Mathematisches Forschungsinstitut Oberwolfach

Report No. 8 / 2001

Mini-workshop: Schur algebras and quantum groups

February 25th - March 3rd, 2001

K. Erdmann
Mathematical Institute
24-29 St. Giles
Oxford OX1 3LB
UK
erdmann@maths.ox.ac.uk

S. Koenig
Department of Mathematics
and Computer Science
University of Leicester
University Road
Leicester LE1 7RH
UK
sck5@mcs.le.ac.uk

Contents

1 The workshop.

This mini–workshop aimed at discussing existent and potential connections between certain algebraic and geometric objects, in particular Schur algebras and quantum groups. The main focus was on the known epimorphisms from type A quantized enveloping algebras to Schur algebras, their restriction to Hall algebras and their potential analogues for other types. A description of these problems and a reading list had been circulated beforehand. At the workshop, there were survey lectures and more specialized presentations, including suggestions to attack the central problems (some of them stimulated by the material distributed earlier). During the discussions, new links were discovered between different approaches, proofs were simplified and a conjecture arose on the type C situation.

The schedule was very flexible and no time constraints were imposed on speakers; this format turned out to be very popular both with speakers and audience. There was general agreement that mini–workshops provide a very welcome new format for stimulating research meetings.

There have been 15 participants from Australia, France, Germany, the UK and the USA. 27 per cent of the participants were female, the majority of the participants was younger than 35.

2 The subject area.

Schur algebras in arbitrary characteristic (over infinite fields) were introduced 1980 in J.A.Green's influential monograph [17] on polynomial representations of the general linear group. Schur himself had considered the characteristic zero situation in his thesis. Green also introduced integral Schur algebras and insisted on characteristic free notions (eg codeterminant basis for Weyl modules). Schur algebras over finite fields started to be used only very recently (by topologists in the context of functor cohomology, or topological Hochschild cohomology, see [16]).

Classical Schur algebras of type A, as introduced by Green, cover the polynomial representation theory of the general linear group over an infinite field. Donkin [10, 11] introduced more general Schur algebras which cover the rational representation theory of reductive groups. These algebras somehow lack the nice combinatorial theory of the type A–situation where the symmetric group (which on that occasion does not wear its usual Weyl group hat) is of much help. There is some information available on Brauer algebras, which replace the symmetric group in types B and C, but not much, and type D seems to be completely unknown. Analogues of many of Green's main results (standard basis with multiplication formula, bases for Weyl modules, Schur functor, etc) seem to be unknown in the other types.

Green's and Donkin's Schur algebras are made for defining characteristic. In non–defining characteristic (where the group is defined over a field of characteristic p and representations are taken over a field of characteristic different from p) one has to use another kind of Schur algebra, the quantized one, which has been introduced by Dipper and James [7] in type A by Hecke algebra combinatorics instead of symmetric group combinatorics. More generally, Dipper, James and Mathas [9] defined cyclotomic quantized Schur algebras, which contain type A and also type B and C. These gadgets correspond to cyclotomic Hecke algebras, which come from complex reflection groups (instead of crystallographic Coxeter groups), but no Schur algebras are known for the other cases occuring there.

2.1 Hyperalgebras and quantum groups

Associated with a semisimple group is a semisimple complex Lie algebra which has a universal enveloping algebra. This algebra has an integral form which in characteristic p defines the hyperalgebra. Universal enveloping algebras can be quantized and then again have integral forms (Lusztig's version). Another way to construct these quantum groups is through Ringel's Hall algebras [29] which in the Dynkin case give the positive part of the quantum group, and in the affine case something bigger than that.

2.2 Epimorphisms

Carter and Lusztig [5] observed that the hyperalgebra maps onto each Schur algebra (again in type A). Beilinson, Lusztig and MacPherson [3] used the analogue in the quantized version to view the quantized enveloping algebra as a subalgebra of a projective limit of quantized Schur algebras (where the map $S(n, r+n) \to S(n, r)$ from degree $r+n$ to degree r is 'cancelling the determinant'). They described the map in an explicit, but not very pleasant way. Jie Du worked this out in more detail. R.M.Green [19] found a precise description of the kernel of this map (giving a basis of the kernel as a vector space). This description looks much nicer if one restricts the map to the positive part of the quantum group. Then the kernel has as a basis those elements of the PBW basis which have a sum of root multiplicities bigger than r – where r is the degree of the Schur algebra (Corollary 2.4. in [19]).

