# Mathematisches Forschungsinstitut Oberwolfach 

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# Gewöhnliche Differentialgleichungen: Spectra and Nonlinear Differential Equations 

March 18th - March 24th, 2001

The present conference was organized by J. Mawhin (Louvain-la-Neuve), K. Schmitt (Salt Lake City), and H.O. Walther (Gießen). It was attended by 43 participants from 10 European countries (Belgium, Czech Republic, Germany, France, Great Britain, Italy, the Netherlands, Slovakia, Spain, Switzerland), from Russia, Australia, Canada, Chile, and the U.S.A. From the 43 participants 25 had not been present at the meeting two years before.

34 lectures were held, all well attended. 22 of the lecturers had not given a lecture at the previous meeting.

The lectures can be divided into 3 major groups: 1. The largest group of lectures presented new developments in variational methods and their application to ordinary and partial differential equations. The results and phenomena discussed were multiplicity of ground states, complicated periodic and otherwise oscillating solutions, chaotic dynamics, prescribed numbers of zeros and nodal domains, existence of peak solutions, location of peaks, and bifurcation from isolated and extended parts of spectra. Among the differential equations considered were Hill's equation, an Emden-Fowler equation, Schrödinger equations, the $p$-Laplacian, and generalizations of the latter. The methods used were often based on mountain pass geometry, also in situations with less smoothness and coercivity than usual. An Implicit Function Theorems with loss of derivatives (as in Nash-Moser theory) was employed, and tools from algebraic topology (computation of homotopy and homology groups) applied.

A common feature seen here is that variational methods are now developing into tools which can capture quite complicated dynamical properties and structures of solution sets, far beyond mere existence and multiplicity of stationary and periodic solutions.
2. Another large group of lectures was devoted to the long term behavior of solutions in particular classes of ordinary, functional, and partial differential equations and other systems. Presented were results about convergence to single points in the presence of manifolds of equilibria of reaction-diffusion equations, normal forms for systems with lines of equilibria, the relation between quasiperiodic solutions of O.D.E's and associated P.D.E's, chaos in delay D.E's, desynchronization and stable periodic motion in neural networks with delays, symmetry breaking bifurcation in low-dimensional systems, and periodic points, periods, and global analytic center manifolds for discrete dynamics in 2 complex variables. Further lectures in this group addressed the Swift-Hohenberg equation with noise, and the solution of a precise equation for water waves, a dispersion relation, and stability.

Notable new or improved tools were a guiding functional related to momenta for a reaction-diffusion equation, a new variant of the Lefschetz fixed point theorem which gave sharper information about complicated periodic trajectories of delay D.E's, and the methods used for 2-dimensional complex dynamics, which have nothing in common with the emerging complex-analytic approaches based on notions generalized from the onedimensional case.
3. A third group of lectures concentrated on spectra of linearized problems. Studied were spectral properties which presumably let spiral solutions of reaction-diffusion systems bifurcate and decompose, and create superspirals seen on a larger scale. Another phenomenon presented occurs in functional differential equations with delayed and advanced arguments, where there is no solution semigroup on the full state space, but forward and backward semigroups exist in subspaces defined by spectral decomposition. (The F.D.E's in question arise in the study of travelling waves for nonlinear lattice differential equations.) Floquet multipliers of time-periodic delay D.E's were investigated, with an eye to the still far aim of a Floquet theory on the whole state space for suitable classes of such equations.

A few, very interesting lectures addressed problems not yet mentioned. Via a transformation new and shorter proofs of Cafarelli-Kohn-Nirenberg inequalities were obtained, with optimal constants. This is related to variational methods and ground states of boundary value problems. A sequence of interpolating operators from nonlinear elliptic equations (e.g., the Gel'fand equation) to Monge-Ampére equations helped to understand better the differences in the solution sets of these equations. The problem to characterize which boundary conditions for elliptic operators yield generators of contracting semigroups of positive operators was studied in the particularly difficult non-transversal case, were subtle nonlocal operators come into play.

During the week the organizers continued the discussion of current trends in O.D.E's and related systems, and their presentation in Oberwolfach.

It is a pleasure to mention the friendly hospitality of the members and staff of the institute. During the week they also had to take care of the sick - a virus attacked almost a third of the participants, for a day or two. - Due to weather conditions the excursion had to be replaced by a visit of the mineral museum in Oberwolfach. The rain ended with the meeting.

# Abstracts 

## David Arcoya

Mountain pass for functionals with noncoercive part

(joint work with L. Baccardo, L. Orsina)

We report some joint results with L. Baccardo und L. Orsina (University of Roma, Italy) about the existence of critical points for some variational problems. In the classical $C^{1}$ theory, the ellipticity of the differential operator implies that the main part of the functional is coercive. In contrast, the new feature here is the lack of coercivity of this main part. Furthermore, an additional difficulty is arising from the non- $C^{1}$-differentiability of the functionals. A model for integralfunctionals with main part having degenerate coerciveness is given by

$$
J(v)=\frac{1}{2} \int_{\Omega} \frac{|\nabla v|^{2}}{(b(x)+|v|)^{2 \alpha}}-\frac{1}{m} \int|v|^{m}, \quad v \in H_{0}^{1}(\Omega)
$$

with $0<\alpha<\frac{1}{2}, 0<\beta_{1} \leq b(x) \leq \beta_{2}, 1<m<2^{*}(1-\alpha)\left(2^{*}=\frac{2 N}{N-2}, N \geq 3\right)$. Notice that in this case, $\alpha=0\left(1<m<2^{*}\right)$ corresponds to the subcritical semilinear case. Our results can be considered as an extension to quasilinear boundary value problems of the semilinear previous ones. Specifically, we prove the existence of a global minumum provided that $1<m<2(1-\alpha)$. For $m \in\left(2,2^{*}(1-\alpha)\right)$ we apply a new version of the Mountain Pass Theorem. Here, the main difference with the semilinear framework is the verfication of the Palais-Smale condition. Indeed, in contrast with the semilinear case where the $L^{\infty}$-boundness of the limit points (the critical points) is obtained by using the equation that they satisfy, here, in order to verify the compactness condition it is crucial to show previously that the limit point is bounded. The cases $2(1-\alpha)<m<\min \left\{2^{*}(1-\alpha), 2\right\}$ and $m>2^{*}(1-\alpha)$ are also discussed.

