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Geometric Rigidity and Hyperbolic Dynamics

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This conference was the second on Geometric Rigidity and Hyperbolic Dynamics being held in Oberwolfach. It was organized by W. Ballmann (Bonn), A. Katok (PennState) and G. Knieper (Bochum).
The lectures covered recent developments on rigidity and ergodic theory of group actions, and on hyperbolic as well as partially hyperbolic dynamics. Connections to symplectic geometry were also treated.
The stimulating discussions and the marvellous working conditions provided by the Institute of Oberwolfach created a lively scientific atmosphere.
In particular, the participation of many young researchers and mathematicians from such diverse areas such as topology, probability theory and geometry shows that rigidity theory and hyperbolic dynamics are growing and active topics with connections to many mathematical disciplines. Certainly, this conference will be of great influence to further developments and will enhance the research in this field.

Abstracts

The Mok-Siu-Yeung-Bochner Formula

Scot Adams, University of Minnesota

Following work of Mok, Siu and Yeung (Invent. Math. 1993) we show that a nonlinear analogue of the Matsushima Bochner formula implies that up to diffeomorphism a higher rank locally irreducible compact symmetric space of nonpositive curvature admits only one nonpositively curved metric of unit volume, a result due to Gromov.

On the mixing property of hyperbolic flows

Martine Babillot, University P. and M. Curie, Paris

One has the following lemma from ergodic theory:
Lemma (F. Parreau) Let A be an abelian group acting on a Borel standard group (X, m) in a measure preserving way. If for some function $\varphi \in L^2_0(X, m)$ and some sequence $t_n \to +\infty, \varphi \circ T_{t_n} \not\to 0$ in the weak L^2 topology, then there exists a sequence $s_n \to \infty$ and a nonconstant function ψ such that both $\varphi \circ T_{s_n}$ and $\varphi \circ T_{-s_n}$ convergence weakly in L^2 to ψ.
This allows to use Hopf-like arguments for proving the mixing property of hyperbolic flows with respect to measures which have a product structure w.r.t. the stable and unstable foliations. The lemma ist used to prove:
Theorem: If (M, g) is a compact, non positively curved manifold of rank 1, then Knieper's measure of maximal entropy is mixing.

Quasisymmetric spheres and quasi-Möbius group

Mario Bonk, University of Michigan

Suppose Γ is a Gromov hyperbolic group whose boundary at infinity $\partial_\infty\Gamma$ is homeomorphic to the standard 2-sphere $\mathbb{S}^2$. Is $\partial_\infty\Gamma$ quasisymmetric to $\mathbb{S}^2$? In this case the action at Γ on $\partial_\infty\Gamma$ is essentially conjugate to a Möbius group action on $\mathbb{S}^2$? This leads to the general characterization problem at $\mathbb{S}^2$ up to quasisymmetry. In joint work with B. Kleiner we showed that if a metric space X homeomorphic to $\mathbb{S}^2$ is locally linearly connected and Ahlfors 2-regular, then X is quasisymmetric to $\mathbb{S}^2$. Our methods can be applied to certain spaces with Hausdorff dimension $Q > 2$.

A brief survey of partial hyperbolicity and stable ergodicity

Keith Burns, Northwestern University, Evanston, USA

A diffeomorphism $f : M$ is partially hyperbolic if there are a Tf-invariant splitting

$$TM = E^u \oplus E^c \oplus E^s$$

and $\alpha < 1 < \beta$ such that if v_u, v_c, v_s are nonzero vectors in E^u, E^c, E^s respectively, then

$$\frac{||Tfv_s||}{||v_s||} < \alpha < \frac{||Tfv_c||}{||v_c||} < \beta < \frac{||Tfv_u||}{||v_u||}.$$

Suppose now that M is compact, f is C^2, and f preserves a smooth measure μ on M; the subset of the space PH of partially hyperbolic diffeomorphisms with these properties will be denoted PH^2_μ.
Main conjecture: (Pugh and Shub) Ergodicity holds on an open and dense subset of PH^2_μ.
$f \in PH$ is accessible if any two points are joined by a "us-path", composed of parts of unstable and stable manifolds.
The main conjecture splits into to halves.
Conjecture 1 (Pugh and Shub) If $f \in PH^2_\mu$ is accessible, then f is ergodic.
Conjecture 2 (Pugh and Shub) Accessibility holds on an open and dense subset of PH.
Substantial partial results towards both of these conjectures have been proved in the last decade, notably by Pugh and Shub. The lecture surveyed these result.