R.M.Green also defined affine q–Schur algebras [20]. Varagnolo and Vasserot [32] have used affine q–Schur algebras (for the cyclic quiver of type A) in their proof of LLT–conjecture through Lusztig's q–conjecture. Their approach is based on Hall algebras, and they are using the Hall algebra version of Green's Corollary 2.4. Standard basis elements of the Hall algebra are representations of the corresponding quiver (with respect to some fixed orientation), and the condition then reads that representations with not more than r indecomposable direct summands are mapped to basis elements of the Schur algebra and the other basis elements are in the kernel.

Recently, Doty and Giaquinto [13, 14] found a nice description of the kernel as an ideal; at present their result only covers $GL(2)$ (classical and quantized), but they hope to cover the general situation in a similar way.

References

[1] S. Ariki, *On the decomposition numbers of the Hecke algebra of $G(m, 1, n)$.* J. Math. Kyoto Univ. **36** (1996), no. 4, 789–808.

[2] S.Ariki, *Cyclotomic q-Schur algebras as quotients of quantum algebras.* J. Reine Angew. Math. 513 (1999), 53–69.

[3] A. A. Beilinson, G. Lusztig and R. MacPherson, *A geometric setting for the quantum deformation of* GL_n. Duke Math. J. **61** (1990), no. 2, 655–677.

[4] K. Bongartz, *On degenerations and extensions of finite dimensional modules.* Adv. Math. 121 (1996), 245-287.

[5] R. Carter and G. Lusztig, *On the modular representations of the general linear and symmetric groups.* Math. Z. 136 (1974), 193–242.

[6] N. Chriss and V. Ginzburg, *Representation theory and complex geometry.* Birkhäuser Boston, Inc., Boston, MA, 1997. x+495 pp.

[7] R. Dipper and G.D. James, *The q-Schur algebra,* Proc. London Math. Soc. (3) **59** (1989), 23-50.

[8] R. Dipper, *Polynomial representations of finite general linear groups in the non-describing characteristic,* Progress in Mathematics **95**, Birkhäuser Verlag Basel (1991), 343-370.

[9] R. Dipper, G. James and A.Mathas, *Cyclotomic q-Schur algebras.* Math. Z. 229 (1998), 385–416.

[10] S. Donkin, *On Schur algebras and related algebras. I.* J. Algebra 104 (1986), 310–328.

[11] S. Donkin, *On Schur algebras and related algebras. II.* J. Algebra 111 (1987), 354–364.

[12] S. Donkin, *The q–Schur algebra.* viii+232 pp, Cambridge University Press (1999).

[13] S. Doty and A.Giaquinto, *Presenting Schur algebras as quotients of the universal enveloping algebra of gl(2).* Preprint math.RT/0010005.

[14] S. Doty and A.Giaquinto, *Presenting quantum Schur algebras as quotients of the quantized universal enveloping algebra of gl(2).* Preprint math.RT/0011164.

[15] J. Du, *q-Schur algebras, asymptotic forms, and quantum* SL_n. J. Algebra 177 (1995), 385-408.

[16] E.Friedlander and A.Suslin, *Cohomology of finite group schemes over a field.* Invent. Math. 127 (1997), 209–270.

[17] J.A. Green, *Polynomial representations of* GL_n. Lecture Notes in Mathematics, 830. Springer-Verlag, Berlin-New York, 1980. vi+118 pp.

[18] J.A. Green, *Hall algebras, hereditary algebras and quantum groups.* Invent. Math. 120 (1995), 361–377.

[19] R.M. Green, *q-Schur algebras as quotients of quantized enveloping algebras.* J. Algebra 185 (1996), 660-687.

[20] R.M.Green, *The affine q-Schur algebra.* J. Algebra 215 (1999), 379–411.

[21] H. Knight and A. Zelevinsky, *Representations of quivers of type A and the multisegment duality.* Adv. Math. 117 (1996), no. 2, 273–293.

