Mathematisches Forschungsinstitut Oberwolfach

Report No. 2 / 2001

Combinatorial Convexity and Algebraic Geometry

January 14th – January 20th, 2001

The conference was organized by Victor V. Batyrev (Tübingen), Peter McMullen (London), Tadao Oda (Sendai) and Bernard Teissier (Paris).

51 scientists participated. During the five days of the conference 18 main talks were given, and several study groups were organized. Recent developments in combinatorics as well as algebraic geometry were presented, with special focus on their interplay.

The organizers and participants thank the 'Mathematisches Forschungsinstitut Oberwolfach' for making this conference possible.

The abstracts are listed in the order the talks were given. Abstracts of the informal talks are added at the end.

Abstracts

Polynomial equations with generic Newton polyhedra

Askold Khovanskii

Consider a system of equations $P_1(x) = \cdots = P_n(x)$, $x \in (C^*)^n$, where the P_i are Laurent polynomials whose Newton polyhedra are located sufficiently generally with respect to each other. In the two-dimensional case, it means that two Newton polygons do not have parallel sides with identically directed outer normals.
It turns out that such a system of equations is very simple: it resembles in many ways just one polynomial equation in one unknown. Using the geometry of Newton polyhedra, one can explicitly sum the values of any given Laurent polynomial T over the roots of the system. One also can compute the product of all roots of the system in the group $(C^*)^n$. This last computation involves what are called Parshin's symbols, and is related to Parshin-Kato theory.

The ℓ-vector of general and balanced simplicial complexes

Gabor Hetyei

This talk explores the properties of an invariant that seems to be associated to many linear (in)equalities of various classes of simplicial complexes. Its flag version, the flag ℓ-vector, provides the sparsest known basis to describe all linear inequalities that hold for the flag f-vector of the order complex of an arbitrary graded poset. The formula that connects it to the flag h-vector is an involution. The entries of the flag h-vector may be defined as the number of different shelling components in the lexicographic shelling of the order complex of a CL-shellable poset (such posets were studied by Björner and Wachs.) This definition inspires a "planar analogue" of the flag h-vector, and other generalizations. For each analogue, the connection formula with the flag ℓ-vector remains an involution.
A slightly modified version of the flag ℓ-vector yields the cd-index of an Eulerian poset. An easy new proof of the existence of the cd-index may be given using the modified flag ℓ-vector.
Finally, the non-flag version of the ℓ-vector is connected to the Hilbert series of the local cohomology module of the Stanley-Reisner ring of a simplicial complex.
Most of these results are joint work with Louis Billera and Margaret Bayer; some of it is work in progress.

Algebraic Morse Theory, Stratified Toroidal Varieties and Factorization of birational maps

Jaroslaw Włodarczyk

We develop a Morse-like theory in order to prove the Weak Factorization Theorem, which states that any birational map between smooth complete algebraic varieties over an algebraically closed field can be factored into a sequence of blow-ups and blow-downs. In the above theory, the Morse function is replaced by a K^*-action. The critical points of the Morse function correspond to fixed point components of the action. The homotopy type changes when we pass through the critical points. Analogously, in the algebraic setting "passing through" the fixed points of the K^*-action induces some simple birational

transformations called blow-ups, blow-downs and flips. They are analogous to spherical modifications.
In classical Morse Theory, by means of the Morse function we can decompose the manifold into elementary pieces—"handles". In the algebraic Morse theory, we decompose a birational map into blow-ups and blow-downs.
The important technical tool used in the proof is the theory of Stratified Toroidal Varieties, generalizing the theory of Toroidal Embeddings.

Vertex algebras associated to toric varieties

Lev. A. Borisov

Let M and N be dual free abelian groups, and let Σ be a complete rational polyhedral fan in N that defines a toric variety $\mathbb{P}_\Sigma$. To this set of combinatorial data one can associate a vertex algebra V with a conformal structure.
To construct V, one first considers a certain degeneration $\mathrm{Fock}^{\Sigma}_{M\oplus N}$ of the standard lattice vertex algebra $\mathrm{Fock}_{M\oplus N}$ constructed for the lattice $M\oplus N$ with the natural quadratic form. This degeneration is defined by Σ, and is analogous to the degeneration of $\mathbb{C}[N]$ in which the product of two monomials is redefined to be zero, unless both degrees lie in the same cone of Σ. Afterwards, the vertex algebra V is obtained from $\mathrm{Fock}^{\Sigma}_{M\oplus N}$ as cohomology with respect to a certain explicitly written differential. This differential depends on a generic choice of complex numbers, one for each one-dimensional cone of Σ. In the smooth case, this choice is irrelevant.
The algebra V is the cohomology of a certain sheaf of vertex algebras on $\mathbb{P}_\Sigma$. When $\mathbb{P}_\Sigma$ is smooth, this sheaf is the chiral de Rham complex constructed by Malikov, Schechtman and Vaintrob. The string-theoretic cohomology of the toric variety $\mathbb{P}_\Sigma$ can be recovered as a certain cohomology of V. In addition, the graded dimension of V is an elliptic genus of $\mathbb{P}_\Sigma$, and has some modular properties. It would be interesting to see what other information is encoded in this vertex algebra. Recently, Malikov and Schechtman have shown that, in the smooth case, if one uses undeformed $\mathrm{Fock}_{M\oplus N}$ in place of $\mathrm{Fock}^{\Sigma}_{M\oplus N}$, then one can recover the quantum cohomology ring of $\mathbb{P}_\Sigma$.

4-Polytopes and 3-Spheres with Fat Face Lattices

Günter M. Ziegler

(joint work with David Eppstein, UC Irvine)

We introduce and study the parameter *fatness*, $\frac{f_1+f_2}{f_0+f_3}$, for the classes of

- rational 4-dimensional convex polytopes,
- 4-dimensional convex polytopes,
- 3-dimensional CW-spheres with the intersection property, and
- Eulerian lattices of rank 5.

It is not clear whether fatness is bounded at all on any of these classes.
Here we construct examples of

- rational 4-dimensional convex polytopes of fatness larger than $5-\varepsilon$,
- 4-dimensional convex polytopes of fatness larger than 5.01, and
- 3-dimensional CW-spheres with the intersection property of fatness larger than $6-\varepsilon$.

This implies counter-examples to conjectured f-vector inequalities of Bayer (1988) and of Billera & Ehrenborg (1999).
Most of our examples are constructed using the "Eppstein construction": as the convex hull of a 4-polytope with all ridges tangent to S^3, and its polar. This construction has a close connection with ball packings in S^3. Their study should lead to an infinite family of 2-simple 2-simplicial polytopes.
Finally, we present the f-vector of an Eulerian lattice of rank 7 (the face lattice of a simplicial 5-dimensional manifold) that is not the f-vector of a 5-dimensional CW-sphere with the intersection property. This separates the classes of f-vectors of $(d-1)$-dimensional spheres and of Eulerian lattices of rank $d+1$, for $d=6$. No such separation is known for $d=4$.

Group completions via Hilbert schemes

Michel Brion

For a given torus T, there is no canonical complete toric T-variety; but if T acts faithfully on a projective algebraic variety X, then one obtains two equivariant completions $\mathcal{H}_{T,X}$ (resp. $\mathcal{C}_{T,X}$) of T: the set of graphs of elements of T, together with their limits as closed subschemes (resp. cycles) in $X \times X$. More generally, for every connected algebraic group G of automorphisms of X, this defines two projective, $G \times G$-equivariant completions $\mathcal{H}_{G,X}$ and $\mathcal{C}_{G,X}$ of G, together with a surjective morphism $\mathcal{H}_{G,X} \to \mathcal{C}_{G,X}$, the restriction of the Hilbert-Chow morphism. If G is the full connected automorphism group of X, then $\mathcal{H}_{G,X}$ (resp. $\mathcal{C}_{G,X}$) turns out to be the unique irreducible component of the Hilbert scheme (resp. Chow variety) of $X \times X$ containing the diagonal.
If $X = \mathbf{P}^n$ and $G = \mathrm{Aut}(X) = \mathrm{PGL}(n+1)$, M. Thaddeus showed that $\mathcal{H}_{G,X} = \mathcal{C}_{G,X}$ is smooth, and is the classical space of "complete collineations". We generalize this to all homogeneous spaces, as follows.