Growth of conjugacy classes in Gromov hyperbolic groups

Michel Coornaert, University of Strasbourg
(joint work with G.Knieper)

Let Γ be a group acting properly and cocompactly by isometries on a proper geodesic δ-hyperbolic metric space X whose boundary contains more than two points. Let $P(t)$ denote the number of conjugacy classes of primitive elements $\gamma \in \Gamma$ such that $\inf_{x\in X} d(x,\gamma x) \leq t$. We prove that there are positive constants A, B, h and t_0 such that

$$Ae^{ht}/t \leq P(t) \leq Be^{ht} \text{ for all } t \geq t_0.$$

Coarse geometric perspective on negatively curved manifolds and groups

Alex Furman, University of Illinois at Chicago, USA

Let (X,g) be a compact negatively curved manifold. For a point $x \in \tilde{X}$ consider the metric $d_{g,x}$ on $\Gamma = \pi_1(X)$ defined by $d_{g,x}(\gamma_1,\gamma_2) = dist_{\tilde{g}}(\gamma_1 x, \gamma_2 x)$ where $\tilde{g}$ is the lift of g to the universal cover $\tilde{X}$ of X. Then $d_{g,x}$ is a left-invariant metric, quasi-isometric to any word metric on the Gromov hyperbolic group Γ. Moreover for $x, y \in \tilde{X}$, $|d_{g,x} - d_{g,y}| \leq 2diam(X)$. Motivated by this example we study the following general setup: Γ is a non elementary Gromov hyperbolic group, D_Γ the collection of all left invariant metrics d on Γ which are quasi-isometric to a word metric, $D_\Gamma = D_{\Gamma/\sim}$ where $d \sim d'$ in D_Γ if $d - d'$ is bounded. Given a class $\delta = [d] \in D_\Gamma$ we define notion of growth, marked length spectrum, Bowen-Margulis measure, cross-ratio etc. associated with $\delta \in D_\Gamma$, in a way which generalizes the geometric concepts known for (X,g). In this general coarse-geometric setting we give certain characterizations of lattices and locally symmetric metrics in purely coarse-geometric terms.

Lipschitz foliations and critical regularity

Boris Hasselblatt, Tufts University Medford

There are three results, obtained recently, that demonstrate familiar themes and may point to new directions in investigations of optimal and critical regularity on invariant subbundles and foliations in hyperbolic dynamical systems. The central theme is that relations between contraction and expansion rates determine the regularity these invariant structures have (usually measured by a Hölder exponent), and that when this level of regularity is exceeded, the regularity is substantially higher and one obtains additional structural information.

Theorem I: Symplectic C^1-perturbations of the map on the 4-torns defined by $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^{1+[2/\epsilon]}$ either have subbundles that are almost nowhere, ϵ-Hölder, or the subbundles are C^1 with Hölder derivative.

Theorem II. (with P. Foulon, Strasbourg) For a volume preserving Anosov flow on a 3-manifold the sum of the (strong) stable and unstable bundles is Zygmund regular, and the following are equivalent:

- An additive cocycle (longitudinal Katok-Anosov-Moser cocycle) is trivial.
- The sum of the bundles is C^1.
- The sum of the bundles is C^∞.
- The flow is a suspension or a contact flow.

This is closely related to a joint work of Hurder and Katok (Publ. IHES, 1990).

Theorem III. (with J. Schmeling) For a linear solenoid (Smale attractor, DE map) the unstable holonomies are Lipschitz off a set of a priori smaller Hausdorff dimension than the attractor.

A consequence is that the dimension of the hyperbolic set is the sum of the dimensions of (typical) stable and unstable slices. We expect a result of that type in general (topological Eckmann-Ruelle conjecture).

Rigidity of joinings for higher rank Abelian actions

Boris Kalinin, University of Michigan
(joint work with Anatole Katok)

We investigate joinings of strongly irreducible totally nonsymplectic Anosov $\mathbb{Z}^k, k \geq 2$ the actions by toral automorphisms. We show that the existence of a nontrivial joining has strong implications for these actions, in particular, that the restrictions of the actions to a finite index subgroup of $\mathbb{Z}^k$ are conjugate over $\mathbb{Q}$. We also obtain a discription of the joining measures modulo the classification of zero-entropy measures for the actions.