[22] A. Lascoux, B. Leclerc and J.–Y. Thibon, *Hecke algebras at roots of unity and crystal bases of quantum affine algebras.* Comm. Math. Phys. 181 (1996), 205–263.

[23] G. Lusztig, *Quivers, perverse sheaves, and quantized enveloping algebras.* J. Amer. Math. Soc. **4** (1991), no. 2, 365–421.

[24] G. Lusztig, *Introduction to quantum groups.* Progress in Mathematics, **110**. Birkhäuser Boston, Inc., Boston, MA, 1993. xii+341 pp.

[25] A. Mathas, Lecture Notes on: *Hecke algebras and Schur algebras of the symmetric groups.* American Mathematical Society (1999).

[26] C. Moeglin and J.-L. Waldspurger, *Sur l'involution de Zelevinski.* J. Reine Angew. Math. 372 (1986), 136–177.

[27] L.G. Peng and J. Xiao, *Triangulated categories and Kac-Moody algebras.* Invent. math. 140 (2000), 563-603.

[28] M. Reineke, *On the coloured graph structure of Lusztig's canonical basis.* Math. Ann. 307 (1997), 705-723.

[29] C.M.Ringel, *Hall algebras and quantum groups.* Invent. Math. 101 (1990), 583–591.

[30] C.M. Ringel, *The multisegment duality and the preprojective algebras of type A.* AMA Algebra Montp. Announc. 1999, no. 1, Paper 2, 6 pp. (electronic).

[31] B. Sevenhant and M. Van den Bergh, *On the double of the Hall algebra of a quiver.* J. Algebra 221 (1999), 135-160.

[32] M. Varagnolo and E. Vasserot, *On the decomposition matrices of the quantized Schur algebra.* Duke Math. J. 100 (1999), no. 2, 267–297.

[33] J. Xiao, *Drinfeld double and Ringel-Green theory of Hall algebras.* J. Algebra 190 (1997), 100-144.

[34] A.V. Zelevinsky, *Induced representations of reductive p-adic groups. II. On irreducible representations of* GL(n). Ann. Sci. École Norm. Sup. (4) 13 (1980), 165–210.

[35] G. Zwara, *Degenerations of finite-dimensional modules are given by extensions.* Compositio Math. 121 (2000), 205–218.

3 Abstracts.

(In the order of the talks.)

Introduction to q-Schur algebras
Richard Dipper, Stuttgart

This was an introductory survey talk on the construction of $q-$Schur algebras and some basic facts on their representations.

Hecke algebras $\mathcal{H}_{R,q}(\Sigma_r) = \mathcal{H}$ associated with symmetric groups Σ_r are remarkable q-deformations of the group algebras $R\Sigma_r$. They are generated by elements T_i $(1 \leq i \leq r-1)$ subject to relations $T_iT_j = T_jT_i, \quad T_iT_{i+1}T_i = T_{i+1}T_iT_{i+1}$ and $(T_i+1)(T_i-q) = 0$ for $1 \leq i, j \leq r$ and $|i-j| \geq 2$. Here $q \in R$ is assumed to be invertible. Let V be an n-dimensional R-free $R-$module, the action of Σ_r on $V^{\otimes r}$ by place permutation can be q-deformed to produce an action of the Hecke algebra $\mathcal{H}$ on $V^{\otimes r}$. The q-Schur algebra $\mathcal{S} := \mathcal{S}_{R,q}(n,r)$ is defined to be the centralizing R-algebra of this $\mathcal{H}$-action on tensor space $V^{\otimes r}$.

One has that $V^{\otimes r}$ as an $\mathcal{H}$-module is the direct sum $\oplus_{\lambda \in \Lambda(n,r)} M^\lambda$ of 'permutation type' modules M^λ. Here $\Lambda(n,r)$ is the set of compositions of r into n parts. This gives rise to the standard basis $\{\phi^d_{\lambda,\mu} : \lambda, \mu \in \Lambda(n,r), d \in \mathcal{D}_{\lambda,\mu}\}$ of the q-Schur algebra, where $\mathcal{D}_{\lambda,\mu}$ is a certain set of double coset representatives on Σ_r.

