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Stable Laws, Processes and Applications

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The present conference was organized by Jan Rosinski (Knoxville), Gennady Samorodnitsky (Ithaca) and Werner Linde (Jena). The 23 talks that were delivered during the conference covered many aspects of the theory of stable measures and processes. Moreover, several aspects of applications were presented. In the following we include the abstracts of the talks in alphabetical order.

Abstracts

Some aspects of selfdecomposability

OLE E. BARNDORFF-NIELSEN (AARHUS)

After a brief recall of various key characterisations of selfdecomposability (in the sense of Paul Lévy), results from three recent investigations, where selfdecomposability has played a seminal role, were reported:

- Multivariate subordination was introduced in [BNPS] and, in particular, it was shown there that, subject to some mild regularity conditions, operator selfdecomposability of the subordinator together with operator stability of the subordinand implies operator selfdecomposability of the subordinated process.
- In the type of stochastic volatility models discussed in a series of papers ([BNS1], [BNS2] and [BNS3]), selfdecomposability is a key concept. For these models, and indeed for substantial generalisations thereof, a mixed normal limiting behaviour of arbitrary powers of absolute returns has been established in [BNS4].
- The paper [BNT1] introduced the concept of selfdecomposability in free probability and contributed to the study of the close analogy between properties of infinite divisibility and Lévy processes in classical probability and in free probability. The paper [BNT2] surveys and improves on these results and moreover presents a free analogue of the Lévy-Ito representation.

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[BNS1] Barndorff-Nielsen, O.E. and Shephard, N. (2001a): Modelling by Lévy processes for financial econometrics. In O.E. Barndorff-Nielsen, T. Mikosch and S. Resnick (Eds.): *Lévy Processes - Theory and Applications.* Boston: Birkhäuser. Pp. 283-318.

[BNS2] Barndorff-Nielsen, O.E. and Shephard, N. (2001b): Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial economics (with Discussion). *J. R. Statist. Soc.* B. **63**, 167-241.

[BNS3] Barndorff-Nielsen, O.E. and Shephard, N. (2001c): Econometric analysis of realised volatility and its use in estimating stochastic volatility models. *J. R. Statist. Soc.* **B 64**. (To appear.)

[BNS4] Barndorff-Nielsen, O.E. and Shephard, N. (2001d): Realised power variation and stochastic volatility models. (Submitted.)

[BNT1] Barndorff-Nielsen, O.E. and Thorbjørnsen, S. (2001a): Selfdecomposability and Lévy processes in free probability. *Bernoulli* (To appear.)

[BNT2] Barndorff-Nielsen, O.E. and Thorbjørnsen, S. (2001b): Lévy laws and processes in free probability. (Submitted.)

Entrance laws of semi–stable Markov processes, exponential functionals of Lévy processes and some factorizations of the exponential laws

JEAN BERTOIN (PARIS VI)

(joint work with Marc Yor and M. E. Caballero)

We consider the asymptotic behavior of semi-stable Markov processes valued in $]0,\infty[$ when the starting point tends to 0. The entrance distribution is expressed in terms of the exponential functional of the underlying Lévy process which appears in Lamperti's representation of a semi-stable Markov process. In the case when the underlying Lévy process is a subordinator (i.e. has increasing sample paths, the moments of the exponential functional can be computed explicitly, and this yields a remarkable factorization of the exponential variable.

Potential theory of Schrödinger operator based on fractional Laplacian

Tomasz Byczkowski (Wroclaw)

In our talk we review some recent results concerning potential theory of symmetric (rotationally invariant) α-stable processes and of Feynman-Kac semigroups based on them. Among the most important topics we discuss the Boundary Harnack Principle and the Conditional Gauge Theorem for Lipschitz domains. Results we mention were proved by K. Bogdan, in some joint papers by K. Bogdan and T. Byczkowski and also in the papers of Z.-Q. Chen and R. Song.