Let X be a projective variety, homogeneous under a connected group G of automorphisms. Let T be a maximal torus of G, with Weyl group W.
(i) $\mathcal{H}_{T,X} = \mathcal{C}_{T,X}$ is smooth, with fan defined by the chambers of the finite reflection group W.
(ii) $\mathcal{H}_{G,X} = \mathcal{C}_{G,X}$ is smooth; it is the "wonderful completion" of the semisimple adjoint group G constructed by De Concini and Procesi.
(iii) Every point of $\mathcal{H}_{G,X}$ is a reduced, Cohen-Macaulay subscheme of $X \times X$.

Reference: math.AG/0010215

Haiman's $n!$ Theorem and $(n+1)^{n-1}$ Theorem

Bernd Sturmfels

We present the recent spectacular work of Mark Haiman at the interface of Combinatorics and Algebraic Geometry. His $n!$ *Theorem* can be stated in four different guises:

(1) **Combinatorics**: If λ is a partition of n and $\{(p_1, q_1), \ldots, (p_n, q_n)\}$ its diagram, that is, the associated order ideal in $\mathbf{N}^2$, then the determinant $\Delta_\lambda := \det\big(x_i^{p_j} y_i^{p_j}\big)_{1 \leqslant i,j \leqslant n}$ is a homogeneous polynomial in $2n$ variables. The vector space $\mathbf{C}[\partial_{x_1}, \ldots, \partial_{y_n}] \cdot \Delta_\lambda$ consisting of all successive partial derivatives of this polynomial has dimension $n!$.

(2) **Representation Theory**: The *Hilbert-Frobenius series* of the $\mathbf{Z}^2$-graded S_n-module $\mathbf{C}[\partial_{x_1}, \ldots, \partial_{y_n}] \cdot \Delta_\lambda$ is a symmetric polynomial. Its expansion into Schur functions is

$$H_\lambda(q,t;x_1,\ldots,x_n) \quad = \quad \sum_{\mu \vdash n} K_{\mu\lambda}(q,t) \cdot s_\mu(x_1,\ldots,x_n),$$

where the $K_{\mu\lambda}(q,t)$ are the *Kostka-Macdonald coefficients*. The coefficient of $q^i t^j$ in $K_{\mu\lambda}$ equals the multiplicity of the irreducible S_n-module indexed by μ in the graded component of bidegree (i,j). Hence the H_λ coincide with the *Macdonald polynomials*.

(3) **Algebraic Geometry**: The space $(\mathbf{C}^2)^n$ of n labeled points in the affine plane $\mathbf{C}^2$ and the Hilbert scheme H_n of colength n subschemes in $\mathbf{C}^2$ both admit natural morphisms onto the space $S_n(\mathbf{C}^2)$ of n labeled points in $\mathbf{C}^2$. The *isospectral Hilbert scheme* X_n, defined as the reduced fiber product of these two morphisms, is Cohen-Macaulay, Gorenstein and normal. In particular, $X_n \to H_n$ is a flat morphism of degree $n\,!$.

(4) **Commutative Algebra**: For any space E and integers $l,n \geqslant 0$, the *polygraph* is

$$Z(n,l) \quad := \quad \big\{ \, (P,Q) \in E^n \times E^l \, : \, \{P_1,\ldots,P_l\} \subseteq \{Q_1,\ldots,Q_n\} \, \big\}.$$

Let $E = \mathbf{C}^2$, and fix coordinates $P_i = (x_i, y_i)$ for $P \in (\mathbf{C}^2)^n$. Then $Z(n,l)$ is an affine variety in $\mathbf{C}^{2l+2n}$, namely, it is the union of n^l linear subspaces of dimension $2n$. The coordinate ring of $Z(n,l)$ is a free module over the polynomial subring $\mathbf{C}[x_1,\ldots,x_n]$.

The flatness theorem in (3) implies that $X_n \to H_n$ can be identified with the universal family of Nakamura's *Hilbert scheme of orbits* $(\mathbf{C}^2)^n//S_n$. Using recent work of Bridgeland, King and Reid on the *categorical McKay correspondence*, Haiman proves that the higher cohomology of certain natural bundles on the Hilbert scheme $H_n = (\mathbf{C}^2)^n//S_n$ vanishes. He then applies his vanishing theorems to solve a long-standing problem in Combinatorics. The resulting $(n+1)^{n-1}$-*Theorem* states the following: if I is the ideal generated by all homogeneous S_n-invariants of positive degree in $R = \mathbf{C}[x_1,\ldots,x_n,y_1,\ldots,y_n]$, then R/I is a vector space of dimension $(n+1)^{n-1}$, the number of labeled tress on n nodes.

Polyhedral K_2

Winfried Bruns

Some of our joint work with J. Gubeladze (Tbilisi) follows the principle *from vector spaces to polytopal algebras:* we try to transfer notions defined in terms of finite-dimensional K-vector spaces and K-linear maps to polytopal K-algebras $K[P]$ and their morphisms. Here K is a field, and P is a lattice polytope. The algebra $K[P]$ has the lattice points of P as generators, subject to the binomial relations that represent their affine dependences. The bridge between vector spaces and polytopal algebras is formed by two facts: (i) a vector space is regarded as the degree 1 component of its symmetric algebra, which upon the choice of a basis is isomorphic to $K[X_1,\ldots,X_n]$ ($n = \dim V$); (ii) $K[X_1,\ldots,X_n] = K[\Delta_{n-1}]$ for the unit $(n-1)$-simplex Δ_{n-1}.
We construct a polytopal variant of classical K-theory in which $\mathrm{GL}_n(K)$ and the group $\mathrm{E}_n(K)$ of elementary matrices are replaced by $\mathrm{gr.aut}(K[P])$ and its elementary subgroup. The vehicle by which we can pass to stable, perfect groups is the so-called doubling spectrum of a lattice polytope. For a class of lattice polytopes that we call balanced we can establish a full variant of Milnor's construction of K_2: the polytopal Steinberg group is the universal central extension of the stable elementary group, and one is justified to call

its center $K_2(P)$. (The field of coefficients can be replaced by an arbitrary commutative ring.)
We have classified all balanced polytopes of dimension 2. For a subclass of the class of balanced polytopes (of arbitrary dimension) one can also define higher K-groups.

Fiber Polytopes and the generalized Baues problem

Vic Reiner

This talk surveys the background and known results on the generalized Baues problem (GBP) of Billera and Sturmfels. The GBP concerns the topology of a partially ordered set $\omega(P \to Q)$ associated to any linear surjection $P \to Q$ of polytopes. The elements of the poset, roughly speaking, are polytopal subdivisions of Q which are projections of polytopal subcomplexes of the boundary of P, and the order relation is by refinement. The subposet $\omega_{coh}(P \to Q)$ consisting of the *coherent* subdivisions (*i.e.*, those induced by a certain geometric construction) is known by a theorem of Billera and Sturmfels to be the poset of proper faces of a polytope of dimension $\dim P - \dim Q$, called the *fiber polytope* $\Sigma(P \to Q)$. Consequently, the poset $\omega_{coh}(P \to Q)$, or rather its order complex, is homeomorphic to a $(\dim P - \dim Q - 1)$-sphere. The GBP asks whether $\omega(P \to Q)$ has the same homotopy type.
This is known to be true for $\dim Q = 1$ and for $\dim P - \dim Q \leqslant 2$, but false in general by a counterexample of Rambau and Ziegler. Nevertheless, there are interesting and important special cases of the problem which remain open.