Next, a cellular basis of $\mathcal{H}$, the Murphy basis, is constructed, which is indexed by pairs of standard λ-tableaux, where λ runs through the partitions of r. This can be extended to obtain the semistandard basis of $\mathcal{S}$, which is labelled by pairs of semistandard λ-tableaux of type $\mu \in \Lambda(n,r)$. Here λ runs through $\Lambda^+(n,r)$, the set of partitions of r into at most n parts. This basis is cellular too. As a consequence we define for each partition $\lambda \in \Lambda^+(n,r)$ the Weyl module W^λ and the irreducible $\mathcal{S}$-module $F^\lambda := W^\lambda/\text{rad}W^\lambda$ (when R is a field). Then the set of all F^λ for $\lambda \in \Lambda^+(n,r)$ is a complete set of pairwise non-isomorphic irreducible $\mathcal{S}$-modules, and the matrix $[d_{\lambda,\mu}]$ describing the composition multiplicities of the F^μ in W^λ is upper unitriangular and is a square matrix. Thus $\mathcal{S}$ is quasi-hereditary.

q-Schur algebras as quotients of quantized enveloping algebras
Richard M. Green, Lancaster

This talk summarized the results in the paper "q-Schur algebras as quotients of quantized enveloping algebras". The main result is an explicit description of the kernel of the map from $U(gl_n)$ to the q-Schur algebra $S_q(n,r)$.

The affine q-Schur algebra
Richard M. Green

This talk discussed connections with Du's IC basis for the q-Schur algebra, Lusztig's algebra $\dot{U}$ and affine q-Schur algebras, and furthermore affine Schur-Weyl duality.

Hall algebras, geometry of quiver representations and qunatum groups
Markus Reineke, BUGH Wuppertal

This is an introductory talk on the Hall algebra approach to quantum groups, which was used is later talks to study the epimorphisms from quantum groups to Schur algebras using the geometry of representations of quivers. First, some basic results on the representation theory of Dynkin quivers are reviewed, concentrating on Gabriel's Theorem classifying the indecomposable representations, and on the representation-directedness of the path algebra. Next, some basic properties of varieties of representations are discussed, and K. Bongartz' description of the orbit closures in these varieties is given. C. M. Ringel's Hall algebra approach to quantum groups is recalled by realising the positive part of a quantized enveloping algebra via a convolution product on functions on representation varieties. Using this approach, PBW type bases for quantum groups are constructed, and G. Lusztig's canonical basis is defined in terms of perverse sheaves on representation varieties.

Hall algebras, geometry of quiver representations and quantum groups, II
Thomas Brüstle, Bielefeld

We discuss different aspects of the epimorphism ψ from the quantum group U_q of SL_n to the $q-$Schur algebra $S_q(n,r)$. In particular,we give an explicit description of the restriction of ψ to the Hall algebra U_q^+: Essentially, the morphism ψ associates to a module M its projective resolution, in case M lies not in the kernel. The kernel is described as the ideal of the Hall algebra formed by isoclasses of modules having more than r direct summands. We show that this description does not work for other types than the linear oriented quiver of type A_n.

Some finite dimensional quotients of the quantum group $U_q(n^+)$
Markus Reineke

Let $\theta_d : \mathcal{U}_q(gl_n) \to S_q(n,d)$ be the natural epimorphism from the quantum group to the q-Schur algebra. The kernel of its restriction to $\mathcal{U}_q(n^+)$ is an ideal $I(d)$ which is compatible with a PBW basis and Lusztig's canonical basis; if the PBW basis is parametrized by isoclasses of representations of an equioriented quiver of type A, it is spanned by those representations which have more than d indecomposable direct summands.

Starting from this observation, we consider two-sided ideals of $\mathcal{U}_q(n^+)$ in type A, D, E which have finite dimensional quotients, and are compatible with a PBW basis and with the canonical basis. We describe those additive functions f defined on isoclasses of representations of quivers for which the subspaces $I_f(d)$, spanned by PBW basis elements corresponding to isoclasses of representations M such that $f(M) \geq d$, are such 'good' ideals. Among those functions, there is a unique choice f_0 for which all the $I_{f_0}(d)$ are minimal. For equioriented type A, the ideal defining (part of) the q-Schur algebra is recovered. Thus, the ideals $I_{f_0}(d)$ could possibly serve as candidates for generalizing the definition of q-Schur algebras.