Censored stable processes

Zhen–Qing Chen (Seattle)

A censored stable process Y in a Euclidean open set D is a symmetric stable process which is not allowed to make jumps outside D. In this talk, we discussed some recent results on the basic properties of censored stable processes. An almost sharp answer is given to the question of when the process Y has finite lifetime, and if it is finite, whether the left limit of Y_t at its lifetime exists under Euclidean topology. We presented boundary Harnack inequality for harmonic functions in bounded $C^{1,1}$-smooth open set D, as well as the sharp two-sided estimates of Green functions of Y and the identification of Martin boundary of Y in D. Pure jump Girsanov transform of Y and its application to censored relativistic stable processes were described. Hardy inequality for censored stable processes is also mentioned.

Limit theory for some non-linear time series models including GARCH and stochastic volatility models

Richard A. Davis (Fort Collins)

In deriving limit theory for the sample autocorrelation function (ACF) of a time series with either an infinite fourth moment or an infinite second moment, two conditions are typically imposed. The first is a regular variation condition that requires the finite dimensional distributions to be jointly regularly varying. The second is a mixing condition specifying the rate and nature at which events become asymptotically independent. It can be shown that a large class of nonlinear time series models, including those arising from a stochastic recurrence equation such as GARCH, and stochastic volatility (SV) models satisfy these conditions. The limit distribution for the sample ACF can be described as the ratio of two stable distributed random variables. Interestingly, the behavior of the sample ACF for GARCH and SV models are vastly different. In the SV case, the limit behavior is comparable to that of the sample ACF of an iid sequence with heavy tails. In particular, the rate of convergence to 0 in probability is faster the heavier the tail. For GARCH processes, the behavior is just the opposite. The sample ACF does not even converge to a constant in probability for sufficiently heavy tails. The contrasts in sampling behavior of the ACF for GARCH and SV models will be illustrated via simulation.

Domains of attraction of stable distributions in Euclidean space

Laurens de Haan (Rotterdam)

Well-known necessary and sufficient conditions are derived for the domains of attraction using characteristic functions and regular variation rather than the theory of infinite-divisible distributions. The conditions in terms of the characteristic function are less well-known. The higher-dimensional case is unfinished as yet.

On one–dimensional stochastic equations driven by symmetric stable processes

Hans–Jürgen Engelbert (Jena)

We study stochastic equations

$$X_t = x_0 + \int_0^t b(u, X_{u-})\, dZ_u, \quad t \geq 0,$$

driven by one-dimensional symmetric stable processes Z of index α with $0 < \alpha \leq 2$. Here $b : [0, +\infty) \times R \to R$ denotes a measurable diffusion coefficient and $x_0 \in R$ is the initial value. As special cases for the driving process Z, Brownian motion ($\alpha = 2$) and the Cauchy process ($\alpha = 1$) are included. We are interested in general conditions for existence and uniqueness of weak solutions. Our main results generalize recent results of P.A. Zanzotto who dealt with homogeneous diffusion coefficients b. The basic tool is time change of symmetric stable processes. Using the property that appropriate time changes of stochastic integrals with respect to symmetric stable processes are again symmetric stable processes with the same index, we present a new approach which completely unifies the treatment of two quite different cases: the continuous case ($\alpha = 2$) and the purely discontinuous case ($0 < \alpha < 2$).

Limit distributions of Studentized means

F. Götze (Bielefeld)
(joint work with G. P. Chistyakov)

Let X, X_j, $j \in \mathbb{N}$, be independent, identically distributed random variables with a probability distribution F. It is shown that Student's statistic of the sample $\{X_j\}_{j=1}^n$ has a limit distribution G such that $G(\{-1, 1\}) \neq 1$, if and only if:
1) X is in the domain of attraction of a stable law with some exponent $0 < \alpha \leq 2$,
2) $\mathbf{E}X = 0$ if $1 < \alpha \leq 2$;
3) if $\alpha = 1$, then X is in the domain of attraction of Cauchy's law and Feller's condition holds: $\lim_{n\to\infty} n\mathbf{E}\sin(X/a_n)$ exists and is finite, where a_n is the infimum of all $x > 0$ such that

$$nx^{-2}\big(1 + \int_{[-x,x)} y^2\, F\{dy\}\big) \leq 1\ .$$

If $G(\{-1, 1\}) = 1$, then Student's statistic of the sample $\{X_j\}_{j=1}^n$ has a limit distribution if and only if $\mathbf{P}(|X| > x)$, $x > 0$, is a slowly varying function.