## Matthias Büger

## How diffusion may change the dynamics of reaction-diffusion-systems

We discuss two examples of r-d-equations in which the dynamics differs essentially from the underlying ODE: We construct a vectorfield $g: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ which contains no periodic cycles, but $u_{t}=d A u+g(u)$ (with Dirichlet boundary conditions on a smooth and bounded domain $\Omega$ ) has periodic solutions. The second example - and the main part of the talk - is about the r-d-system $\frac{d}{d t} u_{k}=d A u_{k}+c_{k} u_{k} f(t, x, u), k=1, \ldots, m, x \in \Omega \subset \mathbb{R}$ bounded and smooth, $d>0, f$ smooth and periodic in $t$. Such systems occur in population genetics if we look at a species with $m$ phenotypes which differ only in the time they need for growing up, maturing, reproducing themselves, and finally dieing. If in the ODE $u$ does not tend to zero then for each $k$ with $u_{k}(0)>0, u_{k}$ does not tend to zero (i.e. the whole population becomes extinct or all phenotypes survive). However, the symmetric diffusion term has the consequence that at most one phenotype may survive. These examples show that the diffusion term may cause an essential change in the dynamics.

## Anna Capietto

## Superlinear indefinite equations on the real line and chaotic dynamics

(joint work with W. Dambrosio, D. Popini)

We study existence and multiplicity of solutions to a second order equation of the form

$$
\begin{equation*}
x^{\prime \prime}+c x^{\prime}+q(t) g(x)=0, t \in(a, b) \tag{1}
\end{equation*}
$$

where $-\infty \leq a<b \leq+\infty, c$ is a real constant, $q:(a, b) \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions. Multiplicity is investigated by searching solutions with suitable prescribed nodal properties. The main feature of the problem is that of being 'superlinear indefinite’, i.e.

$$
\lim _{|x| \rightarrow+\infty} \frac{g(x)}{x}=+\infty
$$

and that the function $q$ is allowed to have (infinitely many) zeros on $(a, b)$. The problem of the existence of globally defined solutions to (1) is motivated by the fact that the superlinearity of $g$ implies a blow up phenomenon in the intervals where $q$ is negative. Nevertheless, our main result guarantees the existence of a double sequence of globally defined solutions to (1) having also a prescribed (arbitrarily large) number of zeros in the intervals where $q$ is positive and at most one zero in every interval where $q$ is negative. Moreover, as a consequence we show that some features of chaotic dynamics can be found in the particular case when in equation (1), $q$ is $\omega$-periodic and $c=0$.

## Pavel Drabek

## Some qualitative properties of quasilinear boundary value problems with the $p$-Laplacian

Let $\lambda_{1}>0$ be the principal eigenvalue of the $p$-Laplacian $\Delta_{p}$ on the bounded domain $\Omega \subset \mathbb{R}^{N}$ with smooth boundary $\partial \Omega, \varphi_{1}>0$ be an associated eigenfunction, $1<p<2$. We consider the boundary value problem
(1) $\Delta_{p} u+\lambda_{1}|u|^{p-2} u=h$ in $\Omega, u=0$ on $\partial \Omega$
and corresponding energy functional

$$
\begin{equation*}
E_{h}(u)=\frac{1}{p} \int_{\Omega}|\nabla u|^{p}-\frac{\lambda_{1}}{p} \int_{\Omega}|u|^{p}+\int_{\Omega} h u, u \in W_{0}^{1, p}(\Omega) . \tag{2}
\end{equation*}
$$

We prove that for $h \in C(\bar{\Omega}), \int_{\Omega} h \varphi_{1}=0$, the energy functional (2) is unbounded from below and has a local saddle point geometry. Moreover, we characterize the set of all $h \in C(\bar{\Omega})$ for which the boundary value problem (1) has at least one solution. Also more precise results for $N=1$ and general $p>1$ are discussed.

## Yihong Du

## Realization of prescribed patterns in some simple semilinear equations

We demonstrate that positive solutions with quite arbitrary patterns can be found in problems of the type
(1) $\quad-\Delta u=f_{\varepsilon}(x, u), x \in \Omega ; B u=0, x \in \partial \Omega$,
where the boundary operator $B$ can be Dirichlet type, Neumann type, or Robin type. Moreover, the nonlinearities can be chosen from well-known models and the patterned solutions are globally asymptotically stable. To be more precise, we show that given a bounded domain $\Omega$ in $R^{N}(N \geq 2)$ with smooth boundary $\partial \Omega$, and an arbitrary set of finitely many disjoint closed subdomains $D_{1}, \ldots, D_{m}$ of $\Omega$, we can find a simple continuous function $f(x, u, \varepsilon)=f_{\varepsilon}(x, u)$ (sublinear in nature) such that the boundary value problem (1) has a unique positive solution $u_{\varepsilon}$ and for $\epsilon$ small, $u_{\varepsilon}$ has peaks concentrating exactly in $D_{1} \cup \ldots \cup D$. Moreover, any positive solution (regard-less of the initial value) of the parabolic problem

$$
u_{t}-\Delta u=f_{\varepsilon}(x, u), x \in \Omega, t>0 ; B u=0, x \in \partial \Omega, t>0
$$

satisfies

$$
\lim _{t \rightarrow \infty} u(x, t)=u_{\varepsilon}(x), \text { uniformly for } x \in \bar{\Omega} .
$$

## Bernold Fiedler <br> Takens-Bogdanov bifurcations without parameters <br> (joint work with S. Liebscher)

Bifurcation theory addresses vector fields $\dot{x}=f(x, \lambda)$, foliated by constant parameter $\dot{\lambda}=0$, and usually with a trivial equilibrium (manifold) $0=f(0, \lambda)$, parametrized by $\lambda$. Bifurcation without parameters addresses such equilibrium manifolds losing normal hyperbolicity, and dropping the requirement of foliation by a constant 'parameter' $\lambda$. We discuss the three types of dynamics arising in the Takens-Bogdanov case, where $\operatorname{dim} \lambda=2$ and a double eigenvalue zero occurs normally to the equilibrium surface. Skipped applications of bifurcations without parameters include competition in chemostats [Farkas] (1984), uncoupling coupled oscillators [Alexander \& Auchmuty] (1986), central difference discretizations of systems of balance laws [F. \& L.] (2001), oscillators, stable, non-Lax shocks of systems of balance laws [F. \& L.], [L.] (2000).