Cyclotomic q–Schur algebras.
Andrew Mathas, University of Sydney

This talk was a survey of the representation theory of the cyclotomic q–Schur algebras studied by Dipper, James and Mathas. These algebras were introduced because of the success of the Schur algebras and the q–Schur algebras of Dipper and James. The q–Schur algebra can be defined as the endomorphism algebra of a certain module for the Iwahori–Hecke algebra of the symmetric group; similarly, a cyclotomic q–Schur algebra is the endomorphism ring of a module for the Ariki–Koike algebras — or the cyclotomic Hecke algebras of type $G(r,1,n)$.

The Ariki–Koike algebra $\mathcal{H}$ is a cyclotomic algebra of type $G(r,1,n)$, and it becomes the Iwahori–Hecke algebra of type **A** or **B** when $r = 1$ or 2 respectively. By working over a ring R which contains a primitive rth root of unity, and by specializing the parameters appropriately, the Ariki–Koike algebra turns into the group algebra $R(C_r\mathcal{S}_n)$ of the wreath product of the cyclic group C_r of order r with the symmetric group $\mathcal{S}_n$ of degree n.

For each multicomposition λ of n there is an interesting right ideal M^λ of $\mathcal{H}$. The cyclotomic q–Schur algebra is the endomorphism algebra $\mathcal{S} = \mathrm{End}_{\mathcal{H}}\Big(\oplus_\lambda M^\lambda\Big)$. Under the specialization above where $\mathcal{H} \cong R(C_r \wr \mathcal{S}_n)$, the module M^λ becomes a module induced from a subgroup of the form $(C_r \times \cdots \times C_r) \times \mathcal{S}_\lambda$.

This talk described how to construct a cellular basis for the cyclotomic q–Schur algebra. The first step was to give a cellular basis for $\mathcal{H}$. This basis can be modified to give a basis of the ideals M^λ; in particular, this shows that each M^λ has a "Specht series" (that is, a filtration with each subquotient isomorphism to a Specht module). Finally, the bases of the M^λ can be "lifted" to give a basis of $\mathcal{S}$.

As a consequence, for each multipartition λ there is a Weyl module W^λ and W^λ has simple head F^λ; further, $\{F^\lambda\}$ is a complete set of pairwise inequivalent irreducible $\mathcal{S}$–modules. Using the cellular structure of $\mathcal{S}$, it is now easy to see that the cyclotomic q–Schur algebra is quasi–hereditary.

Generators and relations for Schur algebras
S. Doty, Loyola University Chicago.

This is a report of some recent joint work with Giaquinto. We obtain a new presentation of Schur algebras via generators and relations. The presentation is compatible with the usual presentation of the universal enveloping algebra of the general linear Lie algebra. We also give an explicit description of the integral form of the Schur algebra as an analogue of Kostant's integral form of U, and find new integral bases of this integral form.

This talk outlined the important steps of the proof of these results in the classical case.

Generators and relations for Schur algebras, continued
A. Giaquinto, Loyola University, Chicago.

This discusses aspects of the proof which are different for the quantum case as compared with the classical case. This makes essential juse of some explicit commutation formulas (which are fortunately available, from work of N. Xi).

On Oehms' construction of symplectic q-Schur algebras
Anton Cox, City University, London.

This talk gave be a survey of the thesis of Oehms, in which he constructs candidates for the title of symplectic q-Schur algebras. We outlined the principle ideas behind this construction, and the results thus obtained.

Intersection of opposed real big cells
Robert Marsh, University of Leicester

Let G be a connected linear algebraic group. Any flag variety has many different cell decompositions, the so-called 'Bruhat decompositions'. There is precisely one for each Borel subgroup; and furthermore there is always a unique open dense orbit called the big cell. In this joint work with Konstanze Rietsch, we fix two opposite Borel subgroups and study the intersection of the two resulting big cells.

More precisely, we are interested in the real points of this variety, in the case where everything is split over the real numbers. There has been some recent work determining the number of its connected components and their Euler characteristics (Rietsch, Shapiro-Shapiro-Vainshtein, SSV- Zelevinsky). Such characteristics associate an as yet unexplained integer to each element in the canonical basis. We show how to compute the Euler characteristics of the individual connected components of the intersection of two opposed big cells in the real flag variety of type G_2, by employing the 'Chamber Ansatz' of Berenstein-Fomin-Zelevinsky.

