Mathematisches Forschungsinstitut Oberwolfach

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Topologische Methoden in der Gruppentheorie

January 28th – February 3rd 2001

The meeting was organized by H. Abels (Bielefeld), P. Kropholler (QMWC, London) and K. Vogtmann (Cornell Univ.)

This meeting continued a series of meetings on topological methods in group theory, taking place about every 4 years. The previous one was in December 1995. At the present meeting new breakthroughs and important advances in the field were presented and discussed. They included the following topics: The recent proof by Krammer and Bigelow that braid groups are linear, new applications of the combinatorial Morse theoretic techniques of Bestvina and Brady, groups acting on cubical complexes — other than the ones studied by Bestvina and Brady —, automorphisms of free groups, splittings of finitely generated groups motivated by the Jaco–Shalen–Johannson decomposition of three–manifolds, L^2–cohomology and bounded cohomology, group actions on buildings, algebraic geometry over the free group and others.

The subject is growing and making progress. Accordingly, there were lively discussions and strong scientific interaction between the participants of the meeting.

List of talks

Monday, January 29

AM

9:00 – 10:00	R. Bieri *Topological properties of SL_2–actions on the hyperbolic plane*
10:15 – 11:15	A. Zuk *On a conjecture of Atiyah*
11:30 – 12:30	K. Brown *The coset poset*

PM

4:00 – 5:00	M. Sapir *Finitely presented non–amenable groups without free subgroups*
5:15 – 6:15	M. Burger *Bounded continuous cohomology and applications*

Tuesday, January 30

AM

9:00 – 10:00	J. Berrick *Acyclic groups and wild arcs*
10:15 – 11:15	B. Rémy *Compactifying trees*
11:30 – 12:30	M. Davis *Mock reflection groups*

PM

4:00 – 5:00	D. Krammer *Braid groups are linear*
5:15 – 6:15	P. Papasoglu *Quasi–isometry invariance of group splittings and JSJ decompositions*

Wednesday, January 31

AM

9:00 – 9:45	I. Leary *Some groups of type VF*
10:00 – 10:45	K.-U. Bux *Generalized weight tests for presentation 2–complexes*
11:00 – 11:45	J. Meier *Duality groups*

Thursday, February 1

AM

9:00 – 10:00	S. Bigelow	*(Correction to) Braid Groups are linear*
10:15 – 11:15	M. Lustig	*The structure of an automorphism of* F_n
11:30 – 12:30	A. Karlsson	*Kobayashi–type metrics and dynamics of endomorphisms*

PM

4:00 – 5:00	L. Mosher	*Quasi–actions on trees*
5:15 – 6:15	W. Lück	*The relation between the Baum–Connes Conjecture and the Trace Conjecture*

Friday, February 2

AM

9:00 – 10:00	R. Charney	*Metric characterizations of spherical and Euclidean buildings*
10:15 – 11:15	W. Ballmann	*On the rank problem in non–positive curvature*
11:30 – 12:30	Z. Sela	*Diophantine geometry over groups and the elementary theory of a free group*

PM

4:00 – 5:00	M. Bridson	*Rigidity and torsion in* $Aut(F_n)$ *and* $Out(F_n)$
5:15 – 6:15	M. Bestvina	*Bounded cohomology of subgroups of mapping class groups*

Abstracts

Some group theoretic applications of twin buildings

Peter Abramenko

Let G be a (possibly twisted) Chevalley group of classical type and of rank $r \geq 1$. Then $G(\mathbb{F}_q[t])$ is of type F_{r-1} and not of type F_r if $q \geq 2^{2r-1}$ (Abels/A. 1987 for the special case $G = SL_{r+1}$ and A. 1995 for the general case). The same result without any restrictions concerning q can possibly be proved by Behr if he succeeds in completing his current research program.

Question: What is the precise finiteness length of $\Gamma = G(\mathbb{F}_q[t, t^{-1}])$?

Conjecture: Γ is of type F_{2r-1}, not of type F_{2r}
(settled only for $r = 1$; $SL_2(\mathbb{F}_q[t, t^{-1}])$ is f.g., not f.p. (Stuhler 77)).
$\Gamma = G(\mathbb{F}_q[t, t^{-1}])$ is a typical example of a group acting "nicely" on a locally finite twin building (Δ_+, Δ_-)
(in this case $\Delta_\pm$ is the Bruhat–Tits building of $G(\mathbb{F}_q((t^{\pm 1}))))$. Another example: $\Gamma = \mathcal{G}(\mathbb{F}_q)$, $\mathcal{G}$ = Kac–Moody group of compact hyperbolic type.

Conjecture: If a group Γ acts "nicely" on a "sufficiently thick" locally finite twin building (Δ_+, Δ_-) with $d := \dim \Delta_\pm$, then Γ is of type F_{2d-1} but not of type F_{2d}.
$\underline{d = 1}$ (twin tree case): Conjecture is true (A. 98; thick is sufficient)
$\underline{d > 1}$: there exists a program of a proof which is carried out in most details; one topological result is still missing.

Recent Counter–Example (in the compact hyperbolic situation): for not "sufficiently thick" twin buildings. $\mathcal{G}$ = Kac-Moody group of hyperbolic type (4,4,4) (i.e. all off-diagonal entries of the Coxeter matrix are 4), $\mathbb{F}_q = \mathbb{F}_2 \Longrightarrow \mathcal{G}(\mathbb{F}_q)$ is even $\underline{\text{not } F_2}$ ($\Longrightarrow$ not $F_{2d-1} = F_3$).
Background: Construction of non–standard Moufang twin buildings.

On the rank problem in non–positive curvature

Werner Ballmann

The rank rigidity for compact Riemannian manifolds of non–positive (sectional) curvature asserts that for any such manifold, M, either the universal covering space, $\widetilde{M}$, is a Riemannian product, or $\widetilde{M}$ is a symmetric space of rank ≥ 2, or else the so–called geometric rank of M is equal to one. In the latter case, some important characteristics of M resemble those of manifolds of negative curvature.
There should be a similar picture in the larger class of compact (singular) spaces of non–positive curvature. Rank rigidity is known for 2–dimensional polyhedra with piecewise smooth metrics of non–positive curvature. In addition, there are some partial results in higher dimension.
I will describe the state of the affairs.

Growth of Lie algebras and parabolic spaces, and counting of normal subgroups

Laurent Bartholdi

Let G be a finitely generated group acting on a d-regular rooted tree, and fix an infinite ray e in the tree. The associated *parabolic space* is $X = G/\operatorname{stab}_G(e)$, with metric induced by the word metric on G. The *growth* of X is the function $g(n) = \#\{x \in X \mid d(x,1) = n\}$. Let $\{\gamma_n(G)\}_{n\in\mathbb{N}}$ be the lower central series of G, and let $\mathcal{L}(G) = \bigoplus_{n\geq 1} \gamma_n(G)/\,\gamma_{n+1}(G)$ be the corresponding Lie ring, first considered by Wilhelm Magnus. Its *growth* is the function $f(n) = \operatorname{rank}(\gamma_n/\gamma_{n+1})$, where $\operatorname{rank}(A)$ is the minimal number of generators of the abelian group A.

Branch groups, as defined by Rostislav Grigorchuk, are those groups acting on rooted d-regular trees that contain a subgroup K itself containing K^d, where the d copies of K act on the d subtrees below the root, and all inclusions have finite index.

Theorem 1. *Let G be a branch group with the notation as above. The growth of X is larger than that of $\mathcal{L}(G)$; more precisely, there is a constant C such that*

$$f(1) + \cdots + f(n) < C(g(1) + \cdots + g(n))$$

holds for all $n \in \mathbb{N}$.

This result follows from a complete description of the Lie algebra of Branch groups. The structure is explicit, and for the most famous examples, to wit the Grigorchuk group $\mathfrak{G}$ and the Gupta-Sidki group Γ, the growth functions are as follows:

Theorem 2.

group	$\sum_{i=1}^n f(i)$	$\sum_{i=1}^n g(i)$
$\mathfrak{G}$	$\sim n$	$\sim n$
Γ	$\sim n^{\log_{1+\sqrt{2}}(3)}$	$\sim n^{\log_2(3)}$

The first row was proven by Rostislav Grigorchuk and myself. The second row is new and answers a long-standing question by Said Sidki.

Normal subgroups of G are naturally associated to ideals in $\mathcal{L}(G)$, which can be described and counted using the methods used in the above results:

Theorem 3. *The number of normal subgroups of $\mathfrak{G}$ of index 2^n is contained between $n\log_2(n)/7$ and $n\log_2(n)/4$, and is an odd number.*

The asymptotics answer a question by Alex Lubotzky.

The above results appear in my preprint "Lie algebras and growth of branch groups", available on the web at

`http://arxiv.org/math.GR/0101222`.

Higher finiteness properties of S-arithmetic groups over function fields

Helmut Behr (Frankfurt am Main)

$[F : \mathbb{F}_q(t)] < \infty,\ O_S \subset F\quad S$-arithmetic subring, $s = \# S$,

G almost simple algebraic group, defined over F,

$r = \operatorname{rank}_F G,\ r_v = \operatorname{rank}_{F_v} G,\ F_v$ completion of $F,\ v \in S$,

$\Gamma\quad S$-arithmetic subgroup of G.

Γ is of type $F_n :\iff \exists\, K(\Gamma,1)$ with finite n-skeleton.
Question: Γ is of type F_{n-1}, but not F_n iff $r > 0$ and $\sum_{v\in S} r_v = n$?

Known results: The answer is yes in the following cases:

(a) $n = 1$ or 2: Finite generation and finite presentability,
H. Behr: Crelle Journal 495 (1998), 79–118.
(b) $G = SL_2$: see U. Stuhler in Inv.math. 57 (1980), 263–281.
(c) G classical, $O_S = \mathbb{F}_q[t],\ q \geq 2^{2n-1}$:
see P. Abramenko, Springer Lecture Notes 1641 (1996),
and H. Abels in Israel J. Math. 76 (1991), 113–128.

Thus there are some open problems (for $G \neq SL_2,\ n \geq 3$):

i) Is the assumption $q \geq 2^{2n-1}$ necessary?
ii) Treat other rings than $F_q[t]$ for $s = 1$ and moreover for $s > 1$, for instance: Is $SL_3\ (\mathbb{F}_q[t,t^{-1}])$ of type F_3?
iii) How to treat non-split groups?

I shall sketch a program to attack this question with some new methods, old in other contexts.

