Mathematisches Forschungsinstitut Oberwolfach

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Aperiodic Order

5–12 May 2001

Organised by Michael Baake (Greifswald), Jean V. Bellissard (Toulouse) and Robert V. Moody (Edmonton), the workshop on Aperiodic Order was the third major mathematics meeting devoted entirely to this subject, the previous being at the Fields Institute in 1995 and a workshop at the MFO in 1998.

Aperiodic order deals with extended mathematical and physical (atomic) structures that have long-range order but which are not (necessarily) based on periodic repetition. The concept of order is one that grows ever more subtle with time, both because Nature manifests ordered structures in an infinitude of different ways (for most of which we still have incredibly little understanding) and because of the increasing sophistication of symmetry, self-similarity, and invariance within mathematics itself.

The discoveries of physical quasicrystals and of aperiodic tilings served to define the new area in the early 1980's. Since then the subject has deepened considerably and the present workshop, with its concentration on homological methods, dynamical systems, C^*-algebras, and statistical mechanics, brought the subject to a new level of maturity.

The newness of the subject and the extraordinary breadth and depth of the disciplines that (unexpectedly) enter it made this an exciting workshop for everyone. There is no one person who "knows it all", and over and over again we heard from our participants how important this week had been in putting the diverse aspects of the subject into perspective. We were gratified to find that for more than a third of our participants this was their first time at Oberwolfach, and for many the first time to interact personally with people whom they had known only through their work and reputations before. The atmosphere was one of great mutual support and intense interest.

The days were run with two full hour lectures starting the morning and afternoon sessions, and 45-minute lectures otherwise, with the usual deviation on Wednesday. We also had several evening sessions for priming material or extensions of ideas developed in the lectures. Another important aspect of the workshop was the extensive use of posters. All attendees were asked to produce posters of their recent work and these quickly spilled beyond the usual poster space. They became a useful focal point for people during the breaks, and in the scheduled interaction periods.

As usual, the infrastructure and support-staff at the MFO were exemplary. The workshop was a highly enjoyable and successful event.

Edited by Uwe Grimm

Programme

Monday, 07.05.2001

9.00		Opening address
9.15	H.-O. Georgii	Percolation and the number of phases in the Ising model
10.15		Coffee break
10.45	A. C. D. van Enter	Non-periodic long-range order in lattice models
11.30		Short break
11.45	R. Mosseri	Entropy of random tilings
12.30		Lunch
14.00		Posters and Discussion
15.30		Afternoon tea
16.15	J. A. Propp	Random tilings
17.15		Short break
17.30	J.-P. Allouche	Combinatorics on words and physics
18.30		Dinner

Tuesday 08.05.2001

9.00	D. Damanik	Spectral theory of one-dimensional Schrödinger operators with low-complexity potentials
10.00		Coffee break
10.30	D. Lenz	Uniform ergodic theorems and applications
11.15		Short break
11.30	I. Krasovsky	On the measure of the spectrum of quasiperiodic operators
12.30		Lunch
14.00		Posters and Discussion
15.30		Afternoon tea
16.15	I. Guarneri	Anomalous transport with quasiperiodic Hamiltonians
17.15		Short break
17.30	H. Schulz-Baldes	Transport in polymer chains
18.30		Dinner
20.00	T. Janssen	Scale space groups

Wednesday 09.05.2001

9.00	B. Solomyak	Pure point spectrum for Delone sets
9.45	V. Elser	Exceptionally symmetric fundamental regions for the root lattices in 2D
10.30		GROUP PHOTOGRAPH
10.35		COFFEE BREAK
10.50	P. Gummelt	Concepts for random ensembles of overlapping clusters
11.35	C. Radin	On aperiodicity as optimization
12.20		LUNCH
13.30		EXCURSION
18.30		DINNER
20.00	R. V. Moody	Oberwolfach in Canada? A status report
20.15	L. Danzer	When are inflation species linearly repetitive?
20.45	L. Danzer	Portraits of mathematicians

Thursday 10.05.2001

9.00	C. Skau	Orbit equivalence and ordered K-theory
10.00		COFFEE BREAK
10.30	J. Kellendonk	Topological invariants of aperiodic point sets
11.15		SHORT BREAK
11.30	J. Hunton	Cohomology of projection method tilings
12.30		LUNCH
14.00		Posters and Discussion
15.30		AFTERNOON TEA
16.15	E. A. Robinson	Tilings corresponding to non-Pisot matrices
17.15		SHORT BREAK
17.30	N. M. Priebe	Substitution sequences in $\mathbb{Z}^d$ with non-simple Lebesgue spectral component
18.30		DINNER
20.00	I. Putnam	An informal introduction to C^*-algebras, dynamical systems and K-theory

Friday 11.05.2001

9.00	B. Kümmerer	Quantum Markov processes and coding theory
10.00		Coffee break
10.30	R. A. Rebolledo	Stochastic models for open quantum systems
11.15		Short break
11.30	D. Spehner	Kinetic approach to hopping transport
12.30		Lunch
14.30	P. Kramer	From the dual geometry of lattices to quasiperiodic sections and covering
15.15		Closing remarks
15.30		Afternoon tea
16.00		Posters and Discussion
18.30		Dinner

Abstracts

Combinatorics on words and physics

Jean-Paul Allouche

The first part of this talk consists of a brief survey of examples of the use of combinatorics on words in physics: Penrose's seminal paper; the paper of Mendès-France and the author on the one-dimensional Ising model (at imaginary temperature) and the Rudin-Shapiro sequence; Kohmoto's and Ostlund's results on the Fibonacci masses and springs; trace equations; discrete Schrödinger equations with aperiodic potentials.

The second part concentrates on the discrete Schrödinger operator, giving first a result of Hof-Knill-Simon that links the existence of arbitrarily long palindromes in the sequence of potentials to the singular continuous nature of the spectrum of the operator. We then (joint work with M. Baake, J. Cassaigne and D. Damanik) introduce the notion of "palindrome complexity" of a sequence on a finite alphabet, and study classical sequences and families of sequences from this point of view.

