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Representations of Finite Groups

March 25th – March 31st, 2001

The meeting was organized by M. Broué (Paris), R. Dipper (Stuttgart), B. Külshammer (Jena), and G.R. Robinson (Birmingham). In eight lectures of 50 minutes length each and 26 short contributions of 30 minutes each, results and developments of the past few years as well as challenges for the future were outlined. Representation theory of finite groups has grown into a huge area touching and interrelating to many other fields in mathematics. It was felt that the special format of organizing the talks chosen for the meeting was particularly adequate for this meeting. For most talks 30 minutes were sufficient to present the most important ideas. More fundamental developments were presented in long lectures, held mostly by younger participants. Still the total number of hours of lectures was low, so that there was plenty of free time left for discussions.
One of the neighbouring areas which has influenced representation theory of finite groups and vice versa in recent years is the theory of algebraic and quantum groups. There were several lectures demonstrating this influence:
J.Brundan and A.Kleshchev reported in two talks on their joint work on representations of the double covers of symmetric groups. They used and generalized ideas of Lascoux, Leclerc and Thibon and of Grojnowski to relate these representations to those of Hecke-Clifford superalgebras (introduced by them) and crystals af a certain affine Kac-Moody Lie algebra. In his lecture on graded cyclotomic Hecke algebras A.Ram (on joint work with A.Shepler) classified all algebras obtained by applying Drinfeld's construction to complex reflection groups. Lusztig showed that geometric information contained in affine Hecke algebras can be recovered from the corresponding graded Hecke algebra. Drinfeld's construction defines an object similar to a semidirect product of a polynomial ring $S(V)$ and the complex group algebra of any subgroup of $GL(V)$ and is a generalisation of Lustzig's construction, which also applies to complex reflection groups. G.Malle gave a lecture on his joined work with M.Geck and L.Iancu on generic degrees associated with cyclotomic Hecke algebras. He and his collaborators proved a formula which he had conjectured on the Oberwolfach conference in 1996. A.Mathas presented in his talk a completely different and independent proof.
There were several talks on Morita equivalences of blocks of special groups. Thus J.Chuang gave a long and A.Hida a short lecture on Morita Theorems for blocks of symmetric and general linear groups. These results are important for proving Broue's conjecture for these groups.
Broue's conjecture remains a very active area. In several talks progress was reported for

some special types of groups. In a long talk R.Rouquier presented ideas concerning block gradings which seem to be relevant for the proof of the abelian defect group conjecture.
Another highlight of the conference were definitely the talks on work of C.Bonnafe and R.Rouquier. They showed, using deep geometrical ideas, that each p-block of a finite reductive group for a nondescribing prime p is Morita equivalent to an isolated block. For example for general linear groups the isolated blocks are precisely the unipotent ones. As a consequence this reduces the questions of classifying irreducibles and calculating decomposition numbers entirely to isolated blocks.
An interesting recent development in block theory has been the work of M. Linckelmann, which defines a cohomology ring for a general block B by making use of fusion in the category of B-subpairs and stability conditions on the usual cohomology ring of the defect group. This opens up several possible directions of research. For example, it becomes possible to discuss varieties for modules outside the principal block.
Another avenue of exploration, discussed by Linckelmann at this meeting, is the possibility of defining an analogue of a classifying space for a general block (that is, a topological space with the same (localized at p) cohomology as the block). Linckelmann reported on ongoing joint work with P.J. Webb, where, so far, an object with the desired properties can be found in the category of spectra.
Endo-permutation modules play an important role in representation theory, in particular in the structure of nilpotent blocks and of blocks of p-solvable groups. They also appear naturally in equivalences between module categories. The Dade group D(P) of a finite p-group P parametrizes the isomorphism classes of indecomposable endo-permutation modules. By a result of Puig, D(P) is a finitely generated abelian group. In the 1999 Oberwolfach conference, Bouc and Thévenaz had reported on their joint work culminating in the computation of the torsion free part of D(P). Since then, considerable progress has been achieved towards the calculation of the torsion subgroup of D(P).
In two lectures at this conference, Thevenaz and Carlson reported on recent joint work, mainly on endo-trivial modules. These can be viewed as building blocks of arbitrary endo-permutation modules. One particularly striking result is that, for the prime 2, the subgroup T(P) of D(P) consisting of endo-trivial modules is now completely known. This is an important step towards a complete description of D(P) in general.
One of the most intriguing problems in block theory is Dade's conjecture which unifies earlier conjectures such as Alperin's weight conjecture and the Alperin-McKay conjecture. Dade's conjecture exists in several variants. Recently, Dade's projective conjecture (DPC) has attracted most of the attention. Robinson has proved a number of reduction theorems for DPC which include a solution for the class of p-solvable groups.
During the conference, Eaton reported on further progress, obtained in collaboration with Robinson, on the structure of a minimal counterexample to DPC. In another talk, An gave an overview concerning the related project aiming at the verification of Dade's conjecture for groups which are close to simple.
It seems to be widely believed that Dade's conjecture will be a consequence of the vanishing of the Euler characteristic of a chain complex connecting certain Grothendieck groups (whose origin is still a mystery). In his talk, Boltje explored consequences of the existence of such a chain complex which are supported both by computer calculations and by theoretical arguments involving (among others) Broue's abelian defect group conjecture.
Other active areas of representation theory as integral representation theory, Mackey algebras, cohomological questions and derived equivalences were highlighted in further lectures

and it was demonstrated that representation theory of finite groups is a broad very active area of mathematics, and progress often comes where interaction with neighbouring fields is vivid.
The free time allowed for many fruitful and exciting discussions between the participants. Old cooperations could be continued, new teams emerged, and this will certainly lead to further progress in the future.

Abstracts

Dade's Conjectures for Some Finite Groups

J. An

In 1992, E.C. Dade generalized the Knörr-Robinson form of Alperin's weight conjecture and introduced a conjecture, Dade's ordinary conjecture. In 1994 and 1995, he raised several conjectures and announced that his final (= reductive) conjecture can be proved by verifying it for all the non-abelian finite simple groups. Two more versions of his conjecture were introduced by Gary K. Schwartz in 2000.
Dade's final (or other forms of) conjecture has been verified/investigated for 20 sporadic finite simple groups and several finite simple groups of Lie type.

1. Current Works
Dade's conjecture has been verified for the following cases:
(a) Sporadic Simple Groups
Except the groups HN, Th, Ly, J_4, B and M, 19 sporadic simple groups have been verified for the final conjecture and the radical chains has been classified for Fi_{24}.
(b) Simple Groups of Lie type
$L_2(q)$ (final=final conjecture); $L_3(q)$ (final, $p|q$); $\mathrm{Sz}(2^{2n+1})$ (final); $L_n(q)$ (ordinary $p|q$); $G_2(q)$ (final $p \not| q$, $q \neq 2,4$; and $p|q$, $2,3|q$); ${}^3D_4(q)$ (final, $p \not| q$); ${}^2G_2(3^{2n+1})$ (final); ${}^2F_4(2^{2n+1})$ (ordinary, $p \neq 2$); ${}^2F_4(2)'$ (final); A_n (ordinary, abelian defect groups);
(c). Other Groups
S_n (ordinary); $\mathrm{GL}_n^\epsilon(q)$ (ordinary, $p|q$); $\mathrm{GL}_n^\epsilon(q)$ (invariant, $p \not| q$); $\mathrm{Sp}_{2n}(q)$ and $\mathrm{SO}_m^\epsilon(q)$ (ordinary, $p \not| q$); p-soluble (projective); $G/O_p(G)$ TI Sylow p-subgroup (projective);
(d) General Results
Cyclic defect blocks (final); Tame blocks (invariant ordinary); Abelian defect unipotent blocks (ordinary); Principal abelian defect 2-block (ordinary); Principal abelian defect 3-block (ordinary); Abelian defect blocks with special inertial quotients (ordinary).

2. Main Reductions in the Verifications of Dade's Conjecture for Classical Groups in Non-defining Characteristics
Let $G = \mathrm{GL}_n(q)$, $\mathrm{GL}_n^-(q) = \mathrm{U}_n(q^2)$, $\mathrm{Sp}_{2n}(q)$, $\mathrm{SO}_{2n+1}(q)$, $\mathrm{SO}_{2n}^\pm(q)$. Suppose p is odd.
First Reduction:
Let R be a p-subgroup of G, $A(R)$ the intersection of all maximal normal abelian subgroups of R and

$$\mathcal{P}(R) = \Omega_a(A(R)) = \langle y \in A(R) : |y| = r^a \rangle,$$

where a is the multiplicative order of q^2 modulo p when $G = \mathrm{Sp}, \mathrm{SO}$, and while $G = \mathrm{GL}^\epsilon$, a is the multiplicative order of ϵq modulo p. Define

$$\mathcal{CR}(G) = \{C \in \mathcal{R}(G) : \mathcal{P}(P_i) = P_i \ \forall\ i\},$$

where $\mathcal{R}(G)$ is the set of radical chains of G.
Then Dade's conjecture for G can be reduced to the chains of $\mathcal{CR}(G)$.
Second Reduction:

Let B be a block of G, V the underlying space of G, D a defect group of B and $C_V(D)$ the fixed-point subspace of D. If $C \in \mathcal{R}(G)$ with final subgroup R, then define $C_V(C) = C_V(R)$. Let

$$\mathcal{CR}^*(B) = \{C \in \mathcal{CR}(G) : C_V(C) = C_V(D)\}.$$

Then Dade's conjecture for B can be reduced to the chains of $\mathcal{CR}^*(B)$.

Third Reduction:

Suppose $B \subseteq \mathcal{E}_p(G,(s))$ for some semisimple p'-element of G^*. If $G = \mathrm{Sp}$ or SO, then let $t \in G^*$ such that

$$m_{X-1}(t) = m_{X-1}(s) + m_{X+1}(s)$$

and $m_\Gamma(t) = m_\Gamma(s)$ for all $\Gamma \neq X \pm 1$. If $G = \mathrm{GL}^{\pm}$, then set $t = s$. Then $L^* = C_{G^*}(t)$ is a regular subgroup of G^*, and R_L^G is a perfect isometry between $\mathcal{E}_p(L,(s))$ and $\mathcal{E}_p(G,(s))$. Then Dade's conjecture for blocks can be reduced to the principal block B_0 or isolated blocks of G using R_L^G.