I. Reduction theory of arithmetic groups

There exist two versions of reduction theory (also for number fields). The first one constructs a fundamental ("Siegel-")domain for the action of Γ:
For $\Gamma = G(\mathbb{F}_q[t]),\ G$ Chevalley group, this is a polyhedral cone C in an apartment of the affine Bruhat–Tits–building X for $G(F_v)$. The proofs of the results above use filtrations of C and X and a criterion of K. Brown.
The second version defines "reduction of points in X with respect to parabolic subgroups of G" (cf. G. Harder in Inv.math. 42 (1977), 135–175), which may be viewed as points of the building X_0 at infinity and allows to define the unstable region X' of X (cf. D. Grayson in Springer Lecture Notes 966 (1980), 69–90, and also in Comm.math.Helv. 59 (1984), 600–634, using ideas of Serre, Quillen and Stuhler). X' has a cover, whose nerve is given by the spherical Tits building X_0 for G such that X' has the homotopy type of a bouquet of $(r-1)$-spheres. In the number field case X' may be retracted to its "inner" boundary, but this is not true for function fields! Therefore we need also

II. Buildings with opposition

There is a natural notion of "opposition" in a spherical building X_0 and one can consider the simplicial complex Opp X_0 of pairs of opposite simplices. This complex has the same homotopy type as X_0 itself, shown by R. Charney for the group $G = GL_n$ (see Inv.math. 56 (1980), 1–17), by Lehrer and Rylands for groups of type A_n and C_n (see Math.Ann. 296 (1993), 607–624), and finally in general by A. von Heydebreck (Dissertation Frankfurt 2000) — with completely different proofs using combinatorial methods or homological algebra or the geometry of buildings. I need a version which is also valid over rings; this was proved only in the first case.

III. Special case: G Chevalley group, $\#\mathbf{S} = 1$

In order to answer the question in this case, I construct a space $\widetilde{X'}$ by splitting up X' into apartments in such a way that $\widetilde{X'}$ has a cover whose nerve is $\operatorname{Opp} X_0$. This space can be retracted to its boundary $\partial\widetilde{X'} = \widetilde{Y}$, but $\widetilde{Y}/\Gamma$ is not yet compact; for this purpose I need a subcomplex of the same homotopy type where opposition is defined with respect to Γ. Then we have the

Proposition: If G is an almost simple Chevalley group of rank $r > 0$ over F, then a S-arithmetic subgroup Γ for $\#S = 1$ is of type F_{r-1}.
Conjecture: Γ is not of type F_r.

Empty shell

A. J. Berrick

A (discrete) group G is *acyclic* if $H_i(G; \mathbb{Z}) = 0$ for all $i \geq 1$. In particular, acyclic groups are perfect.
Prominent among examples of acyclic groups in the literature are those with binate structure. G is *binate* if to each finitely generated subgroup H one can associate a homomorphism $\varphi : H \to G$ and $u \in G$ such that for all $h \in H$

$$h = [u,\, \varphi(h)].$$

Examples include acyclic groups used by J Mather (for classifying foliations), Wagoner (for higher algebraic K-theory), Kan & Thurston and Baumslag, Dyer & Heller (for modelling homotopy types by groups) and de la Harpe & McDuff's large automorphism groups. These groups have no nontrivial finite quotient, although every groups is 2-step subnormal in a binate group.
Examples of acyclic groups without binate structure have been more sporadic. The first announced as acyclic were those of Baumslag & Gruenberg's commutator subgroups of certain two-generator, one-relator groups and Epstein's fundamental group of an open 3-manifold (1967). Another is Higman's four-generator, four-relator group, constructed so as to have no nontrivial finite quotient. The talk described joint work with Yan-Loi Wong (to appear in *Proc London Math Soc*) relating these sporadic examples, as follows.

Theorem 1. (Acyclic groups of Baumslag-Gruenberg type) *Let*

$$B = \langle x_n \mid r(x_n, x_{n+1}, \ldots, x_{n+k}) \rangle_{n \in \mathbb{Z}}$$

where r is a word in the free group of rank $k+1$. Then the following statements are equivalent.

(a) *r has exponent sum zero in k of its variables, and exponent sum ± 1 in the remaining variable.*
(b) *B is a perfect group.*
(c) *Define $G = \big\langle x, y \mid r(x, yxy^{-1}, \ldots, y^k x y^{-k}) \big\rangle$. Then*
  (i) *G_{ab} is infinite cyclic,*
  (ii) *B is isomorphic to the commutator subgroup of G, and*
  (iii) *B is acyclic.*

Theorem 2. (Acyclic groups of Higman type) *Let*

$$C_m = \langle x_n \mid r(x_n, x_{n+1}, \ldots, x_{n+k}) \rangle_{n \in \mathbb{Z}/m}$$

where r is a (cyclically reduced) word (involving both x_n and x_{n+k}) in the free group of rank $k+1$, and $m \geq 2k$. Then the following statements are equivalent.

(a) *r has exponent sum zero in k of its variables, and exponent sum ± 1 in the remaining variable.*
(b) *C_m is a perfect group.*
(c) *Define $G_m = \big\langle x, y \mid r(x, yxy^{-1}, \ldots, y^k x y^{-k}),\ [x, y^m] \big\rangle$. Then*
  (i) *$(G_m)_{ab}$ is infinite cyclic,*
  (ii) *C_m is isomorphic to the commutator subgroup of G_m, and*

(iii) C_m *is acyclic.*

Higman's group is the example with $m = 4$ and $r(x_n, x_{n+1}) = x_n[x_n, x_{n+1}]$, where subscripts are to be read in the cyclic group $\mathbb{Z}/4$.

Theorem 3. (Generalization of Epstein and Fox & Artin (1948) constructions) *Let X be a closed connected orientable 3-manifold with infinite cyclic cover $S^2 \times \mathbb{R}$. Let λ be a smooth knot in X such that $[\lambda]$ generates $H_1(X)$. Then the connected infinite cyclic cover of $X \setminus \lambda$ is the complement of a wild arc κ in S^3 with the following properties.*

(i) $S^3 \setminus \kappa$ *is aspherical;*

(ii) $\pi_1(S^3 \setminus \kappa)$ *is the commutator subgroup of* $\pi_1(X \setminus \lambda)$ *with*

$$\pi_1(X \setminus \lambda)/\pi_1(S^3 \setminus \kappa) \cong \mathbb{Z};$$

(iii) $\pi_1(S^3 \setminus \kappa)$ *is acyclic.*

Remarks 4.

1. *Epstein's group is* $\pi_1(L) = \left\langle z_i \mid z_i = [z_i, z_{i+1}^{-1}][z_i, z_{i-1}] \right\rangle_{i \in \mathbb{Z}}$ *, which is evidently of Baumslag-Gruenberg type.*
2. *A* stitch *like Fox's (1949) gives rise to a wild arc κ in S^3; here* $\pi_1(S^3 - \kappa) = \langle b_n \mid b_n = [b_{n-1}, b_n^{-1}][b_n, b_{n+1}^{-1}] \rangle_{n \in \mathbb{Z}}$ *, also of Baumslag-Gruenberg type.*
3. *Such groups are residually finite; do they always have $\mathfrak{A}_5$ as a quotient?*

Bounded cohomology of subgroups of mapping class groups

Mladen Bestvina and Koji Fujiwara

We show that every subgroup of the mapping class group $MCG(S)$ of a compact surface S is either virtually abelian or it has infinite dimensional second bounded cohomology. As an application, we give another proof of the Farb-Kaimanovich-Masur rigidity theorem that states that $MCG(S)$ does not contain a higher rank lattice as a subgroup.
Mladen Bestvina (Latex file of the whole paper is available from the Los Alamos server).

Topological properties of SL_2–actions of the hyperbolic plane

Robert Bieri (Frankfurt)

This is joint work with Ross Geoghegan. Every action $\rho : G \to \text{Isom}(M)$ of a group G on a proper CAT(O)–space M imposes a certain structure on the boundary ∂M, which can be encoded in a sequence of subsets

$$\partial M \supseteq \Sigma^0(\rho) \supseteq \Sigma^1(\rho) \supseteq \cdots \supseteq \Sigma^n(\rho) \supseteq \ldots .$$

The definition uses controlled homotopy over M: We choose a contractible free G–CW–complex X, and a G–map $h : X \to M$; then we say that X is *controlled coarsely* $(n-1)$–*connected over the endpoint* $e \in \partial M$, (CC^{n-1} over e) if each cocompact G–subspace $K \supseteq X$ is contained in a cocompact G–subspace $K' \subseteq K$ with the following property: there is a bag $\lambda \geq 0$ such that for each $i < n$ every singular i–sphere of K over the horoball HB_e (at e) dies in K' over the λ–neighbourhood of HB_e.

Invariance Theorem: This is independent of the choice of X and $h : X \to M$, so that $\Sigma^n(\rho) := \{e \in \partial M \mid X \text{ is } CC^{n-1} \text{ over } e\}$ is an invariant of ρ.

In the lecture I sketched how we compute $\Sigma^n(\mu_m) \subset \partial\mathbb{H}^2$ for the Möbius action $\mu_m : SL_2\left(\mathbb{Z}\left[\frac{1}{m}\right]\right) \to \mathrm{Isom}(\mathbb{H}^2)$ on the hyperbolic plane. We find

Theorem:

$$\Sigma^n(\mu_m) = \begin{cases} \partial\mathbb{H}^2, & \text{if } n < \pi(m) \\ \partial\mathbb{H}^2 - (\mathbb{Q} \cup \infty), & \text{if } n \geq \pi(m), \end{cases}$$

where $\pi(m)$ stands for the number of different primes dividing m. The method applies also to other S–(arithmetic Fuchsian) groups.

Braid groups are linear

Stephen John Bigelow

Vaughan Jones has shown how to obtain representations of the braid group B_n corresponding to the irreducible representation of the symmetric group S_n, indexed by Young diagrams. These can be used to define the Jones and HOMFLY polynomials of a knot or link. The definitions are very natural to an expert in subfactors, but somewhat mysterious to a topologist. I will propose a more topological definition of these representations following work of Ruth Lawrence.

Rigidity and Torsion in Aut(F_n) and Out(F_n)

Martin R. Bridson

The following results are motivated by the desire to extend the analogy betweeen $\mathrm{Aut}(F_1)/\mathrm{Out}(F_1)$ and lattices in higher–rank Lie groups. These results are reminiscent of the rigidity properties of such lattices, but they are proved by elementary means, in particular a detailed analysis of the torsion in $\mathrm{Aut}(F_n)$.
The first two results are part of joint work with Karen Vogtmann.

