Mathematisches Forschungsinstitut Oberwolfach

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Arnold Conjecture and Floer Homology

October 8th – October 12th, 2001

The Arbeitsgemeinschaft was organized by Dietmar Salamon (Zürich) and Matthias Schwarz (Leipzig).

The course of this Arbeitsgemeinschaft was a series of 16 1-hour presentations with subsequent discussions. The main object of the meeting was Floer homology in the symplectic context. The first talks were focussed on an introduction, providing the necessary analytical concepts for defining Floer homology. Whereas Floer homology was at first motivated by the proof of the Arnold conjecture, the focus was soon directed towards the more general version provided for arbitrary symplectic diffeomorphisms rather than merely nondegenerate Hamiltonian ones. The central object was the presentation of Floer homology as a functor (cf. the exposition in the program for this meeting). After discussing the relation of the ring structure on Floer homology with the quantum cohomology ring the emphasis of the last third of the meeting was laid on very recent application of Floer homology in view of Lagrangian intersections and Seidel's results based on the generalized Dehn twist.

The stimulating talks presented a fairly comprehensive introduction into the concept of Floer homology and provided access to very promising and recent developments in the field. They gave rise to fruitful discussions, in particular thanks to a very interested group of participants comprising also a fairly high proportion of extremely motivated graduate students and young researchers.

The lively atmosphere and inspiring environment of this meeting gave rise to a number of new contacts, and stimulating substantial impact promising further progress.

Abstracts

Lecture 1, Introduction: Arnold Conjecture, Symplectic Fixed points and Lagrangian Intersections

Joa Weber

In the first part we introduce symplectic manifolds and their Lagrangian submanifolds, and we show that the former are necessarily even-dimensional and orientable. We discuss some basic examples such as $(\mathbb{R}^{2n}, \omega_0), (T^*L, \omega_{\text{can}} = -d\lambda), (M \times M, -\omega \times \omega)$ and give examples of Lagrangian submanifolds in each case, such as graphs of closed 1-forms on L and graphs of symplectomorphisms of (M, ω). We state Darboux's Theorem and the Lagrangian neighbourhood theorem and define Hamiltonian symplectomorphisms.

In the second part we state and discuss the weak / strong Arnold Conjecture for symplectic fixed points in the degenerate as well as the nondegenerate case. A proof of the strong Arnold Conjecture is given for Hamiltonian symplectomorphisms which are sufficiently close to the identity. Here we use the facts from the first part. Finally, the Arnold Conjecture for Lagrangian intersections is briefly mentioned.

Lecture 2: Morse Homology

Ilya Dogolazky and Anna Pratoussevitch

We introduce the Morse complex and Morse homology, and we discuss some examples (different Morse functions on S^2, the 2-dimensional real projective space and the 2-dimensional torus). In the second part we introduce index pairs, describe the Morse complex with the help of index pairs and sketch the proof of the following

Theorem:

> The Morse complex is a chain complex ($\partial \circ \partial = 0$). The homology of the Morse complex is isomorphic to the singular homology of the manifold.

Lecture 3: Introduction to the concept of Floer homology

Annette Huber

In the first part of this overview we give an axiomatic characterization of Floer homology. It is a map

$$HF_*\colon \left\{ \begin{array}{c} \text{symplectic diffeom.} \\ \text{of } (M,\omega) \end{array} \right\} \quad \to \quad \left\{ \begin{array}{c} \text{fin. gen. } \mathbb{Z}/2 \text{ -graded,} \\ \Lambda\text{-modules} \end{array} \right\}$$

with Λ a principal ideal domain. The axioms are naturality, isotopy, identity, Lefschetz property, dimension, duality and product. In general, Λ has to be chosen as a Novikov ring.

In the second part, the construction of Floer homology for Hamiltonian diffeomorphisms was sketched under the simplifying assumptions $c_1|_{\pi_2(M)} = [\omega]|_{\pi_2(M)} = 0$. In particular, the moduli space of connecting orbits was defined and key properties listed.