Geometric interpretation of arithmetic codes for geodesics

Svetlana Katok, Penn State, PA, USA

Each oriented geodesic γ on the modular surface $M = PSL(2,\mathbb{Z})\backslash\mathcal{H}$ can be represented as a doubly–infinite sequence of segments between successive returns to a particular cross–section of the unit tangent bundle SM, $P \cup Q$, where P consists of all tangent vectors with base points in the right circular arc of the boundary of the standard fundamental region $F = \{z \in \mathcal{H} \mid |z| \geq 1,\ |\Re z| \leq \frac{1}{2}\}$ for $PSL(2,\mathbb{Z})$ such that corresponding geodesics have both end points positive and therefore go in the positive (clockwise) direction, and Q consists of all tangent vectors with base points in the right vertical side of F pointed inwards, i.e. in the negative direction.

Each segment of γ is $PSL(2,\mathbb{Z})$–equivalent to a *reduced* geodesic γ_i in $\mathcal{H}$ from u_i to w_i i.e. such that $0 < u_i < 1$ and $w_i > 1$. We assign to γ_i a sequence $(\dots n_{-3}, n_{-2}, n_{-1}, n_0, n_1, n_2 \dots)$ $(n_i \geq 2)$ obtained from "-" continuous fraction expansions of $w_i = (n_1, n_2 \dots)$ and $\frac{1}{u_i} = (n_0, n_{-1}, n_{-2}, \dots)$. Then the next segment γ_{i+1} produces the same coding sequence (shifted one symbol to the left). We call the coding sequence of any $PSL(2,\mathbb{Z})$–equivalent to γ reduced geodesic the *arithmetic code* of γ. Closed geodesics have periodic coding sequences, and this theory may be considered to be an extension of the reduction theory for closed geodesics. Conversely, each reduced geodesic in $\mathcal{H}$ either enters F through P, or is equivalent to a geodesic entering F through Q.

This allows to represent the geodesic flow $\{\varphi_t\}$ on SM symbolically as a special flow over the space of doubly-infinite sequences on the infinite alphabet $\mathcal{N} = \{n \geq 2\}$ with the Tykhonov product topology, $X := \mathcal{N}^{\mathbb{Z}} = \{x = (n_i)_{i=-\infty}^{\infty} \mid n_i \in \mathcal{N},\, i \in \mathbb{Z}\}$, with the ceiling function $f(x)$ equal to the time of the first return to $P \cup Q$. Let σ be the left shift of X given by $(\sigma x)_i = n_{i+1}$. We give an explicit formula for $f(x)$.

Theorem 1. *Let $x \in X$, $x = (\dots, n_0, n_1, n_2, \dots)$, and $w(x)$ and $u(x)$ be the end points of the corresponding geodesic $\gamma(x)$, where $w(x) = (n_1, n_2 \dots)$ and $\frac{1}{u(x)} = (n_0, n_{-1}, n_{-2}, \dots)$ are "-" continuous fraction expansions. Then $f(x) = 2\log w(x) + \log g(x) - \log g(\sigma x)$ with $g(x) = \frac{(w(x)-u(x))\sqrt{w(x)^2-1}}{w(x)^2\sqrt{1-u(x)^2}}$, i.e. is cohomologous to $2\log w(x)$.*

We call a geodesic *positive* if all segments comprising it begin and end on the set P. A positive geodesic avoids the set Q, its arithmetic code counts the number of successive times it hits the right vertical side of F in the positive direction, and hence coincides with its *geometric code* in the sense of Morse, and they are characterized by the following property:

Theorem 2. *A geodesic γ is positive if and only if its arithmetic code $(\dots n_{-3}, n_{-2}, n_{-1}, n_0, n_1, n_2 \dots)$ does not contain 2 and the following pairs: $\{3,3\}$, $\{3,4\}$, $\{4,3\}$, $\{3,5\}$, and $\{5,3\}$, i.e. $\frac{1}{n_i} + \frac{1}{n_{i+1}} \leq \frac{1}{2}$.*

This follows from my earlier result (*Geom. Dedicata*, **63** (1996), 123-145) for closed geodesics and a continuity argument.

The set of positive geodesics in SM is obtained from the set X with the help of an infinite matrix A of zeros and ones, where $A(i,j) = 0$ when the pair $\{i,j\}$ is prohibited:

$$X_A = \{x \in X \mid A(n_i, n_{i+1}) = 1\},$$

and the restriction of the left shift σ to X_A is a countable Markov chain.