Theorem 1: *If $1 \geq 3$ then Out(F_n) and Aut(F_n) are co–Hopfian and all of their automorphisms are inner.*

Theorem 2: *If $n < 1$ then any homomorphism φ : Aut$(F_1) \to$ Out(F_n) has image $\{1\}$ or $\mathbb{Z}_2$.*

(There are several related results concerning quotients of $\mathrm{Aut}(F_1)$.)

Theorem 3: *If Aut(F_1) acts by isometries on a complete CAT(O)–space of dimension d and no point is fixed by the whole group, then $n \geq 2(d+2)$.*

The coset poset

Kenneth S. Brown

For a finite group G and a non-negative integer s, let $P(G,s)$ be the probability that a randomly chosen ordered s-tuple from G generates G. Philip Hall gave an explicit formula for $P(G,s)$, exhibiting the latter as a finite Dirichlet series $\sum_n a_n n^{-s}$, with $a_n \in \mathbb{Z}$ and $a_n = 0$ unless n divides $|G|$. For example,

$$P(A_5, s) = 1 - \frac{5}{5^s} - \frac{6}{6^s} - \frac{10}{10^s} + \frac{20}{20^s} + \frac{60}{30^s} - \frac{60}{60^s}.$$

In view of Hall's formula, we can speak of $P(G, s)$ for an arbitrary complex number s. The reciprocal of this function of s is sometimes called the *zeta function* of G.
The work described in this talk arose from an attempt to understand the value of the zeta function at $s = -1$. More precisely, I wanted to explain some surprising divisibility properties of $P(G, -1)$, which is an integer, that I observed empirically. For example,

$$P(A_5, -1) = 1 - 25 - 36 - 100 + 400 + 1800 - 3600 = -1560,$$

which is divisible by $60 = |A_5|$. Similarly, $P(A_6, -1)$ is divisible by $|A_6|$, while $P(A_7, -1)$ is divisible by $|A_7|/3$.
The main theorem is a general divisibility result of this sort. The theorem specifies, for each prime p, a power p^a that divides $P(G, -1)$; the exponent a is defined in terms of the p-local structure of G. The precise statement is somewhat technical and will be omitted.
Perhaps more interesting than the result itself is the nature of the proof, which is topological. The starting point is an observation of S. Bouc, giving a topological interpretation of $P(G, -1)$. Consider the *coset poset* $\mathcal{C}(G)$, consisting of proper cosets xH ($H < G$, $x \in G$), ordered by inclusion. Recall that we can apply topological concepts to a poset $\mathcal{P}$ by using the simplicial complex $\Delta(\mathcal{P})$ whose simplices are the finite chains in $\mathcal{P}$. In particular, we can speak of the Euler characteristic $\chi(\mathcal{P}) := \chi(\Delta(\mathcal{P}))$ and the reduced Euler characteristic $\tilde{\chi}(\mathcal{P}) := \chi(\mathcal{P}) - 1$. Bouc's observation, then, is that

$$P(G, -1) = -\tilde{\chi}(\mathcal{C}(G)).$$

This makes it possible to study divisibility properties of $P(G, -1)$ by using group actions on $\mathcal{C}(G)$ and proving the contractibility of certain fixed-point sets. The group we use is G, acting by conjugation, or $(G \times G) \rtimes \mathbb{Z}_2$, acting by translation and inversion.
Having studied the Euler characteristic of the coset poset, one naturally wants to go further and study its homotopy type. Our results here are meager, but we show, for example, that $\mathcal{C}(G)$ has the homotopy type of a bouquet of spheres if G is solvable. The dimension of the spheres is $d - 1$, where d is the number of complemented chief factors of G, and the number of spheres is $(-1)^d P(G, -1)$.
There remain many open questions about the coset poset.

Generalized Weight Tests for Presentation 2-Complexes

Kai-Uwe Bux and Steve Gersten

Let $\mathcal{P} = \langle x_1, \ldots, x_m \mid R_1, \ldots, R_n \rangle$ be a finite group presentation, K the associate presentation 2-complex. This complex has one vertex, a loop for each generator, and a disk for each relator glued in so that the relation can be read of its boundary. We fix an orientation on each 2-cell such that the relator is read along the boundary in positive direction.
We denote by L the link of the vertex in K. This graph has two vertices for each generator in $\mathcal{P}$ and one edge for each corner of a relator disk in K. Therefore, we can think of the edges in L as different colours assigned to all the corners of relator disks.
Given a spherical diagram $D : \mathbb{S}^2 \to K$ we pull back the colouring to the corners in D. A vertex $v \in D$ is called *monochromatic* if all surrounding corners are of the same colour – this is, the induced map $Lk(v) \to L$ maps all of $Lk(v)$ to one edge in L. The presentation $\mathcal{P}$ is called *monochromatic* if every reduced spherical diagram $\mathcal{D} : \mathbb{S}^2 \to K_P$ contains at least two monochromatic vertices.
We make the following

Conjecture 1. *Every one relator presentation of the trivial group is monochromatic.*

We develop a sufficient condition for monochromaticity of a presentation in form of a generalized weight test, which we call an *M-test* for obvious reasons. It can be checked mechanically whether a presentation admits an M-test or not. In support of our conjecture, a computer has verified that all one relator presentations of the trivial group with relator length up to 19 have M-tests and are therefore monochromatic.
M-tests can also be used to identify some classes of monochromatic presentations. In particular one can prove the following theorems.

Theorem 2. *If $\mathcal{P}$ is a one relator presentation and the link graph $L_{\mathcal{P}}$ contains a cut edge, then $\mathcal{P}$ admits an M-test.*

Theorem 3. *If $\mathcal{P} = \langle x \mid w(x)\rangle$ where xx occurs precisely once in the cyclic word w, then $\mathcal{P}$ has an M-test.*

This includes a result of R. Fenn and C. Rourke about the presentations

$$\langle x \mid x^{-1}xx^{-1}x \dots x^{-1}xx\rangle$$

which plays a central role in their account on Klyachko's car lemma. In fact, one can use M-tests to reprove Klyachko's car lemma and hence his:

Theorem 4. *The Kervaire Conjecture holds true for torsion free groups, i.e., if G is a non–trivial torsion free group, t a generator of an infinite cyclic group, and $w \in G \star \langle t\rangle \setminus G$, then the natural homomorphism*

$$G \to G \star \langle t\rangle / \langle\langle w\rangle\rangle$$

is injective.

We mention that M-tests can also be used to prove some special cases of the Whitehead conjecture.
For each geometric edge $e \in L$, there are two directed edges e^+ traversing the corner corresponding to e in positive direction with respect to the fixed orientation of the ambient 2–cell and e^- traversing its corner in the other direction.
We will take the set of directed edges as a common set of vertices for the construction of two directed graphs. The graph Γ^b contains an edge from $e_1^{\varepsilon_1}$ to $e_2^{\varepsilon_2}$ if the composition $e_1^{\varepsilon_1} \circ e_2^{\varepsilon_2}$ is a directed path in L. Note that, for every vertex v in a spherical diagram D, the map $Lk(v) \to L$ induces a directed circle in Γ^b.
In the other directed graph Γ^r an edge points from $e_1^{\varepsilon_1}$ to $e_2^{\varepsilon_2}$ if the following hold:

- $\varepsilon_1 = \varepsilon_2$.
- The underlying edges e_1 and e_2 represent adjacent corners in one relator disk.
- With respect to the orientation of the relator disk induced by $\varepsilon_1 = \varepsilon_2$, the corner e_1 precedes e_2.

We think of these graphs as subgraphs of one directed graph $\Gamma = \Gamma^b \cup \Gamma^r$. The edges in Γ^b are coloured black whereas the edges in Γ^r are red.
Note that the map $e^{\varepsilon} \mapsto e^{-\varepsilon}$ on the vertices of Γ induces a colour preserving, orientation reversing involution $\phi : \mathcal{E}(\Gamma) \to \mathcal{E}(\Gamma)$. This is to say: if there is an edge from $e_1^{\varepsilon_1}$ to $e_2^{\varepsilon_2}$ then there is an edge of the same colour from $e_2^{-\varepsilon_2}$ to $e_1^{-\varepsilon_1}$. We extend φ to directed edge paths $P = \vec{e}_1 \circ \dots \circ \vec{e}_r$ in Γ by

$$\phi(P) := \phi(\vec{e}_r) \circ \dots \circ \phi(\vec{e}_1).$$

Given a real valued function $W : \mathcal{E}(\Gamma) \to \mathbb{R}$ assigning weights to edges of Γ we define the *total weight* of the edge path P to be the sum $T_W(P) := \Sigma_{i=1}^r W(\vec{e}_1)$.

Definition 5. *An* M-test *is a real valued weight function* $w : \mathcal{E}(\Gamma) \to \mathbb{R}$ *satisfying the following axioms:*

1. *Every black loop has weight* 0.
2. *For every closed edge path* P *of black edges that passes through at least two different vertices of* Γ, *we have*
$$T_w(P) + T_w(\phi(P)) \geq 4\pi.$$
3. *For every red closed edge path* P, *we have* Γ, *we have*
$$T_w(P) + T_w(\phi(P)) \geq 4\pi.$$
4. *For closed edge path* $P = \vec{e}_1 \circ \cdots \circ \vec{e}_4$ *of length* 4 *with alternating colours, we have*
$$T_w(P) + T_w(\phi(P)) \leq 4\pi.$$

We mention that a slight change in the above definition of allows for constructing an asphericity tests, which we call an *A-test*.
The main result is

Theorem 6. *If a presentation* $\mathcal{P}$ *admits an M-test* $w : \mathcal{E}(\Gamma) \to \mathbb{R}$, *then any reduced spherical diagram* $D : \mathbb{S}^2 \to K_{\mathcal{P}}$ *has at least two monochromatic vertices.*

Metric Characterizations of Spherical and Euclidean Buildings

Ruth Charney

Abstract: A building is a simplicial complex with a covering by Coxeter complexes (called apartments) satisfying certain combinatorial conditions. A building whose apartments are spherical (resp. Euclidean) Coxeter complexes has a natural piecewise spherical (resp. Euclidean) metric with nice geometric properties. We show that we can recognize when a piecewise spherical or piecewise Euclidean complex is a building by a few simple metric properties and that all of the combinatorial information can be retrieved from these properties. For example, we prove that a piecewise spherical complex which is CAT(1) and has the property that every geodesic segment can be locally geodesically continued in a non-empty, discrete set of directions, is isometric to a spherical building. (joint work with Alexander Lytchak)

Mock reflection groups

Michael W. Davis

Abstract: This is a report on some joint work with Tadeusz Januszkiewicz and Rick Scott. It turns out that there is a rich class of examples of nonpositively curved closed manifolds which are tiled by either permutohedra or associahedra. Such examples arise as certain blow-ups of $\mathbf{R}P^n$ of projective hyperplane arrangements associated to finite reflection groups. The universal covers of such examples yield tilings of $\mathbf{R}^n$ by permutohedra or associahedra. The group of symmetries A of such a tiling of the universal cover is generated by involutions, but in general it is not a reflection group, rather it is a "mock reflection group". I explain these examples, give a presentation for the groups A and discuss some of their properties.