## J.-P. Gossez

## Asymmetric elliptic problems with indefinite weights

(joint work with M. Arias, J. Campos, M. Cuesta)

We prove the existence of a first nontrivial eigenvalue for the following asymmetric elliptic problem:

$$
\left\{\begin{array}{c}
-\Delta_{p} u=\lambda\left[m(x)\left(u^{+}\right)^{p-1}-n(x)\left(u^{-}\right)^{p-1}\right] \text { in } \Omega \\
u=0 \text { on } \partial \Omega
\end{array}\right.
$$

A first application is given to the description of the beginning of the Fučik spectrum with weights for the $p$-Laplacian. Another application concerns the study of nonresonance for the problem

$$
\left\{\begin{array}{c}
-\Delta_{p} u=f(x, u) \text { in } \Omega \\
u=0 \text { on } \partial \Omega .
\end{array}\right.
$$

One feature of our nonresonance condition is that they involve eigenvalues with weights rather than pointwise restrictions.

## Hans-Peter Heinz

## Nonlinear eigenvalue problems admitting eigenfunctions with prescribed geometric properties

We consider nonlinear eigenvalue problems of variational structure which can be written in the form

$$
A u+B(u) u=\lambda u
$$

with linear operators $A$ and $B(u)$, where $B(u)$ depends nonlinearly on the function $u$. For each fixed $u$ in an appropriate set of functions the linear problem

$$
A v+B(u) v=\lambda v
$$

is assumed to admit a sequence of eigenvalues

$$
\mu_{1}(u) \leq \mu_{2}(u) \leq \ldots
$$

Also we assume that the associated functional $\psi$ admits critical values

$$
C_{1} \leq C_{2} \leq \ldots
$$

of Ljusternik-Schnirelman type. We give sufficient conditions which guarantee that for given $n \in \mathbb{N}$ there exists a solution $(u, \lambda)$ such that $\psi(u)=C_{n}$ and $\lambda=\mu_{n}(u)$. This is illustrated by an application to nodal solutions of radially symmetric nonlinear Schrödinger equations. Further applications are sketched.

## Jon Jacobsen

## Nonlinear problems associated with the $\kappa$-Hessian operators

For $\Omega \subset \mathbb{R}^{N}$ a bounded domain we consider problems of the form

$$
\left\{\begin{array}{cc}
S_{k}\left(D^{2} u\right)=f(\lambda, u) & x \in \Omega \\
u=0 & \partial \Omega
\end{array}\right.
$$

where
$S_{k}\left(D^{2} u\right)=S_{k}\left(\lambda\left[D^{2} u\right]\right)=\sum_{1 \leq i_{1}<\ldots<i_{k} \leq N} \lambda_{i_{1}} \ldots \lambda_{i_{k}}$ is the $k$ th-symmetric
polynomial acting on the eigenvalues of the symmetric matrix $D^{2} u$. For instance, we prove for the analogue of the Liouville-Gelfand problem with $f(\lambda, u)=-\lambda e^{u}$, and $\Omega=B(0,1)$, the solution continua emanating with $\lambda>0$ from $\left(\lambda,\|u\|_{\infty}\right)=(0,0)$ are asymptotic to $\{0\} \times[0, \infty)$ for $N \leq 2 k$, asymptotic to the vertic line given by $\lambda=(2 k)^{k}(N-2 k)$ for $2 k<N<(2 k+8)$ and oscillating with respect to this line, and monotonically asymptotic to the same line if $N \geq 2 k+8$. Related problems for $\Delta_{p}$ are considered.

## Louis Jeanjean

## Elliptic problems with asymptotically linear nonlinearities on $\mathbb{R}^{N}$

## (joint work with K. Tanaka)

We consider the problem of finding a nontrivial solution of

$$
-\Delta u+v(x) u=f(u) \text { on } \mathbb{R}^{N}, N \geq 2
$$

where we assume
(v1) $v \in C\left(\mathbb{R}^{N}, \mathbb{R}\right), \exists \alpha>0: v(x) \geq \alpha, \forall x \in \mathbb{R}^{N}$
(v2) $\lim _{|x| \rightarrow \infty} v(x)=v(\infty)$ exists and $v(x) \leq v(\infty), x \in \mathbb{R}^{N}$
Moreover we require $f: \mathbb{R}^{+} \rightarrow \mathbb{R}$ satisfies
(f1) $\frac{f(s)}{s} \rightarrow 0$ as $s \rightarrow 0^{+}$
(f2) $\frac{f(s)}{s} \rightarrow a<+\infty$ as $s \rightarrow+\infty$
(f3) Defining $\sigma: \mathbb{R}^{+} \rightarrow \mathbb{R}$ with $\sigma(s)=f(s) s-2 F(s)\left(F(s)=\int_{0}^{s} f(t) d t\right)$ we have

$$
\begin{gathered}
(i) \sigma(s) \geq 0, \forall s \geq 0 \\
(i i) \exists \delta>0: \frac{2 F(s)}{s^{2}} \geq v(\infty)-\delta \Longrightarrow \sigma(s) \geq \delta
\end{gathered}
$$

Our result is the following
Theorem Assume $\left(v_{1}\right)-\left(v_{2}\right),\left(f_{1}\right)-\left(f_{3}\right)$ hold and that $a>\inf \Sigma(-\Delta+v(x))$ ( $\Sigma$ denotes the spectrum of $-\Delta+v(x)$ on $H^{1}\left(\mathbb{R}^{N}\right)$ ). Then (0.1) has a nontrivial positive solution.
Remarks: Under our assumptions the natural functional I associated to ( 0.1 ) has a mountain pass geometry. To get a nontrivial solution the two main difficulties to overcome are the lack of a priori bounds for Palais-Smale (PS) sequences, due here to the fact that the nonlinearities are asymptotically linear and the lack of compactness at infinity. In contrast to cases where a smooth natural constraint can be introduced the problem of compactness is more serious here, because, under $\left(f_{3}\right)$ there is no obvious strict inequality between the 'energy' of our problem and the one of the 'problem at infinity'.

## Klaus Kirchg ÄSSner

## Dispersive dynamics of water waves

Two dimensional potential flow in a layer with free upper surface and driven by gravity is analyzed. It is shown that, if $\lambda=g h / c^{2}<1$, the quiescent state is asymptotically stable, whereas, to $\lambda>1$ it is unstable ( $\mathrm{g}=$ gravity, $\mathrm{h}=$ mean height of free surface, c velocity of observer). Existence and stability is proven - for the first time - by using a Paley-Wienertype theorem and a detailed discussion of the dispersion relation $(s+\sigma)^{2}=\sigma \tan \sigma$, where s is the (complex) frequency and $\sigma$ the complex wave-number. It is also shown that, if $\lambda>1$, two branching points of the DR appear on the $(i \beta)$-line, which leads to the instability of the quiescent state. Travelling waves are constructed via space like dynamics.