Final Calculations:

Dade's conjecture is verified for the principal block or isolated blocks of G using the results of Fong, Srinivasan and Olsson, and direct calculations.

Representation types of Hecke algebras of type B

S. Ariki

One basic question in modular representation theory is to ask whether an algebra has finitely many indecomposable modules or not. Since Hecke algebras of type A and type B appear in modular representation theory of finite classical groups of Lie type, these algebras deserve study. Let k be an algebraically closed field. We denote by $\mathcal{H}_n^0$ and $\mathcal{H}_n$ the Hecke algebras of type A and type B respectively, with quadratic relations $(T_i - q)(T_i + 1) = 0$ $(1 \le i < n)$ and $(T_0 - Q)(T_0 + 1) = 0$ (in type B case). We assume that $q = \sqrt[e]{1}$ with $e \ge 3$ for simplicity.

For $\mathcal{H}_n^0$, Uno has already shown that $\mathcal{H}_n^0$ is representation-finite iff its principal block is so, and this occurs precisely when $n < 2e$ is satisfied. Further, Dipper and James have shown that if $-Q$ is not a power of q, then $\mathcal{H}_n - mod$ is Morita equivalent to $\oplus_{n_1+n_2=n}\mathcal{H}_{n_1} - mod \otimes \mathcal{H}_{n_2} - mod$. Hence we may renormalize T_0 and assume that the quadratic relation of T_0 is $(T_0 - 1)(T_0 - q^f) = 0$ where $0 \le f < e$. The block which contains the representation $T_0 \to 1, T_i \to q$ $(1 \le i < n)$ is called the principal block of $\mathcal{H}_n$.

To determine representation types, we have to know Loewy structure of some (or all) PIMs. To do this, we use several results. First result is to know decomposition numbers. [1]

Theorem 1 (A) *Assume that $q = \sqrt[e]{1}$ with $e \ge 3$ and $-Q = q^f$ as above. Then we have the following isomorphism of $\mathfrak{g}(A_{e-1}^{(1)})$-modules.*

$$\oplus_{n\ge 0} K_0(\mathcal{H}_n - proj) \simeq V(\Lambda_0 + \Lambda_f) \tag{1}$$

If we further assume that the characteristic of the ground field k is 0, PIM's correspond to the canonical basis under this isomorphism.

[1] This theorem is the first theorem which shows the existence of the crystal structure (induced by the canonical basis) on the set of simple modules.

Recall that we have Specht module theory for $\mathcal{H}_n$ (Dipper, James and Mathas), and D^λ is non-zero if and only if λ is a Kleshchev bipartiton (A). [2]

Lemma 1 (A-Mathas) *Let (K, R, k) be a modular system, λ be a Kleshchev bipartition, P_K^λ and P_k^λ be corresponding PIMs in characteristic 0 and positive characteristic. If P_K^λ corresponds to $f_{i_1} \cdots f_{i_n} v_{\Lambda_0+\Lambda_f} \in V(\Lambda_0 + \Lambda_f)$, then $[P_k^\lambda]$ is the modular reduction of $[P_K^\lambda]$.*

Using this theorem and lemma, we may compute decomposition numbers in positive characteristic for principal blocks.
The other results we use are

- Specht modules have simple heads,
- PIMs have Specht filtrations,

and elementary facts about symmetric algebras.
The main theorem is the following.

Theorem 2 (A-Mathas) *Assume that $q = \sqrt[e]{1} (e \geq 3), -Q = q^f$ as before. Then*

(1) *If $n \geq e$, then $\mathcal{H}_n$ has infinite representation type.*

(2) *If $n < e$, then the principal block is representation finite.*

Unlike the case of Hecke algebras of type A and the group case, the principal block does not determine the representation type. In fact we have

(3) If $f = 1$ (or $f = e - 1$), then $\mathcal{H}_n$ is representation finite if and only if $n < min(6, e)$.

We also remark that $f = 0$ case gives an example that the crystalized Cartan matrix does not give the Loewy structure of PIMs. This may be explained by the fact that T_0 has a multiple root in this case, and degeneration occurs.

References

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[2] Using this labelling of simple modules, we may speak of modular branching rule $soc(res(D^\lambda)) = \oplus_{A:goodnode} D^{\lambda\setminus A}$. The link between modular branching rule and the crystal graph was first observed by Kleshchev, and implicit in Rouquier's remark in the LLT paper. We need recent result of Grojnowski to prove this.

Prime power degree representations of the symmetric groups

C. Bessenrodt

In 1998, Zalesskii proposed to classify all instances of irreducible complex characters of prime power degree for the quasi-simple groups; in recent joint work of his with Malle the quasi-simple groups with the exception of the alternating groups and their double covers were dealt with.

In the talk I reported on recent joint work with Balog, Olsson and Ono resp. Olsson; we have solved the problem of classifying the irreducible characters of prime power degree for the symmetric and alternating groups resp. their double covers. It turned out that in each of the families S_n, A_n, $\tilde{S}_n$, $\tilde{A}_n$ apart from 'obvious' characters of prime power degree only a short list of accidental extra cases for small n occurred. More precisely, the 'generic' results for S_n and $\tilde{S}_n$ are:

Theorem (Balog, B., Olsson, Ono 2000). Let $n \geq 10$, p a prime. Let λ be a partition of n. Then the irreducible character $[\lambda]$ of S_n is of p-power degree > 1 if and only if $n = p^r + 1$ and $\lambda = (n-1,1)$ or $(2,1^{n-2})$ (in this case $[\lambda](1) = p^r$).

Theorem (B., Olsson 2000). Let $n \geq 9$, p a prime. Let λ be a partition of n into distinct parts. Then the spin character $\langle\lambda\rangle$ of $\tilde{S}_n$ is of p-power degree > 1 if and only if $p = 2$, and $\lambda = (n)$ (i.e. the character is a basic spin character, and then $\langle n\rangle(1) = 2^{[\frac{n-1}{2}]}$) or $n = 2 + 2^a$ for some $a \in \mathbb{N}$, and $\lambda = (n-1,1)$ (here $\langle n-1,1\rangle(1) = 2^{a+2^{a-1}}$).

This second result is deduced from a result that classifies partitions λ of prime power skew degree $g_\lambda =$ the number of shifted standard λ-tableaux.

Apart from combinatorial techniques, the main ingredients in the proofs of these results are the knowledge of degree formulae, information on minimal degrees and new results on the distribution of primes in consecutive integers (as well as computer calculations for 'mid-sized' n). In the talk, an outline of the proof was presented, and it was indicated that some of the methods developed for this proof could also be used for very different problems.

As an application of the classification result for the alternating groups, a conjecture of Huppert was proved, providing a contribution to the proof of his characterisation of the groups $PSL_2(q)$ by the set of their character degrees.

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A first step towards a structural version of Alperin's weight conjecture and Dade's conjecture

R. Boltje

Alperin's weight conjecture and Dade's conjectures state that, for a finite group G and a p-block B of positive defect,

$$\sum_{\sigma \in \mathcal{P}/G} (-1)^{|\sigma|} f(G_\sigma) = 0\,,$$

where $\mathcal{P}$ is the G-set (via conjugation) of chains $\sigma = (P_0 < \cdots < P_n)$ of non-trivial p-subgroups of G (including the empty chain), $|\sigma| = n$ is the length of σ, G_σ the stabilizer of σ, and f is a function that counts certain irreducible characters of G_σ that depend on B. We want to revive the idea that there should exist an exact chain complex C_* with $\dim C_n = \sum_{\sigma\in\mathcal{P}_n/G} f(G_\sigma)$, where $\mathcal{P}_n$ denotes the set of chains of length n. In general, for a sequence $(c_n)_{n\in\mathbb{Z}}$ of nonnegative integers almost all of which are zero, an exact chain complex with $\dim C_n = c_n$ exists if and only if

$$c_n - c_{n+1} + c_{n+2} - + \cdots \geq 0$$

for all $n \in \mathbb{Z}$. Using GAP this condition is verified for the numbers $c_n = \sum_{\sigma\in\mathcal{P}_n/G} f(G_\sigma)$ in many cases, giving more evidence to the above idea. For blocks B with cyclic defect groups such chain complexes exist and have a number of additional nice properties. More generally, this holds if B has abelian defect groups and if Broué's conjecture is true for all subgroups G_σ.

Geometric methods in representation theory of finite reductive groups

C. Bonnafé

Let G be a connected reductive group defined over a finite field of characteristic p and cardinality q, and let $F : G \to G$ be the corresponding Frobenius endomorphism. We are interested in the representation theory of the finite group G^F on an algebraically closed field of any characteristic different from p.

Representations in characteristic 0. We are particularly interested in the computation of the character table of G^F, at least from a theoretical point of view. One of the main conjectures concerning this problem is Lusztig conjecture on character sheaves. More precisely, this conjecture says that the orthogonal basis of the space Class(G^F) of class functions on G^F given by almost characters, and the one given by characteristic functions of character sheaves coincide (up to a diagonal matrix). Thanks to T. Shoji, this conjecture received a positive answer provided that the center of G is connected (and some mild restrictions on p). An important example of a group with non-connected center is the special linear group. We are working on this group since 1993 and we got a series of results which are steps in the way to Lusztig conjecture.

In our most recent work [1], we got a precise version of Digne, Lehrer and Michel's theorem on Lusztig restriction of Gel'fand-Graev characters. This work relies on the deep analysis of the endomorphism algebra of an induced cuspidal character sheaves, particularly whenever the cuspidal character sheaf is supported by the regular unipotent class. As a consequence of this analysis, we get that, for *any* reductive group G, for p good for G, and for q large enough, then Lusztig restriction of a Gel'fand-Graev character is an *explicitly defined* Gel'fand-Graev character. The difference with Digne, Lehrer and Michel's theorem is in the "explicitly defined".