Spectral theory of one-dimensional Schrödinger operators with low-complexity potentials

David Damanik

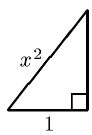
We discuss a combinatorial point of view in the spectral theory of discrete one-dimensional Schrödinger operators $H = \Delta + V$ in $\ell^2(\mathbb{Z})$. Namely, in the case where the potential V takes finitely many values, we consider the complexity function $p : \mathbb{N} \to \mathbb{N}$ associated with V, where $p(n)$ is given by the number of subblocks of V having length n. The point of view we propose is the following: the more complex the potential (i.e., the faster p grows), the more singular the spectral type of H. This tendency has been proven to be correct for extremal complexity behavior. That is, no growth of p implies purely absolutely continuous spectrum, slow growth implies purely singular continuous spectrum, and maximal growth leads to pure point spectrum (at least almost surely with respect to a canonical probability measure). For intermediate complexity, however, only few results are known.

When are inflation-species linearly repetitive (ℓR)?

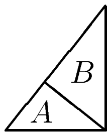
Ludwig Danzer

Given a finite family $\mathcal{F} := \{T_1, \ldots, T_k\}$ of prototiles and an inflation-factor η ($\eta > 1$), such that for each λ, ηT_λ is dissected into congruent copies of some $\mathcal{F}$-tiles ($\lambda = 1, 2, \ldots, k$). This cluster consisting of $a_{1\lambda}$ tiles of type T_1, …, $a_{\kappa\lambda}$ tiles of type T_κ, …, $a_{k\lambda}$ tiles congruent to T_k, is called $\mathrm{infl}(T_\lambda)$. The species $\mathcal{S}$ of all global tilings $\mathcal{P}$, such that every cluster which occurs in $\mathcal{P}$ has a congruent copy in some $\mathrm{infl}^n(T_\lambda)$, is denoted by $\mathcal{S}(\mathcal{F}, \mathrm{infl})$.

<u>Example:</u>

$\mathcal{F} := \{A, B\}$, $A :=$ 

, $B := xA$, $\eta = x \ (= \sqrt{\tau}\,)$,

$$\mathrm{infl}(A) := B, \quad \mathrm{infl}(A) :=$$ 

$$, \quad M = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$

If the $(k \times k)$-matrix $M_{\mathrm{infl}} := (a_{\kappa\lambda})$ is primitive, $\mathcal{S}$ is *weakly repetitive*. That is to say: for every cluster $\mathcal{C}$ from $\mathcal{S}$, there is a radius $P_{\mathcal{C}}$ such that $\mathcal{C}$ has a congruent copy in every tiling $\mathcal{P}$ in $\mathcal{S}$ in every ball of radius $P_{\mathcal{C}}$. If the set $\{P_{\mathcal{C}} \mid \mathcal{C} \text{ fits into a ball of radius } \varrho\}$ is bounded, say by $P(\varrho)$, $\mathcal{S}$ is called *repetitive* (there are inflation-species with primitive M_{infl}, and hence minimal $\mathcal{F}$, which are *not* repetitive; e.g. the example above with x^4 instead of x^2 and $\mathcal{F} = \{A, B, C, D\}$).
Theorem If an *inflation*-species $\mathcal{S}$ is repetitive, it is even *linearly repetitive*. In other words: then there are constants λ_0, λ_1 depending on $\mathcal{S}$ only (not on ϱ), such that $P(\varrho) \leq \lambda_0 + \lambda_1 \varrho$.
The proof consists of several rather trivial steps dealing with some other properties a species of tilings can have (in particular being of locally finite complexity (LFC)).

Exceptionally symmetric fundamental domains for the root lattices in 2D

Veit Elser

It is possible to construct fundamental domains for the root lattices $\mathbb{Z}^2$ and A_2, whose symmetry groups contain as *proper* subgroups their corresponding Weyl groups.

After arguing why such regions cannot be connected, a construction in the "impasto" style of Renaissance painting was presented. In brief: layer upon layer of "paint" involving ever diminishing 8-gons ($\mathbb{Z}^2$) and 12-gons (A_2) are applied to the "canvas" ($\mathbb{R}^2$). The resulting fundamental domains are fractal and are preserved by the non-crystallographic reflection groups $\bullet\!\overset{8}{\text{---}}\!\bullet$ ($\mathbb{Z}^2$) and $\bullet\!\overset{12}{\text{---}}\!\bullet$ (A_2). As an application, these regions define bijections of two copies of their root lattices, one rotated relative to the other: $\{\mathbb{Z}^2, R(\frac{\pi}{4})\mathbb{Z}^2\}$ and $\{A_2, R(\frac{\pi}{6})A_2\}$. The bijection for the A_2 case provides a new construction of the window of the quasiperiodic square-triangle tiling.

Non-periodic long-range order in statistical mechanics: Lattice model examples and concepts

Aernout C. D. van Enter

In statistical mechanics a generally used definition of long-range order is the occurrence of multiple Gibbs or ground-state measures. In case a translation-invariant measure has a non-trivial tail-decomposition into extremal elements which are non-periodic (necessarily uncountably many), non-periodic long-range order occurs. In my talk I reviewed some partial results on toy models in which the existence of such non-periodic long-range order can be proven. All this is restricted to lattice models. In one dimension at positive temperatures existence results can be obtained from the Israel-Bishop-Phelps theorem. The interaction is long-range and is shown to exist in a rather non-constructive way. In three dimensions, by stabilization of a one-dimensional zero-temperature construction by adding nearest-neighbour ferromagnetic interactions in the second and third direction, an existence result for relatively short-range interaction is obtained. The long-range order is Thue-Morse like.

Furthermore, I discussed the diffraction spectrum. By considering the spectrum of the generator of translations for the Thue-Morse system one can distinguish between a singular continuous atomic diffraction spectrum and a pure point molecular diffraction spectrum.

Non-periodic substitution systems are also a source of computable overlap distributions. Non-trivial Purini overlap distributions occur for the Toeplitz system (discrete, based on an ultrametric structure) and the Fibonacci system (continuous), whereas the Thue-Morse systems results in an almost surely trivial overlap.

I reviewed joint work with Clifford Gardner, Bert Hof, Jacek Miękisz, Charles Radin, Miloš Zahradník and Boguslaw Zegarlinski.

Percolation and the number of phases in the 2D Ising model

Hans-Otto Georgii

Gibbs measures serve as models for the equilibrium states of physical systems consisting of many interacting compounds. Of particular interest is the non-uniqueness of Gibbs measures which corresponds to the phenomenon of phase transitions. This phenomenon can often been understood in terms of the formation of infinite clusters in suitable random graphs: such infinite clusters serve as a link between individual and collective behavior. After a general introduction to the fundamentals of the theory, the geometric aspects of phase transitions are illustrated by the two-dimensional Ising model. A recent streamlined proof of the by now classical result that in this model only two phases exist is presented. (Joint work with Y. Higuchi, Kobe).