On the Hanna Neumann Conjecture

Warren Dicks

§ 1. The group theory

Let G be a free group, and let H and K be finitely generated subgroups of G.
Let $\mathrm{rk}(G)$ denote the rank of G, and let $\tilde{\mathrm{r}}(G)$ denote $\max\{\mathrm{rk}(G)-1,0\}$.
Let

$$\sum : = \sum_{HgK\in H\backslash G/K} \tilde{\mathrm{r}}(H^g\cap K),$$

where the summation is over the set of (H,K) double cosets in G, with each double coset HgK contributing $\tilde{\mathrm{r}}(H^g\cap K)$, a value which does not depend on the choice of representative g of the double coset.
In 1956, Hanna Neumann [4] conjectured that

$$\tilde{\mathrm{r}}(H\cap K)\le \tilde{\mathrm{r}}(H)\,\tilde{\mathrm{r}}(K).$$

In 1990, Walter Neumann [5] introduced the formally stronger statement

$$\sum \le \tilde{\mathrm{r}}(H)\,\tilde{\mathrm{r}}(K),$$

currently referred to as the Strengthened Hanna Neumann Conjecture.
Walter Neumann [5] then showed that modifying the techniques of Hanna Neumann [4] yielded $\sum \le 2\,\tilde{\mathrm{r}}(H)\,\tilde{\mathrm{r}}(K)$; in particular, $\sum$ is finite. Similarly, he showed that modifying the 1971 arguments of R˙ G˙ Burns [1] yields

$$\sum \le \max\{2\,\tilde{\mathrm{r}}(H)\,\tilde{\mathrm{r}}(K)-\tilde{\mathrm{r}}(H),2\,\tilde{\mathrm{r}}(H)\,\tilde{\mathrm{r}}(K)-\tilde{\mathrm{r}}(K)\}.$$

In particular, the Strengthened Hanna Neumann Conjecture holds in the cases where both the subgroups have rank two.
In 1992 and 1996, G˙ Tardos [7], [8] improved this to

$$\sum \le \max\{\tilde{\mathrm{r}}(H)\,\tilde{\mathrm{r}}(K),2\,\tilde{\mathrm{r}}(H)\,\tilde{\mathrm{r}}(K)-\tilde{\mathrm{r}}(H)-\tilde{\mathrm{r}}(K)\}.$$

In particular, the Strengthened Hanna Neumann Conjecture holds in the cases where one of the subgroups has rank two, or both have rank three.
Relatively recently, Ed Formanek and I [3] improved this to

$$\sum \le \tilde{\mathrm{r}}(H)\,\tilde{\mathrm{r}}(K)+\max\{\tilde{\mathrm{r}}(H)-2,0\}\max\{\tilde{\mathrm{r}}(K)-2,0\}.$$

In particular, the Strengthened Hanna Neumann Conjecture holds in the cases where one of the subgroups has rank three.
The foregoing is a condensed description of the progress to date, and omits mention of important work of many mathematicians.

§ 2. The topological methods

In 1983, Stallings [6] showed that it was fruitful to consider the Hanna Neumann conjecture from the viewpoint of pullbacks of immersions of finite graphs.
In 1994, in [2], I built on his work and considered pushouts of immersions of finite graphs, codifying some of the information in terms of finite, simple-edged, bipartite graphs, as

follows. (Here "simple-edged" means that there is at most one edge joining any pair of vertices.)

We can associate with the above groups a finite, simple-edged, bipartite graph D with $m\colon = 2\,\tilde{\mathrm{r}}(H)$ red vertices, $n\colon = 2\,\tilde{\mathrm{r}}(K)$ yellow vertices, and $p\colon = 2\sum$ edges. Moreover, we can embed D in three finite, simple-edged, bipartite graphs A, B, C in such a way that the finite, bipartite "amalgamated graph"

$$(A \vee_D B) \vee (B \vee_D C) \vee (C \vee_D A)$$

is simple-edged and can be expressed as the union of two disjoint subgraphs which are isomorphic to each other (as bipartite graphs). (Here $\vee$ denotes the disjoint union, and $\vee_D$ denotes the the disjoint union amalgamating the two copies of D.)

The Amalgamated Graph Conjecture is the conjecture that the conditions on D given in the preceding paragraph imply that $p \leq \frac{1}{2}mn$. It was shown in [2] that this is equivalent to the Strengthened Hanna Neumann Conjecture.

Notice that if D is connected then the amalgamated graph must have an odd number of components. Notice also that the conditions imply that the amalgamated graph has an even number of components. Thus D is not connected, so $p \leq \max\{mn - m, mn - n\}$. This is Burns' result [1].

If $p > \frac{1}{2}mn$, then D has so many edges that one of the connected components is huge, that is, has more than half of the edges of D, more than half of the red vertices of D, and more than half of the yellow vertices of D. Hence the amalgamated graph has three distinguished components, and if we allow certain "weak" amalgamations to be pulled apart, the rest of the amalgamated graph could be rearranged to form two disjoint isomorphic graphs.

This suggests that we try to find a sequence of notions of atomic factorizations of graphs which are all delicate enough to ensure that D has one huge atomic factor and the amalgamated graph has three distinguished atomic factors, but coarse enough to allow the non-distinguished atomic factors of the previous level to break into pairable pieces. This gives the idea of our (technical) proof [3] that

$$p \leq \frac{1}{2}mn + \frac{1}{2}\max\{m-4, 0\}\max\{n-4, 0\}.$$

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Proper actions of lattices on contractible manifolds

Mark Feighn

Abstract: In joint work with Mladen Bestvina, it is shown that a lattice in a semisimple Lie group G cannot act properly discontinuously on a contractible manifold of dimension smaller than that of G/K where K is a maximal compact subgroup of G.

ℓ^2 invariants for groups and equivalence relations

Damien Gaboriau

Measure Equivalence (ME) between countable groups is a measurable analogue of Quasi-Isometry. M. Gromov gave the following criterion:

Criterion [Gromov ('93)] *Two finitely generated groups Γ_1 and Γ_2 are quasi-isometric iff there exist commuting, continuous actions of Γ_1 and Γ_2 on some locally compact space M, such that the action of each of the groups is properly discontinuous and has a* compact *fundamental domain.*

Similarly,

Definition (Gromov ('93)) *Two countable groups Γ_1 and Γ_2 are* Measurably Equivalent (ME) *iff there exist commuting, measure preserving, free actions of Γ_1 and Γ_2 on some Lebesgue measure space (Ω, m) such that the action of each of the groups admits a* finite measure *fundamental domain.*

Some results and examples

- Standard examples of ME groups are given by lattices (= discrete, finite covolume subgroups) Γ_1 and Γ_2 in the same Lie group G. The space (Ω, m) is (G, Haar) and the lattices act by left (resp. right) multiplication on G.
- ME is an equivalence relation on countable groups.
- Results of Dye ('59), Ornstein-Weiss ('81) and for the most general case Connes-Feldman-Weiss imply that the ME class of $\mathbb{Z}$ (the group of integers) consists in all infinite amenable groups.
- A. Furman, improving R. Zimmer's superrigidity for cocycles, showed that for higher rank simple Lie group G, the collection of all its lattices (up to finite groups) forms a single ME class.
- The ME class of the free group $\mathbf{F}_2$ on two generators contains all finitely generated (non cyclic) free groups, all compact surface fundamental groups, free products of a finite number of amenable groups (excluded $\mathbb{Z}/2\mathbb{Z} * \mathbb{Z}/2\mathbb{Z}$), all lattices in $\mathrm{SL}_2(\mathbb{Q}_p)$,...

To each countable group Γ is associated a sequence of numbers $\in [0, \infty]$ called its ℓ^2 *Betti numbers* $(\beta_n(\Gamma))_{n\in\mathbb{N}}$ that are defined using the ℓ^2 chains of CW-complexes on which Γ acts.

Theorem [G.] *If Γ_1 and Γ_2 are measurably equivalent, then they have proportionnal ℓ^2 Betti numbers.*

More precisely *if (Ω, m) is a measure equivalence between them, and D_1 (resp. D_2) is the fundamental domain of the action of Γ_1 (resp. Γ_2), then for all $n \in \mathbb{N}$:*

$$m(D_2).\beta_n(\Gamma_1) = m(D_1).\beta_n(\Gamma_2).$$

Corollary

- *Lattices in different* $\mathrm{Sp}(n,1)$ *are not ME*
- *Lattices in different* $\mathrm{SU}(n,1)$ *are not ME*
- *Lattices in different* $\mathrm{SO}(2n,1)$ *are not ME*
- *Direct products of a different number of free groups are not ME*

Corollary *Lattices in the same locally compact second countable group have proportional* ℓ^2 *Betti numbers. The ratio is given by the ratio of the covolumes.*

Related statement *If* $\beta_1(\Gamma) > 0$ *then every finitely generated normal subgroup* $N \lhd \Gamma$ *either is finite or has finite index.*
Suppose Γ *has a normal subgroup* N *with infinite amenable quotient* Γ/N*. If for some* n*,* $\beta_n(N)$ *is finite then* $\beta_n(\Gamma) = 0$*.*

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Thompson's Group and non–positive curvature

Ross Geoghegan

I will outline the recent thesis of my student Dan Farley. The theorem is that all diagram groups (in the sense of Kilibarda and Guba–Sapir) which are defined by finite semigroup presentations act freely and properly discontinuously on locally finite CAT(O) complexes. One of those groups, defined by the semigroup presentation $\langle x \mid x = x^2 \rangle$ is Thompson's Group F. Farley also shows that all such groups have type F_∞.