Semistability on vector spaces and on groups – An overview

W. Hazod (Dortmund)

Assume $\mathbb{G}$ to be a simply connected nilpotent Lie group with Lie algebra $\mathfrak{G} = \mathbb{V}$, a finite dimensional vector space. Let $X_{n,k} = a_n(Y_k)$ be an array of group-valued random variables as before and let $\overset{o}{X}_{n,k} = \overset{o}{a}_n(\overset{o}{Y}_k)$ denote the corresponding array on the tangent space $\mathfrak{G}$ defined by $\exp(\overset{o}{Y}_n) = Y_n$, where $\overset{o}{a}_n$ denotes the differential of a_n defined by $\exp(\overset{o}{a}_n(\cdot)) = a_n(\exp(\cdot))$. We have $a_n \in \mathrm{Aut}(\mathbb{G})$ and $\overset{o}{a}_n \in \mathrm{Aut}(\mathfrak{G})$, the group of Lie algebra automorphisms $\subseteq \mathrm{GL}(\mathbb{V})$. (Note that exp: $\mathfrak{G} \to \mathbb{G}$ is a C^∞ -isomorphism, and $\mathrm{Aut}(\mathbb{G})$ and $\mathrm{Aut}(\mathfrak{G})$ are isomorphic.)

Thus to the array $X_{n,k} = a_n(Y_k)$ and to $S_n = \prod_{j=1}^{k(n)} a_n(Y_j)$ - the row product process *on the group* $\mathbb{G}$ - there correspond the array $\overset{o}{X}_{n,k} = \overset{o}{a}_n(\overset{o}{Y}_k)$ and the row sum process $\Sigma_n := \sum_{j=1}^{k(n)} \overset{o}{a}_n(\overset{o}{Y}_j)$ *on the vector space* $\mathbb{V}$, the objects of investigation in the theory of operator limit distributions on vector spaces.

During the last decade it could be shown that limits in distribution of S_n (on $\mathbb{G}$) correspond in a 1-1-way to limits of Σ_n (on $\mathbb{V}$) and vice versa, and moreover there exist a list of "translations" from the group case to the vector space case.

Large weighted occupation measures for certain stable processes

Jim Kuelbs (Madison)

Let $\{X(t) : t \geq 0\}$ be a symmetric stable process of index $\alpha \in (0, 2]$ with stationary independent increments and sample paths in $D[0, \infty)$. We assume $X(0) = 0$ with probability one, and for $t \geq 0$, $s > 0$, define $M(t) = \sup_{0 \leq u \leq t} |X(u)|$ and

$$\eta_s(t) = M(st)/(c_\alpha s/LLs)^{1/\alpha},$$

where the constant $0 < c_\alpha < \infty$ is given by

$$c_\alpha = - \lim_{\epsilon \to 0^+} \epsilon^\alpha \log P(M(1) \leq \epsilon)$$

and $LLs = \max(1, \log(\log s))$. We determine $\limsup_{t\to\infty} \Psi_c(t)$ where

$$\Psi_c(t) = t^{-1} \int_0^t I_{[0,c]}(\eta_s(1)\theta(s/t))ds,$$

for various weight functions θ, and $c > 0$. Proofs depend on weighted small ball probability estimates of the sup-norm of these processes, which are then used to obtain a functional law of the iterated logarithm. The occupation measure results are consequences of the law of the iterated logarithm.

Poisson analogues of Cameron-Martin formula and applications to shifted small balls

M. A. Lifshits (St. Petersburg and Lille)
(joint work with P. Deheuvels and E. Shmileva)

Let $\Pi(t)$ be a standard Poisson process. Consider $U_\rho(t) = \rho^{-1/2} (\Pi(\rho t) - \rho t)$. Recall that empirical processes are better approximated by U_ρ than by Wiener process. It is therefore interesting to study fine properties of U_ρ . Take a shift function f and estimate probabilities of shifted small balls with respect to the uniform norm $\mathbf{P}\left\{\|\mathbf{U}_\rho - \lambda \mathbf{f}\| \leq \mathbf{r}\right\}$ for $r \to 0$, $\rho \to \infty$, $\lambda \to \infty$.