We define the *positive geodesic flow* $\{\varphi_t^+\}$ to be a restriction of the geodesic flow $\{\varphi_t\}$ on SM to the set of vectors tangent to positive geodesics. Using a representation of $\{\varphi_t^+\}$ as a special flow over X_A with the ceiling function $f(x) = 2\log w(x)$, we obtain two–sided estimates for the topological entropy of the positive geodesic flow $h(\{\varphi_t\})$ (understood as supremum of measure–theoretic entropies):

Theorem 3. (joint with B. Gurevich) $0.7771 < h(\{\varphi_t\}) < 0.8161$.

The proof relies on an approximation of the ceiling function by a function depending only on $n_1(x)$, and on an application of results of A.B. Polyakov and S.V. Savchenko.

On arithmeticity of some complex hyperbolic lattices

Bruno Klingler, IPDE, ETH Zürich

By Margulis's superrigidity and arithmeticity theorems, lattices in non-compact simple real Lie groups of real rank ≥ 2 and their finite dimensional representations over any local field are well understood. On the other hand there exist non-arithmetic and arithmetic non-superrigid lattices in the unitary group $PU(2,1)$. In this talk we discuss a conjecture by Rogawski which prodicts that under certain cohomological assumptions a lattice Γ in $PU(2,1)$ is arithmetic. We prove this conjecture in some special cases:

Theorem: Let $\Gamma < PU(2,1)$ be a compact torsion-free lattice such that the complex surface $H = \Gamma \backslash H^2_{\mathbb{C}}$ is a "fake $P^2\mathbb{C}$" (i.e. has same Betti numbers as $\mathbb{P}^2\mathbb{C}$). Then any representation $\rho : \Gamma \to PGL(3)$ which is Zariski dense is superrigid. In particular Γ is arithmetic.

On the geometrization of 3-dimensional orbifolds

Bernhard Leeb, University of Tübingen

The so-called Orbifold Theorem is part of the evidence available for Thurstons Geometrization Conjecture. We explain joint work with M. Boileau (Toulouse) and J. Porti (Barcelona) which implies the Orbifold Theorem. A different proof has been announced by Cooper, Hodgson and Kerckhoff. In our talk, we will focus on a geometric aspect of the argument, namely the study of the local geometry of hyperbolic cone manifolds with cone angles $\leq \alpha < \pi$ and a lower diameter bound. We obtain a thick-thin decomposition which implies thickness and finiteness results as in the case of hyperbolic manifolds.

Kazhdans property (T), L^2-spectrum and isoperimetric inequalities for locally symmetric spaces

Enrico Leuzinger, University of Karlsruhe

Let $V = \Gamma \setminus G/K$ be a Riemannian locally symmetric space with nonpositive curvature and such that the isometry group G of its universal covering space has Kazhdan's property (T). Let λ_0 be the bottom of the L^2-spectrum of the Laplace-Beltrami-Operator if V of $Vol(V) = \infty$ and let λ_1 be the smallest non-zero eigenvalue if $Vol(V) < \infty$. We showed that there is a constant $c(G) > 0$ depending only on G such that $\lambda_0(V) \geq c(G) > 0$ (resp. $\lambda_1(V) \geq c(G) > 0$). This yields new rigidity properties for lattices, linear isoperimetric inequalities for general locally symmetric spaces and estimates for the growth of orbital counting functions.

Quantum unique ergodicity & rigidity

Elon Lindenstrauss, Institute for Advanced Studies, University of Princeton

Rudnick and Sarnak conjectured that if M is a compact manifold of negative curvature and φ_i the eigenfunction of the Laplacian (with corresponding eigenvalues $\lambda_i \to \infty$) then (if φ_i are normalized to have L^2-norm one)

$$|\varphi_i|^2 dvol \to \frac{1}{vol(M)} dvol.$$

If one considers the case of $\Gamma \setminus G/K\Gamma$ with an arithmetic convergence lattice in G (this has been previously considered for $Q = SL_2(\mathbb{R})$ or $SL_2(\mathbb{C})$ but is interesting for all G) and φ_i are assumed also to be Hecke eigenfunctions then this conjecture follows from GRH.
I will show how for $G = SL_2(\mathbb{R}) \times SL_2(\mathbb{R})$ this conjecture (for irred. lattices Γ) will follow from conjectures of Katok-Spatzier, Margulis, Furstenberg about measures invariant under Cartan actions, thus outlining a dynamical approach to this problem.