Secondary polytopes for infinite periodic decompositions

Valery Alexeev

The moduli space of stable toric pairs comes with a moment map whose image is a union of polytopes. These polytopes include all secondary polytopes, but also some "generalized secondary polytopes". If one replaces the torus action by an action of a semiabelian group and allows it to vary, even more general polyhedral bodies appear, this time with infinitely many faces. The purpose of this talk was to introduce such a generalized polytope for every complex of lattice polytopes, including the infinite periodic case; and to discuss relations between these bodies and the Generalized Baues Problem.

K-theory of toric varieties

Joseph Gubeladze

One knows that vector bundles on affine toric varieties are trivial. The naïve higher K-theoretical version of this (phrased in terms of K-homotopy invariance) already fails badly for K_1. A natural substitute is the conjecture on the nilpotency of the multiplicative action of $\mathbb{N}$ on the appropriate nil-K-theory of toric varieties, not necessarily affine. This contains both the K-homotopy invariance of regular schemes due to Quillen and, in the affine situation, the stable version of the triviality of vector bundles.
The conjecture is known to be true for Milnor's K_2, or higher groups when the cones are simplicial.
Recently, using Thomason's Mayer-Vietoris sequences for singular varieties and Suslin-Wodzicki's solution to the excision problem, the conjecture has been reduced to a question on the endomorphism ring of (global sections of) certain rank 2 vector bundle on a certain

quasi-projective toric variety. The ring has a nice explicit description, and is closely related to several standard K-theoretical constructions.
Such a reduction amounts to saying that the obstruction to the validity of the conjecture is concentrated on a single point living in a "critical cone". More precisely, in order to prove the conjecture one has to show that the admissible morphisms of the category of vector bundles on the toric variety of reference can be supported by monomials not in this cone; what we do is avoid all points in the cone except maybe one distinguished point.
This provides a plausible approach to the general case. The remaining step is naturally linked with the Witt vector actions on nil-K-theories and induced filtrations.

The Poincaré series of a quasihomogeneous surface singularity

Wolfgang Ebeling

K. Saito has introduced a duality between polynomials which are products of cyclotomic polynomials. He has shown that V. I. Arnold's strange duality between the 14 exceptional unimodal hypersurface singularities is related to such a duality between the characteristic polynomials of the monodromy operators of the singularities. Moreover, he has observed that the dual polynomials pair together to the characteristic polynomial of an automorphism of the Leech lattice. There is a relation of this duality with the polar duality of certain polytopes.
We consider a normal surface singularity (X, x) with good $\mathbb{C}^*$-action and orbit invariants $\{g; b; (\alpha_1, \beta_1), \ldots, (\alpha_r, \beta_r)\}$. The coordinate algebra A is a graded algebra. We consider the Poincaré series $p_A(t)$ of A. If (X, x) is a Kleinian singularity not of type A_{2n}, then we can show that the Poincaré series $p_A(t)$ is a quotient $p_A(t) = \phi_A(t)/\psi_A(t)$, where $\phi_A(t)$ and $\psi_A(t)$ are the characteristic polynomials of the Coxeter element and the affine Coxeter element respectively of the corresponding root system. This is derived from the McKay correspondence.
In the general case we also write $p_A(t) = \phi_A(t)/\psi_A(t)$ as a quotient of two polynomials which may have common factors. We can show that, if (X, x) is a hypersurface singularity, then $\tilde{\phi}_A(t) := \phi_A(t)/(1-t)^{2g}$ is the dual (in Saito's sense) of the characteristic polynomial of the monodromy operator of (X, x). We can interpret this result in terms of a polar duality between certain Newton polytopes. Similar results can be proved for isolated complete intersection singularities of certain types.
Let $p_A(t) = \phi_A(t)/\psi_A(t)$ be the Poincaré series of a Kleinian singularity, a simply elliptic ICIS, a Fuchsian ICIS, a quasihomogeneous hypersurface singularity in $\mathbb{C}^3$ with Milnor number $\mu = 24$ or a special ICIS in $\mathbb{C}^4$ with $\mu = 25$. We can show that the polynomials $\phi_A(t)$ and $\psi_A(t)$ and their dual rational functions are related to the self-dual characteristic polynomials of automorphisms of the Leech lattice, and all such polynomials can be obtained in a suitable way.

Combinatorial Intersection Cohomology for Fans

Karl-Heinz Fieseler

In the talk, we presented a purely combinatorial approach to the intersection cohomology of a toric variety, based on joint work with G. Barthel, J.-P. Brasselet and L. Kaup. Closely related work has been done by P. Bressler and V. Lunts; *cf.* the abstract of the talk by P. Bressler in this volume.

While the quite general approach via intersection complexes of M. Ishida applies to arbitrary perversities, ours is limited to the middle (self-dual) perversity, and originates in the study of the equivariant intersection cohomology of a toric variety. In fact, we define an "intersection Stanley-Reisner-modul" ${E^\bullet}_\Delta$ for any fan Δ, a module over the homogeneous ring $A^\bullet$ of polynomial functions on the vector space V the fan Δ is living in. It is obtained as the set of global sections of a sheaf $\mathcal{E}^\bullet$ (called a "minimal extension sheaf") on the topological space Δ (the subfans being the open subsets). That sheaf replaces, in the non-simplicial case, the sheaf of Δ-piecewice polynomial functions; it can be constructed successively on the q-skeletons $\Delta^{\leqslant q}$ of Δ, starting with the piecewise polynomials at $\Delta^{\leqslant 2}$. Namely, if $\mathcal{E}^\bullet|_{\Delta^{\leqslant q}}$ is known, we determine ${E^\bullet}_\sigma := \mathcal{E}^\bullet(\sigma)$ for a $(q+1)$-dimensional cone σ as follows. Denote by $\overline{M}^\bullet$ the graded vector space obtained from an $A^\bullet$-module $M^\bullet$ by reducing mod $\mathbf{m} := A^{>0}$. Now take the already constructed module $M^\bullet := {E^\bullet}_{\partial\sigma}$, and set ${E^\bullet}_\sigma := A^\bullet \otimes_{\mathbf{R}} \overline{M}^\bullet$, the restriction map ${E^\bullet}_\sigma \longrightarrow {E^\bullet}_{\partial\sigma}$ being induced by some $\mathbf{R}$-linear section of the residue map $M^\bullet \longrightarrow \overline{M}^\bullet$.
This construction leads to a meaningful intersection cohomology $IH^\bullet(\Delta) := {\overline{E}^\bullet}_\Delta$ for "quasi-convex" fans Δ, *i.e.*, fans Δ generated by their n-dimensional cones ($n = \dim V$) such that the support $|\partial\Delta|$ of their boundary subfan is a real homology manifold, and a combinatorial Poincaré duality holds. For a rational complete fan, the components of the h-vector satisfy $h_i := \dim IH^{2i}(\Delta)$, while the g-vector is determined as $g_i = \dim \overline{E}^{2i}_\sigma$. In order to remove the rationality assumption, it remains to prove a combinatorial version of the Hard Lefschetz theorem.