Anomalous transport in quasiperiodic Schrödinger operators

Italo Guarneri

Schrödinger operators endowed with singular continuous spectra exhibit several peculiarities in the associated wave packet dynamics. These are known from extensive numerical simulations. A short review of such empirical data is given; the possible physical implications are outlined. General results about the dynamical implications of singular continuous spectra, which provide bounds on the exponents ruling the wave packet diffusion in terms of various spectral dimensional, are reviewed. Finally a result is announced, which sets a lower bound on the transport exponents of the "critical" Harper (almost-Mathieu) Hamiltonian, averaged over the phase, in terms of the multifractal dimensions of the density of states. Numerical results available in the literature indicate that this rigorous bound may be optimal.

Concepts for random ensembles of overlapping clusters

Petra Gummelt

Cluster coverings are patterns of overlapping units (so-called clusters) and hence complete the hierarchy of packings and tilings. The question for a single aperiodic planar building block ("einstein") which is still open for conventional tilings is solved within the more general context of coverings. Since almost all definitions and concepts well-established in tiling theory can be also used for cluster coverings, we asked for the application of the well-known tiling model to overlapping units. Focussing on matching rules, we offered two schemes of random covering ensembles which are motivated by tiling theoretical aspects and structure models of quasicrystals, respectively. We illustrated our approach by a detailed analysis of the "random relative" corresponding to the aperiodic decagon.

Cohomology of projection method tilings

John Hunton

We consider cut-and-project point patterns/tilings obtained on $\mathbb{R}^d$ from a rank N lattice Λ, and restrict for now to acceptance domains which are the projection of the unit cell (so polyhedral) for Λ. As discussed in Kellendonk's talk (this workshop; see Kellendonk's abstract), to such a situation we attach a Cantor dynamical system $(X, \mathbb{Z}^d)$, or a locally Cantor system $(V_c, \mathbb{Z}^{n+d})$, where V_c is a "Cantorised" n-dimensional Euclidean space, Cantorised by a family of cutting hyperplanes, and $X = {}^{V_c}/_{\mathbb{Z}^n}$.

To such a set-up a variety of topological invariants can be associated: $H^*(\mathbb{Z}^d; C(X, \mathbb{Z}))$, $K_*(C(X) \rtimes \mathbb{R}^d)$, $K^*_{\text{top}}(\Omega)$, $\check{H}^*(\Omega)$, etc., where Ω is the associated tiling space (hull). By work of Forrest-Hunton [FH], these are all isomorphic as Abelian groups. Recent work of Sadun and Williams shows that a similar situation of equivalent invariants exists for a very wide class of tilings.

Tools of algebraic topology are used to produce a number of results, both theoretical and computational. In particular, writing L_0 for the number of $\mathbb{Z}^{n+d}$-orbits of cut points in V_c, we prove

Theorem L_0 is finite if and only if $H^*(\mathbb{Z}^d; C(X, \mathbb{Z})) \otimes \mathbb{Q}$ is of finite rank/$\mathbb{Q}$.

Almost all such cut-and-project tilings display infinite L_0. As work of Kellendonk and of Anderson-Putnam shows that substitution tilings have *finite* rational rank invariants we obtain

Corollary "Generically", projection tilings are not representable as substitution tilings.

We also see that when $H^*(\mathbb{Z}^d; C(X, \mathbb{Z})) \otimes \mathbb{Q}$ is finite-dimensional, $H^*(\mathbb{Z}^d; C(X, \mathbb{Z}))$ is free Abelian, providing an obstruction to substitution tilings being described as projections: $\check{H}^*(\Omega)$ for substitutions will often contain divisibility.

Formulae describing the invariants for codimensions 1, 2 and 3 are given, as is a formula for the Euler characteristic, i.e., $\text{rk}(K_0) - \text{rk}(K_1)$, for arbitrary codimension patterns.

Scale-space groups

Ted Janssen

Aperiodic crystals are characterized by the fact that the sharp Bragg spots belong to an n-dimensional vector module (rank $n > 3$). They can be considered as intersection of an n-dimensional reciprocal lattice with 3D physical space. Since the relation between the 3D and nD Fourier transforms is 1–1, the space group symmetry of the embedded nD structure is relevant for the 3D crystal.

However, in physical sense there is a difference between the physical and the additional $(n-3)$ dimensions. This consideration has to be taken into account for the choice of the definition of equivalence of space groups. Usually, in 3D, space groups can be calculated as extensions of a point group K by the 3D lattice $\mathbb{Z}^3$. Equivalence classes are orbits of the normalizer of K on the second cohomology group $H^2_\phi(K)$. For nD space groups for quasiperiodic crystals, one has to limit oneself to the action of a subgroup of the normalizer. It is discussed what the options are.

Quasicrystals show often, in addition to the Euclidean point group, scale symmetry. The scale operators can be lifted to nD basis transformations of the lattice. Together with

K, these transformations generate an infinite non-Euclidean point group. These groups can be extended again by $\mathbb{Z}^n$, and, in general, these are non-trivial extensions. The calculation can again be based in H^2_ϕ, but also in H^1_ϕ (for finite groups these are isomorphic). The (nontrivial) extensions are called scale-space groups and maybe used to construct a new type of tilings.

Topological invariants for aperiodic point sets

Johannes Kellendonk

In the cut and projection scheme one constructs point sets by projecting, from a higher-dimensional periodic lattice Λ, the points in a strip which is a thickening of an irrationally placed lower-dimensional linear space onto that linear space E. The thickening is determined by a so-called acceptance domain K. This gives rise to a dynamical system consisting of a space V_c which is obtained from the complementary subspace V (in which K lies) by disconnecting it along the points which are obtained from the boundary of K by translation with a vector of Γ, the projection of Λ onto V along E. The group acting is Γ and we investigated the dynamical invariants of that system. Such invariants are the cohomology of the group Γ with coefficients in $C_c(V_c, \mathbb{Z})$ (integer valued continuous functions over X), the K-theory of the C^*-algebra $C(x) \rtimes \mathbb{Z}^d$ where $\Gamma = \mathbb{Z}^d \oplus \mathbb{Z}^n$, $\mathbb{Z}^n$ spans V, and $X = {}^{V_c}/_{\mathbb{Z}^d}$. We obtain results for $n \leq 3$.