Lecture 4: Floer's connecting orbits

Sebastian Götte

In contrary to ordinary Morse theory, the gradient of the action functional does not define a flow on the space of loops in symplectic manifolds. Instead, one has to solve a PDE.

Thus, Floer theory gives one periodic orbits of a Hamiltonian diffeomorphism, and cylinders $u\colon \mathbb{R} \times S^1 \to M$ satisfying

$$(*) \qquad \partial_s u + J\partial_t u - \nabla H_t = 0,$$

and which sit between periodic orbits. It turns out that these "connecting solutions" are precisely the solutions of $(*)$ of finite energy.

If one has a sequence of finite energy solutions of $(*)$ such that the energy remains bounded, one would like a subseqence to converge to a limit solution. However, there are two phenomena: Concentration of energy at certain points leads to the development of "bubbles", which are pseudo-holomorphic spheres attached to the limit solution. Also, energy disappearing "at infinity" gives rise to new limit solutions that one sees after shifting the s-parameter in $(*)$.

Finiteness of the number of "bubbles" is guaranteed because every solution of $(*)$ and every pseudoholomorphic sphere has energy bounded below by an a priori given quantity.

Lecture 5: Floer homology for symplectic fixed points I: Fredholm theory

Gregor Noetzel

In order to give a manifold structure to the space of "connecting orbits", i.e. the space of solutions of the Floer equation which connect periodic solutions of the Hamiltonian system, one has to express these solutions as zeros of a section of a suitable Banach bundle over a Banach manifold of maps $\mathbb{R} \times S^1 \to M$. Computing the differential at such a solution gives an operator $F_S\colon W^{1,p} \to L^p$ of the form $F_S\xi = \partial_s\xi + J_o\partial_t\xi + S\xi$ where $S \in C^\infty(\mathbb{R} \times S^1, \mathbb{R}^{2n\times 2n})$ with limits $S^\pm(t) = \lim_{s\to\pm\infty} S(s,t)$. In the lecture it is shown that F_S is a Fredholm operator and the index depends only on the "ends" $S^\pm$. The spectral flow is introduced and an example of computation in the case of Morse homology, the finite-dimensional analogue of Floer homology, is given. Then we sketch a proof of

$$\operatorname{ind} F_S = \mu_{cz}(S^+) - \mu_{cz}(S^-)$$

where $\mu_{cz}(\cdot)$ is a suitably defined integer, called Conley-Zehnder-index, which is characterized axiomatically. The proof consists of using the axioms to bring F_S into an easy-to-compute form. Finally it is indicated how one defines the Conley-Zehnder-index for a periodic solution.

Lecture 6: Floer homology for symplectic fixed points II: Transversality

Alberto Abbondandolo

Let (M,ω) be a symplectic manifold, $\{J_t\}_{t\in S^1}$ an almost complex structure compatible with ω, and $\{X_t\}_{t\in S^1}$ symplectic vector fields. Our aim is to show that for a generic choice of $\{J_t\}$, the space $\mathcal{M}(x^-, x^+, X_t, J_t)$ of smooth maps $u\colon \mathbb{R}\times S^1 \to M$ solving

$$(*) \qquad \overline{\partial}_{J,X} u = \partial_s u + J_t(u)(\partial_t u - X_t(u)) = 0, \quad u(s,\cdot) \overset{s\to\pm\infty}{\longrightarrow} x^\pm(\cdot),$$

is a manifold (where $x^\pm$ are nondegenerate 1-periodic solutions of $\dot{x} = X_t(x)$). The map $\overline{\partial}_{J,X}$ can be seen as a smooth section of a Banach bundle $\mathcal{E} \to \mathcal{W}$, and its fiberwise derivative is $D_{u,J} : W^{1,p}(u^*TM) \to L^p(u^*TM)$,

$$D_{u,J}v = \nabla_s v + J_t(u)\nabla_t v + \nabla_v J_t(u) - \nabla_v(J_t X_t)(u)\,.$$