We use the Skorokhod formula for mutual density of the distributions of processes with independent increments instead of Cameron-Martin one's which is suitable for Gaussian case. A reasonably chosen variation of intensity replaces here the Gaussian admissible shifts. We show (with P.Deheuvels) that Wiener-type small ball behavior for high-intensity Poisson process occurs in a larger zone than suggested by classical KMT method. On the other hand, there exists a zone of "intermediate intensities" where Poisson small ball behavior is nontrivial but different from its Wiener counterpart.

Continuity of stochastic integrals with respect to infinitely divisible random measures

Michael B. Marcus (New York)
(joint work with Jan Rosinski)

Let $\mathcal{S}$ be a δ-ring of Borel subsets of a Borel space S with the property that there exists an ascending sequence $\{S_n\} \subset \mathcal{S}$ such that $\bigcup S_n = S$. Let m be a σ-finite Borel measure on S that is finite on sets in $\mathcal{S}$. Let $\{\theta(\cdot, s)\}_{s\in S}$ be a measurable family of Lévy measures on R such that $\int_{|x|>1} |x|\, \theta(dx, s) < \infty$ for every $s \in S$. Assume that

$$\int_{S_n} \int (x^2 \wedge |x|)\, \theta(dx, s) m(ds) < \infty$$

for each n. Then there exists an independently scattered random measure $M = \{M(A) : A \in \mathcal{S}\}$ such that for every $A \in \mathcal{S}$

$$E \exp iuM(A) = \exp \int_A \int_{-\infty}^{\infty} \{e^{iux} - 1 - iux\}\, \theta(dx, s) m(ds).$$

Moreover, $E|M(A)| < \infty$ and $EM(A) = 0$ for every $A \in \mathcal{S}$.

Sufficient conditions for continuity are are given for the mean–zero infinitely divisible process

$$X(t) = \int_S f(t, s)\, M(ds) \qquad t \in T$$

where T is a compact metric space and $f : T \times S \to R$ (or C) is a deterministic function.

Stable laws, fractional derivatives, and anomalous diffusion

Mark M. Merschaert (Reno)

The density $p(x, t)$ of a stable Lévy motion $\{X_t : t \geq 0\}$ solves a fractional partial differential equation where the order of the fractional derivative equals the stable index α. This fractional PDE models anomalous diffusion, in which a passive tracer spreads at a faster rate than the classical diffusion equation predicts. The stable densities also reproduce the skewness and power law tails commonly observed in tracer tests. Operator Lévy motions are governed by more complex fractional PDEs that model multiscaling anomalous diffusion, in which the rate of spreading depends on the coordinate. Note: The slides used to give this talk, including references, can be found at http://unr.edu/homepage/mcubed/MFOoct01.pdf

Is a random vector regularly varying if all linear combinations of its components are regularly varying?

Thomas Mikosch (Copenhagen)

In some joint work with Bojan Basrak (EURANDOM) and Richard Davis (Colorado State, Fort Collins) the following problem arose. We needed to show that the finite-dimensional distributions of a GARCH process (used for modeling financial returns) are multivariate regularly varying which in turn can be used to show that the sample autocorrelation function of such a process has an infinite variance stable limit, provided certain moment conditions are satisfied.

The theory of stochastic recurrence equations which can be used for the squares of a GARCH process gives one that the linear combinations of a lagged vector of the squares are regularly varying (Kesten, 1973, Act. Math.). Thus we were faced with the problem to show that the lagged vector itself is regularly varying. This turns out to be a non-trivial question.

For a vector with arbitrary components and regularly varying linear combinations we solved the problem if the index of regular variation is not an integer. Then it follows that the vector itself is regularly varying. If the components of the vector are non-negative we can show the same result if the index of regular variation is not an even integer. The remaining cases are open.