The K-theory is relevant for the gap-labelling of operators describing electron motion in such an aperiodic set. For generic positions of E and polytopal acceptance domain K, the K-theory is infinitely generated, but, as a result of a conversation on this workshop with Moody and Schulte, we now know that the K_0-group modulo infinitesimals is always finitely generated.

From the dual geometry of lattices to quasiperiodic sections and coverings

Peter Kramer

The Voronoi and the dual Delone complexes of a lattice Λ in E^n provide a hierarchy of dual pairs of boundaries X_j, X_j^* of complementary dimension. Lattice points are dual to Voronoi polytopes and Delone polytopes dual to holes, i.e., vertices of the Voronoi polytopes. Dual tiling theory takes advantage of this richer geometry. Tilings $(\mathcal{T}, \Lambda)$ are projected to $E_\parallel \subset E^n$ from Voronoi boundaries, tilings $(\mathcal{T}^*, \Lambda)$ from Delone boundaries. Here $E^n = E_\parallel + E_\perp$ where $E_\parallel$, $E_\perp$ are invariant under a point subgroup of the holohedry of Λ. Since 1990 we analyze clusters and coverings in tilings in terms of dual tiling theory. As clusters in the dual tilings we propose parallel projections of Delone and of Voronoi polytopes to the tiling space $E_\parallel$. A general construction theorem yields the unique asymmetric filling and the windows of these clusters. The well-known decagon clusters in the 2D Penrose tiling can be identified as Voronoi projections. The dual triangle tiling, projected from the same root lattice A_4, is covered by pentagonal Delone clusters. We explore the tilings of icosahedral point symmetry projected from the 6D root lattice D_6 and study their Delone and Voronoi coverings. We expect that the analysis can yield insight into the structure of quasicrystals.

On the measure of the spectrum of discrete quasiperiodic operators

IGOR KRASOVSKY

We consider the discrete Schrödinger operator

$$(H_{\alpha,\theta}\psi)(n) = \psi(n-1) + \psi(n+1) + f(\alpha n + \theta)\psi(n)$$

on $l_2(\mathbb{Z})$, where $f(x)$ is a real periodic analytic function of period 1. For any irrational α and real θ, we show that, if the corresponding Lyapunov exponent is a.e. positive, then $|S(\alpha,\theta)| = \lim_{n\to\infty} |\cup_{\theta\in\mathcal{R}} S(p_n/q_n,\theta)|$, where $S(\beta,\theta)$ is the spectrum of $H_{\beta,\theta}$, $|S(\beta,\theta)|$ its Lebesgue measure, and p_n/q_n is the sequence of canonical rational approximants to α.

For the almost Mathieu operator ($f(x) = 2\lambda\cos 2\pi x$) it follows that the measure is equal to $4|1-|\lambda||$ for all real θ, $\lambda \neq \pm 1$, and all irrational α. (Joint work with S. Ya. Jitomirskaya, Irvine).

Quantum Markov processes and coding theory

BURKHARD KÜMMERER

In symbolic dynamics, Markov processes are constructed from road-coloured graphs. If the road-colouring has a synchronizing word, then this can be used to represent the Markov process as a factor of a Bernoulli process.

In quantum probability, given a $*$-homomorphism $J : \mathcal{A} \to \mathcal{A}\otimes\mathcal{C}$ for some (C^*-) algebras $\mathcal{A}$ and $\mathcal{C}$ (called "random variable"), a quantum Markov process is obtained from it as follows. Define a $*$-homomorphism

$$T: \begin{array}{ccccccccc} \mathcal{A} & \otimes & \mathcal{C} & \otimes & \mathcal{C} & \otimes & \mathcal{C} & \otimes & \ldots \\ & {\scriptstyle J}\searrow & & {\scriptstyle \mathrm{Id}}\searrow & & {\scriptstyle \mathrm{Id}}\searrow & & {\scriptstyle \mathrm{Id}}\searrow & \\ \overbrace{\mathcal{A} \quad \otimes \quad \mathcal{C}} & & & \otimes & \mathcal{C} & \otimes & \mathcal{C} & \otimes & \ldots \end{array}$$

and random variables $J_0 : \mathcal{A} \ni x \mapsto x\otimes\mathbb{I}\otimes\mathbb{I}\otimes\ldots \in \mathcal{A}\otimes\mathcal{C}\otimes\mathcal{C}\otimes\ldots$ and $J_n := T^n \circ J_0$, then for any state of the form $\varphi\otimes\psi\otimes\psi\otimes\ldots$, for φ and ψ states on $\mathcal{A}$ and $\mathcal{C}$, respectively, $(J_n)_n$ is a Markov process.

For commutative $\mathcal{A}$ and $\mathcal{C}$, the random variable J can be identified with a road-colouring and the above Markov process as the corresponding Markov process obtained from this road-colouring.

Motivated from an interpretation of such a Markov process in terms of open quantum systems we (Kümmerer-Maassen) initiated a systematic study of asymptotic completeness of such a Markov process. It roughly means that for large n, $J_n(x)$ ($x \in \mathcal{A}$) tends to be an element in $\mathbb{I}\otimes\mathcal{C}\otimes\mathcal{C}\otimes\ldots$. (The precise formulation uses the strong operator topology with respect to the above mentioned product state).

Theorem (Gohm/Kümmerer/Lang, 2001): For $\mathcal{A}$, $\mathcal{C}$ commutative and finite-dimensional, asymptotic completeness is equivalent to the existence of a synchronizing word.

We have been able to generalize this to the non-commutative and infinite-dimensional context. This involves a suitable formulation/generalization of the notion of a synchronizing word in this general context, as well as proving an analogous result.

Finally, we show that the micro-maser is a "literal" physical realization of such a Markov process. It is asymptotically complete, and this allows to prepare quantum states of the field mode by sending in suitably prepared atoms. Numerical computations show that this can be done in a way which seems to be experimentally realizable. It opens the prospective of using the micro-maser as a quantum coding machine.

Uniform ergodic theorems and applications

Daniel Lenz

We consider minimal subshifts over a finite alphabet and the associated Schrödinger operators. A necessary and sufficient condition for validity of a uniform subadditive theorem is discussed. This condition is given by uniform positivity of weights. Thus, it is in particular satisfied for all linearly repetitive subshifts (e.g. those arising from primitive substitutions). Furthermore, we study the question whether the spectrum of the associated Schrödinger operators coincides with the set of energies with vanishing Lyapunov exponent.