Quasi-ordinary singularities via toric geometry

Pedro González Pérez

Quasi-ordinary singularities form a class of complex analytic singularities that we can study using the methods of toric geometry. In the hypersurface case, the topological type of these singulaties is completely characterized by the data of a finite number of fractional monomials, the characteristic monomials, that can be associated to a parametrization of the hypersurface. In the irreducible case, it is well known that the normalization is a toric singularity, *i.e.*, the singularity of an affine normal toric variety at its special point. We have built two toric procedures, depending only on the combinatorial data of the characteristic monomials, that provide embedded normalization of the hypersurface in a variety with only toric singularities. We have shown then that solving all these toric singularities provides an embedded resolution of the quasi-ordinary hypersurface.

Quasisymmetric functions and Eulerian enumeration

Louis J. Billera

(joint work with Samuel Hsiao and Stephanie van Willigenburg)

We describe some recent work that links enumeration of chains in (graded) Eulerian posets to questions about representations of quasisymmetric functions. In each of these subjects there is a naturally defined algebra, which turn out to be related. The *peak algebra* Π of Stembridge consists of all quasisymmetric functions arising from so-called "enriched P-partitions". The algebra $A_{\mathcal{E}}$ of the speaker and Liu consists of all chain-enumeration functionals $\sum \alpha_S f_S$ on Eulerian posets. Both algebras have Hilbert series given by the Fibonacci numbers.

Recently, Bergeron, Mykytiuk, Sottile and van Willigenburg have shown that, when equipped with natural coproducts, Π and $A_{\mathcal{E}}$ are dual Hopf algebras. Thus, for a rank $n+1$ graded poset P, the natural quasisymmetric function $F(P) = \sum_{S\subset[n]} f_S(P)\, M_S$ is in Π if and only if P is Eulerian.

A main new result is that, in terms of the standard basis Θ_S for Π, the coefficients of the representation of $F(P)$, for P Eulerian, are given by the coefficients of the **cd**-index of P (actually one-half the **c2d**-index). One corollary is that the quasisymmetric functions $2F(Z)$, for Z the face-lattice of a zonotope (alternately, hyperplane arrangement) form a full-dimensional sublattice in Π.

Coordinate and diagonal subspace arrangements

Volkmar Welker

For a simplicial complex Δ over the ground set $[d] := \{1, \ldots, d\}$ we study two objects:

- the Stanley-Reisner ideal I_Δ of Δ;
- the arrangement V_Δ of coordinate subspaces that define the vanishing locus of I_Δ in real d-space.

A result by Gasharov, Peeva and Welker shows that the cohomology of the complement of V_Δ with coefficients in k and the Tor-groups $\mathrm{Tor}_*^{k[x_1,\ldots,x_d]}(k[x_1,\ldots,x_d]/I_{\Delta^*}, k)$ are isomorphic as k-vectorspaces—here Δ^* denotes the combinatorial Alexander dual of Δ. In parallel work, Babson & Chan (for $\mathrm{char}\, k \neq 2$) and Buchstaber & Panov (in general) show that this indeed is an isomorphism of algebras. For arrangements defined by diagonals $\{x_{i_1} = \cdots = x_{i_l}\}$, an additive result holds for the squarefree multidegree of $\mathrm{Tor}_*^{k[x_1,\ldots,x_d]/I_\Delta}(k,k)$ and the cohomology of the complement, by a result of Peeva, Reiner and Welker. A more general additive result was proven by Buchstaber & Panov.

Hypersurface exceptional singularities

Shihoko Ishii

(joint work with Yuri Prokhorov)

In this paper, we try to determine hypersurface exceptional singularities defined by non-degenerate function. Since an exceptional singularity is log-canonical, under our situation it must be either strictly log-canonical or canonical. For the strictly log-canonical case, the singularity is exceptional if and only if it is purely elliptic of type $(0, d-1)$, where d is the dimension of the singularity . If $d = 2$, then it is a simple elliptic singularity and there are 3-types $\tilde{E}_6$, $\tilde{E}_7$ and $\tilde{E}_8$. If $d = 3$ then it is a simple $K3$-singularity , and isolated weighted homogeneous ones are classified into 95-types by Yonemura. Here we study the other case in which the singularity is canonical. In the 2-dimensional case, canonical singularities are hypersurface and there is a plt blow-up which is given by a weighted blow-up. Moreover, the class of exceptional canonical 2-dimensional singularities is bounded: E_6, E_7, E_8. We generalize these facts to the higher dimensional case. A criterion for a canonical singularity to be an exceptional singularity is obtained by Prokhorov by means of plt blow-up. In order to make use of it, we have to construct a plt-blow-up first. First we prove that there exists a weighted blow-up which gives a plt-blow-up. We also prove that, for every canonical singularity defined by a non-degenerate function, there exists such a singularity defined by a weighted homogeneous polynomial such that the former is exceptional if and only if the latter is exceptional. Thus we can reduce the problem to the weighted homogeneous case. Next, we prove the finiteness of the set of all weights of weighted homogeneous hypersurface exceptional singularities of fixed dimension. Then, we determine all exceptional singularities of the Brieskorn type of dimension 3.

Intersection cohomology for fans

PAUL BRESSLER

(joint work with Valery Lunts)

We consider a fan as a topological space (the open sets are the subfans) equipped with a sheaf of graded algebras (the sheaf of continuous cone-wise polynomial functions graded by assigning degree 2 to the linear functions).

To each fan we associate the category of sheaves of finitely generated graded modules over the structure sheaf which are flabby and locally free. We show that this category is semi-simple (*i.e.*, every object is non-canonically isomorphic to a direct sum of indecomposable objects), and the indecomposable objects are in one-to-one correspondence with the cones of the fan. The unique zero-dimensional cone corresponds to a distinguished indecomposable object.

We prove that, for a complete fan in a vector space, the global sections of an object of our category constitute a free graded module over the graded (by assigning degree 2 to the linear functions) ring of polynomial functions on the vector space. The higher cohomology vanishes due to flabbiness.

Subdivision of fans induces morphisms of associated ringed spaces. We show that our category is stable by direct image under morphisms induced by subdivision, *i.e.*, the direct image of a flabby locally free sheaf is, likewise, flabby and locally free, while the higher direct images vanish due to flabbiness. In particular, the direct image of an indecomposable object is non-canonically isomorphic to a direct sum of indecomposable objects.

We define a contravariant "duality" involution on our category. If the fan is complete, the (free) module of global sections of an object is dual to the module of global sections of the dual object. The distinguished indecomposable object is non-canonically isomorphic to its own dual.

For a complete fan, we define the intersection cohomology of the fan as the reduction modulo the maximal ideal (of polynomials vanishing at the origin) of the module of global section of the distinguished indecomposable object. The intersection cohomology of a complete fan is a finite dimensional graded vector space. We conjecture that the dimensions of the graded components coincide with the generalized h-numbers introduced by R. Stanley. It follows from the results mentioned above that, for complete fans, the dimensions of the graded components of the intersection cohomology satisfy "Poincare duality" and behave in non-decreasing fashion under subdivision.

Abstracts of informal talks

Ordinary Polytopes and h-Vectors

Margaret M. Bayer

My main topic of research is the flag vectors of polytopes. The *h-vector* of a rational polytope, that is, the sequence of middle perversity intersection homology Betti numbers of the toric variety associated to the polytope, is a linear function of the flag vector. One obstacle to finding conditions on flag vectors or h-vectors of polytopes is that we do not know how to generate broad classes of nonsimplicial polytopes. I am currently studying a promising class of (odd-dimensional) nonsimplicial polytopes, the "ordinary polytopes", introduced by Bisztriczky and further studied by his student Dinh. These polytopes are "multiplicial", that is, all faces are multiplexes, a type of nonsimplicial polytope that generalizes simplices. The ordinary polytopes include as a special case cyclic polytopes, which played a central role in the study of f-vectors of simplicial polytopes, and in other research on polytopes. The h-vectors of ordinary polytopes range from the h-vectors of multiplices, which are of the form $(1, k, k, k, \ldots, k, 1)$, to those of cyclic polytopes, in which all entries are maximal for the given h_1. This suggests that ordinary polytopes can be used to generate a rich variety of flag vectors and h-vectors. With Carl Lee, I am studying how to generalize to ordinary polytopes the Billera-Lee construction on cyclic polytopes, which was used to prove the sufficiency of the McMullen conditions for h-vectors of simplicial polytopes.

