consequence that it is polynomial; and, last, when $l = 1$, it is trivial.

Tomographic reconstruction algorithms – implemented and used at the Bundesanstalt für Materialforschung und -prüfung (BAM)

Jürgen Goebbels

Using complete data sets for the 2D case filtered backprojection, iterative techniques (modified ART) and the Region-of-Interest method (ROI) are implemented and demonstrated for some examples. For the 3D or cone beam geometry up to now only the Feldkamp Algorithm was applied. With the newest apparatus, developed at BAM, a spatial resolution of 2 μm could be reached, shown on porous materials like Al-foam and ceramic membranes. In the case of an incomplete data set the Maximum Entropy Method is used together with a priori knowledge and restrictions to the defect space. The method developed for process tomography is based on the image representation as a stochastic dynamic system together with a Kalman filter theory approach in the state space.

Radon transforms over finite fields

Eric Grinberg

We replace $\mathbb{R}^n$ by a vector space over a finite field, define a natural analog of the Radon transform and consider the main problems of tomographic integral geometry: injectivity, range characterization and admissibility. There are many results and many open problems in each category. The discrete results have found continuous applications and conversely. Some results are analogous to the continuous variant while others differ. The proofs sometimes involve counting arguments and sometimes not; in the latter case a modification of the continuous proof may yield its finite counterpart.

Nonlinear techniques in combinatorial optimization

Martin Grötschel

This presentation gives a survey on the theoretical approaches and algorithmic techniques used today in combinatorial optimization. It sketches special purpose methods (such as shortest path, min-cost flow algorithms, etc.) and linear programming techniques (cutting planes, branch-and-cut, ...). The focus, however, is on nonlinear programming theory and algorithms and their utilization in combinatorial optimization. The connection between submodularity and convexity will be outlined as well as the use of duality results. Lagrangian relaxation of integer programs and the (natural) exploitation of subgradient algorithms will be explained as well as more recent relaxation techniques such as semidefinite programming. The main aim of this talk is to make the non-mathematicians in the audience aware of the mathematical modeling techniques available today and of existing connections between the various fields of optimization.

Electron tomography in structural biology: Conditions, applications, problems. Part 2: Data evaluation

Reiner Hegerl

Mainly four tasks arise from the data evaluation in electron tomography of biological objects. (i) Projection alignment means that the images have to be shifted and rotated such that they refer to a common coordinate system. (ii) The 3D reconstruction is usually performed by weighted backprojection. Problems arise from incomplete data (large angular increment, limited tilt range), image data resulting from different contrast mechanisms (phase contrast approximation distorted by multiple scattering, inelastic scattering, absorption contrast, etc.), and noise. (iii) The visualisation of large volumes containing complex structures, e.g. cell organelles, requires signal enhancement and image segmentation. Noise can be removed by non-linear anisotropic diffusion, a method that preserves structural features much better than conventional filter techniques like low-pass filtration or the median filter. When the structure of macromolecules is investigated, one prefers to combine the 3D reconstructions of many individual molecules of the same type by averaging – after appropriate alignment – in order to obtain a significant model. (iv) Finally the quality of the reconstruction has to be checked. Given the size of the reconstruction volume and the number and directions of projections, theoretical limits for the attainable resolution can be estimated, e.g. by calculating a 3D point spread function. In the case of macromolecules, the resolution can be assessed on the basis of statistics, e.g. by the Fourier shell correlation function.

Discrete tomography with absorption

Attila Kuba

A family of new kind of discrete tomography problems, called Emission Discrete Tomography (EDT), is introduced: the reconstruction of discrete sets from their absorbed projections (line sums). The absorbed line sum of the discrete set F along line l is $\sum_{F \cap P} e^{-\mu \cdot x(P)}$, where $x(P)$ denotes the distance of the point P from the detector. A 2D uniqueness problem for discrete sets (or equivalently, for binary matrices) is discussed, and a necessary and sufficient condition is given for binary matrices being uniquely determined with respect to the absorbed row and column sums when the absorption coefficient is $\mu = \log((1+\sqrt{5})/2)$.