A free group generated by a three state automaton

R. Grigorchuk

There is a canonical way to generate by a finite automaton a group or a semigroup. Automata groups possess many interesting and unusual properties as among them there are infinite torsion groups, groups of intermediate growth, amenable but not elementary amenable groups etc.
The negative solution of the strong Atiyah Conjecture on L^2–Betti numbers was recently done on the base of the automata presentation of the lamplighter group (joint result of R. Grigorchuk, P. Linnell, T. Schick and A. Zuk).
Here is a remarkable problem: which groups have finite automata presentations? In joint work with A. Zuk we answer positively a question of Brunner–Sidki and show that the 3–state automaton of Aleshin generates a free group of rank 3. In the proof a new notion of dual automaton and of double reduced transitivity is used.

Recent results concerning the geometric invariants of metabelian groups

Jens Harlander

Let G be a group and X a $K(G,1)$-complex with finite m-skeleton. A character $\chi : G \to \mathbb{R}$ gives rise to a hight function $h : \tilde{X} \to \mathbb{R}$ on the universal covering of X. The geometric invariant $\Sigma^m(G) \subseteq Hom(G, \mathbb{R})$ (Bieri-Strebel 1980, Bieri-Neumann-Strebel 1987, Bieri-Renz 1988) consists of the set of characters for which the positive half $h^{-1}[0,\infty)$ is (m-1)-connected. These invariants originated in the work of Bieri-Strebel (1980) on finitely generated metabelian groups G where it was shown that $\Sigma^1(G)$ contains the information as to whether G is finitely presented. In general the Σ-invariants contain complete information about the finiteness-type of normal subgroups above the commutator subgroup.

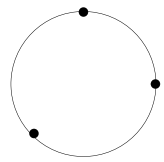

FIGURE 1. $\Sigma^1(G)^c$ for the metabelian group $G = \langle a, x, y \mid [x, y],\ a^x a^y a,\ [a, {}^w a] \rangle$ (w ranges over all words in x, y) shows that G can be finitely presented but is not of type F_3.

Although the Σ-invariants have undergone quite an evolution since 1980 and have been investigated for many different classes of groups, some fundamental open questions remain in the metabelian setting. Two conjectures, the F_m-conjecture and the Σ^m-conjecture, stand out. They can be loosely formulated as follows:

F_m-conjecture: If G is a finitely generated metabelian group then $\Sigma^1(G)$ contains the information as to whether G is of type F_m.

Σ^m-conjecture: If G is a metabelian group of type F_m then $\Sigma^m(G)$ can be obtained from $\Sigma^1(G)$ by a simple process.

The F_2-conjecture is true (Bieri-Strebel 1980) and both conjectures are known to be true for metabelian groups of finite rank (Aberg 1986 (F_m-conjecture), Meinert 1996 (Σ-conjecture)). Although considerable extensions of Abergs work exist (Noskov 1993, Kochloukova 1997) the general case seems to be still far off. In low dimensions some recent progress has been made. The F_3-conjecture (Bieri-Harlander 1999), the Σ^2-conjecture (Kochloukova 1998) and the Σ^3-conjecture (Harlander-Kochloukova 2000) have all been confirmed in the split case. If asked to speak at the meeting I will report on these recent low dimensional results.

One relator products of groups

JIM HOWIE

(Joint with Robert Shwarz)

Once upon a time, long long ago, in a galaxy far from here, I proved some theorems about groups constructed in the following way.
Let G_1, G_2 be groups, $W \in G_1 \star G_2$ be a cyclically reduced word of length ≤ 2, and $m \leq 2$ an integer. Then let

$$G = \frac{G_1 \star G_2}{N(W^m)}$$

(where $N(\cdot)$ denotes normal closure).
If m is big enough (which in practice means $m \geq 4$ unless further restrictions are placed on G_1, G_2 and/or W) then things like the Freiheitssatz ($G_1 \hookrightarrow G \hookleftarrow G_2$) are true. Moreover, the "obvious" construction yields a $K(G,1)$–space, and so $H^*(G)$ is "essentially" made up of $H^*(G_1) \oplus H^*(G_2) \oplus H^*(\mathbb{Z}_m)$, where $\mathbb{Z}_m$ is generated by W.
There are some exceptions to the latter statement, in cases where G is "induced" from some finite triangle group presentation – i.e. $W \equiv U \cdot V$ where $U^p = V^q = 1$ and $\frac{1}{p} + \frac{1}{q} + \frac{1}{m} > 1$.

But in this case we can recover the situation by showing that the "obvious" pushout diagram

$$\begin{array}{ccc} \mathbb{Z}_p \star \mathbb{Z}_q & \longrightarrow & \text{Triangle}(p,q,m) \\ \downarrow & & \downarrow \\ G_1 \star G_2 & \longrightarrow & G \end{array}$$

is "Mayer–Vietoris" (translates to a pushout of $K(\pi,1)$–spaces, and so induces Mayer–Vietoris sequences in (co)–homology.
This work is an attempt to improve the bound $m \geq 4$ to $m \geq 3$. We cannot do so in full generality, but we can prove suitable theorems if we restrict to the case where G is indeuced (in the same sense) from a one–relator product of finite cyclic groups (a *generalized triangle group*), i.e. if $\exists U, V \in G_1 \star G_2$ with $U^P = V^q = 1$ and $W = W'(U,V)$, then $G = \frac{G_1 \star G_2}{N(W^3)}$ is *nice*, in the sense that

(i) the Freiheitssatz holds ($G_1 \hookrightarrow G \hookleftarrow G_2$)
(ii)
$$\begin{array}{ccc} \mathbb{Z}_p \star \mathbb{Z}_q & \longrightarrow & \frac{\mathbb{Z}_p \star \mathbb{Z}_q}{N(W'^3)} \\ \downarrow & & \downarrow \\ G_1 \star G_2 & \longrightarrow & G \end{array}$$
is Meyer–Vietoris.

Applications of bounded cohomology to rigidity and to foliations

Alessandra Iozzi

Recently a systematic theory of continuous bounded cohomology for locally compact groups using homological methods has been developed by Burger and Monod [3], and has proven to have far reaching and very diverse applications.
In this report I want to give a few examples to illustrate how this theory can be used to obtain both rigidity results for actions of finitely generated groups, and a vanishing theorem for the tangential cohomology of some amenable foliations.

Rigidity results. (Joint with M. Burger, [2], [1], [8]) We shall define invariants associated to a continuous representation $\pi : \Gamma \to H$, where Γ is a finitely generated group and H is an appropriate topological group, via the interplay between the pull-backs of bounded cohomology classes and of ordinary cohomology classes of H. We specialize the discussion to two particular cases, where $H = SU(1,n)$ and where $H = \mathrm{Homeo}_+(S^1)$, the groups of orientation preserving homeomorphisms of the circle. In the first result, information will be obtained by the vanishing of an appropriate cohomology class. Namely, let $\omega_n \in H^2_c(SU(1,n), \mathbb{R})$ be the class defined by the Kähler form. For every continuous homomorphism $\pi : \Gamma \to SU(1,n)$, we get a bounded class $\pi^*(\omega_n) \in H^2_b(\Gamma, \mathbb{R})$. Then we have:

Theorem 1. $\pi^*(\omega_n) = 0$ *if and only if either* $\pi(\Gamma)$ *fixes a point in the boundary of* n*-dimensional complex hyperbolic space* $\mathbb{H}^n_{\mathbb{C}}$*, or* $\pi(\Gamma)$ *leaves a totally real subspace of* $\mathbb{H}^n_{\mathbb{C}}$ *invariant.*

If on the other hand we specialize Γ to be a lattice in $SU(1,m)$, $m < n$, then we can get information exactly from the opposite situation, that is, roughly speaking, from the maximality of the invariant. Namely, let $M = \Gamma\backslash\mathbb{H}^m_{\mathbb{C}}$ be a finite volume hyperbolic manifold and assume that either $m \geq 2$ or M is compact (otherwise $H^2(M,\mathbb{R}) = 0$), so that the L^2-cohomology $H^2_{(2)}(M)$ of M injects into $H^2_{dR}(M) \simeq H^2(\Gamma,\mathbb{R})$. If $\pi : \Gamma \to SU(1,n)$ is a representation, ω_M is the Kähler class on M and $< \cdot,\cdot >$ is the standard inner product in $H^2_{(2)}(M)$, we observe that $\pi^*(\omega_n) \in H^2_{(2)}(M)$, so that it makes sense to consider $< \pi^*(\omega_n), \omega_M >$.

Theorem 2. *Let $m \geq 2$. Then $\left|\frac{<\pi^*(\omega_n),\omega_M>}{<\omega_M,\omega_M>}\right| \leq 1$, and equality holds if and only if π is equivariant with respect to an isometric embedding $\mathbb{H}^m_{\mathbb{C}} \hookrightarrow \mathbb{H}^n_{\mathbb{C}}$.*

Since by purely topological methods one can see that $\left|\frac{<\pi^*(\omega_n),\omega_M>}{<\omega_M,\omega_M>}\right|$ is constant on connected components of the representation variety $\mathrm{Rep}(\Gamma, SU(1,n))$, we can conclude the following:

Corollary 3. *There are no non-trivial deformations of Γ in $SU(1,n)$.*

Observe that this extends a result of Goldman and Millson [5] who proved the theorem in the cocompact case. Moreover, the requirement that $m \geq 2$ is necessary, as Gusevskii and Parker [7] constructed examples of (non-cocompact) lattices in $SU(1,1)$ which have quasi-Fuchsian deformations in $SU(1,2)$.
With the same methods we can tackle also problems in which H is not necessarily a linear group, giving for instance a functorial proof of Milnor-Wood inequality ([10], [11]) and of a theorem by Matsumoto [9]. To this extent, let Σ_g be a compact orientable surface of genus $g \geq 2$ and fundamental group Γ and let $\pi : \Gamma \to \mathrm{Homeo}_+(S^1)$ be a homomorphism. If $e \in H^2(\mathrm{Homeo}_+(S^1),\mathbb{Z})$ is the Euler class, then $\pi^*(e) \in H^2(\Gamma,\mathbb{Z}) \simeq H^2(\Sigma_g,\mathbb{Z})$ measures the obstruction to lifting the Γ-action to $\widetilde{S^1} \to S^1$ and defines the Euler number by $\mathrm{eu}(\pi) = (\pi^*(e), [\Sigma_g])$, where $[\Sigma_g] \in H_2(\Sigma_g,\mathbb{Z})$ is the fundamental class of Σ_g.