Despite its little difference with Digne, Lehrer and Michel's version, this theorem is necessary for computing without ambiguity the character table of the special linear group. Indeed, in our thesis, we have computed the Lusztig functor in the special linear group with a little imprecision which was due to the fact that, at the time the thesis was written,

only Digne, Lehrer and Michel's theorem was available. Now, this gap has been filled, and the march through Lusztig conjecture should only be a question of time, since only technical questions must be answered (but specialists know that the technic in the special linear group is not a recreative matter).
Along this march, we got some particular results about groups of type A. First, Mackey formula holds in type A without restriction on p or q (*cf.* [2]). Also, the twisting operator has been computed for this type [3], provided that the Frobenius endomorphism acts trivially on the center of G.
More generally, we are interested in reductive groups with non-connected center. The theorem on Gel'fand-Graev characters is an example of a problem occuring only for these groups. Also, we prove Mackey formula in type C without restriction on p or q, and in general, we are now able to show that it holds for every group certainly if $q \geq 13$. However, the methods involved in these demonstrations are not satisfying.

Representations in characteristic $\ell \neq p$. Let K be an algebraic closure of the ℓ-adic field, let R be its valuation ring, and let k be its residue field (k is an algebraic closure of the field with ℓ elements). The theory of representations of the algebras kG^F or RG^F has grown surprisingly in the last two decades, starting with the work of P. Fong and B. Srinivasan, and continued in the nineties by many people. Many fundamental questions on modular representations of finite abstract groups lead to beautiful conjectures involving the geometry associated to the algebraic group G (Deligne-Lusztig varieties, braid groups...). One of the main problems is the relation between the representation theory of a block and its Brauer correspondent. Broué's famous conjecture predicts that they should be derived equivalent whenever the defect group is abelian.
With R. Rouquier, we proved a result providing a reduction argument for all these questions. Let us describe it in a few words. Let s be a semisimple element of the dual group G^{*F^*}, which is of order prime to ℓ. Let L be an F-stable Levi subgroup of a non-necessarily F-stable parabolic subgroup of G, and let $L^* \subset G^*$ be a dual of L. We assume that $s \in L^{*F^*}$. To the pair (L, s) (respectively (G, s)) is associated a sum of blocs $B(L, s)$ (respectively $B(G, s)$) of RL^F (respectively RG^F).

Theorem [4]. *If $C_{G^*}(s) \subset L^*$, then the Lusztig functor R_L^G induces a Morita equivalence between the algebras $B(L, s)$ and $B(G, s)$.*

If the center of G is connected, then this theorem reduces important questions (decomposition matrices, Broué's conjecture...) to the case of isolated blocks.
To prove this theorem, we followed the strategy imagined by M. Broué, relating this Morita equivalence to a geometric question on certain local systems on the Lusztig variety defining the Lusztig functor R_L^G.

References

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The p-blocks of the Mackey algebra

S. Bouc

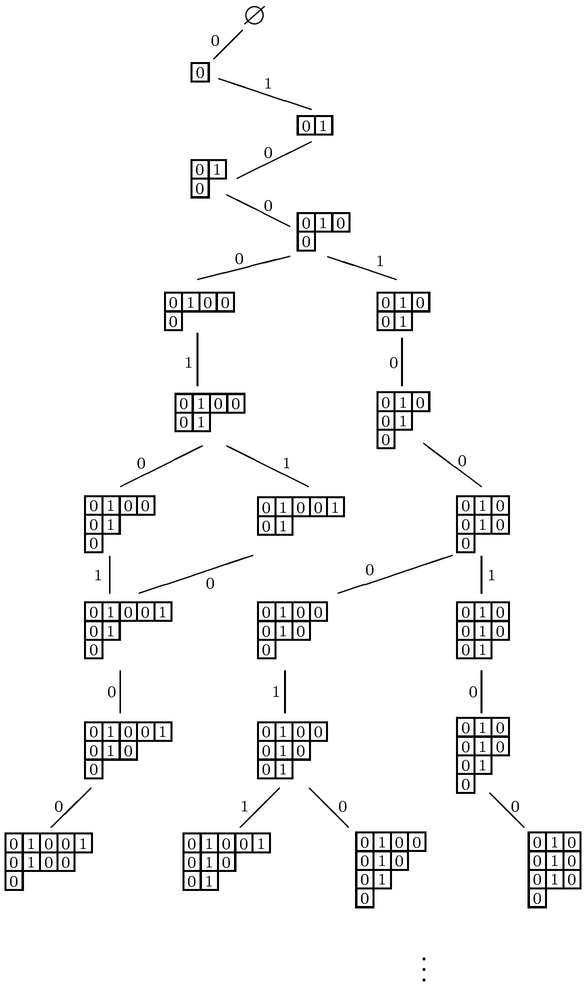
Let p be a prime number, and $\mathcal{O}$ be a complete discrete valuation ring of characteristic 0 with residue field of characteristic p. Let G be a finite group, and denote by $\mu_{\mathcal{O}}(G)$ the Mackey algebra of G over $\mathcal{O}$.
This work states explicit formulae for the block idempotents of $\mu_{\mathcal{O}}(G)$, in terms of the blocks of the group algebra $\mathcal{O}G$, and the $\mathcal{O}$-lifts of their images by Brauer morphisms.
The proof uses the natural ring homomorphism from the crossed Burnside ring $B^c_{\mathcal{O}}(G)$ to the center of the Mackey algebra, and a description of the prime spectrum and block idempotents of $B^c_{\mathcal{O}}(G)$.
Some consequences on Mackey functors can be deduced from these formulae. In particular, one can show that a block b of G and the corresponding block b^{μ} of the Mackey algebra have the same defect groups.

Hecke-Clifford superalgebras, crystals of type $A^{(2)}_{2\ell}$ and modular branching rules for $\widehat{S}_n$

J. Brundan and A. Kleshchev

One of the most exciting developments in type A representation theory in the last few years is the discovery of an intimate connection between the representations of the symmetric group in characteristic p and the highest weight module $V(\Lambda_0)$ of the affine Kac-Moody algebra of type $A^{(1)}_{p-1}$, where Λ_0 is the zeroth fundamental dominant weight of $A^{(1)}_{p-1}$. For example, the modular branching rules for the symmetric group correspond exactly to the crystal graph of this highest weight module, see [K] and [MM] respectively. This coincidence was first observed by Lascoux, Leclerc and Thibon [LLT], who went on to formulate a precise conjecture, later proved (in greater generality) by Ariki [A1], relating the canonical basis coefficients of $V(\Lambda_0)$ to decomposition numbers, not of the symmetric group in characteristic p but rather of the associated Hecke algebra at a pth root of unity over $\mathbb{C}$. More recently, Grojnowski [G] has revealed a purely algebraic way to relate the representation theory of the symmetric group to the highest weight module $V(\Lambda_0)$. For example, the coincidence between the modular branching rules and the crystal graph structure is fully explained in this approach. More generally, there is a family of *cyclotomic Hecke algebras* or *Ariki-Koike algebras* for each dominant integral weight λ of $A^{(1)}_{p-1}$, and the representation theory in this family is intimately related to the highest weight module $V(\lambda)$. To complete the picture here, further work of Ariki [A2] has given a very explicit combinatorial description of the crystal graph of $V(\lambda)$ in terms of certain multipartitions.
Leclerc and Thibon [LT] have also noticed similarities between the fundamental representation $V(\Lambda_0)$ of the *twisted* affine Kac-Moody algebra $A^{(2)}_{p-1}$ and the combinatorics underlying the modular representation theory of the double covers $\widehat{S}_n$ of the symmetric group in odd characteristic p. In particular they conjectured that certain partitions that arise naturally in the Lie theoretic construction of $V(\Lambda_0)$ – so-called *restricted p-strict partitions* – should also label the irreducible representations of $\widehat{S}_n$ in characteristic $p > 2$. A suitable labelling

set before then was only known for $p = 3, 5$ (by work of Andrews, Bessenrodt, Morris and Olsson [ABO, BMO]). In [BK1], we gave a construction valid for all odd p of the irreducibles, fulfilling Leclerc and Thibon's hope regarding the labelling. Our approach there involved Sergeev's superalgebra analogue of Schur-Weyl duality [S], ultimately relating representations of $\widehat{S}_n$ to the supergroup $Q(n)$.
In new work [BK2], reported at the present meeting, we have succeeded in extending the arguments of Grojnowski [G] to the twisted case, replacing the cyclotomic Hecke algebras with new algebras called cyclotomic Hecke-Clifford superalgebras. These are certain finite dimensional quotients of superalgebras introduced in [JN]. Again, there is one family of cyclotomic Hecke-Clifford superalgebras for each dominant integral weight λ of $A_{p-1}^{(2)}$. In particular, we obtain an algebraic construction purely in terms of the representation theory of Hecke-Clifford superalgebras of the plus part $U_{\mathbb{Z}}^+$ of the enveloping algebra of the Kac-Moody algebra $A_{p-1}^{(2)}$, as well as of Kashiwara's highest weight crystals $B(\infty)$ and $B(\lambda)$ for each dominant weight λ. The crystal $B(\lambda)$ gives a natural parametrization of the irreducible representations of the corresponding family of Hecke-Clifford superalgebras, the edges in the crystal graph describe the modular branching rules, etc....



Taking the special case $\lambda = \Lambda_0$ in the main results, we obtain applications for the modular representation theory of the double covers $\widehat{S}_n$ of the symmeric groups. This exploits an explicit combinatorial description of the crystal $B(\Lambda_0)$ discovered recently by Kang [Kan]. In particular, the parametrization of irreducibles, classification of blocks and analogues of the modular branching rules of the symmetric group for the double covers over fields of odd characteristic are deduced. For example, the crystal graph $B(\Lambda_0)$ of $A_2^{(2)}$ is listed above. This underlies the representation theory of $\widehat{S}_n$ in characteristic 3. The vertices of the nth level of the graph parametrize the irreducible (spin) representations of $\widehat{S}_n$, and the edges of the graph describe the irreducible $\widehat{S}_{n-1}$ (resp. $\widehat{S}_{n+1}$) representations that appear in the socle of the restriction (resp. induction) of the corresponding irreducible representation to $\widehat{S}_{n-1}$ (resp. to $\widehat{S}_{n+1}$).