This can be shown to hold if and only if the Lyapunov exponent exists uniformly. In this case the Lyapunov exponent is continuous.

Based on these results we infer zero Lebesgue measure spectrum for a large class of Schrödinger operators related to one-dimensional quasicrystals, including all aperiodic Schrödinger operators associated to primitive substitutions.

Random tilings and partitions

Remy Mosseri

Interesting sets of random tilings with rhombi, rhombohedra (and their higher dimensional analogues), and with given fixed boundary conditions, can be put in 1-to-1 correspondence with partitions: well known cases are lozenge tilings inside a hexagon, and planar partitions. We discuss here other cases which belong to two different families:

— n-dim. tilings (with n-dim. rhombohedra) corresponding to standard (hyper)solid partitions. They allow for a representation as an n-dim. facetted membrane in an $(n+1)$–dim. hypercubic lattice (so-called codim.-one case).

— 2d rhombus tilings which can be lifted as a 2-dim. facetted membrane in a $(2+d)$-dim. hypercubic lattice (codim.-d case), corresponding to "generalized" partitions.

The tiling configuration space $\mathcal{C}$ is a (very) high-dimensional polytope (convex in the codim.-1 case) whose integral volume leads to the "entropy" of the tiling set. We describe a decomposition of $\mathcal{C}$ into "normal simplices", which greatly reduce the complexity in computing the integral volume (the main difficulty here being not to overcount points on "face" sharing simplices). Exact and approximate enumeration formulas are given, as well as some numerical estimates. In particular, an exact (new) formula enumerates the total number of simplices in the 2d case (of general codimension). The question of "dynamical" properties (path count) on $\mathcal{C}$ is also addressed.

Substitution sequences in $\mathbb{Z}^d$ with non-simple Lebesgue spectral component

Natalie M. Priebe

We present a construction of d-dimensional substitution sequences for which the continuous part of the spectrum is generated by measures equal to Lebesgue measure. A special case is the Rudin-Shapiro substitution sequence. All substitution sequences of this type are shown to factor onto $\mathbb{Z}^d$ sequences of $\{+1, -1\}$ which have Lebesgue diffraction measure. A point of interest in the construction is the essential use of Hadamard matrices.

Random tilings

Jim Propp

I give a survey of recent work on random tilings that focusses on very simple two-dimensional tiling models (namely, tiling models such as domino- or lozenge-tiling models that are dual to dimer models on regular grids) in the presence of general boundary conditions, where "random" means "chosen from the uniform distribution". The precise shape of the boundary of a region can have a profound impact on the behavior of a random tiling of the region, not just near the boundary but far inside it as well, leading to situations in which coarse-grained quantities (such as the density of tiles of a particular shape and orientation) are locally constant but vary over macroscopic distances. Typically, the region being tiled splits into macroscopic sub-regions, each of which is one of three types: *frozen regions* where the boundary effects are so strong that the tiling is periodic; *tropical regions* throughout which the tiling is non-periodic and all coarse-grained quantities are constant; and *temperate regions* throughout which coarse-grained quantities vary continuously as one travels a macroscopic distance. The occurrence of these domains, and the shapes of the boundaries that separate them, are related to phase transitions of the model in the presence of a suitable external field, which in turn arise from the existence of a height-representation for the tiling model.

Informal introduction to C^*-algebras, dynamics and K-theory

Ian Putnam

Assuming no prior background in C^*-algebras or K-theory, I will present some basic features of both, and how they may be applied in dynamics. This highlights A. Connes' program of non-commutative geometry.

On aperiodicity as optimization

Charles Radin

We consider the problem of optimally dense packings of the hyperbolic plane $\mathcal{H}^2$ with disks of fixed radius R. We use the metrizable topology on the space Ω of packings of $\mathcal{H}^2$ in which convergence means uniform convergence on compact subsets of $\mathcal{H}^2$; Ω is compact in this topology. For fixed origin $O \in \mathcal{H}^2$ we define $A = \{\omega \in \Omega\colon\ O \text{ is inside a disk in } \omega\}$. For a (Borel) probability measure μ, invariant under rigid motion, we define the "density" $D(\mu)$ as $\mu(A)$. We prove there exists ergodic μ_R such that $D(\mu_R) = \sup_\mu D(\mu)$, and define "optimally dense" ω as those whose orbit closure is the support of some μ_R. Also we prove that, with the exception of countably many R, *no* optimally dense packing can have symmetry group with compact fundamental domain, i.e., the packings are "aperiodic". We emphasize that, as in statistical mechanics, the natural object which solves our problem is a *probability measure* on packings. Although this seems necessary in $\mathcal{H}^2$, this approach is also fruitful for packings in Euclidean spaces.

Quantum noises, particles and master equation

Rolando Rebolledo

The basic tools of quantum stochastic calculus, as developed by Hudson and Parthasarathy, are explained in view of its application to quantum transport.

The first part of the lecture introduces quantum noises and explores their connection with Wiener and Poisson processes.

A system of quantum particles is then analyzed through a quantum stochastic differential equation which leads to the quantum flow of electronic transport. The associated quantum dynamical semigroup is obtained by projecting the quantum flow on the initial space of the dynamics.

In the last part of the lecture, the analysis of equilibrium is performed. Namely, it is shown that a stationary Gibbs density matrix exists if and only if the transport rates satisfy a system of balance equations.

Tilings corresponding to non-Pisot matrices and explicit construction of Markov partitions

E. Arthur (Robbie) Robinson

Let A be a $d \times d$ integer matrix with 2-dimensional expanding subspace W^+. Consider substitutions θ on d symbols having "structure matrix" A (symbols like 2^{-1} are allowed). These are free group endomorphisms. We think of A as an Abelianization of θ and θ as a "non-Abelianization" of A. Let $P_1, \ldots, P_d$ be projections of the standard basis to W^+. In a natural way, each substitution θ defines a piecewise linear boundary curve made from these vectors. The goal is to tile the inside of each curve using the $\binom{d}{2}$ rhombic prototiles with $P_1, \ldots, P_d$ as edges (using only positive tiles!). If this is possible we can get a tiling substitution Θ and iterate it to get a family of self-affine tilings of $\mathbb{R}^2$.