The Global Dimension of $S(2,r)$ and $S(3,r)$
Alison Parker, QMW London.

We first look at quasi-hereditary algebras and review the notions of good filtration dimension and Weyl filtration dimension. This then gives us an upper bound for the global dimension. We outline how to calculate the good filtration dimension and Weyl filtration dimension for all irreducible modules for $S(2,r)$ and $S(3,r)$ (r arbitrary). We then show that the global dimension is twice the maximum of the good filtration dimensions. To do this we introduce modules of the form $\nabla(\mu)^F \otimes L(\mu)$ (μ dominant. ν p-restricted, $\nabla(\mu)$ the induced module). The induced modules $\nabla(\lambda)$ are filtered by such modules and we give explicitly the filtration for SL_2 and SL_3.

Structure constants of q-Schur algebras (type A)
Richard M. Green

This was a contribution which was inspired by the lecture(s) of M. Reineke. It presented a method how to determine structure constants for $S_q(n,r)$ by using the positive part $S_q^+(3n,r)$. That is, all structure constants of q-Schur algebras are in principle obtainable from the Hall algebra approach.

Having seen this, Markus Reineke was able to give an argument in terms of the Auslander-Reiten quiver which explains why this is possible.

$\mathbf{Z}\mathcal{S}_n$ and modular morphisms
Matthias Künzer, Bielefeld

We consider the integral group ring of the symmetric group on n letters $\mathbf{Z}\mathcal{S}_n$ as a subring of a direct product of matrix rings over $\mathbf{Z}$ via the restriction of the rational Wedderburn isomorphism to the integral situation, that is

$$(*) \qquad \mathbf{Z}\mathcal{S}_n \longrightarrow \prod_{\nu \vdash n} (\mathbf{Z})_{n_\nu \times n_\nu},$$

given by sending a group element to the tuple of its operations on the integral Specht modules with respect to chosen $\mathbf{Z}$-linear bases. The image Λ of the embedding $(*)$ is thus an *isomorphic copy* of the integral group ring we are interested in.

Once suitable bases for these integral Specht modules chosen, we can read off a presentation of $\mathbf{Z}_{(p)}\mathcal{S}_n$ ($\simeq \Lambda_{(p)}$) by quiver and relations, where p is a prime dividing $n!$. Reducing modulo p, we may then derive a presentation by quiver and relations of the modular group ring $\mathbf{F}_p\mathcal{S}_n$. Moreover, we can read off the multiplicities of the Specht modules in the indecomposable projectives of $\mathbf{Z}_{(p)}\mathcal{S}_n$. But so far, such a suitable choice has only been achieved for n small.

The cokernel of the embedding $(*)$ has cardinality $\sqrt{n!^{n!}/\prod_\nu n_\nu^{\binom{n_\nu}{2}}}$. In particular, our embedding is not an isomorphism for $n \geq 2$, and we can ask for necessary conditions on a tuple of integral matrices to lie in Λ. Given $\mathbf{Z}$-free $\mathbf{Z}\mathcal{S}_n$-modules X and Y, an integer $m \geq 2$ and a $\mathbf{Z}\mathcal{S}_n$-linear map $X/mX \xrightarrow{f} Y/mY$, this yields a such a necessary condition on a tuple of matrices to lie in Λ. For instance, if $X = S^\lambda$ and $Y = S^\mu$ are Specht modules, with operating matrices $\rho^\lambda(\sigma)$ and $\rho^\mu(\sigma)$ of $\sigma \in \mathcal{S}_n$ respectively, the morphism property of (a $\mathbf{Z}$-linear integral inverse image of) f reads $f \circ \rho^\lambda(\sigma) \equiv_m \rho^\mu(\sigma) \circ f.$. Hence, given a tuple of integral matrices $(\xi^\nu)_\nu$, we obtain the necessary condition $f \circ \xi^\lambda \equiv_m \xi^\mu \circ f$. It can be shown that there is a system of such modular morphisms that yields a condition which is necessary and sufficient for a tuple of matrices to lie in Λ. In general, however, such a system involves not only Specht modules.