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Endotrivial Modules in Characteristic Two

J.F. Carlson

Let G be a finite group and k a field of characteristic $p > 0$. A finitely generated kG-module M is an endotrivial module if $\mathrm{Hom}_k(M, M) \cong M \otimes M^*$ is the direct sum of a trivial kG-module and a projective kG-module. The tensor product with an endotrivial module induces a self equivalence of the stable category of kG-modules modulo projectives. The endotrivial modules also form the building blocks for the Dade group of endopermutation modules. Throughout this work we assume that the group G is a p-group.

We say that two endotrivial modules M and N are equivalent if $M \oplus P \cong N \oplus Q$ for some projective modules P and Q. The equivalence classes of endotrivial modules form a group under tensor product. A theorem of Dade shows that for G an abelian p-group, the indecomposable endotrivial modules are translates $\Omega^n(k)$ of the trivial module k. The torsion free part of the group is detected on restriction to elementary abelian subgroups of rank two. The rank of the torsion free group has been determined by the work of Alperin. In earlier work, Thevenaz and I showed that the torsion endotrivial modules are detected on extraspecial and almost extraspecial subgroups of the group G. In addition, we were able to eliminate some cases when the prime p is odd. More recently we can show, in the case that $p = 2$, that among extraspecial and almost extraspecial groups only the quaternion group has nontrivial torsion endotrivial modules. With some additional work it can be proved that the group of torsion endotrivial modules is trivial for all 2-groups G which are not quaternion semi-dihedral or cyclic.

The proof involves a large amount of group cohomology. It is obtained by deriving a contradiction based on the dimension of a nontrivial indecomposable torsion endotrivial module if one exists. On the one hand, the dimension of such a module must be bounded from above, in terms of the group cohomology. Recent work has allowed us to sharpen the bound. On the other hand, an analysis of the support varieties of certain submodules of such a module permits the construction of a large torsion endotrivial module whose dimension exceeds the upper bound.

Currently we are attempting to apply the same techniques to the remaining cases for p odd. Some new results have been obtained and there seems to be some promise of a complete solution.

Symmetric groups, wreath products, and canonical bases

J. Chuang

Radha Kessar and I have found p-blocks of the symmetric groups which are Morita equivalent to the principal p-block of the wreath product $S_p \wr S_w$ where $w < p$. This can be regarded as a first step in proving Broué's Abelian defect group conjecture for symmetric groups. To complete a proof, one needs to show that blocks of symmetric groups with isomorphic defect groups are derived equivalent. Jeremy Rickard has a partial solution to this problem, which shows that Broué's conjecture is true, for example, for blocks with defect groups of order less than or equal to p^5.

In another direction Kai Meng Tan and I have used these Morita equivalences to calculate decomposition numbers, radical filtrations of projective modules, and radical and Jantzen filtrations of Specht modules. We also make the connection with canonical bases via LLT's philosophy. Here are the details:

Fix an integer $n \geq 2$ and a nonnegative integer w. Let κ be an n-core with the following property: κ has no addable nodes with n-residue i and at least $w-1$ removable nodes with n-residue i, for each $i = 1, \ldots, n-1$.

Theorem 1 (with Radha Kessar) *Suppose $n = p$ is prime and $w < p$. Then the p-block of the symmetric groups with core κ and weight w is Morita equivalent to the principal p-block of $S_p \wr S_w$.*

Let λ and σ be partitions with n-core κ and n-weight w, and suppose that σ is n-regular. Let $(\lambda^0, \ldots, \lambda^{n-1})$ be the n-quotient of λ and let $(\emptyset, \sigma^1, \ldots, \sigma^{n-1})$ be the n-quotient of σ (the first component is $\emptyset$ because σ is n-regular).
We define a polynomial

$$d_{\lambda\sigma}(q) = \sum_{\substack{\mu^1,\ldots,\mu^{n-1} \\ \nu^0,\ldots,\nu^{n-2}}} \left(\prod_{i=0}^{n-1} c_{\mu^i \nu^i}^{(-1)^i \lambda^i} \right) \left(\prod_{i=1}^{n-1} c_{\nu^{i-1} \mu^i}^{(-1)^i \sigma^i} \right) q^{|\nu|},$$

where in the first product we put $\mu^0 = \emptyset$ and $\nu^{n-1} = \emptyset$, we use a negative sign to indicate the conjugate of a partition, and $|\nu| = |\nu^1| + \cdots + |\nu^{n-1}|$.

Theorem 2 (with Kai Meng Tan)

1. *The $d_{\lambda\sigma}(q)$'s are coefficients of canonical basis elements of the basic representation of $\widehat{sl_n}$ written in terms of the defining basis of the Fock space representation. (Therefore by Ariki's Theorem the $d_{\lambda\sigma}(q)$'s evaluated at $q = 1$ are decomposition numbers for Hecke algebras at complex n-th roots of 1.)*
2. *Suppose $n = p$ and $w < p$. Then the $d_{\lambda\sigma}(q)$'s evaluated at $q = 1$ are decomposition numbers for symmetric groups.*
3. *Suppose $n = p$ and $w < p$. Then for the symmetric groups, the multiplicity of D^σ in the j-th Jantzen layer of S^λ is equal to the coefficient of q^j in $d_{\lambda\sigma}(q)$. Moreover, the Jantzen and radical filtrations of S^λ coincide (up to shift by $|\lambda^0|$).*

Characters and Hecke algebras of type A

S. Donkin

Let $H(r)$ be the type A Hecke algebra of degree r over a field k, with parameter $q \in k$. This paper was motivated by a conjecture of Andrew Mathas asserting that for q a primitive lth root of 1, the determinant of the Cartan matrix of $H(r)$ is a power of l. We prove here that the determinant is a positive divisor of a power of l. The result is analogous to the well known result that the determinant of the Cartan matrix of the group algebra kG of a finite group G over a field of characteristic p is a power of p. In fact this is much more than an analogy. Our method is to associate to each finite dimensional projective $H(r)$-module a character of the symmetric group then study some properties of the Cartan matrix using character theory exactly as in the general finite group case. An amusing feature is that we, in effect, study the modular character theory of symmetric groups, but with the characteristic of the modular field in Brauer's theory replaced by an arbitrary positive integer l.

On a minimal counterexample to Dade's projective conjecture

C. Eaton

Dade's projective conjecture ([2]) states that for any p-block B with non-normal defect groups of a finite group G with $O_p(G) \leq Z(G)$, and any $\lambda \in \mathrm{Irr}\,(O_p(Z(G)))$ and $d \in \mathbb{N}_0$, we should have

$$\sum_{\sigma \in \mathcal{C}(G|O_p(G))/G} (-1)^{|\sigma|} k_d(G_\sigma, B, \lambda) = 0.$$

Here $\mathcal{C}(G|O_p(G))$ is the set of chains

$$O_p(G) = Q_0 < \cdots < Q_n$$

of p-subgroups of G, and $|\sigma| = n$. We let G act on the chains by term-wise conjugation, and $\mathcal{C}(G|O_p(G))/G$ is a set of G-conjugacy class representatives, G_σ is the chain stabilizer. $k_d(G_\sigma, B, \lambda)$ is the number of irreducible characters of G_σ belonging to Brauer correspondents of B for which $\chi(1)_p p^d = |G|_p$ and $(\chi, \lambda^G) \neq 0$.
This is a refinement of the Knörr-Robinson reformulation ([3]) of Alperin's weight conjecture ([1]), and as such implies the weight conjecture.

Robinson has demonstrated that this conjecture holds for p-solvable groups ([4]), which involves showing that a minimal counterexample has $O_{p'}(G) \leq Z(G)$. This means that in a minimal counterexample the generalized Fitting subgroup $F^\star(G)$ is a central product of $Z(G)$ and the components (quasi-simple subnormal subgroups) of G. This suggests two problems concerning the structure of a minimal counterexample:

(I) Show that G permutes the components transitively,

(II) Show that the components are normal.

If both these problems are solved then it follows that a minimal counterexample is a covering group of an automorphism group of a simple group.

• The author together with Geoffrey Robinson have successfully completed problem (I). This uses recent results of Robinson ([5]) as well as Clifford-theoretic techniques analogous to the Fong correspondences.

• The author and Burkhard Höfling have studied the special case of problem (II) where $G = H \wr S_n$ for some H and n. A stronger result is proved that if Dade's projective conjecture holds for H then it also holds for G. It may be hoped that this will indicate how a solution to problem (II) may proceed.

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Modular Representations of the Affine Hecke Algebra

J. Graham

(joint work with G. Lusztig, A. Cox and P. Martin)

Let k be an algebraically closed field and $q \in k^\times$. Let $\hat{H}_n$ denote the affine Hecke algebra associated to the general linear group GL_n. Bernstein and Zelevinski and others have defined "standard modules" for $\hat{H}_n$. Given semisimple $S \in \mathrm{GL}_n(k)$ and nilpotent matrix $N \in \mathrm{M}_n(k)$ such that $N^S = q^2N$, one may construct $\hat{H}_n$-module $V_{S,N}$. An important problem is to determine the multiplicity of irreducibles in $V_{S,N}$. When k is the complex numbers this is well understood. If q is a root of unity the question reduces to determining the decomposition matrix of the Ariki-Koike Hecke algebra for the parameters which Ariki treats in [*On the decomposition numbers of the Hecke algebra of* $G(m,1,n)$, 1996]. We determine the decomposition numbers in arbitrary characteristic and arbitrary q for $V_{S,N}$ when N has (at most) two blocks.

Units of p-power order in principal blocks of p-constrained groups

M. Hertweck

The result presented in my talk is related to work of Roggenkamp and Scott (on automorphisms of the principal p-block of a p-constrained group), and depends again on Weiss' results on p-permutation modules.
Let G be a finite group which has a normal p-subgroup N with $\mathrm{C}_G(N) \leq N$, and let R be a p-adic ring. The group of units in RG of augmentation 1 is denoted by $\mathrm{V}(RG)$.