On string-theoretic E-functions

Dimitrios I. Dais

• Let X be a normal complex variety, *i.e.*, a normal, integral, separated scheme of finite type over $\mathbb{C}$. Suppose that X is $\mathbb{Q}$-Gorenstein, *i.e.*, that a positive integer multiple of its canonical Weil divisor K_X is a Cartier divisor. Then X is said to have at most *log-terminal* singularities if there exists an *snc*-desingularization $\varphi\colon \widetilde{X} \longrightarrow X$, *i.e.*, a desingularization of X whose exceptional locus $\mathfrak{Exc}(\varphi) = \cup_{i=1}^{r} D_i$ consists of **s**mooth prime divisors $D_1, D_2, \ldots, D_r$ with only **n**ormal **c**rossings, such that the "discrepancy" w.r.t. φ, which is the difference between the canonical divisor of $\widetilde{X}$ and the pull-back of the canonical divisor of X, is of the form

$$K_{\widetilde{X}} - \varphi^* (K_X) = \sum_{i=1}^{r} a_i \; D_i$$

with all the a_i's > -1.

• Allowing the existence of log-terminal singularities, in order to pass to stringy invariants, one essentially takes into account the "discrepancy coefficients".

Definition. Let X be a normal complex variety with at most log-terminal singularities, $\varphi\colon \widetilde{X} \longrightarrow X$ an snc-desingularization of X as above, $D_1, D_2, \ldots, D_r$ the prime divisors of the exceptional locus, and $I := \{1, 2, \ldots, r\}$. For any subset $J \subseteq I$ define

$$D_J := \begin{cases} \widetilde{X}, & \text{if } J = \emptyset \\ \bigcap_{j \in J} D_j, & \text{if } J \neq \emptyset \end{cases} \qquad \text{and} \qquad D_J^{\circ} := D_J \smallsetminus \bigcup_{j \in I \smallsetminus J} D_j \; .$$

The algebraic function

$$E_{\mathrm{str}}(X;u,v) := \sum_{J\subseteq I} E\left(D_J^{\circ};u,v\right) \prod_{j\in J} \frac{uv-1}{(uv)^{a_j+1}-1}$$

(under the convention $\prod_{j\in J} = 1$ if $J=\emptyset$, and $E(\emptyset;u,v) := 0$) is called the *string-theoretic E-function* of X. (The usual E-polynomials are defined as in [2]).

The main result of [1] says that:

Theorem. *$E_{str}(X;u,v)$ is independent of the choice of the snc-desingularization $\varphi : \widetilde{X} \longrightarrow X$.*

Definition. The rational number

$$e_{\mathrm{str}}(X) := \lim_{u,v\to 1} E_{\mathrm{str}}(X;u,v) = \sum_{J\subseteq I} e\left(D_J^{\circ}\right) \prod_{j\in J} \frac{1}{a_j+1}$$

is called the *string-theoretic Euler number* of X. Moreover, the *string-theoretic index* $\mathrm{ind}_{\mathrm{str}}(X)$ of X is defined to be the integer

$$\mathrm{ind}_{\mathrm{str}}(X) := \min\left\{ l\in\mathbb{Z}_{\geqslant 1} \;\middle|\; e_{\mathrm{str}}(X)\in \frac{1}{l}\mathbb{Z} \right\} .$$

• All $\mathbb{Q}$-Gorenstein toric varieties X have at most log-terminal (but not necessarily isolated) singularities, $\mathrm{ind}_{\mathrm{str}}(X)=1$, and $e_{\mathrm{str}}(X)$ is equal to the nomalized volume of the defining fan. Moreover, $E_{\mathrm{str}}(X;u,v)$ is a polynomial for Gorenstein toric varieties X, but this is not true for non-toric Gorenstein singularities.

• In the papers [4] and [3] we deal with the evaluation of the E_{str}-functions and string-theoretic Euler numbers $e_{\mathrm{str}}(X)$ for the underlying spaces X of three-dimensional **A**-**D**-**E** singularities, and of $\mathbf{A}_{n,\ell}^{(r)}$-singularities

$$x_1^{n+1} + x_2^{\ell} + x_3^{\ell} + \cdots + x_{r+1}^{\ell} = 0,$$

and emphasize some distinctive features of the computational methodology, in connection with a conjecture of Batyrev [1] about the boundedness of $\mathrm{ind}_{\mathrm{str}}(X)$.

References

[1] Batyrev V.V.: *Stringy Hodge numbers of varieties with Gorenstein canonical singularities*, in "Integrable Systems and Algebraic Geometry", Proceedings of the Taniguchi Symposium 1997, Eds. M.-H. Saito, Y. Shimizu & K. Ueno; World Scientific, (1998), pp. 1-32.

[2] Batyrev V.V., Dais D.I.: *Strong McKay correspondence, string-theoretic Hodge numbers and mirror symmetry*, Topology **35**, (1996), pp. 901-929.

[3] Dais D.I.: *On the string-theoretic Euler number of a class of absolutely isolated singularities.* preprint, math.AG/0011118, (2000).

[4] Dais D.I., Roczen M.: *On the string-theoretic Euler number of 3-dimensional A-D-E singularities,* preprint, math.AG/0011117, (2000).

Some surprising similarities between matroids and shifted families

Art Duval

A recursion due to Kook expresses the Laplacian eigenvalues of a matroid M in terms of the eigenvalues of its deletion $M-e$ and contraction M/e by a fixed element e, and an error term. We show that this error term is given simply by the Laplacian eigenvalues of the relative family $(M-e, M/e)$.

We further show that, by suitably generalizing deletion and contraction to arbitrary families, the Laplacian eigenvalues of shifted families satisfy this exact same recursion.

Remarks on the compactification of toric varieties

Günter Ewald

There exists in $\mathbb{R}^3$ a polygon P with 14 rational vertices that has the following property. Looking from 0 each vertex is "hidden behind" an edge. We present explicitly the vertices of P. This P can be used to construct a toric variety with peculiar compactification properties.

Equivariant embeddings into smooth toric varieties

Jürgen Hausen

By a theorem of J. Włodarczyk, a normal variety X can be embedded into a (normal separated) toric variety if and only if any two points of X have a common affine neighbourhood. We consider some problems arising from Włodarczyk's result. The first one concerns singularities: On the one hand, one would like to get rid of the assumption of X being normal; on the other hand it is important to know when one can choose a smooth ambient space. Among other things, we prove:

Theorem. *An irreducible variety X admits a closed embedding into a smooth toric variety if and only if X is 2-divisorial, i.e., any pair $x, x' \in X$ has a common affine neighbourhood of the form $X \setminus \operatorname{Supp}(D)$ with an effective Cartier divisor D on X.*

A second problem is to ask for equivariant embeddings with respect to actions of connected linear algebraic groups. For quasiprojective varieties, Sumihiro's Equivariant Embedding Theorem guarantees existence of a locally closed equivariant embedding into a projective space. The equivariant version of our above result gives a generalization.