## Bernhard Lani-Wayda

## A generalized Lefschetz fixed point theorem and symbolic dynamics in delay equations

(joint work with R. Szrednicki)
We prove a version of the Lefschetz fixed point theorem for compositions of maps in normed spaces, with the domains not necessarily invariant. The theorem is then applied to return maps in delay-equations with periodic feedback of the type $\dot{x}(t)=f(x(t-1))(f$ sine-like). We thus obtain symbolic dynamics for such equations.

## Vy Khoi Le

## Nontrivial solutions of quasi-linear equations with slowly growing principal parts

We are concerned with the existence of nontrivial solutions of quasi-linear elliptic equations of the form:

$$
-\operatorname{div}(a(|\nabla u|) \nabla u)=g(x, u) \text { in } \Omega
$$

with Dirichlet boundary condition $u=0$ on $\partial \Omega$, in the case $\psi(t)=a(t) t$ has very slow growth $\left(\psi(t) \ll t^{\varepsilon}, \forall \varepsilon>0\right)$. Our main tool is a version of the Mountain Pass Theorem for variational inequalities, without the Palais-Smale condition, in Orlicz-Sobolev spaces.

## Stanislaus Maier-Paape <br> Pattern formation below criticality forced by noise <br> (joint work with D. Blömker)

We address the question of pattern formation of equations of Swift-Hohenberg type perturbed by additive noise $\xi$

$$
\partial_{t} u=-\varepsilon^{2}\left(\Delta+\frac{1}{2} \varepsilon^{-2}\right)^{2} u-u^{3}+\xi, u(0)=0 .
$$

This equation is stable in the absence of noise. Nevertheless, the stochastic forcing is strong enough to drive the solutions with high probability into regions where patterns are observed, which usually occur in spinoidal decomposition.

## Luisa Malaguti

## Travelling wavefronts in reaction-diffusion equations with convection effects and non-regular terms

(joint work with C. Marcelli)
We look for wavefront solutions of the following reaction-diffusion equation

$$
\begin{equation*}
\frac{\partial u}{\partial t}+\frac{\partial H(u)}{\partial x}=\frac{\partial}{\partial x}\left[d(u) \frac{\partial u}{\partial x}\right]+g(u), u=u(t, x) \tag{1}
\end{equation*}
$$

which takes into account both a non-linear convection effect given by $H(u)$ and densitiy dependent diffusion due to $d(u)$. A solution of (1) is called a travelling wave solution
(t.w.s.) if $u(t, x)=U(z)$ with $z=x+c t$; in this case the real constant $c$ stands for the wave speed. We assume
(2) $g(u)<0$ for $u \in] 0,1[$ and $g(u) \equiv 0$ otherwise
and investigate the existence of monotone t.w.s. connecting the two stationary states $u \equiv 0$ and $u \equiv 1$, i.e.,

$$
\text { (3) } \lim _{z \rightarrow+\infty} U(z)=1 \text { and } \lim _{z \rightarrow-\infty} U(z)=0 \text {. }
$$

Let $D^{+}(d g)(u)$ and $D_{+}(d g)(0)$ denote the upper and lower right Dini-derivatives of $d g$ at the point $u$, respectively. Set $h(u)=\frac{d H(u)}{d u}$.
Theorem Assume that $h, g$ and $d$ are continuous in $[0,1]$, with $\min d(u)>0$ and take $g$ satisfying (2). Then, if $D^{+}(d g)(0)<+\infty$, there exists a constant $c^{*}$ with

$$
2 \sqrt{D_{+}(d g)(0)}-h(0) \leq c^{*} \leq 2 \sqrt{\sup _{s \in] 0,1]}} \frac{d(s) g(s)}{S}-\min _{u \in[0,1]} h(u)
$$

such that (1) admits montone solutions satisfying (3) if and only if $c \geq c^{*}$. Moreover the solution is unique up to a translation of the origin. Whereas if $D_{+}(d g)(0)=+\infty$, problem (1) does not admit monotone solutions for any wave speed $c$. The proof combines comparison-type techniques and phase plane arguments. We remark that we are able to treat any continuous non-linearity $g$. In particular, we can study the case when $g(0)=0$, implying that the origin of the $\left(U, U^{\prime}\right)$ phase plane is a degenerate equilibrium point.

## John Mallet-Paret

## Exponential Dichotomies for Mixed-Type Functional Differential Equations via Holomorphic Factorization

(joint work with S. Verduyn Lunel)
We study a class of mixed-type linear functional differential equations of which

$$
\dot{x}(t)=a x(t)+b x(t-1)+c x(t+1)
$$

is the simplest (although multiple and distributed shifts are permitted). Assume hyperbolicity, namely $\Delta(i s) \neq 0 \forall s \in \mathbb{R}$, where $\Delta(\lambda)=\lambda-a-b e^{-\lambda}-c e^{\lambda}$.
Theorem 1 Given $\lambda_{0} \in \mathbb{C}$ there exist measures $d \eta_{-}(\theta)$ and $d \eta_{+}(\theta)$ on $[-1,0]$ and $[0,1]$ such that $\Delta(\lambda)\left(\lambda-\lambda_{0}\right)=\Delta_{-}(\lambda) \Delta_{+}(\lambda)$ where

$$
\Delta_{-}(\lambda)=\lambda-\int_{-1}^{0} e^{\lambda \theta} d \eta_{-}(\theta), \quad \Delta_{+}(\lambda)=\lambda-\int_{0}^{1} e^{\lambda \theta} d \eta_{+}(\theta)
$$

are the characteristic functions of a retarded and advanced equation. The factorization is unique up to 'root swapping' between $\Delta_{ \pm}$. If $n_{ \pm}=$the number of roots of $\Delta_{ \pm}$with $\mp \operatorname{Re} \lambda>0$, and $n_{0}=0$ or 1 as $\operatorname{Re} \lambda_{0}<0$ or Re $\lambda_{0}>1$ respectively, then $n_{\#}=n_{+}-n_{-}+n_{0}$ is independent of the factorization.