Theorem A. *Any finite p-group in $\mathrm{V}(RG)$ which normalizes N is conjugate to a subgroup of G in the units of RG.*

Theorem B. *Any finite p-group in $\mathrm{V}(RG)$ which centralizes N is contained in N.*

Corollary. *If G is p-constrained with $\mathrm{O}_{p'}(G) = 1$, then any central unit of finite p-power* *order in* $\mathrm{V}(RG)$ *is trivial, i.e. contained in* $\mathrm{Z}(G)$.

Some special arguments are needed in the $p = 2$ case. The following result is used, which holds for any finite group G and a 2-adic ring R. If $u \in \mathrm{V}(RG)$ with $u^2 = 1$ and $u \in 1+2(2RG+R[T_2]+R)$, where $R[T_2]$ denotes the R-linear span of the set of involutions of G, then $u = 1$.

Morita Equivalent Blocks of Finite General Linear Groups in Non-defining Characteristic

A. Hida

(joint work with H. Miyachi)

Let kG be the group algebra of a finite group G over an algebraically closed field k of characteristic $l > 0$. Let S_n be the symmetric group of degree n. Chuang and Kessar defined a certain l-core ρ and proved the following.

Theorem 1 ([2]) *Suppose that* $w < l$. *Let* $B_{\rho,w}$ *be the block of* kS_n *with* l*-core* ρ *and weight* w, *where* $n = |\rho| + lw$. *Then* $B_{\rho,w}$ *and the principal block of* $k(S_l \wr S_w)$ *are Morita equivalent.*

Corollary 2 ([1]) *Suppose that* $2 < l$. *Broué's conjecture is true for blocks of symmetric groups of weight 2. Namely, let* B *be a block of symmtric group with weight* 2, *then* B *and the Brauer correspondent of* B *are derived equivalent.*

We will show that similar results hold for unipotent blocks of a general linear group $GL_n(q)$ where $l \nmid q$. Let $e = e(q)$ be the multiplicative order of q in k. Let $B_{\rho,w}(q)$ be the unipotent block of $kGL_n(q)$ with e-core ρ and weight w. Let $H(q) = GL_{e(q)}(q) \wr S_w$.

Theorem 3 *Suppose that* $w < l$. *Then* $B_{\rho,w}(q)$ *and the principal block* $B_0(kH(q))$ *of* $kH(q)$ *are Morita equivalent.*

Corollary 4 *Broué's conjecture is true for unipotent blocks of finite general linear groups of weight* 2 *in non-defining characteristic.*

Let $r(q)$ be the largest integer r such that $l^r | q^{e(q)} - 1$. Let q' be a power of a prime. If $e = e(q) = e(q')$ and $r(q) = r(q')$, then $B_0(kH(q))$ and $B_0(kH(q'))$ are Morita equivalent. Let τ be an arbitrary e-core. Then we have the following result.

Theorem 5 *If* $e(q) = e(q')$, $r(q) = r(q')$, $w \leq 5$ *and* $w < l$, *then* $B_{\tau,w}(q)$ *and* $B_{\tau,w}(q')$ *are Morita equivalent.*

Remark. In [3], W.Turner proved similar results.

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Imprimitive irreducible representations of finite quasi-simple groups

G. Hiss

(joint work with W. Husen and K. Magaard)

An irreducible representation of a finite group G on a vector space V is *imprimitive*, if it is induced from a proper subgroup of G. This is equivalent to the statement that there is a direct decomposition

$$V = \bigoplus_{i=1}^{m} V_i$$

of V with $m > 1$ such that the summands V_i are permuted by the action of G.
In joint work with William Husen and Kay Magaard we have determined the ordinary irreducible imprimitive representations of the finite quasi-simple groups. Somewhat surprisingly, the proportion of imprimitive representations is rather large. For example, if $G = SL_n(q)$, the number of ordinary irreducible representation of G is of the form

$$q^{n-1} + \text{lower terms in } q.$$

Among these,

$$\left(1 - \frac{1}{n}\right) q^{n-1} + \text{ lower terms in } q$$

are imprimitive.
Our work is motivated by the program of describing the maximal subgroups of linear groups such as $GL(V)$. Aschbacher's approach divides the general problem into various partial problems, one of which is the following: When is the normalizer of an absolutely irreducible quasi-simple subgroup G of $GL(V)$ maximal? In general this is not the case when the embedding of G into $GL(V)$ is imprimitive.

Hochschild Cohomology of Tame Blocks

T. Holm

To any associative algebra Λ one can associate the Hochschild cohomology ring $HH^*(\Lambda) := \oplus_{i\geq 0} Ext^i_{\Lambda\otimes\Lambda^{op}}(\Lambda,\Lambda)$ (multiplication given by Yoneda product). For group algebras or blocks a complete description of the ring structure seems to be known only in few cases (e.g. abelian groups, blocks with cyclic defect groups).
In the talk we discuss Hochschild cohomology groups for blocks of tame representation type (i.e., with defect groups dihedral, semidihedral or generalized quaternion). Such

blocks have at most three simple modules. For blocks with one or three simple modules we present a complete answer for the additive structure of Hochschild cohomology. This relies on our classification of algebras of dihedral, semidihedral and quaternion type up to derived equivalence. Since Hochschild cohomology is invariant under derived equivalence it suffices for the general case to consider one particular block in each derived equivalence class. Standard examples are provided by blocks of linear groups like GL, SL, PSL and extensions of these, for which then explicit calculations are carried out.

On blocks with symmetric stable center

R. Kessar

Let G be a finite group, k an algebraically closed field of prime characteristic p, and b a block of kG. The stable center, $\bar{Z}(kGb) =: Tr_1^G(kGb)$ of kGb is an invariant of the Morita-stable category of kGb., i.e. a stable equivalence of Morita type between two block algebras induces an isomorphism between the corresponding stable centers. We study $\bar{Z}(kGb)$ under the assumption that it is a symmetric algebra and we show that when b is the principal block of kG, then $\bar{Z}(kGb)$ is symmetric if and only if the Sylow p-subgroups of G are abelian and the centralisers of non-identity p-elements of G are p-nilpotent. We also obtain necessary and sufficient conditions in the general case. These results extend earlier work of Okuyama and Tsushima who showed that the center $Z(kGb)$ of kGb is a symmetric algebra if and only if b is a nilpotent block with abelian defect groups.
This is joint work with Markus Linckelmann.

Blocks of the Cyclotomic Hecke Algebras of $G(d, 1, r)$ and $G(d, d, r)$

S. Kim

(joint work with M. Broué)

In the work of G. Lusztig on the irreducible characters of the reductive groups over the finite fields, the notion of "family of characters" of the corresponding Weyl groups, plays a fundamental role. Lusztig defines the partition into families of the set of irreducible characters of a Weyl group W by using the asymptotic based ring associated to the Kazhdan–Lusztig basis of the Hecke Algebra of W.

Recently, we have interest in generalizing the notion of family of characters to the complex reflection groups , or more precisely to various types of Hecke algebras associated to the complex reflection groups.
The principal obstacle for this generalisation is that we don't have (or not yet ?) the Kazhdan–Lusztig basis for the complex reflection groups (not of Coxeter). However, the results of Gyoja , completed by a recent work of R. Rouquier , gives a substitute for the definition of families.
Rouquier shows in fact that the families of $Irr(W)$, W a Weyl group, are just the blocks of $Irr(H(W))$, $H(W)$ the classical Hecke algebra, over an appropriate ring. This definition can be generated easily to all the cyclotomic algebras of complex reflection groups and this definition is the one we refer and use in our work.

For the "Ariki–Koike" cyclotomic Hecke algebras, we obtain in particular, thanks to a conjecture of Graham and Lehrer which is proven by Grojnowski , the generalization of the Lustig families of the spetsial algebras.
More generally, we treat here the case of arbitrary "cyclotomic algebra" – which covers in particular the known cases (for the Weyl groups) under the name of "Hecke algebra with unequal parameters".

Theorem for $G(d,1,r)$: *Two irreducible characters ξ_λ, ξ_μ are in the same family if and only if the corresponding symbols, λ, μ have the same contents (that is, same entries with the same multiplicities, possibly arranged in different rows). Here, the shape of the symbols correponds to the chosen variables for the Hecke algebra of the group $G(d,1,r)$.*

The complex reflection groups $G(d,d,r)$ may be viewed as generalizing the Weyl group of type D_r (which are the groups $G(2,2,r)$), and the dihedral groups ($G(d,d,2)$ is the dihedral group of order $2d$). We determine the "families" (Rouquier blocks) of the cyclotomic algebras of the groups $G(d,d,r)$.
We also obtain, in particular by considering the case of algebras called "spetsial", the generalization of the classification of Lusztig of the families of the Weyl group of type D_r (which appears here as a simple application of the Clifford theory of blocks), as well as a new proof of the classification of the families of the dihedral groups obtained "experimentally" by Lusztig , and then by Malle and Rouquier by direct approach on the "Rouquier blocks".

Theorem for $G(d,d,r)$: *Two irreducible characters ξ_λ, ξ_μ are in the same family if and only if the corresponding symbols, in rectangular shape, λ, μ have the same contents (that is, same entries with the same multiplicities, possibly arranged in different rows).*

Broué's conjecture for the 3-dimensional special unitary groups

N. Kunugi

Let G be a finite group. Let $(K, \mathcal{O}, k)$ be a splitting p-modular system for G. We denote by $B_0(G)$ the principal block of $\mathcal{O}G$. We consider the following well-known conjecture.

Conjecture (Broué) Let G be a finite group with abelian Sylow p-subgroup P. Then the principal blocks $B_0(G)$ and $B_0(N_G(P))$ are splendid derived equivalent.