It is shown how non-unique binary matrices can be generated from elementary switching patterns. Also a polynomial time complexity algorithm is given to reconstruct hv-convex discrete sets from their absorbed row and column sums.

Principles of reconstruction algorithms

Alfred K. Louis

The aim of the presentation is to show some important steps in the derivation of reconstruction algorithms in (continuous) tomography. Starting from the parallel geometry, which is used also in discrete tomography, we compute the adjoint operator and derive consistency conditions and an inversion formula. For developing algorithms we start from the approximate inverse where we compute instead of f a smoothed version of f, called $f_\gamma(x) = \langle f, e_\gamma(x,\cdot)\rangle$.
The auxiliary problem $A^*\psi_\gamma(x) = e_\gamma(x,\cdot)$ is solved leading to $S_\gamma g(x) = (g, \psi_\gamma(x))$, the so-called approximate inverse with reconstruction kernel ψ_γ. Fast methods depend on the exploitation of invariants of the operator. If we use all the invariants of the Radon transform we end up with methods of filtered backprojection type. As example we sketch the derivation of inversion formulae for 3D X-ray CT.

On the computational complexity of determining three-dimensional lattice sets from their two-dimensional X-rays

Alberto del Lungo

A generalization of a classical discrete tomography problem is considered: reconstruct three-dimensional lattice sets from their two-dimensional X-rays parallel to three coordinate planes. This is an open problem raised by Gardner and Gritzman in the book [1]. The aim of the talk is to prove that this generalization is NP-hard. From our reduction it follows that the problem is NP-hard even in the special case where the three-dimensional lattice subsets are 6-connected and convex along the lines parallel to the three axes. We point out that these sets are the natural three-dimensional generalization of horizontally and vertically convex polyominoes.

[1] R.J. Gardner and P. Gritzmann, Uniqueness and Complexity in Discrete Tomography, in Discrete Tomography: Foundations, Algorithms and Applications, editors G.T. Herman and A. Kuba, Birkhäuser, Boston, MA, USA, (1999) 85-113.

Analysis of electron microscopical image contrasts: Trial and error versus inverse problems

Wolfgang Neumann, Kurt Scheerschmidt

High-resolution transmission electron microscopy is a reliable technique providing information about the three-dimensional bulk structure projected along the direction of electron incidence at atomic resolution. However, a direct analysis of such electron micrographs is only possible for very thin and weakly scattering specimens, where a direct relation between the projected structure and specimen exist. The computer simulation of images is therefore the common technique for the interpretation of experimental high-resolution images. The images are calculated on the basis of an assumed structure model for a variety of operating conditions as well as structure parameters. The trial-and-error matching technique is the indirect solution to the direct scattering problem applied to analyse the nature of the object under investigation. Alternatively, inverse problems as direct solutions of electron scattering equations can be deduced using either an invertible linearized Eigenvalue system or a discretized form of diffraction equations. This analysis is based on the knowledge of the complex electron wave at the exit plane of an object reconstructed for the surrounding of single reflections by electron holography or other wave reconstruction techniques. In principle, it enables directly the retrieval of the local thickness and orientation of a sample as well as the refinement of potential coefficients or the determination of the atomic displacements, caused by a crystal defect, relative to the atom positions of the perfect lattice. Considering especially the sample orientation as perturbation the solution is given by a generalized and regularized Moore-Penrose inverse, where the resulting numerical algorithms imply ill-posed inverse problems.

On quasiregular bidimensional sequences

Maurice Nivat
(joint work with Laurent Vuillon)

A bidimensional or 2D sequence $U : \mathbb{Z}^2 \to \{0, 1\}$ is

- *regular* iff it is invariant by two non collinear translations $\vec{u}$ and $\vec{v}$;
- *quasi regular* iff it is invariant by one translation $\vec{u}$.

The $m \times n$ complexity of a 2D sequence U is the number of different $m \times n$ submatrices of U.

We attempt to prove the following conjecture.

Conjecture. If for some m, n the $m \times n$ complexity of a 2D sequence U is $\leq mn$ then it is quasi regular.