We intend to discuss certain modular morphisms between Specht modules, of type 'one-box-shift' and 'two-box-shift', and to explain the consequences for Λ in case $n = 4$.

Generators and relations for symplectic Schur algebras
S. Doty.

In this talk I present a conjecture for the type C analogue of the type A presentation of Schur algebras, obtained recently in joint work with Giaquinto. The conjecture is based on that result, along with results of Donkin in the 1980's. The conjecture has both a classical and a quantum version.

The conjecture was obtained at this workshop.

Determining distant decomposition numbers
Anton Cox

Let G be a semisimple, connected, simply-connected algebraic group over an algebraically closed field of characteristic $p > 0$. In order to determine the characters of the simple modules for G it is enough — by Steinberg's tensor product theorem — to determine the characters for a certain finite set of weights. In principle this allows one to determine the composition multiplicities of simple modules inside Weyl modules, but not in a practical manner. We give an algorithm for determining these decomposition numbers in general from those for another finite set of weights.

There is another such algorithm due to Jantzen, which corresponds to a filtration of the Weyl module. A priori, it is not clear that the stages in our algorithm have such a representation-theoretic interpretation. We shall however show that they can be interpreted in terms of lifts of certain modules from a corresponding family of quantum groups, and also relate them to results of Doty on the structure of symmetric powers.

4 Participants.

Dr. Thomas Brüstle
bruestle@mathematik.uni-bielefeld.de
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
33501 Bielefeld

Prof. Dr. Ruth Corran
rc62@mcs.le.ac.uk
Dept. of Math. and Computer Science
University of Leicester
University Road
GB-LeicesterLE1 7RH

Dr. Anton Cox
a.g.cox@city.ac.uk
Dept. of Mathematics
The City University
Northampton Square
GB-LondonEC1V OHP

Prof. Dr. Richard Dipper
rdipper@mathematik.uni-stuttgart.de
Mathematisches Institut B
Universität Stuttgart
70550 Stuttgart

Prof. Dr. Stephen Doty
doty@math.luc.edu
Math. Science Department
Loyola University of Chicago
Chicago, IL 60626-5385
USA

Prof. Dr. Karin Erdmann
erdmann@maths.ox.ac.uk
Mathematical Institute
Oxford University
24 - 29, St. Giles
GB-OxfordOX1 3LB

Prof. Dr. Anthony Giaquinto
tonyg@math.luc.edu
Math. Science Department
Loyola University of Chicago
Chicago, IL 60626-5385
USA

Dr. Richard Green
r.m.green@lancaster.ac.uk
Dept. of Mathematics
University of Lancaster
Fylde College
Bailrigg
GB-Lancaster, LA1 4YF

Dr. Steffen König
sck5@mcs.le.ac.uk
Dept. of Math. and Computer Science
University of Leicester
University Road
GB-LeicesterLE1 7RH

Dr. Matthias Künzer
kuenzer@mathematik.uni-bielefeld.de
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 25
33615 Bielefeld

Dr. Robert J. Marsh
rjm25@mcs.le.ac.uk
Dept. of Math. and Computer Science
University of Leicester
University Road
GB-LeicesterLE1 7RH

Prof. Dr. Andrew Mathas
mathas@maths.usyd.edu.au
School of Mathematics & Statistics
University of Sydney
SydneyNSW 2006
AUSTRALIA

Dr. Will Turner
turnerw@maths.ox.ac.uk
Mathematical Institute
Oxford University
24 - 29, St. Giles
GB-OxfordOX1 3LB

Alison Parker
a.parker@qmw.ac.uk
School of Mathematical Sciences
Queen Mary and Westfield College
University of London
Mile End Road
GB-London, E1 4NS

Prof. Dr. Michela Varagnolo
Michela.Varagnolo@math.u-cergy.fr
Departement de Mathematiques
Universite de Cergy-Pontoise
Batiment Saint-Martin
2, avenue Adolphe Chauvin
F-95302 Cergy-PontoiseCedex

Dr. Markus Reineke
reineke@math.uni-wuppertal.de
Fachbereich 7: Mathematik
U-GHS Wuppertal
42097 Wuppertal