Let $H = N_G(P)$. The following is an important approach to show existence of derived equivalence between $B_0(G)$ and $B_0(H)$.
(I) Construct a stable equivalence between $B_0(G)$ and $B_0(H)$ from splendid equivalences between $B_0(C_G(Q))$ and $B_0(C_H(Q))$ for all non-trivial subgroups Q of P.
(II) Lift it to a splendid derived equivalence between $B_0(G)$ and $B_0(H)$.
Let M be the Green correspondent of $B_0(G)$ with respect to $(G \times G, \Delta(P), G \times H)$. The bimodule M does not induce a stable equivalence in general. However known examples of derived equivalent blocks which are constructed from a stable equivalence are almost constructed from a stable equivalence by M.
We show the following as an example of derived equivalences constructed from a stable equivalence not coincides the Green correspondence.

Theorem (Kunugi-Waki) Let $G = SU(3, q^2)$ and let $(K, \mathcal{O}, k)$ be a splitting r-modular system for G where $r \geq 5$. We assume that r divides $q+1$. Let P be a Sylow r-subgroup of G (Then $P \cong C_{r^a} \times C_{r^a}$ where r^a is the r-part of $q+1$). Then the principal blocks $B_0(G)$ and $B_0(N_G(P))$ are splendidly derived equivalent.

On spaces and spectra associated with p-blocks

M. Linckelmann

It seems to be a general intuition that there should be associated with a p-block b of a finite group G a topological space, whose meaning for the block should be analogous to what is the classifying space for a group.

In common work with Peter Webb, inspired by results and methods due to Benson-Feshbach and Martino-Priddy (who determine the decomposition of classifying spaces of finite groups in the category of p-complete spectra), we associate with any such p-block a canonical p-complete spectrum $\hat{B}(G, b)$, which appears as a summand of the classifying space BP_+ of a defect group P (viewed as p-complete spectrum). If b is the principal p-block of G then $\hat{B}(G, b)$ coincides with BG_+. It is an open question, whether there is in general a topological space $B(G, b)$ whose p-completion is $\hat{B}(G, b)$. The answer is positive, if $N_G(P)$ controls fusion in b (which is the case whenever P is abelian), and in that case, $B(G, b)$ is just the space $B(P \rtimes E)$, where E is the inertial quotient of b.

Some very recent work of Broto-Levi-Oliver (2000, 2001) indicates the direction, where one might find such spaces: their work considers certain extensions of the local category of b by the functor sending a b-centric subgroup Q of P to its center $Z(Q)$. Under some strong technical hypotheses, we construct such categories. Even though at this point their existence for an arbitrary block is still open, this seems to be an encourageing perspective.

Markov traces and generic degrees

G. Malle

We report on joint work with Meinolf Geck and Lacrimioara Iancu. Let $\mathcal{H}_n$ be the cyclotomic Hecke algebra of the imprimitive complex reflection group $G(r, 1, n)$ over the ring $A = \mathbb{Z}[q^{\pm 1}, u_1^{\pm 1}, \ldots, u_r^{\pm 1}]$. In joint work with Kirsten Bremke and Andrew Mathas it was shown that $\mathcal{H}_n$ is a symmetric algebra over A. Moreover, the symmetrizing trace τ_0 is uniquely determined once an additional symmetry property is assumed, by a result obtained jointly with Broué and Michel. Being a trace form, τ_0 can be decomposed as a linear combination of irreducible characters of $\mathcal{H}_n$ over a splitting field, and the coefficients in this decomposition are called the generic degrees of $\mathcal{H}_n$.
Conjecturally, the generic degrees play an important role in the representation theory of groups of Lie type, for example they should specialize to degrees of unipotent characters of classical groups. Moreover, they should have extensions to sets of so-called unipotent degrees. An explicit knowledge of the generic degrees seems desirable for various applications, for example the determination of "families" (blocks of $\mathcal{H}_n$) for complex reflection groups, as described in the talk by Sungsoon Kim at this conference.

A conjecture on the values of the generic degrees was presented at the Oberwolfach conference in April 1996. This conjecture has now been proved by us, and independently by Andrew Mathas (see his talk). In our approach, we are able to link the generic degrees to weights of Markov traces on $\mathcal{H}_n$. Our proof consists of the following three steps. First, we determine the weights of Markov traces on $\mathcal{H}_n$ for a rather restricted set of parameters. This step follows an idea of Orellana from the case $r = 2$, using a family of homomorphisms from the braid group of type B_n to braid groups of type A_{n+f}. Secondly, by a density argument and explicit computation, we are able to deduce the weights for arbitrary Markov traces. In the case $r = 2$, this had previously been achieved by Iancu. Finally, we verify that τ_0 is a Markov trace on $\mathcal{H}_n$ with respect to explicitly known values of the parameters. Thus, the generic degrees are obtained as specializations of the weights determined before. By previous work of mine, this completes the determination of generic degrees of cyclotomic Hecke algebras for all finite complex reflection groups. The explicit results exhibit some striking and yet unexplained properties of suitable specializations. For example, for given group W, all generic degrees are integral if and only if they all lie in the field of fractions of A. Reflection groups for which this happens are called *spetsial*.

Graded Derived Equivalences

A. Marcus

Let K be a normal subgroup of the finite group H, and regard the group algebra $\mathcal{O}H$ as a G-graded algebra, where $(\mathcal{K}, \mathcal{O}, k)$ is a splitting p-modular system and $G = H/K$. Let b be a G-invariant block of $\mathcal{O}K$ with defect group D, and let b' be the Brauer correspondent of b in $K' = N_K(D)$. The Brauer correspondence induces a bijection $c \leftrightarrow c'$, called the Harris-Knörr correspondence, between blocks of H lying over b and blocks of H' lying over b'.

An interesting question is whether the existence of a Rickard equivalence between $b\mathcal{O}K$ and $b\mathcal{O}K'$ implies that $c\mathcal{O}H$ and $c'\mathcal{O}H'$ are Rickard equivalent.

We approach this problem by means of graded derived equivalences. We prove the following graded version of Rickard's theorem:

If R and S are two G-graded algebras, then the following statements are equivalent.

(i) *There is a G-graded tilting complex $T \in \mathcal{D}(R\text{-Gr})$ and an isomorphism of G-graded algebras $S \to \mathrm{End}_{\mathcal{D}(R)}(T)^{\mathrm{op}}$.*

(ii) *There is a complex X of G-graded (R, S)-bimodules such that $X\overset{\mathbf{L}}{\otimes}_S - : \mathcal{D}(S) \to \mathcal{D}(R)$ is an equivalence.*

(iii) *There is a triangle equivalence between $\mathcal{D}(S)$ and $\mathcal{D}(R)$ preserving graded objects, which is compatible with grade-forgetting and conjugation.*

An extension of Broué's conjecture states that if D is abelian and G is a p'-group, then $b\mathcal{O}H$ and $b'\mathcal{O}H'$ are graded derived equivalent. Relying on constructions due to Rouquier and Okuyama, we show that this conjecture holds in the following cases: D is cyclic; $D \simeq C_3 \times C_3$ and K is simple; $K = \mathrm{SL}_2(p^n)$.

Finally, there are implications towards Dade's inductive conjecture, since a graded equivalence preserves the relevant Clifford theoretical invariants.

Generic degrees for Ariki-Koike algebras

A. Mathas

The Ariki-Koike algebra $H_{r,n}$ is a deformation of the group algebra of $W_{r,n} := (\mathbb{Z}/r\mathbb{Z}) \wr \mathfrak{S}_n$, the wreath product of a cyclic group of order r with a symmetric group of order n. If $r = 1$ or $r = 2$ then $W_{r,n}$ is a Weyl group and $H_{r,n}$ is an Iwahori–Hecke algebras. The Iwahori–Hecke algebras are intimately connected with the representation theory of the finite groups of Lie type. When $r > 2$ then $W_{r,n}$ is a complex reflection group and $H_{r,n}$ is a cyclotomic Hecke algebra (in the sense of Broué and Malle) and *conjecturally* the representation theory of $H_{r,n}$ still plays an important role in the representation theory of the finite groups of Lie type. In particular, when $H_{r,n}$ is semisimple then there is a polynomial D_χ associated to each irreducible character of $H_{r,n}$ and certain specializations of this polynomial should compute the dimensions of irreducible representations of certain finite groups of Lie type. The polynomials D_χ are called the *generic degrees* of $H_{r,n}$ and formulae for these polynomials were conjectured by Malle.

In my talk I showed one way to compute the generic degrees. The idea is to first write down a Wedderburn basis of $H_{r,n}$; this can be done explicitly (in principle) by a cunning use of the Murphy operators in $H_{r,n}$. In particular, this gives a description of the primitive idempotents in $H_{r,n}$; the generic degrees are given by the values of a trace form on these idempotents.

The generic degrees were also computed by Geck, Iancu and Malle.

The centre of the group algebra of a Symmetric group

J. Murray

Suppose that n is a positive integer and that k is a perfect field of characteristic $p > 0$. Let Z denote the centre of the group algebra of the symmetric group S_n over k. So Z has as k-basis the class sums of the conjugacy classes of S_n. As is well known, the conjugacy classes of S_n are in bijection with the partitions of n.

Suppose that g is an element of S_n, and D is a p-defect group of g. We show that $C_{S_n}(D) =< g_p > \times N$, for some group N, where g_p is the p-part of g. This has a number of consequences for the structure of the k-algebra Z, and the centres of the p-blocks of S_n. Let Z_r denote the span of the p-regular class sums of S_n, a k-subspace of Z. Then one consequence is that Z_r forms a subalgebra of Z. Let $\phi : Z \to Z$ be the Frobenius map $\phi(z) = z^p$, for each $z \in Z$. So ϕ is a semi-linear transformation on Z, with respect to the Frobenius automorphism of k. Another consequence is that the image of ϕ is contained in Z_r. Let $\Omega^+ = \sum\{g \in S_n \mid g^p = 1\}$ in kS_n. Then it follows that if $z \in Z$, then $z^p = 0 \iff z\Omega^+ = 0$. This last equality may in fact hold in the centre of the group algebra of an arbitrary finite group.