Theorem. *Let X be a normal 2-divisorial variety with a regular action of a connected linear algebraic group G. Then there exist a smooth toric variety Z endowed with a linear G-action and a G-equivariant closed embedding $X \to Z$.*

Reference: math.AG/0005086

Intersection complexes of fans and toric varieties

Masanori Ishida

We fix a subfield k of the field $\mathbb{R}$ of real numbers. Let Δ be a finite k-fan in $\mathbb{R}^r$. We define the intersection complex $\mathrm{ic}(\Delta)^\bullet$ of middle perversity in an additive category $\mathrm{CGEM}(\Delta)$. By applying a functor Γ, we get a finite complex $\Gamma(\mathrm{ic}(\Delta))^\bullet$ of finite dimesional graded k-vector spaces.

After proving the decomposition theorem of cohomologies for barycentric subdivisions of fans, we get the following theorems.

The First Diagonal Theorem If Δ is complete, then

$$\mathrm{H}^p(\Gamma(\mathrm{ic}(\Delta))^\bullet)_q = \{0\},$$

for $p, q \in \mathbb{Z}$ with $p \neq q + r$.

The Second Diagonal Theorem Let $\pi \subset N_{\mathbb{R}}$ be a k-cone of dimension r. Then

$$\mathrm{H}^p(\Gamma(\mathrm{ic}(F(\pi) \setminus \{\pi\}))^{\bullet})_q = \{0\},$$

if $p + q \geqslant 0$ with $p \neq q + r - 1$, or $p + q \leqslant -1$ with $p \neq q + r$.

Using these theorems, we get the following analogy of the strong Lefschetz theorem for a complete k-fan Δ which has a strongly convex piecewise linear function h. For each $0 \leqslant i < r$, there exists a k-linear map

$$\phi_i\colon\ \mathrm{H}^i(\Gamma(\mathrm{ic}(\Delta))^{\bullet})_{i-r} \longrightarrow \mathrm{H}^{i+1}(\Gamma(\mathrm{ic}(\Delta))^{\bullet})_{i-r+1}$$

defined by h, and this is injective if $i \leqslant (r-1)/2$, and surjective if $i \geqslant (r-1)/2$.

References

Masa-Nori Ishida, Intersection complexes of fans and toric varieties (preprint).

Toric resolution and special representation for McKay correspondence

Yukari Ito

The McKay correspondence for simple singularities was observed by J. McKay in 1979, and has now been developed in several areas, not only in Mathematics, but also in Physics. The McKay correspondence is originally a correspondence between simple Lie algebras of type A, D and E and finite subgroups of $SL(2,\mathbb{C})$. ¿From the view point of singularity theory, this correspondence implies a one-to-one correspondence between non-trivial irreducible representations of a finite subgroup G of $SL(2,\mathbb{C})$ and exceptional divisors of the minimal resolution of $\mathbb{C}^2/G$.

We can see this phenomenon as a numerical correspondence by the relation

$$\text{Cartan matrix} = -\text{intersection matrix}$$

via the corresponding Dynkin diagram.

More geometrically, in 1983 Gonzalez-Sprinberg and Verdier explained this correspondence in terms of vector bundles over the minimal resolution using case by case calculations, and Artin and Verdier gave a conceptual proof in 1985.

Later, Knörrer and Esnault tried to consider the case when the group G is a finite subgroup of $GL(2,\mathbb{C})$. Moreover, Wunram and Riemanchneider defined the special representations, and Wunram proved the one-to-one correspondence between the special representations and the exceptional divisors of the minimal resolution of $\mathbb{C}^2/G$.

In around 1997, Nakamura and the author introduced the G-Hilbert scheme, which is a G-orbits of the Hilbert scheme of points on $\mathbb{C}^n$, and proved that the G-Hilbert scheme of $|G|$ points on $\mathbb{C}^2$ is the minimal resolution of $\mathbb{C}^2/G$ when $G \subset SL(2,\mathbb{C})$. Later, Kido showed that a similar result holds for the cyclic case, and recently A. Ishii proved this for any finite subgroup of $GL(2,\mathbb{C})$.

In this workshop, I would like to introduce the meaning of the special representations in terms of toric geomtry in dimension 2. Of course, all of them are cyclic, but we can obtain the corresponding exceptional divisors from the information of G-Hilbert schemes which can be written as combinatorial data.

As a remark, we would like to compare other generalizations of the McKay correspondence, which gives higher dimensional versions. Most of them used conjugacy classes in spite of representations to make a correspondence with the cohomology of the minimal (resp. crepant) resolutions (*cf.* Ito-Reid, Batyrev-Dais) and their generalized (stringy) cohomology in GL case is different from the usual cohomology of the minimal resolutions even in dimension 2 (*cf.* Batyrev and Konsetvich's motivic integration). On the other hand, our special representaion gives the usual representation in dimension two, but the generalization to the higher dimensional case is still unknown.

Notes on embedding of abelian surfaces into toric 4-folds

Takeshi Kajiwara

We study embedding of abelian surfaces into projective toric 4-folds. This work is in progress. We first remark that a proper toric d-fold ($d \geqslant 3$) contains no abelian subvariety of dimension $d-1$. So, we start to study this case, which is related to the study of vector bundles on toric varieties.

Embedding an abelian surface into 4-dimensional projective space $\mathbb{P}^4$ was first studied by Ramanan, and, by Serre's construction, gives us an indecomposable vector bundle of rank 2 on $\mathbb{P}^4$. Such a vector bundle is well known as a Horrocks-Mumford bundle, which was constructed by using monads.

Sankaran studied the embedding of abelian surfaces into toric 4-folds with Picard number at most 2. We study further such embedding into toric 4-folds. For example, we can show that every embedding of an abelian surface into the product of toric surfaces is trivial, *i.e.*, the abelian surface is the product of elliptic curves which are contained in the toric surfaces. This is a generalization of a result of Hulek.

Cohomology ring of semiample hypersurfaces and nef complete intersections, and string cohomology of orbifolds

Anvar R. Mavlyutov

We have solved the long-standing problem of describing the cohomology of semiample hypersurfaces in complete simplicial toric varieties. The description uses the canonical contraction associated with the semiample divisor. The solution was first obtained for nondegenerate hypersurfaces, and this helped us to understand the situation for all semiample quasismooth hypersurfaces. In the process of calculation, we discovered a new vanishing theorem for semiample divisors. When the divisor is trivial, this vanishing theorem reduces to Danilov's vanishing result of the cohomology of a complete simplicial toric variety. The description of cohomology of nef complete intersections combines beautifully a "Cayley trick" and the canonical contraction. We also present some insight into how the string cohomology of orbifolds can be deduced from the descriptions of the cohomology of resolutions.

Triangulations of simplicial polytopes

Peter McMullen

A triangulation of a simplicial d-polytope P which contains no interior faces of dimension $\lfloor \frac{1}{2}d \rfloor$ is called small-face-free (SFF). If P has a SFF triangulation, then it is unique. The generalized lower bound conjecture (GLBC) states that certain linear combinations $g_r(P)$ of the face-numbers of P are non-negative for $1 \leqslant r \leqslant \frac{1}{2}d$; moreover, equality would occur only when P has a triangulation with no interior $(d-r)$-faces. An important implication for the GLBC is the following. The inequalities $g_r(P) \geqslant 0$ are part of the g-theorem. Using these, and an induction argument on SFF triangulations, it can be shown that the general equality in the GLBC follows from the case $d = 2r$.