In certain cases this result can be derived from the theorem of D. Beauquier and M. Nivat stating that if one can tile the plane by translations of a single finite piece P then there exists a regular tiling of the plane by translations of P and all tilings of P by translations of P are quasi regular.

Up to now the conjecture has been proved by Sander and Tijdeman for $m = 2$ and Epifanio, Koskas and Mignosi if the $m \times n$ complexity is $\leq \alpha mn$ where α is a coefficient unfortunately low ($\sim 1/100$!).

We look at low rectangular complexity 2D sequences: quasi regular ones one obtained as perturbations of regular sequences, in which one breaks one of the 2 translations leaving it invariant. If one breaks the 2 translations then certainly the number of $m \times n$ matrices grows much above mn.

Consistency conditions on phase unwrapping in MRI

Sarah Patch

Ideal magnetic resonance imaging (MRI) systems have uniform background field, i.e., $|B_o| \equiv const.$ To minimize $\nabla|B_o(x)|$ across the imaging volume, we use phase difference images, $\phi(x) \in \mathbb{R}^2$ which we can only measure *mod* 2π.

$$\phi_{meas} = Im(log(e^{i\phi_{true}})) \in S^1 \qquad \text{and} \qquad \phi_{true}\,|B_o| \in \mathbb{R}^1$$

The goal is to recover $\nabla|B_o|$ which is traditionally assumed smooth and slowly varying, *i.e.* $|\phi_{m,n} - \phi_{m,n+1}|, |\phi_{m,n} - \phi_{m+1,n}| < \pi$. Noisy or undersampled images sometimes are inconsistent, much like this toy example, which cannot be *unwrapped.*

$$\left[\begin{array}{cccccc} \pi/2 & 0 & 0 & \pi/2 & & 0 \\ \pi & \pi/2 & 0 & \pi & & \pi/2 \\ \uparrow & \uparrow & & \uparrow & & \\ -\pi/2 & -\pi & -\pi/2 & -\pi/2 & \rightarrow & 3/4\pi \\ & & & & & \uparrow \\ 0 & -\pi/2 & -\pi/2 & -\pi/2 & & -\pi \end{array}\right] \quad \text{arrows denote large jumps}$$

In this talk we pose the problem of finding an efficient way to detect inconsistent data, so that it may be thrown out before fitting ϕ to estimate $\nabla|B_o|$.

Current problems in neutron computed tomography – the physical differences to the X-ray case

Burkhard Schillinger

In the past few years, Neutron Computed Tomography (N-CT) has been developed as an industrial tool in Germany and Switzerland. The neutron interaction with matter is nearly complementary to X-rays: Neutrons penetrate most metals easily but are very sensitive to hydrogen and many other light-weight elements. They are an ideal tool to examine thick metal samples and composite metal-plastic samples such as turbine blades or machine parts with sealants and adhesives.

However, N-CT is currently done with simple backprojection algorithms for X-rays that take into account neither the different energy dependence of the neutron interaction nor the different beam geometry of neutron beams. It is not possible to build a neutron point source, all beams are more or less an approximation to parallel beam geometry done by a diaphragm and long consecutive flight tube. If a neutron guide is used, its exit acts like a divergent area source. Spatial resolution is further limited by the scintillator spot size generated by the nuclear reaction products of the detection reaction. For low-energy neutrons, the absorption reaction is inversely proportional to the square root of the neutron energy, but often scattering is the dominant attenuation process, with a large drop below the energy of the Bragg cutoff. Multiple scattering within the sample generates further deviations from the assumption of exponential attenuation.

In spite of these problems, the results are still amazing, and there is for example a huge interest for examination of turbine blades. Large turbine blades are manufactured as monocrystals and cost up to 20,000 Euro for a single blade. X-rays fail completely, neutrons can penetrate these blades in the transverse direction but fail in the longitudinal direction of the half-moon shape of the blades, as the attenuation is too high. These missing projections make a simple backprojection impossible, but discrete tomography with reduced angular range might help to solve the problem.

Atomic structure of dislocation cores and of the amorphous phase of silicon and germanium: Promising problems for discrete tomography?