In the case $d = 4$, when this works for both A & A^{-1}, we can get an explicit Markov partition for A. Here $\dim(W^+) = \dim(W^-) = 2$ and we call A a "non-Pisot" matrix.

A necessary condition for success is $A^* \geq 0$ where A^* is essentially the second compound of A with sign changes. Sufficient conditions for $A^* > 0$ include A symmetric.

This is joint work with Maki Furukado & Shunji Ito.

Transport in ergodic polymer chains

Hermann Schulz-Baldes

By ergodic polymer chain is meant a covariant family of one-dimensional discrete Schrödinger operators the potential of which consists of two finite building blocks aligned according to a code. The probability measure on code space is supposed to be ergodic with respect to the translations. If the transfer matrices across the two polymers commute at a so-called critical energy, the Lyapunov exponent vanishes. It is then possible to show that the dynamical spread of any localized state has to be faster than diffusively. This even holds if the distribution on code space is of Bernoulli type in which case the spectrum is known to be pure-point for almost every code. The main technical tools for the proof of the above are action-angle or Prüfer variables.

Orbit equivalence and ordered K-theory

Christian Skau

H. Dye proved around 1960 the remarkable result that all ergodic measure preserving dynamical systems $(X, \mathcal{B}, \mu, T)$ are orbit equivalent. (Here μ is a probability measure.) All such systems give rise, via the so-called crossed product construction, to the unique hyperfinite II_1 factor of Murray and von Neumann. This was generalized in the early 1970's by Krieger and Connes, resulting in the celebrated theorem that two ergodic non-singular measurable systems are orbit equivalent if and only if the associated von Neumann algebras are isomorphic. Is there an analogous result in the topological/C^*-algebra setting? Giordano, Putnam and Skau were able to show that for Cantor minimal systems (X, T). Here X is a Cantor set (i.e., X is compact, metrizable, totally disconnected without isolated points — all such sets are homeomorphic) and T is a minimal homeomorphism of X, i.e., every T-orbit is dense in X. The theorem states that Cantor minimal systems (X, T) and (Y, S) are strong orbit equivalent (i.e., the orbit cocycles have each at most one discontinuity point) if and only if the associated C^* algebras $C^*(X, T)$ and $C^*(Y, S)$ are isomorphic. Furthermore, this is equivalent to $K^0(X, T) \cong K^0(Y, S)$ as ordered groups with distinguished order units. Here $K^0(X, T) = C(X, \mathbb{Z})/\partial_T C(X, \mathbb{Z})$, where $\partial_T : C(X, \mathbb{Z}) \to C(X, \mathbb{Z})$ is the coboundary map $\partial_T(f) = f - f \circ T^{-1}$, and $K^0(X, T)$ is endowed with the induced ordering of $C(X, \mathbb{Z})$. The K^0-group is effectively computable for many families of interesting Cantor minimal systems, e.g. primitive substitution minimal systems. The K^0-invariant is independent of spectral and entropy invariants. A complete invariant for orbit equivalent is $\widetilde{K^0(X, T)} = K^0(X, T)/\mathrm{Inf}(K^0(X, T))$, which again is order isomorphic to $C(X, \mathbb{Z})/I(C(X, \mathbb{Z}))$, where $I(C(X, \mathbb{Z})) = \{f \in C(X, \mathbb{Z}) \mid \int_x f \mathrm{d}\mu = 0$ for every T-invariant measure $\mu\}$. Both $K^0(X, T)$ and $\widetilde{K^0(X, T)}$ are simple dimension groups. A new result by Giordano, Putnam and Skau gives an example of a free, minimal $\mathbb{Z}^2$-action of a Cantor set which is orbit equivalent to a Cantor minimal systems (X, T), i.e., to a $\mathbb{Z}$-action. We hope to extend this result to general free, minimal $\mathbb{Z}^n$-actions.

Pure point spectrum for Delone sets

Boris Solomyak

Delone sets are uniformly discrete relatively dense sets in Euclidean space. Such a set can be used as a model for an atomic configuration of a solid. We make some additional assumptions, such as finite local complexity and existence of uniform patch frequencies. There are two kinds of spectra that can be associated to the Delone set. The first is the diffraction spectrum measure obtained as the Fourier transform of the autocorrelation of the sum of delta functions on the Delone set. The second is the dynamical spectrum obtained by considering the "hull" (or the "orbit closure" or the "local isomorphism class") of the Delone set, with the action of $\mathbb{R}^d$ by translations. This system is uniquely ergodic, so there is a unique invariant measure on the space. The associated unitary representation on L^2 has a certain spectral measure, which we refer to as the "dynamical spectrum". In joint work with J.-Y. Lee and R. V. Moody, we show that the diffraction spectrum is pure point if and only if the dynamical spectrum is pure point (the direction from dynamical spectrum to diffraction spectrum was earlier established by S. Dworkin). Then I gave a brief survey of results related to the question: how to check if a given Delone set has pure point spectrum. There are conditions in terms of almost periodicity, but they are not always easy to verify. For substitution Delone sets more precise and "practical" criteria can be given. Finally I addressed the question: is it true that for a substitution Delone set, pure point spectrum implies that it is a regular model set? The latter means that the set can be obtained by a cut and project scheme with a "nice" window. Together with J.-Y. Lee and R. V. Moody we show that the answer is "yes" if the substitution Delone set "lives" on a lattice. Many interesting questions still remain open.

Kinetic approach to hopping transport in strongly disordered solids

Dominique Spehner

Electronic transport in disordered solids in the strong localization regime is studied by means of kinetic models involving quantum or classical noises, which mimic the influence of electron-phonon interactions on the electronic dynamics. The quantum dynamical semi-group describing these dynamics is obtained by averaging over the noise. The stochastic dynamics are given by a stochastic Schrödinger equation involving some jump operators from one localized one-electron state into another. Exchanges of electrons with two baths are also modeled by quantum noises, multiplying creation or annihilation operators in the stochastic Schrödinger equation. We develop a linear response theory to compute the current density when an external uniform electric field is applied in this model. A Kubo-like formula for the conductivity is obtained, valid provided the one-electron eigenfunctions are localized and the eigenenergies of the one-electron Hamiltonian are non-degenerate.