Theorem 2 Let $P=\left\{x_{0} \in C[-1,1] \mid \exists\right.$ bounded solution $x(t)$ for $\left.t \leq 0\right\}$ and $Q=\left\{x_{0} \in\right.$ $C[-1,1] \mid \exists$ bounded solution $x(t)$ for $t \geq 0\}$.
Then $P, Q$ are closed subspaces, and

$$
P \oplus Q=C[-1,1]
$$

Here $x_{t}(\theta)=x(t+\theta), \theta \in[-1,1]$.
Theorem 3 Let $\pi^{-}: C[-1,1] \rightarrow C[-1,0]$ and $\pi^{+}: C[-1,1] \rightarrow C[0,1]$ denote the restriction operators. Then $\pi^{-} \mid Q$ and $\pi^{+} \mid P$ are Fredholm with indices $n_{\#}-1$ and $-n_{\#}$ respectively. Formulas for the dimension of the kernel and codimension of the range are also given.

Theorem 4 Let $\wedge^{\tau}: C_{0}^{1}[-\tau, \tau] \rightarrow C^{0}[-\tau, \tau]$ be given by

$$
\left(\wedge^{\tau} x\right)(t)=\dot{x}(t)-a x(t)-b x(t-1)-c x(t+1), \quad|t| \leq \tau .
$$

If $n_{\#}=0$ or 1 then $\operatorname{ker}\left(\wedge^{\tau}\right)=\{0\}$ for all large $\tau$, and $\left\|\wedge^{\tau} x\right\|_{C^{0}} \geq K\|x\|_{C^{1}}$ for $K$ independent of $\tau$. This result can never hold if $n_{\#} \neq 0$ or 1 . In any case, $\wedge^{\tau}$ is Fredholm with index -1 .

Theorem 5 If $a \in \mathbb{R}$ and $b, c \in \mathbb{R}$ with $b c>0$ then $\operatorname{ker}\left(\wedge^{\tau}\right)=\{0\}$ for every $\tau>0$.

## Anna Marie Micheletti

Multiple solutions for an asymptotically linear problem with nonlinearity crossing a finite number of eigenvalues and application to a beam equation

## (joint work with C. Saccon)

Existence and multiplicity results for solutions of equation of the form $L(u)=f(u)$ in a Hilbert space $H$, where $L$ is a selfadjoint operator and the nonlinear term $f$ is asymptotically linear and interacts only with a finite number of isolated eigenvalues of $L$ have been studied by several authors - we only cite the first works on the topic of Amann and Zehnder and Chang. We study a problem of this type with an additonal parameter $b$ and find multiple solutions for $b$ near some eigenvalues of $L$. To be more precise we consider a linear selfadjoint operator $L: D(L) \rightarrow L^{2}(\Omega)$ where the domain $D(L)$ is a dense subspace of $L^{2}(\Omega), \Omega$ being a bounded open subset of $\mathbb{R}^{N}$. Let us denote by $\sigma(L)$ the spectrum of $L$, we assume that there exist $\underline{\gamma}<\bar{\gamma}$ such that $[\underline{\gamma}, \bar{\gamma}] \cap \sigma(L)=[\underline{\gamma}-\delta, \bar{\gamma}+\delta] \cap \sigma(L)$ (for some $\delta>0$ ) and consists of a finite number of eigenvalues of finite multiplicity. We set $[\underline{\gamma}, \bar{\gamma}] \cap \sigma(L)=\left\{\wedge_{1}, \ldots, \wedge_{k}\right\}$. Moreover the nonlinearity $g: \mathbb{R} \times \bar{\Omega} \times \mathbb{R} \rightarrow \mathbb{R}$ will be continuous and such that

$$
\begin{gathered}
\lim _{|s| \rightarrow+\infty} \frac{g(b, x, s)}{s}=b_{\infty} \in\left[\wedge_{k}, \bar{\gamma}\right], \lim _{s \rightarrow 0} \frac{g(b, x, s)}{s}=b \text { uniformly on } b, x \\
\underline{\gamma} \leq \frac{g\left(b, x, s_{2}\right)-g\left(b, x, s_{1}\right)}{s_{2}-s_{1}} \leq \bar{\gamma} \forall b, x, s_{1}, s_{2} \quad s_{1} \neq s_{2}
\end{gathered}
$$

Under suitable assumptions on $g$ we prove that if there exist at least two eigenvalues $\wedge_{1}<\wedge_{2}$ between $b$ and $b_{\infty}$ then for $b<\wedge_{1}$ and close to $\wedge_{1}$, we have at least four nontrivial solutions of the problem

$$
L u=g(b, \cdot, u)
$$

This result is obtained by studying the geometrical properties of the sublevels of the functional associated with the problem and some properties of its gradient. First we find two different linking type inequalities corresponding to two different decompositions of the space, which provide two nontrivial critical levels. On the other side we get a property of the gradient which allows to use an abstract result and get that each linking inequality gives rise actually to two critical points.

## Roger Nussbaum

## Complex Dynamics for Quadratic Fibonacci Sequences

We are interested in the complex dynamics of he map $\Psi: \mathbb{C}^{2} \rightarrow \mathbb{C}^{2}$ given by

$$
\begin{equation*}
\Psi(w, z)=(z, Q(w, z)), Q(w, z):=b_{11} w^{2}+2 b_{12} w z+b_{22} z^{2}+b_{1} w+b_{2} z+b_{3} \tag{1}
\end{equation*}
$$

The Hénon map $\tilde{\Psi}(w, z)=\left(a w^{2}+b z+c, d w\right), a d \neq 0$, is linearly conjugate to $\Psi(w, z)=$ $\left(z, z^{2}+(b d) w+(a c)\right)$, which is of the form in (1). A great deal of work has been done on the dynamics of complex polynomial automorphisms in $\mathbb{C}^{n}$, sometimes with hyperbolicity assumptions, but we are particularly interested in the case that $\Psi$ may not be an automorphism. Note that the study of the dynamcis of $\Psi$ is equivalent to the study of generalized quadratic Fibonacci sequences of GQF sequences given by

$$
\begin{equation*}
z_{j+2}=Q\left(z_{j}, z_{j+1}\right), j \in \mathbb{N} \cup\{0\} \text { or } j \in \mathbb{Z} \tag{2}
\end{equation*}
$$

For simplicity we only describe some results for the example

$$
\text { (3) } \Psi(w, z)=\left(z, w^{2}+z\right),(w, z) \in \mathbb{C}^{2} \text {. }
$$