Theorem 4. *([10], [11], [9], cf. [8], [1]) $|\,\mathrm{eu}(\pi)| \leq \chi(\Sigma_g)$, and equality holds if and only if π is semiconjugate to the action of Γ on S^1 given by any hyperbolization of Σ_g.*

Tangential cohomology of foliations. In [6], Gromov observed that the bounded cohomology of a manifold with amenable fundamental group vanishes and that the bounded cohomology of a negatively curved manifold surjects (in degree 2 and above) onto the ordinary cohomology, hence showing that these two conditions cannot coexist. However, generalizing the above setup to foliations, one has the following:

Theorem 5. [4] *Let X be a compact foliated topological space which is measurable. Suppose also that there is a leafwise Riemannian metric on X with non-positive curvature along the leaves, such that all leaves have rank at most r everywhere. If $(X,\mathcal{F})$ has an amenable fundamental groupoid, the tangential de Rham cohomology groups vanish in degree $n \geq r+1$.*

While the above theorem is proven with purely differential geometrical methods and exploits directly the amenability of the foliation bypassing any bounded cohomology consideration, with Burger [1] we can also give a proof of a related result for foliated bundles arising from amenable actions which is more in the spirit of Gromov's paper, using the functorial approach to bounded cohomology.

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Kobayashi-type metrics and dynamics of endomorphisms

Anders Karlsson

In order to describe certain aspects of the asymptotic geometry of spaces equipped with a Kobayashi-type distance, the so-called Gromov product seems to be a convenient concept. This is illustrated by the recent work of Balogh and Bonk [BaBo 99] proving Gromov hyperbolicity of Kobayashi's metric on strongly pseudo-convex, bounded C^2-domains, and a joint paper with Noskov [KaNo 00] concerning the asymptotic geometry of Hilbert's metric on domains, convex in a strong sense, as well as arbitrary convex bounded domains. The classical Teichmüller spaces may be viewed as in some sense non-strictly convex domains, and may conceivably admit a similar description of their asymptotic geometry. Note that it seems or is a fact that metric spaces of Kobayashi-type are typically not nonpositively curved in either the sense of Alexandrov or of Busemann.

These descriptions in terms of the Gromov product are useful for analyzing the dynamics of individual and random products of endomorphisms (which are distance non-increasing maps), cf. [Ka 99]. The hope is that this will turn out to be useful also for obtaining information about some infinite automorphisms groups.

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Algebraic convergence of function groups

Gero Kleineidam

This is joint work with Juan Souto. Let $\rho : F_k \to PSL_2(\mathbb{C})$ be a discrete and faithful convex-cocompact representation of the free group of rank k into $PSL_2(\mathbb{C})$. Then $\mathbb{H}^3/\rho(F_k)$

is an (open) handlebody H and the quotient of the discontinuity domain of the action of $\rho(F_k)$ on $\hat{\mathbb{C}}$ is a closed Riemann surface of genus k which can be identified with the boundary ∂H of H.
By Ahlfors-Bers theory, there is a covering map from $\mathcal{T}(\partial H)$, the Teichmüller space of ∂H, to the space of $PSL_2(\mathbb{C})$-conjugacy classes of convex-cocompact representations of F_k into $PSL_2(\mathbb{C})$. The deck transformation group is $Mod_0(H)$, the group of those isotopy classes of diffeomorphisms of H which induce an inner automorphism on $\pi_1(H) = F_k$.
Masur [Mas86] and Otal [Ota88] identified an open subset $\mathcal{O}$ of the Thurston boundary of $\mathcal{T}(\partial H)$ with the property that its quotient by $Mod_0(H)$ may be appended to $\mathcal{T}(\partial H)/Mod_0(H)$ as a sort of "boundary at infinity". $\mathcal{O}$ is called the Masur domain. One says that a sequence (ρ_i) of convex cocompact representations converges into the Masur domain if some sequence in $\mathcal{T}(\partial H)$ representing (ρ_i) converges to an element of the Masur domain.

Thurston's Masur domain Conjecture 1. Let (ρ_i) be a sequence of convex cocompact representations of F_k into $PSL_2(\mathbb{C})$ which converges into the Masur domain. Then (after conjugating) (ρ_i) has a subsequence which converges to a discrete and faithful representation of F_k into $PSL_2(\mathbb{C})$.

Canary [Can93] proved the conjecture under some extra assumption on each ρ_i. Otal [Ota94] proved that the conjecture holds for $k = 2$ and arbitrary sequences converging to a minimal arational element in $\mathcal{O}$. Following the strategy of Otal's proof we give an affirmative answer for arbitrary $k \geq 2$ and sequences converging to minimal arational elements of $\mathcal{O}$ (see [KS00]). This is the generic case.
Our result can be extended to fundamental groups of compression bodies, i.e. boundary connected sums of handlebodies and trivial interval bundles over closed surfaces. In this case, using methods of Otal (see [Ohs]) we show that the manifold obtained in the limit is topologically tame, i.e. homeomorphic to the interior of a compact manifold.

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Braid Groups are Linear

Daan Krammer

Let B_n denote the braid group on n strands. A certain representation $\rho : B_n \longrightarrow GL\left(\frac{n(n-1)}{2}, \mathbb{Z}\left[q^{\pm 1}, t^{\pm 1}\right]\right)$ was shown to be faithful by S. Bigelow by a beautiful topological argument. His proof involves neither generators of the braid group nor a basis of the module. We will present a proof of the faithfulness that does involve these things. As a consequence of our method, we obtain a relation between the exponents of t and the Charney length function in the braid group.

Some groups of type VF

Ian J. Leary, University of Southampton

The talk described joint work with Brita E. A. Nucinkis

Throughout the talk, G denotes a discrete group. The universal proper G-space, $\underline{E}G$, may be defined as a terminal object in the homotopy category of proper G-CW-complexes, where a proper G-CW-complex is by definition a G-CW-complex in which all stabilizers are finite. (Compare with the universal free G-space EG.) There is another description of $\underline{E}G$: a model for $\underline{E}G$ is a G-CW-complex E such that for $H \leq G$, the fixed point set E^H is empty if H is infinite and is contractible if H is finite.
If H is any torsion-free subgroup of G, then any model for $\underline{E}G$ is also a model for EH, and in particular the minimal dimension of a model for $\underline{E}G$ provides an upper bound on the virtual cohomological dimension of G (written vcd$G$) whenever G is virtually torsion-free. A theorem of F. X. Connolly and T. Kożniewski (for groups of finite vcd) and W. Lück (in general) states that there is a finite type $\underline{E}G$ if and only if

(a) G contains only finitely many conjugacy classes of finite subgroups;
(b) for each finite subgroup $P \leq G$, the normalizer $N = N_G(P)$ admits a finite type BN.

For each $n > 0$, we exhibit a group $G = G(n)$ for which vcd$G = 3n$ but such that any model for $\underline{E}G$ has dimension at least $4n$. This answers a question first posed by K. S. Brown. These groups are of type VF or 'virtually of type F', i.e., they contain finite-index subgroups $H \leq G$ such that H admits a finite BH. We also exhibit groups G that are virtually of type F for which (a) does not hold and others for which (b) does not hold. These examples show that the property of having a finite (resp. finite type) model for $\underline{E}$(-) does not pass to finite-index supergroups.
Our construction relies on work of M. Bestvina and N. Brady, who constructed, for each non-empty finite flag complex L, a torsion-free group H_L with the properties that finiteness conditions of H_L are controlled by the connectivity properties of L. We observed that H_L is functorial in L, and that interesting groups could be obtained as semi-direct products $H_L \rtimes Q$, where Q is a finite group of automorphisms of the simplicial complex L. We also rely on work of R. Oliver (describing which finite groups can act on finite contractible complexes with what fixed point sets) and on an easy special case of a theorem of J. S. Crisp (describing the fixed point subgroups in certain Artin groups for finite groups of 'graph automorphisms').
The group $G = H_L \rtimes Q$ is virtually of type F if and only if the finite flag complex L is contractible (by the theorem of Bestvina-Brady). The case when G contains infinitely many conjugacy classes of finite subgroups (i.e., (a) fails for G) corresponds to the case when the fixed-point set L^Q is empty. The case when (b) fails for the normalizer $N_G(Q)$ corresponds to the case when L^Q is non-empty but is not contractible. In the case when L is 3-dimensional, contractible, and contains a 3-simplex in a free Q-orbit each of whose faces is in a non-free Q-orbit, then vcd$G = 3$ but any model for $\underline{E}G$ has dimension at least 4. Direct products of this G produce the examples claimed above.
When L has m vertices, the group $H_L \rtimes Q$ embeds in $SL_{2m}(\mathbb{Z})$, and so these groups also answer a question of M. Bridson, who asked whether (a) holds for every $G \leq SL_N(\mathbb{Z})$ admitting a finite type BG.

Irreducible automorphisms of free groups have North–South dynamics on the boundary of Outer Space

G. Levitt and M. Lustig

Suppose $\Phi \in \mathrm{Out}(F_n)$ is irreducible with irreducible powers. It acts on the boundary of Culler–Vogtmann's outer space with two fixed points T^+, T^-. We show that for any $T \neq T^\pm$ the sequence $\Phi^p(T)$ converges to $T^\pm$ as $p \to \pm\infty$. The main new tool in the proof is the following: given an $\mathbb{R}$-tree T with trivial arc stabilizers, we assign to $X \in \partial F_n$ a point $Q(X)$ belonging to either ∂T or the metric completion of T.
Our result, and those of Bestvina–Feighn–Handel on polynomially growing automorphisms, suggest the following question: Given n, is there an integer M with the following property: for any $\Phi \in \mathrm{Out}(F_n)$, and any T in the boundary of outer space, the sequence $\Phi^p(T)$ has at most M limit points as $p \to +\infty$. The analogous question for the action of the mapping class group on the Thurston boundary of Teichmüller space has a positive answer (by Nielsen–Thurston theory).

The relation between the Baum-Connes Conjecture and the Trace-Conjecture

Wolfgang Lück (Münster)

Abstract: We prove a version of the L^2-index Theorem of Atiyah which uses the universal center-valued trace instead of the standard trace. We construct for G-equivariant K-homology an equivariant Chern character, which is an isomorphism and lives over the ring $\mathbb{Z} \subset \Lambda^G \subset \mathbb{Q}$ obtained from the integers by inverting the orders of all finite subgroups of G.We use these two results to show that the Baum-Connes Conjecture implies the modified Trace Conjecture which says that the image of the standard trace $K_0(C_r^*(G)) \to \mathbb{R}$ takes values in Λ^G. The original Trace Conjecture due to Baum and Connes predicted that its image lies in the additive subgroup of $\mathbb{R}$ generated by the inverses of all the orders of the finite subgroups of G, and has been disproven by Ranja Roy recently.