Titles of poster presentations

M. Baake, M. Höffe, B. Sing
Diffraction theory of perfect and stochastic quasicrystals

M. Baake, R. V. Moody
Almost periodicity and pure point diffraction

C. Bandt
Local geometry of fractals given by tangent measure distributions

C. Bandt, Y. Wang
Disk-like self-affine tiles in $\mathbb{R}^2$

L. Danzer
When are inflation-species linearly repetitive (ℓR)?

A. C. D. van Enter
Non-periodic long-range order in statistical physics. Lattice models, mainly one-dimensional: examples and concepts

A. Forrest, J. Hunton, J. Kellendonk
Cohomology groups for projection point patterns

D. Frettlöh
Inflation tilings with integer factor

F. Gähler
Generation of quasiperiodic order by maximal cluster covering

H.-O. Georgii
Phase transitions and percolation in continuum Ising models

U. Grimm
Counting power-free word in two letters

U. Grimm
Spectral properties of a tight-binding Hamiltonian on the labyrinth tiling

P. Gummelt
Concepts for random ensembles of overlapping clusters

J. Hunton
Comparing canonical projection method patterns and substitution patterns using their cohomology and K-theory

P. Kramer
From the dual geometry of lattices to quasiperiodic sections and covering

P. Kramer
The cover story: Fibonacci, Penrose, Kepler

B. Kümmerer
Quantum Markov processes and coding theory

J.-Y. Lee, R. V. Moody
Lattice substitution systems, model sets, and diffraction

R. Lück, M. Scheffer
A set of colour-symmetric Fibo⊗Fibo patterns with 3^2 colours

P. McMullen, E. Schulte
Abstract regular polytopes (to appear in Cambridge University Press)

R. V. Moody
Basic theorems on model sets

G. Muraz, J.-L. Verger-Gaugry
A compactness theorem for the set of uniformly discrete sets and its subsets: lattices, clusters, model sets and Delone sets

Z. Papadopolos, G. Kasner, P. Kramer
Delone covering of canonical tilings

Z. Papadopolos, O. Ogievetsky
On quasiperiodic space tilings, inflation and Dehn invariants

N. M. Priebe
A method for generalizing the Rudin-Shapiro substitution to higher dimensions using Hadamard matrices and obtaining $\mathbb{Z}^d$ sequences with Lebesgue correlation measure

J. Propp
Conversations I'd like to have this week

M. Reichert, F. Gähler
Monte-Carlo Simulationen zum Clustermodell des Penrose-Tilings

E. A. Robinson
Tilings associated with some non-Pisot matrices

U. Schnell, J. M. Wills, K. Böröczky, Jr.
Packings, parametric density and quasicrystals

D. Spehner
Hopping transport in disordered solids

J. Vidal, R. Mosseri
Generalized Rauzy tilings: construction and electronic properties

Electronic addresses of participants

Jean-Paul Allouche	allouche@lri.fr http://www.lri.fr/~allouche
Michael Baake	mbaake@uni-greifswald.de http://mahler.math-inf.uni-greifswald.de
Christoph Bandt	bandt@uni-greifswald.de
Jean V. Bellissard	jeanbel@irsamc.ups-tlse.fr
Anton Bovier	bovier@wias-berlin.de
David Damanik	damanik@math.uci.edu http://math.uci.edu/~damanik
Ludwig W. Danzer	danzer@math.uni-dortmund.de
Veit Elser	ve10@cornell.edu
Aernout C. D. van Enter	aenter@phys.rug.nl
Dirk Frettlöh	dirk.frettloeh@udo.edu
Franz Gähler	gaehler@itap.physik.uni-stuttgart.de http://www.itap.physik.uni-stuttgart.de/~gaehler
Hans-Otto Georgii	georgii@rz.mathematik.uni-muenchen.de http://www.mathematik.uni-muenchen.de/~georgii
Thierry Giordano	giordano@science.uottawa.ca
Uwe Grimm	u.g.grimm@open.ac.uk http://mcs.open.ac.uk/ugg2
Italo Guarneri	guarneri@uninsubria.it
Petra Gummelt	petra@mail.uni-greifswald.de
Ulrich Hermisson	hermisso@uni-greifswald.de
Moritz Höffe	moritz.hoeffe@uni-tuebingen.de
John Hunton	J.Hunton@mcs.le.ac.uk http://mcs.le.ac.uk/~jhunton
Ted Janssen	ted@sci.kun.nl
Johannes Kellendonk	kellendonkj@cf.ac.uk http://www.cardiff.ac.uk/maths/ people/kellendonk.html
Peter Kramer	peter.kramer@uni-tuebingen.de

Igor Krasovsky ivk@math.tu-berlin.de

Burkhard Kümmerer kuem@mathematik.uni-stuttgart.de

Jeong-Yup Lee jlee@math.ualberta.ca

Daniel Lenz dlenz@math.uni-frankfurt.de
http://www.math.uni-frankfurt.de/~dlenz

Reinhard Lück lueck@mf.mpi-stuttgart.mpg.de

Robert V. Moody rvm@miles.math.ualberta.ca

Remy Mosseri mosseri@gps.jussieu.fr

Zorka Papadopolos zorka.papadopolos@uni-tuebingen.de

Natalie M. Priebe napriebe@vassar.edu

James G. Propp propp@math.wisc.edu
http://www.math.wisc.edu/~propp

Ian Putnam putnam@math.uvic.ca

Charles Radin radin@math.utexas.edu
http://www.math.utexas.edu/users/radin/

Rolando A. Rebolledo rrebolle@puc.cl
http://www.mat.puc.cl/~rrebolle/index.html

E. Arthur Robinson, Jr. robinson@gwu.edu

Max Scheffer max.scheffer@physik.tu-chemnitz.de

Uwe Schnell schnell@mathematik.uni-siegen.de
http://www.math.uni-siegen.de/wills/schnell

Egon Schulte schulte@neu.edu

Hermann Schulz-Baldes schuba@math.uci.edu

Marjorie Senechal senechal@smith.edu

Christian Skau csk@math.ntnu.no

Boris Solomyak solomyak@math.washington.edu

Dominique Spehner spehner@theo-phys.uni-essen.de

Jean-Louis Verger-Gaugry jlverger@ujf-grenoble.fr

Participants

Prof. Dr. Jean-Paul Allouche
Laboratoire de Rech. Informatique
CNRS
Bâtiment 490
91405 Orsay Cedex
FRANCE

Prof. Dr. Michael Baake
Inst. für Mathematik und Informatik
Universität Greifswald
Friedrich-Ludwig-Jahn-Str. 15a
17487 Greifswald
GERMANY