On some new results for multivariate stable laws

Vygantas Paulauskas (Vilnius)

Let G_α, $0 < \alpha < 2$, be a stable distribution on $\mathbf{R}^d$ with spectral measure ρ. In the talk we present two new results, which give the final answer to some problems, known for several decades. The results are contained in joint papers with V. Bentkus and A. Juozulynas [1], [2]. The boundary of a set $A \subset \mathbf{R}^d$ we denote by ∂A, and $(\partial A)^\varepsilon$ is the ε-neighborhood of ∂A. Let $\mathcal{A}_c$ be class of convex subsets of $\mathbf{R}^d$. For any law G we denote $\eta(\mathcal{A}_c, G) = \sup_{\varepsilon>0}\{\sup_{A\in\mathcal{A}_c} G((\partial A)^\varepsilon)/\varepsilon\}$.

Theorem: *For any stable non-degenerate G_α we have the estimate*

$$\eta(\mathcal{A}_c, G_\alpha) \le C(\alpha, d)K(\alpha, \rho)$$

and there are explicit dependence of constants C and K on parameters d, α, and ρ.

The second result concerns the so-called Lévy–LePage representation of stable vectors. It is known that multidimensional stable law G_α admits the following representation of the type $G_\alpha = \mathcal{L}\left(\sum_{j=1}^{\infty} \Gamma_j^{-1/\alpha} X_j\right)$, $0 < \alpha < 2$, where $\Gamma_n = \lambda_1 + \cdots + \lambda_n$, and $\lambda_1, \lambda_2, \ldots$ are i.i.d. random variables with standard exponential distribution. We assume that random vectors $X_1, X_2, \ldots$ are i.i.d. with distribution F, $E|X_1|^\alpha < \infty$, $EX_1 = 0$ if $\alpha > 1$ and independent of $\Gamma_1, \Gamma_2, \ldots$. Denote by F_n the distribution of the sum $S_n = \sum_{j=1}^{n} \Gamma_j^{-1/\alpha} X_j$. In [2] we got optimal estimates of the quantity $\delta_n(\mathcal{B}) = \sup_{A\in\mathcal{B}} |F_n(A) - G_\alpha(A)|$, where $\mathcal{B}$ stands for the class of Borel subsets of $\mathbf{R}^d$.

References

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Multivariate type G and marginal infinite divisibility

Victor Pérez-Abreu (Guanajuato)

A one dimensional random variable x is said to be of type G if in law x is of the form $z\sqrt{s}$ where z and $s > 0$ are independent random variables with s being infinitely divisible and z having the standard normal distribution. In this talk we present two possible multivariate extensions. The first one is to random vectors x of the form $zS^{1/2}$ where z is an m-dimensional standard normal vector independent of the infinitely divisible nonnegative random $m \times m$ matrix S with Lévy measure V with support on the space of positive definite matrices M_m^+. We show that x has an infinitely divisible law with Lévy density $u(x) = \int_{\mathbf{M}_m^+} \varphi_m(x;\Sigma)V(\mathrm{d}\Sigma)$, where $\varphi_m(x;\Sigma)$ denotes the density function of the m-dimensional normal distribution with mean 0 and variance matrix Σ. As examples of such laws we have the multivariate Normal Inverse Gaussian and we show that any multivariate symmetric stable law has the representation $zS^{1/2}$.

The second class consists of those random vectors whose one dimensional marginals are of type G. We first introduce a general concept of marginal infinite divisibility of random matrices which allow us to construct interesting examples.

Decomposition of self-similar stable mixed moving averages

Vladas Pipiras (Boston)

It has been known for some time now that stable non-Gaussian processes with an invariance property are related to non-singular flows. This connection has been established and explored, for example, for stationary stable processes by Rosiński and for self-similar stable processes by Burnecki, Rosiński and Weron. In this talk, we will focus on stable processes which are both self-similar and have stationary increments. These processes are of interest because, in practice, one often wants a model to have these two characteristics. We will show that if X_α is a symmetric α-stable process with stationary increments given by the mixed moving average

$$X_\alpha(t) = \int_X \int_{\mathbb{R}} (G(x, t+u) - G(x,u)) M_\alpha(dx, du), \ t \in \mathbb{R},$$

where $G : X \times \mathbb{R} \mapsto \mathbb{R}$ is a deterministic function and M_α is a symmetric α-stable random measure on $X \times \mathbb{R}$ with the control measure $m_\alpha(dx, du) = \mu(dx)du$, and if X_α is also self-similar, then it is determined by a non-singular flow, a related cocycle and a semi-additive functional. By using the Hopf decomposition of the flow into its dissipative and conservative components, we will establish a unique decomposition in distribution of X_α into two independent processes

$$X_\alpha \stackrel{d}{=} X_\alpha^D + X_\alpha^C,$$

where the process X_α^D is determined by a nonsingular dissipative flow and the process X_α^C is determined by a nonsingular conservative flow.