The structure of an automorphism of F_n

Martin Lustig, Marseille (presently at MPI Bonn)

The solution of the conjugacy problem for automorphisms of F_n (for a rewritten complete proof see [1], [2]) has various ingredients which are useful tools for further purposes. We list some of these tools here:

(1) A new version of train tracks for free groups which contain 2-cells (so called "Nielsen faces").
(2) A uniqueness result about F_n-actions on $\mathbb{R}$-trees that are invariant under a given automorphism.
(3) A "Nielsen-Thurston" decomposition of F_n into finitely many *strata* which is canonically associated to any given automorphism. It can be determined algorithmically. On each lowest stratum the induced outer automorphism has finite order.
(4) The algorithmic construction of a train track morphism between any two train track maps which represent the same automorphism.
(5) A (computable) normal form for polynomially growing automorphisms of F_n.

(6) A complete (and algorithmic) analysis of the delicate problem, how an automorphisms can be built (in infinitely many different ways) out of given "sub-automorphisms" defined on distinct strata of F_n. In particular, we obtain a canonical decomposition of (some power of) any given automorphism into finitely many *commuting subautomorphisms.*

In particular, the following corollaries to the above have already been deduced in [2]:

Theorem 1. *For all $\hat{\alpha} \in Out(F_n)$ the centralizer $Cen(\hat{\alpha})$ in $Out(F_n)$ contains*

$$A(\hat{\alpha}) \oplus \bigoplus_{v \in V(\hat{\alpha})} S_v$$

as subgroup of finite index. Here $A(\hat{\alpha})$ is the free abelian subgroup generated by the commuting subautomorphisms of $\hat{\alpha}$, and each S_v is the centralizer of the finite order automorphism induced by $\hat{\alpha}$ on one of its lowest strata, and of a finite family of characteristic conjugacy classes in that stratum.

Theorem 2. *There are algorithms to solve the following problems:*

(1) *For any $\alpha \in Aut(F_n)$ determine a finite generating system of*

$$Fix(\alpha) = \{w \in F_n : \alpha(w) = w\}.$$

(2) *For any space X with $\pi_1 X \cong F_n$ and any map $f : X \to X$ which induces an automorphism $f_* = \hat{\alpha} \in Out(F_n)$, and for any two fixed points $x = f(x)$, $x = f(x') \in X$, decide whether the points x and x' lie in the same Nielsen fixed point class of f.*

Further applications of the above tools concern work in progress which indicates a solution of the following problems:

- A fast proof of Brinckmann's result that every automorphism of F_n without periodic conjugacy classes is hyperbolic (in Gromov's sense).

- A generalization to arbitrary non-polynomially growing automorphisms of the result known for irreducible automorphisms α with irreducible powers (iwip), that F_n acts discretely on the cartesian product of the forward and the backward limit $\mathbb{R}$-tree of α.

- A complete determination of the dynamics of the homeomorphism of ∂F_n induced by any automorphism of F_n (joint work with G. Levitt).

- A decomposition of any iwip automorphisms into finitely many Stallings folds which preserve the associated train track structure.

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Calculating curvatures in concrete complexes

JON McCAMMOND

There now exists a well–developed theory of nonpositively curved metrical simplical complexes. Unfortunately for the working geometric group theorist, even if someone hands you an explicit finite metric simplicial complex, there did not — until recently — exist an algorithm to determine whether the specific complex is non–positively curved.
In this talk we describe such an algorithm.
(joint work with Murray Elder)

$\mathrm{P}\Sigma_n$ is a duality group

JOHN MEIER

In [3] Bieri and Eckmann introduced a class of groups, called duality groups, whose cohomology behaves similarly to manifold cohomology.

Definition 0.1 (Duality groups)**.** Let G be an FP group of cohomological dimension n. The group G is an *n-dimensional duality group* if there exists a G-module D (called the dualizing module) such that $H^i(G,M) \simeq H_{n-i}(G, D\otimes M)$ for all integers i and for all G-modules M. Equivalently, G is a duality group if its cohomology with group ring coefficients is torsion free and concentrated in dimension n. There is geometric content to this concept: If X is a compact $K(G,1)$ of dimension $d = \mathrm{vcd}(G)$, then one can establish that G is a duality group by showing that $\widetilde{X}$ is $(d-2)$-acyclic at infinity. See [2] and [6] for background on duality groups.

Example 0.2. The simplest examples of duality groups are the free and free abelian groups. It's also known that duality-by-duality groups are duality groups, which gives a quick proof that the braid group B_n is a duality group of dimensional $n-1$. In recent work it has been shown that $\mathrm{Aut}(F_n)$ is a virtual duality group of dimension $2n-2$ [1].

My recent work has concentrated on finding various local conditions that imply that a group is a duality group. In particular I am interested in the situation where a group admits a natural action on a poset, where the isotropy groups are infinite.

Definition 0.3 (Posets)**.** A finite dimensional poset $\mathcal{P}$ is *graded* if all its maximal chains have the same length. If ς is an element of a graded poset $\mathcal{P}$, the *rank* of ς is the length of an unrefinable chain from a minimal element of $\mathcal{P}$ to ς, and the *corank* of $\varsigma \in \mathcal{P}$ is $\mathrm{crk}(\varsigma) \equiv d - \mathrm{rk}(\varsigma)$ where d is the dimension of $|\mathcal{P}|$.

A G-poset $\mathcal{P}$ has a *strong fundamental domain* if there is a subposet $\mathcal{F} \subset \mathcal{P}$ which is a filter (if $\varsigma \in \mathcal{F}$ and $\tau > \varsigma$, then $\tau \in \mathcal{F}$) and which contains unique representatives of each G-orbit in $\mathcal{P}$.

Theorem 0.4. (K. Brown & J.M. [see [4]]) *Let G be a group of type FP, with $cd(G) = d$. Let G act on a graded poset $\mathcal{P}$, whose geometric realization $|\mathcal{P}|$ is contractible, and where there is a strong fundamental domain $\mathcal{F} \subset \mathcal{P}$ that is finite and Cohen-Macaulay. If the stabilizer of each element $\varsigma \in \mathcal{P}$ is a $(d - crk(\varsigma))$-dimensional duality group, then G is a d-dimensional duality group.*

The *pure symmetric automorphism group*, denoted $\mathrm{P}\Sigma_n$, is the subgroup of $\mathrm{Aut}(F_n)$ consisting of automorphisms that send each generator x_i to a conjugate of itself. Theorem 0.4 can be used to establish:

Corollary 0.5. (Brady, McCammond, M., & Miller [4]) *The pure symmetric automorphism group is a duality group of dimension* $n-1$.

In addition to being realizable as a natural subgroup of $\mathrm{Aut}(F_n)$, the group $\mathrm{P}\Sigma_n$ arises as a motion group. The pure braid group can be thought of as the group of motions of n points in the plane; $\mathrm{P}\Sigma_n$ consists of the motions of the trivial n component link in S^3. (See [8].)

Question 0.6. Is the group of motions of n-spheres trivially embedded in S^{n+2} always a duality group?

Perhaps an even more elementary question is

Question 0.7. Are the groups of motions of non-trivial links in S^3 virtual duality groups, for all non-trivial links? In other words, was the assumption that we were working with the trivial n component link necessary?

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Quasi–actions on trees

Lee Mosher

Let Γ be a graph of groups, finite, and with finitely generated vertex and edge groups. Let T be the Bass-Serre tree, and by gluing together Cayley graphs, let $p : X \to T$ be a $\pi_1\Gamma$–equivariant. "tree of spaces", so that the $\pi_1\Gamma$ action on X is properly discontinuous and cocompact.
To study quasi–isometric rigidity properties of $\pi_1\Gamma$, given a self–quasi–isometry $\varphi : X \to X$, we ask: does φ coarsely respect the vertex spaces and edge spaces of X?
In many cases where vertex and edge groups are (coarse) $PD(n)$ groups, we give good answers to this question, producing many new quasi–isometric rigidity theorems.

Folding cube complexes, Coxeter groups and the Haagerup approximation property

Graham A. Niblo and Lawrence D. Reeves

In [3] we showed that any group acting without a global fixed point on a finite dimensional CAT(0) cube complex admits an unbounded conditionally negative kernel, and therefore cannot have Kazhdan's property T. It is not to hard to see that if the action is properly discontinuous and the complex is locally finite then the conditionally negative kernel is proper and so, via a result of Bekka, Cherix and Valette, the group satifies the Haagerup

approximation property. Such a group is said to be T-amenable. A result of Higson and Kasparov [2] then shows that such groups satisfy the Baum-Connes conjecture.
In fact the hypothesis that the cube complex is finite dimensional is superfluous and one purpose of this note is to remove it. We show:

Theorem 1. *Let G be a group acting cellularly on a CAT(0) cube complex X. If the action has an unbounded orbit then G does not have Kazhdan's property T, and if the action is proper then G is T-amenable.*

This result was used in the thesis of Dan Farley [1] to show that Thompson's F–group is T–amenable, and can be applied more generally to Guba and Sapir's class of Diagram Groups.
The method used in this paper links closely with the paper of Higson and Kasparov by showing how to construct a (metrically) proper affine isometric action of the group G on a Hilbert space directly from the description of its action on the hyperplanes of the CAT(0) cube complex. This geometric construction is implicit in the paper [4], but is obscured by the algebraic language we used there. It may be viewed as a generalisation of Serre's folding operation which yields an action on Hilbert space given an action on a tree [5].
Finally we wish to put in print our construction of a CAT(0) cube complex for any finitely generated Coxeter group, closing the circle of ideas begun in [3].

Theorem 2. *Let G be a finitely generated Coxeter group. Then G acts properly bon a finite dimensional, locally finite CAT(0) cube complex.*

It is worth remarking that, as recorded in [6], the action of the Coxeter group on the cube complex is co-compact if and only if G contains only finitely many conjugacy classes of triangle group subgroups. This holds for word hyperbolic Coxeter groups and finitely generated right-angled Coxeter groups.

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Metric Characterizations of Spherical and Euclidean Buildings

Quasi–isometry invariance of group splittings and JSJ decompositions

Panos Papazoglou

We show that a one-ended finitely presented group splits over a 2-ended group if and only if its Cayley graph is coarsely separated by a quasi-line. This implies in particular that splittings over 2-ended groups are invariant under quasi-isometries. We show that JSJ-decompositions are also invariant under quasi-isometries.