Using the fact that Z is the ring of symmetric polynomials over k in the Jucys-Murphy elements, we show that the image of ϕ coincides with Z_r if $p = 2$. This means that, when k has characteristic 2, the squares of the elements of Z are precisely the k-linear combinations of the 2-regular class sums. Let B be a 2-block of S_n, of weight w. M. Enguehard has shown that the isomorphism type of the centre $Z(B)$ of B is determined by w. We show

that the subspace of squares in $Z(B)$ has dimension equal to the number of partitions of w.
Finally we give some lower bounds for the nilpotency of the radical $J(ZB)$ of $Z(B)$, when B is a p-block of S_n. A result of B. Külshammer shows that $z^{p^t} = 1$ for each $z \in J(ZB)$, where p^t is the exponent of a defect group of B. We show that there exists $z \in J(ZB)$ for which $z^{p^{t-1}} \neq 0$.

Clifford groups

G. Nebe

The Clifford group $\mathcal{C}_m$ is defined as the full normalizer of the extraspecial group $E(m) \cong 2_+^{1+2m} \cong D_8\mathrm{Y}\ldots\mathrm{Y}D_8$ in $O(2^m, \mathcal{R})$. This group comes up in connection with many different topics in discrete mathematics. Its ring of invariants is spanned by the complete weight enumerators of (scalar extensions) of binary selfdual codes. There are similar results for $p > 2$ and more general rings. In the talk I will give a numbertheoretic proof of the fact that $\mathcal{C}_m$ (for $m \geq 2$) is a maximal finite subgroup of $GL(2^m, \mathcal{R})$.

On the Brauer-Glauberman correspondence

L. Puig

Let $\mathcal{R}$ be a *Dedekind ring* with a field of quotients $\mathcal{K}$ of characterisitic zero and G a finite group; recall that the $\mathcal{R}$-module of central $\mathcal{R}$-linear forms $Z^\circ(\mathcal{R}G)$ over $\mathcal{R}G$ is a free $Z(\mathcal{R}G)$-module of rang 1 ; consider the subalgebra $Z_{\mathrm{id}}(\mathcal{R}G)$ generated by all the idempotents of $Z(\mathcal{R}G)$. Let π be a set of primes such that $p\mathcal{R} \neq \mathcal{R}$ for any $p \in \pi$; following Brauer, consider the *graded* $\mathcal{R}$-module structure of $Z^\circ(\mathcal{R}G)$ determined by the π-elements of G (i.e. the central $\mathcal{R}$-valued functions defined over the π-*sections* in Brauer's terms)

$$Z^\circ(\mathcal{R}G) = \bigoplus_{u\in U} Z_u^\pi(\mathcal{R}G)$$

where U is a set of representatives for the G-conjugacy classes of π-elements of G , and, for any $u \in U$, respectively denote by

$$\mathfrak{d}_u^\pi \colon Z^\circ(\mathcal{R}G) \to Z_u^\pi(\mathcal{R}G) \quad \text{and} \quad \mathfrak{m}_u^\pi : Z_u^\pi(\mathcal{R}G) \cong Z_u^\pi\big(\mathcal{R}C_G(u)\big)$$

the canonical map and the $\mathcal{R}$-module isomorphism determined by the restriction. Broué's reformulation of the *Brauer Second Main Theorem* finally leads to the following result.

Proposition. *For any π-element u of G , $Z_u^\pi(\mathcal{R}G)$ is a $Z_{\mathrm{id}}(\mathcal{R}G)$-submodule of $Z^\circ(\mathcal{R}G)$, and there is a unique $\mathcal{R}$-algebra homomorphism*

$$\mathfrak{Br}_u^G : Z_{\mathrm{id}}(\mathcal{R}G) \longrightarrow Z_{\mathrm{id}}\big(\mathcal{R}C_G(u)\big)$$

such that $\mathfrak{m}_u^\pi : Z_u^\pi(\mathcal{R}G) \cong \mathrm{Res}_{\mathfrak{Br}_u^G}\Big(Z_u^\pi\big(\mathcal{R}C_G(u)\big)\Big)$ becomes a $Z_{\mathrm{id}}(\mathcal{R}G)$-module isomorphism. Moreover, if $u = vw = wv$ where the orders of v and w are relatively prime, we have

$$\mathfrak{Br}_u^G = \mathfrak{Br}_w^{C_G(v)} \circ \mathfrak{Br}_v^G \qquad .$$

With this result, we are ready to state the existence and uniqueness of the *Brauer-Glauberman correspondence*. Let H be a finite π-group and consider the category formed by the finite groups endowed with an H-action, called *H-groups,* and by the group homomorphisms $\varphi\colon G \to G'$ compatible with the action of H and inducing a bijection $G_{\pi'} \to G'_{\pi'}$, where $G_{\pi'}$ and $G'_{\pi'}$ denote the corresponding sets of π'-elements; moreover, denote by G^H and ${G'}^H$ the corresponding subgroups of H-fixed elements and by $\varphi^H\colon G^H \to {G'}^H$ the induced group homomorphism, which still induces a bijection $(G^H)_{\pi'} \to ({G'}^H)_{\pi'}$. Now, we have two functors from this category to the category of commutative $\mathcal{R}$-algebras, respectively mapping G on $Z_{\mathrm{id}}(\mathcal{R}G)$ and $Z_{\mathrm{id}}\big(\mathcal{R}G^H\big)$, and φ on the obvious $\mathcal{R}$-algebra homomorphisms

$$Z_{\mathrm{id}}(\varphi) : Z_{\mathrm{id}}(\mathcal{R}G') \to Z_{\mathrm{id}}(\mathcal{R}G) \quad \text{and} \quad Z_{\mathrm{id}}(\varphi^H) : Z_{\mathrm{id}}(\mathcal{R}{G'}^H) \to Z_{\mathrm{id}}(\mathcal{R}G^H) \quad ,$$

and we will exhibit a natural map $\mathfrak{Gl}_H$ between them . Note that if K is a normal subgroup of H then G is also a K-group and G^K becomes a H/K-group.

Theorem. *There is a unique correspondence mapping any finite π-solvable group H and any H-group G on a G-natural map*

$$\mathfrak{Gl}_H^G\colon Z_{\mathrm{id}}(\mathcal{R}G) \longrightarrow Z_{\mathrm{id}}(\mathcal{R}G^H)$$

such that $\mathfrak{Gl}_{<u>}^G = \mathfrak{Br}_u^G$ for any π-element u of G and $\mathfrak{Gl}_L^{G^K} \circ \mathfrak{Gl}_K^G = \mathfrak{Gl}_H^G$ for any exact sequence $1 \to K \to H \to L \to 1$ of finite π-solvable groups. Moreover, $\mathfrak{Gl}_H^G(b) = 0$ for any primitive idempotent b in $Z_{\mathrm{id}}(\mathcal{R}G)$ not fixed by H .

Corollary. *Let H be a finite group and G an H-group . There is a unique correspondence mapping any solvable π-subgroup K of H and any subnormal subgroup L of K on an $\mathcal{R}$-algebra homomorphism*

$$\mathfrak{Ro}_L^K : Z_{\mathrm{id}}(\mathcal{R}G^L) \longrightarrow Z_{\mathrm{id}}(\mathcal{R}G^K)$$

such that $\mathfrak{Ro}_L^K = \mathfrak{Gl}_{G^L}^{K/L}$ whenever $L \lhd K$ and that, for any subnormal subgroup M of L , we have $\mathfrak{Ro}_L^K \circ \mathfrak{Ro}_M^L = \mathfrak{Ro}_M^K$.

Remark. Geoffrey Robinson has already proved the existence of this $\mathcal{R}$-algebra homomorphism whenever K is nilpotent. As in his case, this existence allows to consider a *subnormal inclusion* between the *Brauer π-pairs* over $\mathcal{R}G$ which extends the ordinary inclusion between Brauer p-pairs.

Classification of graded Hecke algebras for complex reflection groups

A. Ram

(joint work with A. Shepler)

The graded Hecke algebra for a finite Weyl group is intimately related to the geometry of the Springer correspondence. A construction of Drinfeld produces an analogue of a graded Hecke algebra for any finite subgroup of $GL(V)$. This paper classifies all the algebras obtained by applying Drinfeld's construction to complex reflection groups. By giving explicit (though nontrivial) isomorphisms we show that the graded Hecke algebras for finite real reflection groups constructed in a different way by Lusztig are all isomorphic to algebras obtained by Drinfeld's construction. The classification shows that, even for real reflection groups, there are algebras obtained from Drinfeld's construction which are not graded Hecke algebras as defined by Lusztig.

Some examples of derived equivalence (work of Miles Holloway)

J. Rickard

I want to talk about some applications by my Ph.D. student, Miles Holloway, of the following theorem of mine to verifying some cases of Broué's Abelian Defect Group Conjecture.

Theorem 1 *Let A be a symmetric algebra over a field k (e.g., a block algebra), and suppose that $X_1, \ldots, X_n$ are objects of the derived category $D^b(\mathrm{mod}(A))$ such that*
(i) $\mathrm{Hom}(X_i, X_j[t]) = 0$ *for* $t < 0$,
(ii) $\mathrm{Hom}(X_i, X_j) = \begin{cases} k & \text{if } i = j \\ 0 & \text{otherwise,} \end{cases}$ *and*
(iii) $X_1, \ldots, X_n$ *generate* $D^b(\mathrm{mod}(A))$ *as a triangulated category.*
Then there is a symmetric k-algebra C and an equivalence

$$D^b(\mathrm{mod}(A)) \longrightarrow D^b(\mathrm{mod}(C))$$

taking $X_1, \ldots, X_n$ to the simple C-modules.

Linckelmann's theorem (stating that a stable equivalence of Morita type between two blocks that takes simple modules to simple modules is essentially a Morita equivalence) has been much exploited recently by Okuyama and others to lift stable equivalences of Morita type to equivalences of derived categories. The theorem above gives the following new method of exploiting Linckelmann's theorem.