We know from Lecture 5 that it is a Fredholm operator. In order to apply the implicit function theorem, we need $D_{u,J}$ to be onto. Introducing a space $\mathcal{J}$ of almost complex structures compatible with ω, we will prove that the set $\mathcal{Z} = \{(u,J) \in \mathcal{W} \times \mathcal{J} | \overline{\partial}_{X,J} u = 0\}$ is a manifold. Then every regular value J of the projection $\mathcal{Z} \to \mathcal{J}$ is such that $D_{u,J}$ is onto for every solution u, and the Sard-Smale theorem implies that the set of regular values is residual.

To prove that $\mathcal{Z}$ is a manifold, we see it as the zero-set of the section

$$\mathfrak{J} : \mathcal{W} \times \mathcal{J} \to \mathcal{E}, \quad \mathfrak{J}(u,J) = \overline{\partial}_{J,X} u$$

whose fiberwise derivative is $D\mathfrak{J}(u,J)[v,Y] = D_{u,J}v + Y_t(u)(\partial_t u - X_t(u))$. Its kernel splits, and to prove that it is onto, it is enough to check that the range is closed. Every $\eta \in L^q(u^*TM)$ in the annihilator of such range solves an equation like $\partial_s \eta + J(s,t)\partial_t \eta + C\eta = 0$ and satisfies $\int\int \langle Y_t(u) J_t(u) \partial_s u, \eta \rangle ds dt = 0$. The nontrivial facts:

- Every zero of a solution w of $\partial_s w + J(s,t)\partial_t w + C(s,t) w = 0$, $\partial_s w \not\equiv 0$ is isolated (Unique continuation),
- the set $\{(s,t) \in \mathbb{R} \times S^1 | \partial_s u(s,t) \neq 0\,,\ u(s,t) \notin u(\overline{\mathbb{R}} \setminus \{s\}, t)\}$ is open and dense,

easily imply that $\eta \equiv 0$.

Lecture 7: Floer homology for symplectic fixed points III: Gluing

Katrin Wehrheim

We complete the construction of Floer homology for a symplectomorphism ϕ in the monotone case: The Floer chain complex is generated by the fixed points of $\phi \circ \psi_1$ (where ψ_1 is the time-1-map of the Hamiltonian flow in the Floer equation and the boundary operator ∂ is defined by counting Floer's connection orbits of index 1). By transversality and compactness this is well-defined for generic (H,J) in the Floer equation. It remains to show that $\partial^2 = 0$ for generic almost complex structures J. Equivalently, one has to identify the broken flow lines of index $1+1$ with the ends of the space of Floer's connecting orbits of index 2. This is achieved by a gluing construction. We give this proof and present some technical foundations of general gluing constructions:

- Pregluing constructs approximate solutions of the Floer equation near broken flow lines.
- A quantitative implicit function theorem is used to find exact solutions near approximate solutions of the Floer equation and to obtain a local uniqueness property.
- The 'Linear gluing theorem' establishes the required Fredholm and surjectivity property and uniform estimates for the linearized Floer operator at 'preglued trajectories'.

Lecture 8: Continuation of Floer homology

Ralf Gautschi

Let (ϕ, γ) be a monotone pair, where ϕ is a symplectomorphism and γ a component of the twisted loop space. If $\{H_t^\alpha, J_t^\alpha\}$ and $\{H_t^\beta, J_t^\beta\}$ are regular, there exists an isomorphism

$$\Phi^{\alpha\beta} : HF(\phi, \gamma; \{H_t^\alpha, J_t^\alpha\}) \to HF(\phi, \gamma; \{H_t^\beta, J_t^\beta\})$$

such that

$$\Phi^{\beta\gamma} \circ \Phi^{\alpha\beta} = \Phi^{\alpha\gamma} \quad \text{and} \quad \Phi_{\alpha\alpha} = id\,.$$

The Proof is in 4 steps.