Bando-Calabi-Futaki characters of compact toric manifolds

Yasuhiro Nakagawa

Let X be a compact toric manifold, and L a holomorphic line bundle on X with $c_1(L) > 0$. When L is the anti-canonical line bundle K_X^{-1} of X, we consider the existence problem for an Einstein-Kähler metric whose Kähler form represent the de Rham cohomology class $2\pi c_1(L) = 2\pi c_1(X)$. As to such existence of Einstein-Kähler metrics, an obstruction, which is called the Futaki character, is known. For general L, Bando, Calabi, and Futaki generalized the Futaki character to an obstruction to the existence of constant scalar curvature Kähler metrics whose Kähler form represent the de Rham cohomology class $2\pi c_1(L)$, which we call the Bando-Calabi-Futaki character. We have proved that the Bando-Calabi-Futaki character of a compact toric manifold vanishes on the Lie algebra of the unipotent radical of the automorphism group.

On Minkowski decompositions of polytopes

Gajane Panina

The virtual polytope group was originally introduced by Khovanskii & Pukhlikov (see also McMullen and Morelli). Virtual polytopes in real space $\mathbb{R}^n$ form a group $\mathcal{P}^*$ with respect to Minkowski addition.

A virtual polytope $K \in \mathcal{P}^*$ is called a *k–cylinder* $(k = 1, \ldots, n+1)$, if it is representable as the Minkowski sum of $n-k+1$–dimensional polytopes: $K = \otimes_i K_i$, with $\dim K_i \leqslant n-k+1$.

Consider the following problem.

Given a polytope K, to find whether K belongs to Cyl_k, *i.e.* whether K is decomposable as the Minkowski sum of k-dimensional polytopes.

Its solution is the following. We construct (explicitly) a collection of mutually orthogonal projectors, which are group homomorphisms $\delta_k \colon \mathcal{P}^* \to Cyl_k$, whose sum is the identity operator.

A polytope K belongs to Cyl_k if and only if $\delta_i K = E$ for each $i = 1, ..., k-1$, where E is the unit element of $\mathcal{P}^*$.

These projectors induce the following direct sum decompositions.

$\mathcal{P}^* = \delta_1\mathcal{P}^* \oplus \delta_2\mathcal{P}^* \oplus \cdots \oplus \delta_n\mathcal{P}^*$ and $Cyl_k = \delta_k\mathcal{P}^* \oplus \cdots \oplus \delta_n\mathcal{P}^*$.

Toward the classification of higher-dimensional toric Fano varieties

Hiroshi Sato

A nonsingular toric *Fano* variety is a nonsingular complete toric variety X whose anti-canonical divisor $-K_X$ is ample. The set of isomorphism classes of nonsingular toric Fano d-folds is a finite set for any dimension d. Nonsingular toric Fano d-folds are classified for $d \leqslant 4$, by Batyrev and Watanabe-Watanabe for $d = 3$, and by Batyrev and Sato for $d = 4$. We define two nonsingular toric Fano varieties X_1 and X_2 to be *F-equivalent* if there exists a sequence of equivariant blow-ups and blow-downs from X_1 to X_2 through nonsingular toric *Fano* d-folds. If we get a complete system of representatives for this equivalence relation, then we get the classification of nonsingular toric Fano d-folds. Namely, for a given nonsingular toric Fano d-fold X, we get all nonsingular toric Fano d-folds which are F-equivalent to X by easy calculation.

For $d = 4$, we get a complete system of representatives for F-equivalence relation. By this result, we get the classification of nonsingular toric Fano 4-folds. Actually, we obtained 124 nonsingular toric Fano 4-folds, supplementing Batyrev's list. We also double-checked the result by a computer program.

Valuations, deformations, and toric geometry

Bernard Teissier

Given a nœtherian local integral domain R and a valuation ν of its field of fractions which is non-negative on R, corresponding to an inclusion $R \subset R_\nu$ of R in a valuation ring, I study a geometric specialization of R to the graded ring $\mathrm{gr}_\nu R$ determined by the valuation. If the residue field extension $k = R/m \to R_\nu/m_\nu$ is trivial, this graded ring corresponds to an essentially toric variety, of finite Krull dimension but possibly of infinite embedding dimension; it is of the form:

$$\mathrm{gr}_\nu R = k[(U_i)_{i \in I}]/(U^m - \lambda_{mn} U^n)_{(m,n) \in E}, \quad \lambda_{mn} \in k^*,$$

where I and E are countable sets and U^m denotes a monomial. In order to apply this fact to a characteristic-blind proof of local uniformization by deformation of a partial resolution of singularities of $\mathrm{Specgr}_\nu R$, in the case where R is equicharacteristic and excellent, with k algebraically closed, I explore the following strategy

1) Extend the valuation ν to a valuation $\hat{\nu}$ of a suitable nœtherian ν-adic completion $\hat{R}^{(\nu)}$ of R such that

$$\mathrm{gr}_\nu R = \mathrm{gr}_{\hat{\nu}} \hat{R}^{(\nu)}.$$

2) Obtain a presentation, that is a surjective continuous morphism of k-algebras

$$k\widehat{[(u_i)_{i \in I}]} \to \hat{R}^{(\nu)},$$

where the left term is a suitable *scalewise* completion of the polynomial ring, and the kernel is generated up to closure by elements whose initial forms for the term order w deduced from $\hat{\nu}$ are the binomial equations defining $\mathrm{gr}_\nu R$, that is, which are of the form:

$$u^m - \lambda_{mn} u^n + \sum_{w(s) > w(m) = w(n)} c_s^{(mn)} u^s, \qquad \text{with } c_s^{(mn)} \in k.$$

3) Show that all but finitely many of these equations serve to express the images of u_j's in the nœtherian ring $\hat{R}^{(\nu)}$ in terms of only finitely many of them. Then a toric map which resolves the finitely many binomial equations involving these finitely many variables will uniformize $\hat{\nu}$ on $\hat{R}^{(\nu)}$.

4) Use the excellence of R to lift this to a uniformization of ν on R.

Edited by Heinke Conrads

Participants

Dr. Annette A'Campo-Neuen
annette@math-lab.unibas.ch
annette.ACampo@uni-konstanz.de
Fakultät für Mathematik
Universität Konstanz
D 201
Universitätsstr. 10
78457 Konstanz

Prof. Dr. Valery Alexeev
valery@domovoy.math.uga.edu
Dept. of Mathematics
University of Georgia
Athens, GA 30602
USA

Dr. Klaus Altmann
altmann@mathematik.hu-berlin.de
Inst. für Mathematik
Humboldt-Universität Berlin
Rudower Chaussee 25
10099 Berlin

Prof. Dr. Laura Anderson
landersn@math.tamu.edu
Department of Mathematics
Texas A & M University
College Station, TX 77843-3368
USA

Prof. Dr. Eric Babson
babson@math.washington.edu
Dept. of Mathematics
Box 354350
University of Washington
C138 Padelford Hall
Seattle, WA 98195-4350
USA

Prof. Dr. Gottfried Barthel
Gottfried.Barthel@uni-konstanz.de
Fakultät für Mathematik
und Informatik
Universität Konstanz
D 203
78434 Konstanz

Prof. Dr. Victor V. Batyrev
batyrev@bastau.mathematik.uni-tuebingen.de
Mathematisches Institut
Universität Tübingen
72074 Tübingen

Prof. Dr. Margaret M. Bayer
bayer@math.ukans.edu
Department of Mathematics
University of Kansas
405 Snow Hall
Lawrence, KS 66045-2142
USA

Prof. Dr. Louis J. Billera
billera@math.cornell.edu
Dept. of Mathematics
Cornell University
Malott Hall
Ithaca, NY 14853-4201
USA

Prof. Dr. Anders Björner
bjorner@math.kth.se
Dept. of Mathematics
Royal Institute of Technology
S-100 44 Stockholm