WOLFGANG SCHRÖTER

Due to the biatomic basis of the diamond lattice, dislocations in silicon and germanium can exist in two sets with different core structures and properties. Until today an experimental methods to differentiate between the two sets is missing, so that the fundamentally and technologically important question, which of the two sets is realized in Si and Ge, is still open. For the application of discrete tomography, the fact, that dislocations are topological defects and disturb translation symmetry, might be relevant.

Amorphous silicon and germanium have limited structural similarities to crystals, which lead to short-range and medium-range order with a range of about 1.5nm. However, various structural models have been claimed to be compatible with these characteristics. Also high resolution transmission electron microscopy (HRTEM) has been used to tackle the amorphous structure of Si and Ge. While HRTEM in combination with optical diffractrograms gives good evidence for a submicrocrystallite, i.e. invisible by X-ray diffraction, structure, a recently developed method of variable coherency TEM supports a continuous random network model.

Application of discrete tomography to electron microscopy of crystals

PETER SCHWANDER

In HRTEM (High-Resolution Transmission Electron Microscopy) an application of discrete tomography arises as follows: a parallel beam of electrons is directed at a small piece of a 3D crystal. After passage through the crystal and a high magnification lens system, the electrons form a 2D image. The microscope resolution is sufficient so that individual atom-columns can be resolved, at least for some directions. A technique, named QUANTITEM, deduces a signal from the image that is directly proportional to the number of atoms contained in each atom-column. Thus, line sums, each corresponding to the number of atoms contained in a single atom-column, can be obtained from the image. For physics and materials science it is of great interest to reconstruct crystals consisting of about 10^6 atoms from the measured line sums. This reconstruction problem of discrete tomography brings up mathematical issues of practical relevance, such as uniqueness, computational complexity and algorithms. The problem is finite but NP-complete when more than two projection directions are used. Practical constraints of the measurement technique, such as number and type of projection directions, experimental noise and incomplete data due to limited field of view, must also be taken into account. This presentation attempts to outline the challenges of applying discrete tomography to HRTEM, and, last but not least, stimulate the communication between mathematicians and physicists.

Visualization issues in discrete tomography

THORSTEN THEOBALD

Discrete tomography deals with the reconstruction of crystalline structures from a small number of X-rays. One particular task within the reconstruction process is to visualize the three-dimensional crystal.

In this talk, we investigate the visibility problems that naturally arise when representing the crystal by a set of balls. Algorithmic solutions to these problems require to solve some fundamental geometric problems, such as: Under which conditions do four unit balls in $\mathbb{R}^3$ have only finitely many common tangents? What is the maximum number of tangents in the finite case?

By combining techniques from classical and algebraic geometry we show: if the four centers are not collinear then finiteness is guaranteed, and the maximum number of common tangents is 12. Based on these results we deduce algorithms for the visibility problem.

Algebraic aspects of discrete tomography

Robert Tijdeman
(joint work with Lajos Hajdu (Debrecen, Hungary))

By applying algebraic methods the following becomes clear. The set of all integer solutions with prescribed line sums form a grid on a linear manifold. The grid is generated by a finite set of switching elements which can easily be described in terms of the chosen directions. The 0-1-solutions among the grid elements are the shortest vectors in the grid. The problem is therefore to determine a shortest integer vector among the grid elements. This insight is used in an algorithm. In the numerical experiments with this algorithm, which go up to 20 by 20 for random matrices and to 30 by 30 for examples with clustered 1's, up to now only 0-1-matrices with the right line sums have been obtained.

Electron tomography in structural biology: Conditions, applications, problems. Part 1: Data acquisition