Step I. Construction of a homomorphism on the chain level

$$\varphi^{\alpha\beta} : CF(\varphi, \gamma; \{H_t^\alpha\}) \longrightarrow CF(\varphi, \gamma; \{H_t^\beta\})$$

This involves the analysis of an equation similar to Floer's equation:

$$\partial_s u + J_{s,t}(u)(\partial_t u - X_{H_{s,t}}(u)) = 0, \quad u \colon \mathbb{R}^2 \to M,$$

where $\{H_{s,t}, J_{s,t}\}$ is a homotopy from $\{H_t^\alpha, J_t^\alpha\}$ to $\{H_t^\beta, J_t^\beta\}$.

Step II. $\varphi^{\alpha\beta}$ is a chain map, i.e. $\varphi^{\alpha\beta} \circ \partial^\alpha = \partial^\beta \circ \varphi^{\alpha\beta}$. Similar to the proof that $\partial \circ \partial = 0$, this follows from a gluing theorem.

Step III. If $\{H_{s,t}^0, J_{s,t}^0\}, \{H_{s,t}^1, J_{s,t}^1\}$ are homotopies, there is

$$T \colon CF(\varphi, \gamma; \{H_t^\alpha\}) \longrightarrow CF(\varphi, \gamma; \{H_t^\beta\})$$

such that

$$\varphi_1^{\alpha\beta} + \varphi_0^{\alpha\beta} = \partial^\beta T + T\partial^\alpha\,.$$

The proof requires a parametrized version of the PDE above.

Step IV. That $\Phi^{\beta\gamma} \circ \Phi^{\alpha\gamma} = \Phi^{\alpha\gamma}$ and $\Phi^{\alpha\alpha} = id$ is proved by choosing special homotopies.

Lecture 9: Floer-Cohomology for Weakly-Monotone Manifolds

Daniel Roggenkamp and Anna Wienhard

The construction of Floer-cohomology is extended to the class of weakly-monotone manifolds (i.e. for $A \in \pi_2(M) \quad 3 - n \leq c_1(A) < 0$ implies $\omega(A) \leq 0$). In comparison to the monotone case (i.e. $\omega(A) - \lambda c_1(A) = 0 \quad \forall A \in \pi_2(M)$ for fixed $\lambda \geq 0$) one has to deal with the following additional problems. To obtain energy bounds on the one- and two-dimensional components of the moduli spaces of connecting orbits, one takes into account the trivialization of TM along the contractible one-periodic orbits of the Hamiltonian flow, necessary for the definition of the Conley-Zehnder-Index, i.e. one works over a suitable covering of the loop space. As a consequence the cochain groups resulting from this construction are infinite-dimensional vector spaces over $\mathbb{Z}_2$, but they are finite-dimensional over some Novikov ring.

Imposing stronger, but still generic regularity conditions on the almost complex structure and the Hamiltonian function, one can exclude bubbling-off of J-holomorphic spheres with $c_1 \leq 0$ in the one- and two-dimensional components of the moduli spaces of connecting orbits. Thus, the coboundary operators are well-defined.

Lecture 10: Ring Structure I: Pairs-of -Pants product

Dan Fulea

The talk focused on the introduction of the duality structure and product structure on the Floer-homology $HF((M, \omega), \mathbb{F}_2) = HF(id_M)$. For the construction of the multilinear operators on Floer homology, $O(\Sigma)$ associated to topological surfaces with cylindrical ends, the following steps were discussed:

1. Explicit definition of the model surface Σ (with supplementary fixed structure).

2. Differential geometric Banach manifolds involved and associated Banach bundles. These serve as the definition domain and value domain of an elliptic differential operator.
3. The operator $\bar{\partial}_{J,k}$ seen as a section of a Banach manifold with values in a canonical Banach bundle over it.
4. Solution sets of the non-linear PDE $\bar{\partial}_{J,k}(u) = 0$ are finite dimensional manifolds because of the corresponding Fredholm properties of the linearization $D_u := D_u\, \bar{\partial}_{J,k}$ at a solution u of $\bar{\partial}_{J,k}(u) = 0$. Because of the local character of the problem one has a situation already encountered in previous talks, where analogous $\bar{\partial}$-operators were studied on $\Sigma = \mathbb{R} \times S^1$, the infinite cylinder.
5. The moduli spaces of solutions with further special conditions: $\mathcal{M}(\underline{x}, \underline{y}; J, k, \dots)$, where $\underline{x}, \underline{y}$ fix the boundary conditions for solutions in terms of 1-periodic solutions of the Hamiltonian equation, J is a choice of an almost complex structure J on M and k is a suitable extension of the Hamiltonian vector field $X_H(s,t)$ which is at first only well-defined on the cylindrical ends of Σ.
6. Definition of the operation $O = O(\Sigma, J, \dots)$: For fixed $\underline{x}, \underline{y}$ define the number (mod 2): $< \underline{x}, \underline{y} > := \#\mathcal{M}(\underline{x}, \underline{y}; J, \dots)$. Set $O[\underline{x}] := \sum_{\underline{y}} < \underline{x}, \underline{y} > \cdot [\underline{y}]$.

A partial sketch of the compatibility of the operation O with the boundary operator ∂ of the complex computing the Floer homology, $O\partial = \partial O$ was given.

Lecture 11: Ring Structure II: Quantum Cohomology

Urs Frauenfelder

The quantum cup product is a deformation of the ordinary cup product by interaction with J-holomorphic spheres. It can either be defined using Gromov-Witten invariants or Morse-theoretically as a "spiked-sphere" product.

The spiked-sphere product is defined by counting spheres with two marked points on the unstable manifolds of two critical points and with one marked point on the stable manifold of a critical point. These critical points are associated to three auxiliary, generically chosen Morse functions.

The spiked sphere product corresponds to the Pair-of-Pants product in Floer-homology, the ring structure introduced in the previous lecture.

Lecture 12: Lagrangian intersections

Ursula Hamenstädt

A Lagrangian submanifold L of a (compact) symplectic manifold (M, ω) is monotone if there exists $\lambda > 0$ s.th. $\mu = \lambda\omega$ on $\pi_2(M, L)$, where μ is the Maslov homomorphism and ω is integration by ω. For two monotone Lagrangian submanifolds $L, L_1 \subset M$ which intesect tranversely and such that the generator of the Maslov homomorphism is at least 3 it is possible to construct Floer homology. The chain complex is the free $\mathbb{Z}_2$ -vector space over the intersection points $L \cap L_1$. Connecting orbits are holomorphic discs of bounded energy with respect to some fixed compatible (time dependent) almost complex structure J and of Maslov-index 1. A new phenomenon is bubbling of discs which is prevented for discs of Maslov-index ≤ 2 by the assumption on the Maslov-homomorphism.

The resulting Floer-homology can be used to show (Oh): If ϕ_t is a Hamiltonian isotopy of $\mathbb{C}P^n$ s.th. $\phi_1\mathbb{R}P^n$ intersects $\mathbb{R}P^n$ transversely, then $\#\,\mathbb{R}P^n \cap \phi_1\mathbb{R}P^n \geq n+1$.