We refer to the corresponding quadratic Fibronacci sequences given by $z_{j+2}=z_{j+1}+z_{j}^{2}$ as QF sequences. Note that $\Psi$ in (3) is, in general, a $2-1$ map. The following theorems can be found in an article by S. Greenfield and R. Nussbaum in a recent issue of J. Differential Equations.
Theorem 1 Let $\Psi$ be given by (3). For every $b \in \mathbb{C}^{2} \backslash\{0\}$, there exists a biinfinite sequence $<b_{j} \in \mathbb{C}^{2} \mid j \in \mathbb{Z}>$ such that (1) $b_{0}=b,(2) \Psi\left(b_{j}\right)=b_{j+1} \forall_{j} \in \mathbb{Z}$ and (3) either $\lim _{j \rightarrow-\infty} b_{2 j}=(1-i, 1+i)$ and

$$
\lim _{j \rightarrow-\infty} b_{2 j+1}=(1+i, 1-i) \text { or } \lim _{j \rightarrow-\infty} b_{2 j}=(1+i, 1-i) \text { and } \lim _{j \rightarrow-\infty} b_{2 j+1}=(1-i, 1+i)
$$

Theorem 2 For every prime number $p, \Psi$ has a periodic point of minimal period $p$ and a periodic point of minimal period $2 p$. (Of course, $(0,0)$ is the only fixed point of $\Psi$ )

Theorem 3 (Existence of homoclinic orbits) There exists an open set $U$ in the upper half plane of $\mathbb{C}, 0 \in \partial U$, such that for every $z \in U$ there is a biinfinite QF sequence $<z_{j} \in \mathbb{C} \mid j \in \mathbb{Z}>$ with $z_{0}=z, \lim _{j \rightarrow-\infty} z_{j}=0$ and $\lim _{j \rightarrow+\infty} z_{j}=0$. The sequence $<z_{j}>$ can also be taken so that $0<\arg \left(z_{j}\right)<\arg \left(z_{j+1}\right)<\pi$ for all $j \in \mathbb{Z}$ and $\lim _{j \rightarrow-\infty} \arg \left(z_{j}\right)=0$.

## Rafael Ortega

## Massera's theorem for quasi-periodic differential equations

(joint work with M. Tarallo)
Consider the differential equation

$$
(*) \quad \dot{x}=F\left(\omega_{1} t, \omega_{2} t, x\right)
$$

where $F: T^{2} \times \mathbb{R} \rightarrow \mathbb{R}$ is continuous and $\frac{\omega_{1}}{\omega_{2}} \notin \mathbb{Q}$. Here $T^{2}=\mathbb{R} / \mathbb{Z} \times \mathbb{R} / \mathbb{Z}$ is the 2dimensional torus and a generic point of $T^{2}$ is denoted by $\Theta=\left(\theta_{1}, \theta_{2}\right)$. A solution $x(t)$ of $(*)$ is quasi-periodic if there exists $u \in C\left(T^{2}\right)$ such that

$$
x(t)=u\left(\omega_{1} t, \omega_{2} t\right), t \in \mathbb{R} .
$$

It is easy to prove that $u$ is a solution in $T^{2}$ of the p.d.e.

$$
(* *) \quad \omega_{1} \frac{\partial u}{\partial \theta_{1}}+\omega_{2} \frac{\partial u}{\partial \theta_{2}}=F\left(\theta_{1}, \theta_{2}, u\right) .
$$

We consider a larger class of (possibly discontinuous) solutions of this equation. A function $u: T^{2} \rightarrow \mathbb{R}$ will be called solution of $\left({ }^{* *}\right)$ if it is bounded, upper or lower semicontinuous, has a directional derivative $D_{\vec{\omega}}, \vec{\omega}=\left(\omega_{1}, \omega_{2}\right)$, defined everywhere and bounded such that

$$
\left.D_{\vec{\omega}} u(\Theta)=F \mid \Theta, u(\Theta)\right) \quad \forall \Theta \in T^{2} .
$$

The existence of a bounded solution of $(*)$ is sufficient to guarantee the existence of a solution of $(* *)$ in this sense. Based upon a construction of R.A. Johnson, we present an example of discontinuous solutions of $(* *)$ which helps to understand the relationship between the theory of almost automorphic solutions of $(*)$, as developed by Shen and Yi, and the notion of discontinuous solutions of ( $* *$ ).

## Peter Polacik

## Convergence to a ground state for solutions of parabolic equations on $\mathbb{R}^{N}$

(joint work with J. Busca)
We consider the Cauchy problem for reaction-diffusion equations of the form

$$
\begin{array}{ll}
u_{t}=\Delta u+f(u), \quad u=u(x, t), & x \in \mathbb{R}^{N}, t>0, \\
u=u_{0} \geq 0, & x \in \mathbb{R}^{N}, t=0 \tag{1}
\end{array}
$$

where $f \in C^{1}, f(0)=0, f^{\prime}(0)<0$. Under the assumptions that $u_{0}$ decays to zero exponentially as $|x| \rightarrow \infty,\|u(\cdot ; t)\|_{L^{\infty}\left(\mathbb{R}^{N}\right.}$ stays bounded and $u(x, t) \rightarrow 0$, as $|x| \rightarrow \infty$, uniformly with respect to $t$, we prove the following convergence result:

$$
u(\cdot, t) \rightarrow \varphi \text { as } t \rightarrow \infty\left(\text { with convergence in } H^{1}\right)
$$

where either $\varphi=0$ or $\varphi$ is a positive solution (ground state) of the problem

$$
\begin{array}{ll}
\Delta \varphi+f(\varphi)=0 & \text { on } \mathbb{R}^{N} \\
\varphi(x) \rightarrow 0, & \text { as }|x| \rightarrow \infty \tag{2}
\end{array}
$$

As any solution of (2) is radially symmetric about some point $\xi \in \mathbb{R}^{N}$, the convergence result encompasses the asymptotic symmetrization of bounded positive solutions $u$ of (1). The proof combines the energy and comparison methods with invariant manifold techniques.

## Markus Poppenberg

## Methods for the solution of quasilinear Schrödinger equations

We consider the quasilinear Schrödinger equation

$$
i J_{t} u=-\Delta u+V u-\left(\Delta|u|^{2}\right) u .
$$

In a joint work with Klaus Schmitt and Zhi Qiang Wang the existence of standing wave solutions is proved using different methods from the calculus of variations as direct methods and minimax techniques. Results are discussed in the case $n=1$ and in the radial case for
$n \geq 2$. In addition, some results on local well posedness are proved by using a new Inverse Function Theorem with loss of derivatives in Sobolev spaces.