- Start with a stable equivalence $\underline{\mathrm{mod}}(B) \approx \underline{\mathrm{mod}}(A)$ of Morita type, sending the simple B-modules to $Y_1, \ldots, Y_n$.
- Find objects $X_1, \ldots, X_n$ of $D^b(A)$, satisfying conditions (i), (ii) and (iii) of the theorem, such that X_i is isomorphic in $\underline{\mathrm{mod}}(A)$ to Y_i for $i = 1, \ldots, n$.

If these steps can be carried out, then the derived categories of A and B are equivalent as triangulated categories.
Since, apart from finding the images $Y_1, \ldots, Y_n$ of the simple B-modules under the stable equivalence, all the calculation involved in this procedure involves only A-modules, the computation involved in carrying it out is likely to be manageable when one of the algebras involved (namely A) is relatively small, even if the other algebra is large. This tends to be the case in examples of Broué's Conjecture, where A is a block of a local subgroup.
Using the computer algebra system Magma, Holloway has carried out this procedure for various cases of Broué's Conjecture larger than any of the previously known cases that did not fit into infinite families.
For example,

Theorem 2 (Holloway) *Broué's Abelian Defect Group Conjecture is true in characteristic 5 for*

- *the principal block of the Hall-Janko sporadic group J_2,*
- *the non-principal block of full defect of the double cover $2.J_2$ of J_2,*
- *the principal block of the symplectic group $Sp_4(4)$.*

In other words, these blocks all have derived categories equivalent to those of their Brauer correspondents.

In all of these cases the defect group of the block is elementary abelian of order 25. The fact that there is a stable equivalence of Morita type for such blocks is a special case of a theorem of Rouquier.
In fact, the method of proof makes it clear that in all of these cases the equivalence of derived categories is actually a 'splendid' equivalence, which implies in particular that it lifts from k to a complete discrete valuation ring $\mathcal{O}$ of characteristic zero, with residue field k.

On Integral Representations for SL$(2, q)$

U. Riese

Let G be a finite group of exponent $\exp(G) = g$. By Brauer's celebrated theorem every (irreducible) complex character of G can be written in the gth cyclotomic field $\mathbb{Q}(\mu_g)$. It has been conjectured that there should be even a matrix representation with entries in its ring $\mathbb{Z}[\mu_g]$ of integers. Indeed Cliff, Ritter and Weiss [2] settled this in the case where G is solvable. Knapp and Schmid [4] showed that it (essentially) would suffice to prove this when G is quasisimple.
There is a weaker conjecture dealing with the order of the group in place of the exponent. This has been verified so far for all sporadic groups (and their proper central extensions), some few alternating groups, and some few groups of Lie type of small order [4]. It is natural to begin a systematic investigation by studying the finite groups of Lie type A_1 (following a suggestion by W. Feit).

Theorem. *Let $G = SL(2, q)$ for some prime power $q = p^f$. Every irreducible complex character χ of G can be written in $R = \mathbb{Z}[\mu_n]$ with $n = n(\chi)$ being a proper divisor of $exp(G)$, except possibly when χ is a (cuspidal) character of degree $q - 1$. In the exceptional case χ can be realized over $R[\frac{1}{p}]$ with $n = \frac{1}{p} exp(G)$.*
It is not clear whether (some) characters of degree $q - 1$ are really exceptional (according to the conjectures). Our methods do not give a better result (even replacing n by $|G|$). For large powers $q = p^f$ it seems not unlikely that the characteristic p cannot be avoided as "denominator" (for small rings of realization). However, in the case $q = p$ we get what we wanted:

Corollary. *Every character of $G = SL(2, p)$ can be realized over the ring of integers of the $exp(G)$th cyclotomic field.*
The representations can be always chosen to be stable under any group automorphism leaving the character invariant (which is of relevance when attacking the conjecture(s) for

arbitrary finite groups by induction). We mention that "denominators" can be avoided when enlarging the rings. Iwasawa theory yields that the characters (of any finite group) can be written in the ring of integers of *some* abelian number field [4]. Quantitative results would require detailed informations on class groups, however.
The character table of $\mathrm{SL}(2,q)$ has been computed already by Schur [5]. The Schur indices are all known (see for instance Feit [3]). Matrix representations over certain splitting fields have been constructed by Tanaka [6], but this was not be of use here. We learnt a lot from the construction of the Gelfand representations (as described in [1]), and we do appeal to the work by Ward [7] on the Weil representations of the symplectic group.
Our method goes back to Schur (essentially): Let K be an algebraic number field, R_K its ring of integers and V an absolutely irreducible KG-module affording the character χ. We choose an R_KG-lattice M in V and alter M by multiplication with some nonzero ideal J of R_K in such a way, that the Steinitz class $[JM]$ of JM gets trivial in the class group of R_F where F is a suitable extension field of K. Then $JM \otimes_{R_K} R_F$ is a free R_F-lattice. The main problem is that the class groups of the rings involved are unknown. This difficulty disappears when working with groups having only small prime divisors (e.g. sporadic groups).

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Gradings of Blocks with Abelian Defect

R. Rouquier

Endo-Trivial Modules

J. Thévenaz

(joint work with J. F. Carlson)

Let P be a p-group, let k be a field of characteristic p, and let $T(P)$ be the group of all indecomposable endo-trivial kP-modules, with multiplication induced by tensor product. It is known that $T(P)$ is a finitely generated abelian group (L. Puig), its structure is known when P is abelian (E.C. Dade), and the torsion-free rank of $T(P)$ was recently determined by J.L. Alperin. Assuming that G is not cyclic, we conjecture that the torsion subgroup of $T(P)$ is trivial when p is odd. This turns out to be equivalent to the conjecture that the restriction map $T(P) \longrightarrow \prod_E T(E)$ is injective, where E runs over all elementary abelian subgroups of G of rank 2. When $p = 2$, one certainly needs to add the quaternion group of order 8 to the detecting family. As a main step towards the conjecture, we prove that if P is a non-cyclic p-group, then the restriction map $T(P) \longrightarrow \prod_Q T(Q)$ is injective, where Q runs over all subgroups of P which are either elementary abelian p-groups of rank 2, or extraspecial p-groups, or almost extraspecial p-groups. When p is odd, almost extra-special p-groups can be eliminated from the detecting family. When $p = 2$, recent work allows for a definitive result (see Jon Carlson's talk). For the whole Dade group $D(P)$ of endo-permutation kP-modules, the result implies the injectivity of the restriction–deflation map to all sections of P belonging to the same family of groups as above. Moreover, a complete description of $D(P)$ can also be given when P is dihedral, semi-dihedral, quaternion, or metacyclic.

On simple modules of symmetric groups in characteristic 2

K. Uno

Let S_n be the symmetric group on n letters, and k an algebraically closed field of characteristic 2. For a 2-regular partition λ of n, let D^λ denote the simple kS_n-module corresponding to λ. Note that, however, even $\dim_k D^\lambda$ is not known in general. Suppose that λ has two parts. We determine periodic modules among those D^λ.

Theorem 1 *Let $\lambda = (m+s, m)$ be a 2-regular partition of n. Then, D^λ is non-projective periodic if and only if λ is one of the following:* $(3,2)$, $(4,3)$, $(4,1)$.

First we compute $\dim_k D^{(m+s,m)}$ for small s, $s \geq 3$, using the method given by Erdmann. For example, we have the following. Similar formulae hold for other small s's.

Lemma 1 *Let n be an odd positive integer. Then the following holds.*

$$\nu_2(\dim_k D^{(m+3,m)}) = \begin{cases} (n-5)/4 + \nu_2(n-1) & \text{if } n \equiv 1 \mod 4, \\ (n-3)/4 & \text{if } n \equiv 3 \mod 4. \end{cases}$$

Here $\nu_2(t)$ denotes the exponent of the 2-part of t. It gives an upper bound for n, since the dimensions of periodic modules must be divisible by $2^{[n/2]-1}$. Then small number of modules remain and we can compute their rank varieties to determine whether or not they are periodic.

Next, consider $D^{(m+1,m)}$. Let E be the subgroup of S_{2m+1} generated by the m transpositions $(2i-1, 2i)$, $1 \leq i \leq m$. Then E is an elementary abelian 2-subgroup.

Lemma 2 *The restriction of $D^{(m+1,m)}$ to E is isomorphic to the regular kE-module.*

Taking E as a basis of $D^{(m+1,m)}$, we can compute its rank variety with respect to a direct product of some S_4's. This shows that $D^{(m+1,m)}$ is periodic if and only if $n \leq 7$.

Actions on Ext-algebras coming from equivalences between derived categories

A. Zimmermann

Let A be an R-algebra which is projective as module over the commutative ring R, and let M be an A-module. Let $HD_M(A)$ be the group of self-equivalences of standard type of the derived category which map M to an isomorphic copy. If every A-linear automorphism of M is induced by multiplication by an invertible element of the centre of A, then we show that $HD_M(A)$ acts on the Ext-algebra $Ext^*_A(M, M)$ as group of algebra automorphisms. This action is functorial with respect to base change.
For A being the group ring RG for a finite group, we get $Ext^*_{RG}(M, M) \simeq H^*(G, End_R(M))$. Evidently, the trivial module R satisfies these hypotheses. Moreover, the group of automorphisms of RG preserving the augmentation modulo inner automorphisms is a subgroup of $HD_M(A)$. Even the case $G = C_n \times C_n$ gives interesting phenomenons. In joint work with Eric Jespers we show that the universal coefficient sequence

$$0 \longrightarrow H^{2m}(C_n^2, \mathbb{Z}) \otimes_{\mathbb{Z}} R \longrightarrow H^{2m}(C_n^2, R) \longrightarrow Tor_1^{\mathbb{Z}}(R, H^{2m+1}(C_n^2, \mathbb{Z})) \longrightarrow 0$$

is split exact as sequence of $SL_2(\mathbb{Z})$-modules if and only if $\binom{n}{2} \cdot {}_nR \subseteq n \cdot R$ for ${}_nR$ being the ideal of n-torsion elements in R.
We mention that for R being a field of characteristic p the action of the group of splendid self-equivalences fixing the trivial module commute with restriction and transfer from and to centralizers of p-subgroups.

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