Dieter Typke

Electron tomography is one method among others (e.g. serial section techniques, electron crystallography), however, the most general one, to obtain three-dimensional information of biological objects with the transmission electron microscope (TEM). An TEM bright-field image of a not too thick biological specimen may be considered as a parallel projection (of the electric potential) along the direction of the primary beam. A set of images, recorded at an appropriate set of projection directions (by tilting the object), thus delivers the information for 3D reconstructing the object under study. Severe restrictions for biological tomography are due to the high radiation sensitivity and the low contrast of ice-embedded samples; both together lead to a very low signal-to-noise ratio in tomographic reconstructions. With the advent of microprocessor-controlled TEMs, scientific-grade large-area CCD cameras and sufficiently fast desk-top computers in the late eighties, it was possible to automate tomographic data acquisition. To keep the area of interest in the field of view of the CCD camera and in the desired defocus (slight underfocus) one has to correct for lateral and longitudinal displacements that occur upon tilting, due to the limited accuracy of the goniometer. Due to the automation it became feasible to record tomographic data sets (tilt series) at liquid nitrogen temperature of specimens embedded in vitreous ice. In structural biology, electron tomography is applied to macromolecular as well as to the cellular specimens. The determination of macromolecular structures requires averaging over 102 to 104 particle 3D images. The resolution that can be obtained is in the range of 2 to 4 nm. For investigating ice-embedded supramolecular assemblies, such as whole prokaryotic cells or organelles of eukaryotic cells (of a thickness of several 100 nm up to ca. 1 mm), we now use an intermediate voltage (300 kV) microscope, equipped with an imaging energy filter, which is used in the zero-loss mode, to get rid of the high background of inelastically scattered electrons. A main goal of investigating whole cells is to identify single macro-molecular complexes in their cellular context and to possibly get clues on their function in the cell.

Additive sets, sets of uniqueness, minimal matrices and plane partitions

Ernesto Vallejo

In this talk we introduce the notion of a minimal matrix and explain how it is used to give an algebraic characterization of sets of uniqueness, namely finite sets in $\mathbb{Z}^3$ which are uniquely determined by its slice vectors (also called plane sum vectors). This characterization uses matrices with non-negative integer coefficients and prescribed row and column sum vectors. We also give, using this algebraic language, a new characterization of additive sets. From this we can show that the notions of additivity and of being a set of uniqueness coincide for sets contained in a box of size $2 \times q \times r$, for any positive numbers q, r.

Multi-index problems

Milan Vlach

The multi-index transportation problems of linear programming introduced by Motzkin almost five decades ago are natural extensions of the standard transportation problem. They consist of minimizing a linear or affine function of multiple subscripted variables over a polytope given by the nonnegativity constraints on all variables and the equality constraints involving combinations of different kinds of sums of variables.

Qualitative differences between various types of multi-index problems are demonstrated by distinguishing properties of the basic symmetric cases of the three-index problem. Open problems and applications are discussed.

LP-guided approximation algorithms

Sven de Vries

We study the problem of approximating binary images that are only accessible through few evaluations of their discrete X-ray transform, i.e., through their projections counted with multiplicity along some lines. This inverse discrete problem belongs to a class of generalized set partitioning problems and allows a natural packing relaxation. For this ($\mathbb{NP}$-hard) optimization problems we present various approximation algorithms that are based on greedy-rounding of an LP-solution. We provide theoretical bounds for their performance and report on new computational results. In particular, the corresponding integer programs are solved with only small absolute error for instances up to 250000 binary variables.

Impulses from the theory of inverse problems to discrete tomography

Gerhard-Wilhelm Weber

This talk bases on problems from nonlinear optimization and control, motivated by science and engineering, on their algorithmic treatments by using discrete mathematical methods, and on various applications. Under the perspective of discrete tomography, the talk is devoted to the aspect of inverse problems.

We begin by paying attention to the wide class of generalized semi-infinite optimization problems. For such a problem we state stability properties which carefully utilize the Theorem of Implicit Functions. In this context, some combinatorics reflects topological behaviour. Herewith, we prepare the development of solution algorithms, e.g., by discretization or local linearization, and by taking account of intrinsic polyhedral structures. Being aware of the problem's hardness, techniques from reverse engineering randomization and discrete tomography should be utilized.

Then we turn to optimal control of ordinary differential equations, characterizing global stability again, and to time-minimal control of heating, presenting numerical results based on our semi-infinite problems.

We conclude by further interrelations between continuous and discrete theory, and by three recent investigations in the field of discrete tomography. There is the utilization of wavelets for detecting roughness, and the interpretation and measurement of the atoms distribution by linear codes and optimal experimental designs.

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