## Bryan Rynne

## Half-eigenvalues - an alternative to the Fučik spectrum for discussing semilinear problems with jumping non-linearities

We consider the Sturm-Liouville boundary value problem

$$
\begin{align*}
& -\left(p u^{\prime}\right)^{\prime}+q u=f(x, u)+h, \quad x \in[0, \pi], \\
& c_{00} u(0)+c_{01} u^{\prime}(0)=0,  \tag{1}\\
& c_{10} u(\pi)+c_{11} u^{\prime}(\pi)=0
\end{align*}
$$

$f:[0, \pi] \times \mathbb{R} \rightarrow \mathbb{R}$ is a Caratheodory function with $|f(x, u)| \leq A+B(u)$. Define

$$
a(x)=\lim _{u \rightarrow \infty} \frac{f(x, u)}{u}, b(x)=\lim _{u \rightarrow-\infty} \frac{f(x, u}{u} .
$$

Assume that $a, b \in L^{\infty}(0, \pi)$, but $a \neq b$. Let $u^{ \pm}(x)=\max \{ \pm u(x), 0\}$, and define the set

$$
\Sigma_{H}(L, a, b)=\left\{\lambda \in \mathbb{R}: L u=a u^{+}-b u^{-}+\lambda u \text { has a solution } u \neq 0\right\}
$$

Elements of $\Sigma_{H}(L, a, b)$ are called half-eigenvalues. The set $\Sigma_{H}(L, a, b)$ consists of a sequence

$$
\lambda_{1}^{-} \leq \lambda_{1}^{+}<\lambda_{2}^{-} \leq \lambda_{2}^{+}<\ldots
$$

Consider the inhomogeneous equation

$$
\begin{equation*}
L u=a u^{+}-b u^{-}+\lambda u+h, \quad h \in L^{2}(0, \pi) \tag{2}
\end{equation*}
$$

(A) If $\lambda_{k}^{+}<\lambda<\lambda_{k}^{-}$for some $k \in \mathbb{N}$, then (2) has a solution for all $h \in L^{2}(0, \pi)$.
(B) If $\lambda_{k}^{-}<\lambda<\lambda_{k}^{+}$for some $k \in \mathbb{N}$, then there exists $k_{0}$ such that (1) does not have a solution.

These results extend to the problem (1). The results in (A), (B) extend well-known results obtained using the Fučik spectrum to a larger class of functions $a, b$.

## BJörn Sandstede

## Spiral waves: Stability and bifurcations

(joint work with A. Scheel)
We consider spiral waves, $u(t, r, \varphi)=u_{*}(t, \varphi-c t)$, to reaction-diffusion systems $u_{t}=$ $D \Delta u+f(u)$ posed on $\mathbb{R}^{2}$ or on large disks $B_{R}(0)$. It is shown that the spectrum of the PDE linearized about a spiral wave

$$
\mathcal{L}_{*}=D \Delta+c \partial_{\varphi}+f^{\prime}\left(u_{*}(r, \varphi)\right)
$$

on $\mathbb{R}^{2}$ and $B_{R}(0)$ can be computed from the spectrum of the one-dimensional problem $\mathcal{L}_{\infty}=D \partial_{r r}+c k \partial_{r}+f^{\prime}\left(u_{\infty}(r)\right)$ provided $\left|u_{*}(r, \varphi)-u_{\infty}(r+k \varphi)\right| \rightarrow 0$ as $r \rightarrow \infty$. On $\mathbb{R}^{2}$ , the essential spectrum $\sum_{\text {ess }}$ is the main part of the spectrum, while the spectrum on $B_{R}(0)$ approaches the absolute spectral set $\sum_{a b s}$ as $R \rightarrow \infty$. In general, $\sum_{e s s}$ and $\sum_{a b s}$ are different. Applications to meandering and drifting spirals as well as to far-field and core break up of spirals were given.

## A.L. Skubachevskif

## Generation of contractive semigroups by integro-differential operators

> (joint work with E.I. Galakhov)

We consider an elliptic operator of the second order with integro-differential boundary condition

$$
\begin{equation*}
B u(x)=\gamma(x) u(x)+\int_{\bar{Q}}[u(x)-u(y)] \mu(x, d y)=0(x \in \partial Q) \tag{1}
\end{equation*}
$$

where $\gamma(x) \geq 0(x \in \partial Q), \mu(x, \cdot)$ is a nonnegative Borel measure, $Q \subset \mathbb{R}^{n}$ is a bounded domain with boundary $\partial Q \in C^{\infty}$. It were stated sufficient conditions, under which a closure of an elliptic operator with nonlocal condition (1) is a generator of a Feller semigroup.

## C.A. Stuart

## Buckling of a tapered elastica

Consider a column of variable cross-section in a vertical configuration with gravity acting downwards. It may buckle under its own weight. The simplest geometrically exact model uses the Bernoulli-Euler constitutive relation and leads to the following singular nonlinear boundary value problem:

$$
\left.\begin{array}{c}
\left\{A(t) u^{\prime}(t)\right\}^{\prime}+\mu \sin u(t)=0 \text { for } 0<t<1 \\
u(1)=\lim _{t \rightarrow 0} A(t) u^{\prime}(t)=0 \\
\int_{0}^{1} A(t) u^{\prime}(t)^{2} d t<\infty
\end{array}\right\}
$$

where $A$ is a function, determined by the cross-sections such that $A \in C([0,1]), A(t)>0$ for $t>0$ and, for some $p \geq 0$ and $L \in(0, \infty), \lim _{t \rightarrow 0} \frac{A(t)}{t^{p}}=L$. For $p<2$, the situation is similar to that for a regular Sturm-Liouville problem. At $p=2$ the picture changes. The linearised problem has an interval of essential spectrum and infinitely many branches of solutions of $(*)$ bifurcate from the infimum of this interval. The qualitative properties of these solutions also change.

## Susanna Terracini

## Infinitely many solutions to superlinear fourth order equations

We consider a fourth order class of equations

$$
\text { (E) } \quad u^{I V}-c u^{\prime \prime}=f(x, u)
$$

$c>-\frac{1}{4}$, where $f$ satisfies

$$
\begin{gathered}
\left(f_{1}\right) \quad \lim _{u \rightarrow 0} \frac{f(x, u)}{u}=0 \\
\left(f_{2}\right) \quad \exists \delta, C>0: f(x, u) \cdot u \geq(2+\delta) F(x, u)-C\left(\frac{\partial F}{\partial u}=f\right) \\
\left(f_{3}\right) \quad f(x, u) \cdot u \geq 0
\end{gathered}
$$

We prove that, under $\left(f_{1}\right),\left(f_{2}\right),\left(f_{3}\right),(\mathrm{E})$ has infinitely many $2 \pi$-periodic solutions. More precisely for every $k \geq 1$ there exists at least one solution having exactly $2 k$ zeroes.