Prof. Dr. Christoph Bandt
Inst. für Mathematik und Informatik
Universität Greifswald
Friedrich-Ludwig-Jahn-Str. 15a
17487 Greifswald
GERMANY

Prof. Dr. Jean V. Bellissard
IRSAMC
Université Paul Sabatier
118, route de Narbonne
31062 Toulouse Cedex 04
FRANCE

Dr. Anton Bovier
Weierstraß-Institut für Angewandte
Analysis und Stochastik
Mohrenstr. 39
10117 Berlin
GERMANY

Dr. David Damanik
Dept. of Mathematics
University of California
Irvine, CA 92697
USA

Prof. emer. Dr. Ludwig W. Danzer
Fachbereich Mathematik
Universität Dortmund
44221 Dortmund
GERMANY

Prof. Dr. Veit Elser
Dept. of Physics
Cornell University
Clark Hall
Ithaca, NY 14853-2501
USA

Prof. Dr. Aernout C. D. van Enter
Inst. voor Theoretische Natuurkunde
Rijksuniversiteit Groningen
Nijenborgh 4
9707 AG Groningen
THE NETHERLANDS

Dipl.-Math. Dirk Frettlöh
Fachbereich Mathematik
Universität Dortmund
44221 Dortmund
GERMANY

Dr. Franz Gähler
Institut für Theoretische und
Angewandte Physik
Universität Stuttgart
70550 Stuttgart
GERMANY

Prof. Dr. Hans-Otto Georgii
Mathematisches Institut
Universität München
Theresienstr. 39
80333 München
GERMANY

Prof. Dr. Thierry Giordano
Dept. of Mathematics and Statistics
University of Ottawa
585 King Edward
Ottawa, Ontario K1N 6N5
CANADA

Dr. Uwe Grimm
Applied Mathematics Department
Faculty of Math. and Computing
The Open University
Walton Hall
Milton Keynes MK7 6AA
UK

Prof. Dr. Italo Guarneri
Dip. di Scienze
Univ. dell'Insubria
Via Valleggio 11
22100 Como
ITALY

Dr. Petra Gummelt
Inst. für Mathematik und Informatik
Universität Greifswald
Friedrich-Ludwig-Jahn-Str. 15a
17487 Greifswald
GERMANY

Dr. Ulrich Hermisson
Inst. für Mathematik und Informatik
Universität Greifswald
Friedrich-Ludwig-Jahn-Str. 15a
17487 Greifswald
GERMANY

Dipl.-Phys. Moritz Höffe
Institut für Theoretische Physik
Universität Tübingen
Auf der Morgenstelle 14
72076 Tübingen
GERMANY

Dr. John Hunton
Dept. of Mathematics
University of Leicester
University Road
Leicester LE1 7RH
UK

Prof. Dr. Ted Janssen
Inst. for Theoretical Physics
University of Nijmegen
Toernooiveld
6525 ED Nijmegen
THE NETHERLANDS

Dr. Johannes Kellendonk
School of Mathematics
Cardiff University
Cardiff CF24 4YH
UK

Prof. Dr. Peter Kramer
Institut für Theoretische Physik
Universität Tübingen
Auf der Morgenstelle 14
72076 Tübingen
GERMANY

Dr. Igor V. Krasovsky
Inst. für Mathematik MA 7-2
Technische Universität Berlin
Straße des 17. Juni 136
10623 Berlin
GERMANY

Prof. Dr. Burkhard Kümmerer
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57
70569 Stuttgart
GERMANY

Jeong-Yup Lee, MSc
Dept. of Mathematical Sciences
University of Alberta
Edmonton, Alberta T6G 2G1
CANADA

Dr. Daniel Lenz
Fachbereich Mathematik
Universität Frankfurt
60054 Frankfurt/Main
GERMANY

Dr. Reinhard Lück
MPI für Metallforschung
Seestr. 92
70174 Stuttgart
GERMANY

Prof. Dr. Robert V. Moody
Dept. of Mathematical Sciences
University of Alberta
Edmonton, Alberta T6G 2G1
CANADA

Prof. Dr. Remy Mosseri
Groupe de Physique des Solides
Universites Paris 7 et 6, Tour 23
2, pl. Jussieu
75005 Paris
FRANCE

Dr. Zorka Papadopolos
Institut für Theoretische Physik
Universität Tübingen
Auf der Morgenstelle 14
72076 Tübingen
GERMANY

Dr. Natalie M. Priebe
Dept. of Mathematics
Vassar College
Poughkeepsie, NY 12604
USA

Prof. Dr. James G. Propp
Dept. of Mathematics
University of Wisconsin
Madison, WI 53706
USA

Prof. Dr. Ian F. Putnam
Dept. of Mathematics and Statistics
University of Victoria
Victoria, BC, V8W 3P4
CANADA

Prof. Dr. Charles Radin
Dept. of Mathematics
University of Texas
Austin, TX 78712
USA

Prof. Dr. Rolando A. Rebolledo
Facultad de Matemáticas
Universidad Católica de Chile
Casilla 306
Santiago 22
CHILE

Prof. Dr. E. Arthur R. Robinson, Jr.
Dept. of Mathematics
George Washington University
Washington, DC 20052
USA

Dr. Max Scheffer
Inst. für Physik
Technische Universität Chemnitz
09107 Chemnitz
GERMANY

Dr. Uwe Schnell
Mathematisches Institut
Universität Siegen
Emmy Noether Campus
57068 Siegen
GERMANY

Prof. Dr. Egon Schulte
Dept. of Mathematics
Northeastern University
Boston, MA 02115
USA

Dr. Hermann Schulz-Baldes
Dept. of Mathematics
University of California
Irvine, CA 92697
USA

Prof. Dr. Marjorie Senechal
Dept. of Mathematics
Clark Science Center
Smith College
Northampton, MA 01063
USA

Prof. Dr. Christian Skau
Dept. of Mathematics
Norwegian University of Science
and Technology
7034 Trondheim
NORWAY

Prof. Dr. Boris Solomyak
Dept. of Mathematics
University of Washington
Seattle, WA 98195-4350
USA

Dr. Dominique Spehner
Fachbereich Physik
Universität-GH Essen
45117 Essen
GERMANY

Prof. Dr. Jean-Louis Verger-Gaugry
Institut Fourier
CNRS, BP 74
100, rue des Mathématiques
38402 St. Martin d'Hères
FRANCE