Lecture 13: Oh's spectral sequence and applications

Octav Cornea

Oh's spectral sequence is defined for a monotone Lagrangian $L \subset (M, \omega)$ such that the minimal Maslov number (i.e. as above, the positive generator of the Maslov homomorphism) $\Sigma_L \geq 3$. It has the following structure:

$$E_1 \simeq H^*(L; \mathbb{Z}/2); \qquad d_n : E_r^* \to E_r^{*-r\Sigma_L+1}$$

and it collapses to the Floer cohomology of L, $HF^*(L; M)$. The purpose of this spectral sequence is to describe the contribution of the "long" trajectories to the Floer complex associated to a pair of Lagrangians of type $L_0 = L$, $L_1 = \text{graph}(df)$ with f a sufficently C^2-small Morse function on L. (the "long" trajectories are those that are not completely included in a Darboux neighbourhood of L). I have then discussed an application of this spectral sequence that is due to Biran:
Theorem (Biran)

> If $L \subset \mathbb{C}P^n$ is a Lagrangian submanifold with $H_1(L, \mathbb{Z})$ 2-torsion, then $H^*(L; \mathbb{Z}/2) \simeq H^*(\mathbb{R}P^n; \mathbb{Z}/2)$ as vector spaces and, if n is even, as algebras.

Lecture 14: Generalized Dehn twists

Thilo Kuessner

Let V be a Lagrangian sphere in a symplectic manifold (M, ω). We show that the Dehn twist τ at V can be realized by a symplectomorphism. If $\dim M = 4$ we give an explicit homotopy to see that τ^2 is homotopic to the identity in $\text{Diff}(M)$. Under additional assumptions (see the following lecture) it holds in particular that $\pi_0\text{Symp}(M, \omega) \to \pi_0\text{Diff}(M)$ is not injective.
In the second part of the talk, we showed that this applies to a large number of symplectic $4-$manifolds: If a Kähler manifold E^{2n+2} fibers over the disc D^2 with singularities of complex Morse type, then the fiber (M^{2n}, ω) contains a Lagrangian sphere. Conversely, any Dehn twist at a Lagrangian sphere can be realized as the monodromy of an almost holomorphic fibration with Morse singularities.

Lecture 15: Seidel's exact sequence and symplectic isotopy

Dietmar Salamon

Seidel's exact triangle for a Dehn twist along a Lagrangian 2-sphere $L \subset M^4$ has the form

$$\begin{array}{ccc} HF(id) & \longrightarrow & HF(\tau_L) \\ \nwarrow & & \swarrow \\ & \Lambda \oplus \Lambda & \end{array}$$

Here Λ denotes the universal Novikov ring. Its elements have the form $\lambda = \sum \lambda_\varepsilon t^\varepsilon$, $\lambda_\varepsilon \in \mathbb{Z}_2$ where the sum runs over $\varepsilon \in \mathbb{R}$ and $\#\{\varepsilon \in \mathbb{R} | \lambda_\varepsilon \neq 0, \varepsilon \leq c\} < \infty$ for all $c \in \mathbb{R}$. The term $\Lambda \oplus \Lambda$ replaces the Floer homology group $HF(L, L)$. This exact sequence exists, for example, whenever $c_1(TM) = \lambda[\omega]$, $\lambda \leq 0$.

In his thesis Seidel used the exact sequence to prove that, as a module over $H_*(M,\Lambda)$, the Floer homology of τ_L is isomorphic to the quotient of $H_*(M,\Lambda)$ by the submodule generated by $[pt]$ and $[L]$:

$$HF(\tau_L) \cong \frac{H_*(M;\Lambda)}{\operatorname{span}([pt],[L])} \tag{1}$$

This implies the following
Theorem (Seidel)

> If $b_1(M) = 0, b_2(M) \geq 3, c_1(TM) = \lambda[\omega]$ for some $\lambda \leq 0$, and $L \subset M$ is a Lagrangian sphere, then $\tau_L \circ \tau_L$ is not symplectically isotopic to the identity.

The proof is based on the observation, that, if $\tau_L \circ \tau_L$ were symplectically isotopic to the identity then, since $b_1 = 0$, τ_L would be Hamiltonian isotopic to τ_L^{-1}. This would imply the existence of a nondegenerate pairing on $HF(\tau_L)$ compatible with the module structure over $H_*(M;\Lambda)$.
However, the formula (1) implies that such a pairing cannot exist, when $b_2 \geq 3$.
Corollary (Seidel)

> If M^4 is a complete intersection, other than $\mathbb{C}P^2$ or $\mathbb{C}P^1 \times \mathbb{C}P^1$, then there exists a symplectomorphism $\varphi : M \to M$ which is smoothly, but not symplectically, isotopic to the identity.