## Sjoerd Verduyn Lunel

## Spectral theory for periodic differential delay equations

Consider a delay equation $\dot{x}(t)=B(t) x(t-1), B(t)=B(t+\omega)$. For ordinary differential equations $\dot{x}(t)=B(t) x(t)$, there exists a periodic transformation $P(t), P(t)=P(t+\omega)$, and $y(t)=P(t)^{-1} x(t)$ satisfies an autonomous equation $\dot{y}(t)=R y(t)$ (Floquet theory). In this lecture we address the question whether there exists a corresponding theory for differential delay equations. If there exist not-trivial super-exponential decaying solutions (small solutions), then such a Floquet transformation is not possible. And such solutions can actually exist. Consider, $\dot{x}(t)=\sin (2 \pi t) x(t-1)$ with initial data $x(\theta)=\int_{-1}^{\theta} \sin (2 \pi s) d s,-1 \leq \theta \leq 0$. We show that nontrivial small solutions exist if and only if the eigenvectors and generalized eigenvectors of the monodromy operator are not dense in the space of initial data. (If $\omega=1$, the monodromy operator is given by $T: C[-1,0] \rightarrow C[-1,0],(T \varphi)(\theta)=\varphi(0)+\int_{-1}^{\theta} B(s) \varphi(s) d s$.) We discuss necessary and sufficient conditions for completeness of the eigenvectors of the monodromy map and argue that this is a first step towards a periodic transformation to an autonomous equation.

## Zhi-Qiang Wang

## Weighted Sobolev inequalities and related PDEs

(joint work with F. Catrina)
Consider the following weighted Sobolev inequalities due to Caffarelli, Kohn and Nirenberg in 1984: for all $u \in C_{0}^{\infty}\left(\mathbb{R}^{N}\right)$

$$
\int_{\mathbb{R}^{N}}|x|^{-2 a}|\nabla u|^{2} d x \geq c\left(\int_{\mathbb{R}^{N}}|x|^{-b p}|u|^{p} d x\right)^{2 / p}
$$

for some $c=c_{a, b}>0$, where for $N \geq 3$

$$
-\infty<a<\frac{N-2}{2}, a \leq b \leq a+1, p=\frac{2 N}{N-2+2(b-a)} .
$$

We consider the parameter domain of $a<0$, and discuss the best constants, the extremal functions, and we present the symmetry breaking phenomenon for $a<0$ case, which is an contrast with the well studied $a \geq 0$ domain.

## Tobias Weth

## Nodul solutions to a generalized Emden-Fowler equation

Consider the following equation of Emden-Fowler type:

$$
\begin{aligned}
(G E F)-\Delta u & =w(x)|u|^{p-2} u, \quad u \in D^{1,2}\left(\mathbb{R}^{N}\right) \\
N & \geq 3,2<p<\frac{2 N}{N-2}
\end{aligned}
$$

where $w \in C^{\infty}\left(\mathbb{R}^{N}\right), w \geq 0, w \not \equiv 0$ and there exist constants $c>0, a>2-\frac{(p-2)(N-2)}{2}$ such that

$$
|w(x)| \leq \frac{C}{(1+|x|)^{a}} \quad\left(x \in \mathbb{R}^{N}\right) .
$$

It has been proved by S.B. Tshinanga that (GEF) possesses infinitely many solutions, at least one of them being positive. We introduce a new variational method which allows us to estimate the number of nodal domains of a solution $u$ via its energy value $\psi(u)$, where $\psi$ is the energy functional associated with (GEF). Moreover we prove the existence of a solution having precisely two nodal domains. In the particular case where $w(\cdot)$ is radially symmetric, we establish, for given $n \in \mathbb{N}$, the existence of a solution having precisely $n$ nodal domains. In this situation the growth conditions on $w(\cdot)$ may be relaxed in a reasonable way.

## Jianhong Wu

## Stable phase-locking in delayed neural networks

We report some recent work about the global dynamics of small networks of neurons with delayed feedback $\left[\dot{x}_{i}=-\mu x_{i}+\sum_{j=1}^{n} w_{i j} f\left(x_{j}(t-\tau)\right), 1 \leq i \leq n\right]$. For the case where $n=2$, we describe the bifurcation diagram when the signal transmission delay $\tau$ is increased (joint work with Y. Chen and T. Krisztin). In particular, we give relatively complete information about the global attractor which contains, among others, synchronized periodic orbits, phase-locked periodic orbits and various heteroclinic connections representing the transition from synchronization to phase-locking. For the case where $n=3$, we discuss the existence and global continuation of 8 branches of phase-locked periodic orbits, standing waves and mirror-reflecting waves simultaneously bifurcated from the trivial equilibrium (joint work with L. Huang, S. Huang and T. Faria). Finally, we present an application of H.-O. Walther's method to the construction of a stable phase-locked periodic solution (in preparation).

## Fabio Zanolin

## Subharmonic Solutions for Nonlinear Equations with Indefinite Weight

(joint work with B. Liu)
Consider the scalar second-order nonlinear ODE

$$
\text { (1) } \ddot{x}+q(t) g(x)=0 \text {, }
$$

where $g: \mathbb{R} \rightarrow \mathbb{R}$ is sublinear at infinity and $q(t)$ may change sign. Under the assumptions that $g(s) \cdot s>0$ for $|s|$ large and $g$ is monotone nondecreasing near $\pm \infty$, we prove the existence of infinitely many 'large' subharmonic solutions when $q(\cdot)$ is $T$-periodic with $\int_{0}^{T} q(t) d t>0$. The subharmonic solutions we find are characterized by their nodal properties. In this manner, we can also prove the minimality of their period. Our result applies to equations like

$$
\text { (2) } \ddot{x}+q(t)|x|^{p-1} x=0 ; \quad 0<p<1
$$

(even if no homogeneity is required for 'our' $g$ ). We also prove that if $\int_{0}^{T} q(t) d t<0$, then all the solutions of (2) with sufficiently large initial condition are unbounded.

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