Compactifying Trees

Bertrand REMY

The purpose of these notes is to present a motivating simple case of a joint project with Y. Guivarc'h and J.-Ph. Anker. The project itself concerns the compactification of Bruhat-Tits buildings, that is the metric spaces naturally attached to semisimple groups over non-Archimedean local fields. We deal with the geometric, the Furstenberg, the Guivarc'h and the polyhedral compactifications. Our guideline is the case of symmetric spaces. Though we will state the problems in the context of general buildings, the results stated here will concern trees.

Let X be a (Riemannian, non-compact) symmetric space, a (locally finite) Bruhat-Tits building or an arbitrary (locally finite but not necessarily Bruhat-Tits) biregular tree. We assume we are given an appropriate automorphism group G, which acts by isometries on X. «Appropriate» means that G is the semisimple Lie group defining X when it is a symmetric space or a Bruhat-Tits building. When X is a tree, G is an arbitrary locally ∞-transitive isometry group in the sense of M. Burger and S. Mozes.

As a non-positively curved space, X admits an asymptotic boundary $\partial_\infty X$ defined as a set of equivalence classes of geodesic rays. A classical procedure of gluing $\partial_\infty X$ to X makes $X \sqcup \partial_\infty X$ a compactification of X, which we call the *geometric compactification* $\overline{X}^{\mathrm{geom}}$ of X. The *polyhedral compactification* $\overline{X}^{\mathrm{pol}}$ of X is defined by means of a gluing $\dfrac{G \times \overline{F}}{\sim}$ where $\overline{F}$ is the (simpler) compactification of a maximal isometric copy F of a Euclidean space in X. For a symmetric space such an F is a *maximal flat*, for a Bruhat-Tits building it is an *apartment*, and for a tree it is simply a geodesic line. The definition of this kind of compactification is now well-known or straightforward.

PROBLEM
A. Give sense to the Furstenberg and the Guivarc'h compactifications of X.

Both compactifications rely on the simple idea which consists in defining an embedding of X into a compact metrizable space, and then taking the closure of the image. For Guivarc'h's procedure, the compact space is the set of closed subgroups of G endowed with the topology of Hausdorff convergence on compact subsets. Furstenberg's procedure involves the theory of boundaries of groups. The suitable compact space is that of the probability measures on a non-trivial Furstenberg boundary of G.

PROBLEM
B. Identify the above compactifications.

For higher-rank symmetric spaces, the compactifications are *not* all isomorphic. The Furstenberg, the Guivarc'h and the polyhedral compactifications are G-homeomorphic, but the geometric compactification is different from all the other ones.

PROBLEM
C. Use the three isomorphic compactifications of X to parametrize interesting classes of closed subgroups (from a dynamical point of view for instance).

In the case of symmetric spaces, points in the Guivarc'h compactification represent closed subgroups of G : they are precisely the maximal subgroups enjoying the property of distality (Guivarc'h's theorem). A subgroup is *distal* if the adjoint image of each of its elements has its spectrum contained in the unit circle. Besides, taking the point stabilizers enables to classify a certain class of maximal amenable subgroups (Moore's theorem).

Here is the result for arbitrary biregular trees.

Theorem 1. *Let X be a semi-homogeneous tree and G be a closed locally ∞-transitive group of automorphisms without inversion.*

A) The Furstenberg and Guivarc'h compactifications $\overline{\mathcal{V}}_X^{\mathrm{Furs}}$ and $\overline{\mathcal{V}}_X^{\mathrm{Guiv}}$ of the set of vertices $\mathcal{V}_X$ of X make sense.
B) The following identifications hold : $\overline{X}^{\mathrm{geom}} \simeq \overline{X}^{\mathrm{pol}} =: \overline{X}$ and $\overline{\mathcal{V}}_X^{\mathrm{Furs}} \simeq \overline{\mathcal{V}}_X^{\mathrm{Guiv}} =: \overline{\mathcal{V}}_X$. The closure of $\mathcal{V}_X$ in $\overline{X}$ identifies with $\overline{\mathcal{V}}_X$.
C) An amenable subgroup of G either fixes a vertex $v \in X$, either fixes a boundary point $\xi \in \partial_\infty X$ or stabilizes a geodesic line $L \subset T$.

Point C) was proved by elementary arguments by A. Figá-Talamanca and C. Nebbia. Here it is seen as a straightforward consequence of a measure-theoretic result (the analogue of Furstenberg's lemma for trees) due to Lubotzky-Mozes-Zimmer.

Finitely presented non–amenable groups without free non–cyclic subgroups

Mark Sapir

This is joint work with A. Yu. Olshanskii. We solve the finitely presented version of the von Neumann problem by constructing a finitely presented non–amenable group without non–abelian free subgroups. A finitely generated non–amenable group without free subgroups was constructed by Olshanskii in 1979 and later by Adian. The problem of constructing a finitely presented example was formulated by Grigorchuk and Cohen in 1982. Our group is an ascending HNN extension of a torsion finitely generated group of exponent $n \gg 1$ ($n \sim 10^{10}$). So it is torsion by cyclic and satisfies the identity $[x,y]^n = 1$. This is the first example of a non–elementary finitely presented torsion of bounded exponent by cyclic group.

Diophantine geometry over groups and the elementary theory of a free group

Zlil Sela

Abstract: We study sets of solutions to systems of equations over a free group, projections of such sets, and elementary sets defined over a free group. The structure theory we obtain enables us to answer some problems of A. Tarski, and classify those f.g. groups that are elementary equivalent to a free group.

Free subgroups of word–hyperbolic groups

Richard Weidmann
(joint with Ilya Kapovich)

Abstract: We give a proof of the following result which has been stated by Gromov in his original paper on hyperbolic groups.

Theorem 1. *For any $n \in \mathbb{N} \exists C = C(n)$ with the following property. Suppose that $H = \langle g_1, \ldots, g_n \rangle$ acts by isometries on a δ–hyperbolic space (X,d) ($\delta > 0$). Then one of the following holds:*

(i) H is free on X.

(2) $(g_1, \ldots, g_n)$ is Nielsen equivalent to $(f_1, \ldots f_n)$ such that $d(f_1 y, y) < \delta \cdot c$ for some $y \in X$.

This has been known for $n = 2$ (T. Delzant) and $n = 3$ (M. Camb). An alternative proof has been announced by G. Aijantseva.

On a question of Atiyah

ANDRZEJ ŻUK

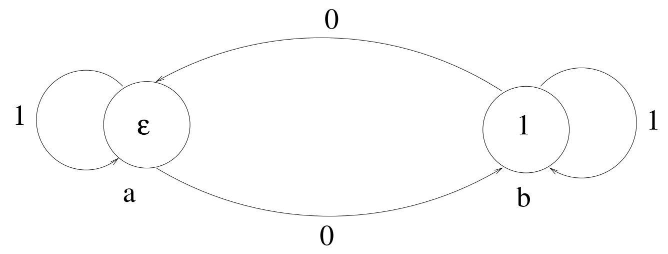

FIGURE 1. The automaton generating the lamplighter group

We present a computation of the spectral measure of a random walk on the lamplighter group and its relation to the Atiyah conjecture about the range of L^2-Betti numbers.

Theorem 1 (Grigorchuk, Żuk [2]). *Let G be a group defined by an automaton as in Figure 1 with a system of generators a and b. Then G is isomorphic to the lamplighter group $\mathbb{Z}_2 \wr \mathbb{Z}$. The spectrum of the Markov ope- rator M on G is equal to $[-1, 1]$. The finite dimensional approximations M_n of M have the spectrum:*

$$\left\{ \cos\left(\frac{l}{q}\pi\right); l \in \mathbb{Z}, q = 1, \ldots, n \right\}.$$

The spectral measure of M is discrete and is equal to

$$\overline{\sigma}\left(\frac{1}{\pi}\arccos(x)\right),$$

where $x \in [-1, 1]$ and for $z \in [0, 1]$

$$\overline{\sigma}(z) = \sum_{q=2}^{\infty} \frac{\#\{p; (p,q) = 1 \text{ and } \frac{p}{q} \leq z\}}{2^q - 1}.$$

Atiyah [1] introduced for a closed Riemannian manifold (M, g) with universal covering $\widetilde{M}$ the analytic L^2-Betti numbers $b^p_{(2)}(M, g)$ which measure the size of the space of harmonic square-integrable p-forms on $\widetilde{M}$. Let $k_p(x, y)$ be the (smooth) integral kernel of the orthogonal projection of all square integrable forms onto this subspace. On the diagonal, the fiber-wise trace $tr_x k_p(x, x)$ is defined and is invariant under deck transformations. It therefore defines a smooth function on M, and Atiyah sets $b^p_{(2)}(M, g) := \int_M tr_x k_p(x, x)\, dx$. By a result of J. Dodziuk this does not depend on the metric and can be determined in combinatorial terms.

Let Γ be a group. Denote with $fin^{-1}(\Gamma)$ the additive subgroup of $\mathbb{Q}$ generated by the inverses of the orders of the finite subgroups of Γ. Note that $fin^{-1}(\Gamma) = \mathbb{Z}$ if and only if Γ is torsion free. We deal with the following conjecture:

Conjecture 2. *If M is a closed Riemannian manifold with fundamental group Γ, then $b^p_{(2)}(M) \in fin^{-1}(\Gamma)$. If Γ is torsion free, this specializes to $b^p_{(2)}(M) \in \mathbb{Z}$.*

In [1] it is only asked whether the L^2-Betti numbers are always rationals, and integers if the fundamental group is torsion free. Later, this question was popularized as the Atiyah conjecture, and also gradually was made precise in the way we formulate it in Conjecture 2 (for a history of this question see the survey paper [4]). The conjecture is proved in many important cases but Theorem 1 enables one to prove the following:

Theorem 3 (Grigorchuk, Linnell, Schick, Żuk [3])**.** *Let the group G be given by the presentation*

$$G = \langle a, t, s \mid a^2 = 1, [t, s] = 1, [t^{-1}at, a] = 1, s^{-1}as = at^{-1}at \rangle.$$

The group G is metabelian and therefore elementary amenable. Every finite subgroup of G is an elementary abelian 2-group, in particular the order of every finite subgroup of G is a power of 2. There exists a closed manifold M of dimension 7 with $\pi_1(M) = G$ such that the third L^2-Betti number

$$b^3_{(2)}(M) = \frac{1}{3}.$$

Conjecture 2 predicts that the denominator is a power of 2 and thus the manifold M is a counterexample.

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