Lecture 16: Graded Lagrangian submanifolds

Matthias Schwarz

Using an additional structure on Lagrangian submanifolds, a so-called grading (extending the notion of oriented Lagrangian Manifolds) Seidel extended the construction of Floer homology $HF(L_1, L_2)$ so that one obtains an absolute grading rather than a purely relative one as in Floer's original setup. This allows finer distinctions of Hamiltonian isotopy classes of embedded Lagrangian submanifolds, in particular in view of the generalized Dehn twist. Based on graded Lagrangian manifolds, Seidel has given a new and also more general proof for the following Theorem:
Theorem(Seidel)

> Let (M^{2n}, ω) be a compact symplectic manifold with contact type boundary, assume n even and ω exact, $2 \cdot c_1(M,\omega) = 0$. If M contains an (A_3)-configuration of Lagrangian spheres (l_0, l_1, l_2), $l_i : S^n \hookrightarrow M$ then M contains infinitely many symplectically knotted spheres, i.e. which are pairwise not Lagrangian isotopic, but which are pairwise isotopic.

An (A_k)-configuration, $k \geq 2$ is a collection of Lagrangian spheres $(l_1, \dots, l_k)$ s.t. for $L_i = l_i(S^n)$:

1. $L_i \cap L_j = \emptyset$ for $|i-j| \geq 2$,
2. $|L_i \cap L_{i\pm1}| = 1$,
3. $L_i \pitchfork L_{i\pm1}$ intersect transversely.

The proof of the above theorem is then based on showing that for an (A_3)-confiuration

$$L_1^{(k)} := \tau_{l_2}^{2k}(L_1) \not\sim_\omega L_1\,,$$

i.e. not Hamiltonian isotopic, although by Haefliger's version of the h-principle $\tau_{l_2}^{2k} \circ l_1 \sim l_1$, isotopic through embeddings. Here τ_{l_2} is the generalized Dehn twist along l_2.

The new idea is to see that although original Floer homology gives

$$\mathbb{Z} \cong HF(L_0, L_1) \cong HF(L_0; L_1^{(k)}) \cong HF(L_1, L_2) \cong HF(L_1^{(k)}, L_2)$$

the refined version for graded Lagrangians allows to distinguish different degrees in Floer homology, namely

$$\begin{aligned} HF(\tilde{L}_0, \tilde{L}_1^{(k)}) &= \mathbb{Z}_2 \quad \text{exactly in degree } 0, \\ HF(\tilde{L}_1^{(k)}, \tilde{L}_2) &= \mathbb{Z}_2 \quad \text{exactly in degree } 2k(1-n), \end{aligned}$$

which implies $k = 0$ if $L_1^{(k)} \sim_\omega L_1$.

Edited by Matthias Schwarz

Participants

Prof. Dr. Alberto Abbondandolo
abbo@sns.it
Scuola Normale Superiore di Pisa
Piazza dei Cavalieri 7
I-56126 Pisa

Peter Albers
albers@mathematik.uni-leipzig.de
Fakultät für Mathematik/Informatik
Universität Leipzig
Augustusplatz 10/11
04109 Leipzig

Dr. Jean-Francois Barraud
Jean-Francois.Barraud@agat.univ-lille1.fr
Universite de Lille I
U.F.R. de Mathematiques Pures
et Appliquees
F-59655 Villeneuve d' Ascq Cedex

Igor Burban
burban@mathematik.uni-kl.de
Fachbereich Mathematik
Universität Kaiserslautern
Erwin-Schrödinger-Straße
67653 Kaiserslautern