Super-rigidity for quasi-morphisms of general irreducible lattices

Nicolas Monod, ETHZ, ETH-Zürich, Switzerland

Joint work with M. Burger. We prove super-rigidity for the second bounded cohomology of a lattice $\Gamma < G = G_1 \times \ldots \times G_n$ in a product of general locally compact groups, under the irreducibility assumption $\overline{\mu_j(\Gamma)} = G_j$ for all j. This includes s-arithmetic groups, Kac-Moody groups, and lattices in products of trees.
As a result, we obtain rigidity for: (1) actions on the circle, (2) representations in the mapping class group of a compact surface, (3) representations in $PSU(n, 1)$. (Taking into account work of Shalom, Ghys, Bernina-Fujiwara, Burger-Iozzi).
Three main steps in our work are: (1) establish a functorial theory of continuous bounded cohomology (with coefficients), (2) constract, for arbitrary (compactly generated) locally compact groups, an amenable doubly ergodic boundary, (3) an improved higher degree Lyndon-Hochschild-Serre sequence.

Counting hyperbolic manifolds

Shahar Mozes, Hebrew University, Jerusalem, Israel

Joint work with Mark Burger, Tsachik Gelander and Alex Lubotzky.
Fix $n \geq 4$ and let $p(V) = \sharp$ {complete hyperbolic manifolds of volume $\leq V$}. We show that there exists constants $c_1, c_2 > 0$ such that for V sufficiently large

$$c_1 V \log V \leq \log p(V) \leq c_2 V \log V$$

Semigroup actions on $\mathbb{T}^n$

Roman Muchnik, Yale University

Let S be a semigroup of non-singular integer $n \times n$ matrices and $G = \langle S \rangle \subset GL(n, \mathbb{Q})$ be the group generated by S. There is a natural action of S on the n-dimensional torus $\mathbb{T}^n$. If G is not virtually cyclic, G acts strongly irreducibly on $\mathbb{Q}^n$ and for all $0 \neq x \in \mathbb{R}^n, S_x$ is unbounded, then for every $a \in \mathbb{T}^n \setminus \mathbb{Q}^n, Sx$ is dense in $\mathbb{T}^n$. Clearly, the converse is true. I give an outline of the proof of this theorem. As a corollary, we obtain that every minimal closed S-invariant subset $M \subseteq \mathbb{T}^n$ is G-invariant, when $S \subset SL(n, \mathbb{Z})$ acts irreducibly on $\mathbb{T}^n$.

Pointwise ergodic theorems for semisimple group actions

Amos Nevo, Technion, Haifa, Israel

Let G be a connected Lie group, $K \subset G$ a compact subgroup. Given an invariant Riemannian metric on G/K, let $B_t = \{g \in G \mid \; d(gK, K) \leq t\}$, and let β_t be the probability measure on G, whose density w.r.t. Haar measure is given by $\frac{1}{Haar(B_t)} X_{B_t}$.
Ball averaging problem: Given an ergodic measure preserving action of G on a probability space, is β_t a pointwise ergodic family in L^1, namely satiesfies, $\forall f \in L^1$, and a.e. $\lim\limits_{t \to \infty} \beta_t f(x) = \int\limits_X f dm$ where $\beta_t f(x) = \int\limits_G f(g^{-1}x) d\beta_t(g)$?
This problem ist open for all non amenable connected Lie groups (and most amenable groups of exponental growth as well). We consider the case of semisimple groups, for the

choice K = maximal compact subgroup, and the Riemannian metric associated with the Killing form, and show:

(1) β_t is a pointwise ergodic family in $L^p, \forall p > 1$, for every semisimple G.
(2) If G is simple, and the norm of β_1 as an operator on the space $L^2_0(X) = \{f \in L^2(X) \mid \int_x f dm = 0\}$ is strictly less than one, then

$$|\beta_t f(x) - \int_X f dm| \leq C_p(x, f) \exp(-\theta_p t)$$

for a.e. $x \in X$, and every $f \in L^p(X), p > 1$, where $\theta_p > 0$. If G has property $(T), \theta_p$ depends only on G.
(3) The same conclusion holds for a large family of averages on G as in (2), not necessarily radial, satisfying only mild growth and continuity conditions.

Thm 1 is partly joint work with E. Stein. Thm 2 is joint work with G. Margulis and E. Stein.