Prof. Dr. Lev A. Borisov
lborisov@columbia.edu
Department of Mathematics
Columbia University
Mailcode 4432
2990 Broadway
New York, NY 10027
USA

Prof. Dr. Paul Bressler
bressler@ihes.fr
Institut des Hautes Etudes
Scientifiques
Le Bois Marie
35, route de Chartres
F-91440 Bures-sur-Yvette

Prof. Dr. Michel Brion
Michel.Brion@ujf-grenoble.fr
Laboratoire de Mathematiques
Universite de Grenoble I
Institut Fourier
B.P. 74
F-38402 Saint-Martin-d'Heres Cedex

Prof. Dr. Winfried Bruns
winfried@mathematik.uni-osnabrueck.de
Fachbereich Mathematik/Informatik
Universität Osnabrück
Albrechtstr. 28
49076 Osnabrück

Dr. Heinke Conrads
heinke.conrads@ruhr-uni-bochum.de
Fachbereich Mathematik
Ruhr-Universität Bochum
Universitätsstr. 150
44801 Bochum

Prof. Dr. Dimitrios I. Dais
ddais@cc.uoi.gr
Department of Mathematics
University of Ioannina
45110 Ioannina
GREECE

Prof. Dr. Art Duval
artduval@math.utep.edu
Department of Mathematical Sciences
University of Texas at El Paso
El Paso, TX 79968-0514
USA

Prof. Dr. Wolfgang Ebeling
ebeling@math.uni-hannover.de
Institut für Mathematik
Universität Hannover
Postfach 6009
30060 Hannover

Prof. Dr. Günter Ewald
ewaldfamily@t-online.de
Fachbereich Mathematik
Ruhr-Universität Bochum
44780 Bochum

Dr. Karl-Heinz Fieseler
khf@math.uu.se
Department of Mathematics
University of Uppsala
P.O. Box 480
S-75106 Uppsala

Dr. Jonathan Fine
j.fine@pmms.cam.ac.uk
jfine@active-tex.demon.co.uk
203 Coldhams Lane
GB-Cambridge CB1 3HY

Dr. Pedro Gonzalez-Perez
gonzalez@math.jussieu.fr
U.F.R. de Mathematiques
Case 7012
Universite de Paris VII
2, Place Jussieu
F-75251 Paris Cedex 05

Dr. Joseph Gubeladze
jgubelad@mathematik.uni-osnabrueck.de
Fachbereich Mathematik/Informatik
Universität Osnabrück
Albrechtstr. 28
49076 Osnabrück

Dr. Jürgen Hausen
juergen.hausen@uni-konstanz.de
Fakultät für Mathematik
und Statistik
Universität Konstanz
Fach D 203
78457 Konstanz

Dr. Gabor Hetyei
hetyei@renyi.hu
ghetyei@email.uncc.edu
Dept. of Mathematics
University of North Carolina
at Charlotte - UNCC
Charlotte, NC 28223
USA

Prof. Dr. Takayuki Hibi
hibi@math.sci.osaka-u.ac.jp
Department of Mathematics
Graduate School of Sciences
Osaka University
Toyonaka 560-0043
JAPAN

Prof. Dr. Masanori Ishida
ishida@math.tohoku.ac.jp
Mathematical Institute
Graduate School
Tohoku University
Aoba
Sendai 980-8578
JAPAN

Prof. Dr. Shihoko Ishii
shihoko@math.titech.ac.jp
Department of Mathematics
Graduate School of Sciences
Tokyo Institute of Technology
Tokyo 152-8551
JAPAN

Prof. Dr. Yukari Ito
yukari@compo.metro-u.ac.jp
Department of Mathematics
Faculty of Sciences
Tokyo Metropolitan University
Hachioji 192-0397
JAPAN

Dr. Jerzy Jurkiewicz
jurkiewi@mimuw.edu.pl
Instytut Matematyki
Uniwersytet Warszawski
ul. Banacha 2
00-913 Warszawa
POLAND

Prof. Dr. Takeshi Kajiwara
kajiwara@math.tohoku.ac.jp
Mathematical Institute
Tohoku University
Aramaki, Aoba-Ku
Sendai, 980-8578
JAPAN

Prof. Dr. Jean Michel Kantor
kantor@math.jussieu.fr
29 Rue Lacepede
F-75005 Paris

Prof. Dr. Ludger Kaup
Ludger.Kaup@fmi.uni-konstanz.de
Fakultät für Mathematik
und Informatik
Universität Konstanz
D 203
78434 Konstanz

Prof. Dr. Askold Khovanskii
askold@math.toronto.edu
Department of Mathematics
University of Toronto
100 St. George Street
Toronto, ONT M5S 3G3
CANADA

Prof. Dr. Carl W. Lee
lee@ms.uky.edu
Dept. of Mathematics
University of Kentucky
715 POT
Lexington, KY 40506-0027
USA

Prof. Dr. Vincent Maillot
vmaillot@math.jussieu.fr
U.F.R. de Mathematiques
Case 7012
Universite de Paris VII
2, Place Jussieu
F-75251 Paris Cedex 05

Dr. Eugeny Materov
eugeny.materov@uni-tuebingen.de
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10
72076 Tübingen

Dr. Anvar Mavlutov
anvar@mpim-bonn.mpg.de
Max-Planck-Institut für Mathematik
Vivatsgasse 7
53111 Bonn

Prof. Dr. Peter McMullen
p.mcmullen@ucl.ac.uk
Department of Mathematics
University College London
Gower Street
GB-London, WC1E 6BT

Dr. Yasuhiro Nakagawa
nakagawa@math.tohoku.ac.jp
Mathematical Institute
Tohoku University
Aramaki, Aoba-Ku
Sendai, 980-8578
JAPAN

Prof. Dr. Tadao Oda
odatadao@math.tohoku.ac.jp
Mathematical Institute
Graduate School
Tohoku University
Aoba
Sendai 980-8578
JAPAN

Dr. Gajane Panina
panina@mail.iias.spb.su
Steklov Institute of Mathematics
Academy of Sciences
Fontanka 26
191011 St. Petersburg
RUSSIA

Prof. Dr. James Pommersheim
jpommersheim@pomona.edu
320 N. Harvard Ave.
Claremont, CA 91711
USA

Prof. Dr. Victor Reiner
reiner@math.umn.edu
Department of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.
Minneapolis, MN 55455
USA

Hiroshi Sato
96m16@math.tohoku.ac.jp
Mathematical Institute
Faculty of Science
Tohoku University
Aoba
Sendai 980-77
JAPAN

Prof. Dr. Uwe Storch
Uwe.Storch@ruhr-uni-bochum.de
Fachbereich Mathematik
Ruhr-Universität Bochum
44780 Bochum

Prof. Dr. Bernd Sturmfels
bernd@math.berkeley.edu
Department of Mathematics
University of California
Berkeley, CA 94720
USA

Prof. Dr. Bernard Teissier
teissier@math.jussieu.fr
Equipe "Geometrie et dynamique"
Institute Mathematique de Jussieu
175 rue de Chevalaret
F-75013 Paris

Prof. Dr. Volkmar Welker
welker@mathematik.uni-marburg.de
FB Mathematik und Informatik
Universität Marburg
Hans-Meerschwein-Str. Lahnbg.
35032 Marburg

Prof. Dr. Jaroslaw Wlodarczyk
wlodar@math.purdue.edu
Dept. of Mathematics
Purdue University
West Lafayette, IN 47907-1395
USA

Prof. Dr. Günter M. Ziegler
ziegler@math.tu-berlin.de
Fachbereich Mathematik
Technische Universität Berlin
Straße des 17. Juni 136
10623 Berlin