Prof. Dr. Octavian Cornea
cornea@gat.univ-lille1.fr
UFR de Mathematiques
Universite de Lille I
F-59655 Villeneuve d'Ascq. Cedex

Mihai Damian
damian@math.u-strasbg.fr
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes
F-67084 Strasbourg Cedex

Prof. Dr. Christopher Deninger
deninge@math.uni-muenster.de
Mathematisches Institut
Universität Münster
Einsteinstr. 62
48149 Münster

Ilya Dogolazky
ilyad@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
53115 Bonn

Urs Frauenfelder
ufrauenf@math.ethz.ch
ETH Zürich
Hofstr.24
CH-8136 Gattikon

Dr. Jan Fricke
fricke@uni-greifswald.de
Inst. für Mathematik und Informatik
Universität Greifswald
Friedrich-Ludwig-Jahn-Str. 15a
17489 Greifswald

Dan Fulea
fulea@euklid.math.uni-mannheim.de
Oppauer Str. 37
68305 Mannheim

Ralf Gautschi
ralf.gautschi@math.ethz.ch
ETH Zürich
Abteilung für Mathematik
CH-8092 Zürich

Dr. Sebastian Goette
sebastian.goette@uni-tuebingen.de
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10
72076 Tübingen

Basak Zehra Gurel
basak@math.ucsc.edu
Department of Mathematics
Baskin Engineering Building 379
University of California
Santa Cruz CA 95064
USA

Prof. Dr. Ursula Hamenstädt
ursula@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
53115 Bonn

Prof. Dr. Annette Huber-Klawitter
huber@mathematik.uni-leipzig.de
Mathematisches Institut
Universität Leipzig
Augustusplatz 10/11
04109 Leipzig

Dr. Bruno Klingler
klingler@math.polytechnique.fr
Institut des Hautes Etudes
Scientifiques
Le Bois Marie
35, route de Chartres
F-91440 Bures-sur-Yvette

Dr. Bernd Kreussler
kreusler@mathematik.uni-kl.de
Fachbereich Mathematik
Universität Kaiserslautern
Erwin-Schrödinger-Straße
67653 Kaiserslautern

Thilo Kuessner
thilo@whitney.mathematik.uni-tuebingen.de
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10
72076 Tübingen

Prof. Dr. Herbert Kurke
kurke@mathematik.hu-berlin.de
Institut für Mathematik
Humboldt-Universität
10099 Berlin

Ursula Beate Ludwig
uludwig@mathematik.uni-leipzig.de
Mathematisches Institut
Universität Leipzig
Augustusplatz 10/11
04109 Leipzig

Dr. Boudewijn Moonen
B.Moonen@ipbi.ipb.uni-bonn.de
Institut für Photogrammetrie
Universität Bonn
Nussallee 15
53115 Bonn

Gregor Noetzel
ngregor@web.de
Mathematisches Institut
Universität Leipzig
Augustusplatz 10/11
04109 Leipzig

Dr. Anna Pratoussevitch
anna@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
53115 Bonn

Daniel Roggenkamp
roggenka@th.physik.uni-bonn.de
Universität Bonn
Physikalisches Institut
Nußallee 12
53115 Bonn

Prof. Dr. Dietmar Salamon
salamon@math.ethz.ch
Mathematik Departement
ETH-Zentrum
Rämistr. 101
CH-8092 Zürich

Prof. Dr. Matthias Schwarz
mschwarz@mathematik.uni-leipzig.de
Mathematisches Institut
Universität Leipzig
Augustusplatz 10/11
04109 Leipzig

Dr. Joachim Weber
joa@math.ethz.ch
ETH Zürich
Abteilung für Mathematik
CH-8092 Zürich

Katrin Wehrheim
wehrheim@math.ethz.ch
Stüssistrasse 101
CH-8057 Zürich

Anna Wienhard
wienhard@math.uni-bonn.de
Mathematisches Institut
Universität Bonn
Beringstr. 1
53115 Bonn