Spherical means and transversal measures of foliations in symmetric spaces

Norbert Peyerimhoff, Ruhr-Universität Bochum, Germany

One approach to prove convergence of spherical means to a space mean in certain non-positively curved compact spaces M is to use unique ergodicity of the foliation $\mathcal{F}$ given by the projection of the unit normal vectors of horospheres in the universal covering $\tilde{M}$. In he case of higher rank symmetric spaces all different isometry types of horospheres are obtained by considering only the horospheres orthogonal to directions in a fixed spherical Weyl chamber $\mathcal{C}^1$. A lower estimate for the Cheeger isoperimetric constant of any horosphere depending on its normal direction in $\mathcal{C}^1$ is given. Consequences of this estimate are, e.g., that the horosphere normal to the barycentric direction is not quasiisometric to any non-barycentric horosphere and that the foliated compact space $(\mathcal{M}, \mathcal{F})$ ($\mathcal{M} \subset SM$ the set of barycentric directions) admits an invariant transversal measure. A result of Veech can be used to derive the uniqueness of this invariant transversal measure (up to a constant factor). This implies, in turn a convergence result for averages w.r.t. the outward unit normal vectors of increasing geodesic spheres to a particular space mean supported in the barycentric directions.

Symplectic rigidity and hyperbolicity in contact dynamics

Leonid Polterovich, Tel Aviv University

A theorem of Moser guarantees that every diffeomorphism of a closed manifold can be isotoped to a volume-preserving one. It turns out that this statement cannot be the extended into contact category. For instance, every contactomorphism of the standard contact 3-torus $(\mathbb{T}^3(\theta, q_1, q_2), \xi = Ker(\cos\theta dq_1 + \sin\theta dq_2))$ which induces the automorphism $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ in $H_1(\mathbb{T}^3)$ is dissipative. The proof is based on Lagrangian intersection theory. Analogous results hold true for general contact manifolds.

Floer Homology and Hamiltonian Dynamics

Matthias Schwarz, Universität Leipzig, Germany

Initially, Floer homology was developed to prove existence of fixed points of non-degenerate Hamiltonian automorphisms of closed symplectic manifolds. Under the milder assumption of weak non-degeneracy, i.e. for all $x \in Fix\phi$ not all Floquet multipliers are 1, Salamon and Zehnder showed the existence of infinitely many periodic points of $\phi \in Ham(M, w)$. In this talk another method based also on Floer homology is introduced in order to prove existence of infinitely many nontrivial, geometrically distinct periodic points without any non-degeneracy assumption, however presently under different dynamical assumptions. The idea is to determine "homologically visible" critical levels in the action spectrum by means of Floer homology and to analyze their behaviour under iterations of ϕ. This leads to a biinvariant metric d on the group of Hamiltonian automorphisms. If $\lim_{n\to\infty} \frac{d(id,\phi^n)}{n} = 0$, ϕ has infinitely many nontrivial periodic points.

Schreier's theorem for isometry groups of the hyperbolic space

Yehuda Shalom, Hebrew University, Jerusalem, Israel

Schreier's theorem is a classical result in combinatorial group theory: Let $e \neq N \lhd F$ be an infinite index normal subgroup of a free group. Then N is <u>not</u> finitely generated. We prove the following extensions:

<u>Theorem A:</u> Let $\Gamma < Iso(\mathbb{H}^n_{\mathbb{R}}) \cong SO(n,1)$ be a f.g. non-amenable discrete group with $\delta(\Gamma) < 2$, where δ is the critical exponent. Then Schreier's theorem holds for Γ.

Another result we prove about the structure of groups with $\delta(\Gamma) < 2$ is:

<u>Theorem B:</u> Every such Γ admits $\Gamma_0 \subset \Gamma$, with $[\Gamma : \Gamma_0] < \infty$ which surjects onto $\mathbb{Z}$. Moreover $\dim(Hom(\Gamma_0, \mathbb{R})) \to 0$ when $\Gamma_0 \to \{e\}$ along finite index subgroups.

Theorem A fails in general when $\delta(\underline{\Gamma}) = 2$ (lattices in $SL_2(\mathbb{C})$).

Theorem B for $\delta(\Gamma) = 2$ would imply a celebrated conjecture of Thurston.

Length spectrum invariants via variational methods

Karl Friedrich Siburg, Ruhr-Universität Bochum, Germany

We investigate length spectrum invariants in two different settings: the geodesic flow on a surface near an elliptic closed geodesic, and billiards in a strictly convex domain. We introduce the minimal action function from Aubry-Mather theory as a "universal" length spectrum invariant, which encompasses many known invariants and contains a lot of geometric information about the metric/domain. By writing geometric data in terms of the minimal action, the proof of their length spectrum invariance becomes almost tautological.

Edited by Gerhard Knieper, Karl Friedrich Siburg

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