Small and large time scale analysis of a network traffic model

SIDNEY RESNICK (ITHACA)
(joint work with Krishanu Maulik)

Empirical studies of the internet and WAN traffic have observed multifractal behavior at time scales below a few hundred milliseconds. There have been some attempts to model this phenomenon, but there is no model to connect the small time scale behavior with behavior observed at large time scales of bigger than a few hundred milliseconds. There have been separate analyses of models for high speed data transmissions, which show that appropriate approximations to large time scale behavior of cumulative traffic are either fractional Brownian motion or stable Lévy motion, depending on the input rates assumed. We bridge this gap and develop and analyze a model offering an explanation of both the small and large time scale behavior of a network traffic model based on the infinite source Poisson model. Previous studies of this model have usually assumed that transmission rates are constant and deterministic. We consider a non-constant, multifractal, random transmission rate at the user level which results in cumulative traffic exhibiting multifractal behavior on small time scales and self-similar behavior on large time scales.

Transience levels for Lévy processes with application to stable Lévy processes

KEN-ITI SATO (NAGOYA)
(joint work with Toshiro Watanabe)

Let $\{X_t\colon t \geq 0\}$ be a transient Lévy process on the d-dimensional Euclidean space. Let $L(B_a)$ be the last exit time of the process from the open ball B_a centered at 0 of radius a. Existence of $E[L(B_a)^\eta]$, the η-order moment of $L(B_a)$, is discussed. The set $\mathcal{T}$ of $\eta \geq 0$ such that $E[L(B_a)^\eta]$ is finite is called the transience level of $\{X_t\}$. 1. It is shown that $\mathcal{T}$ does not depend on a. 2. Under the assumption of strong non-lattice, the criterion for $\eta \in \mathcal{T}$ is given in terms of the logarithmic characteristic function $\psi(z)$ of X_1. 3. It is shown that, if $d \geq 3$ and $\{X_t\}$ is nondegenerate, then $\mathcal{T}$ includes $[0, d/2-1)$. 4. Let $d = 1$ and let X_1 have finite $(1+\delta)$-order moment for some $\delta > 0$. Description of $\mathcal{T}$ by properties of the Lévy measure ν of $\{X_t\}$ is given. 5. Complete description of $\mathcal{T}$ for transient stable Lévy processes for $d = 1$ and partial description of that for $d \geq 2$ is given.

Drift transforms and Green function estimates of discontinuous processes with applications to stable processes

RENMING SONG (URBANA)
(joint work with Zhen-Qing Chen)

Let E be a Lusin space and let m be a σ-finite Borel measure on E with $\mathrm{supp}[m] = E$. Let $X = (X_t,\, \mathbb{P}_x,\, x \in E)$ be a transient irreducible Borel right process on E. Suppose we have another transient Borel right process $\widehat{X} = (\widehat{X}_t,\, \widehat{\mathbb{P}}_x,\, x \in E)$ on E which is a strong dual of X with respect to the measure m. Under this strong duality assumption, X has a Green function which we denote by $G(x, y)$. Let (N, H) be a Lévy system for X and F a bounded function on $E \times E$ vanishing on the diagonal with $\inf_{x,y\in E} F(x,y) > -1$. Under some natural assumptions,

$$t \mapsto \sum_{0<s\le t} F(X_{s-}, X_s) - \int_0^t \int_E F(X_s, y) N(X_s, dy) dH_s$$

is a local martingale. Its Doleans-Dade exponential is

$$M_t = \exp\left(\sum_{s\le t} \ln(1 + F(X_{s-}, X_s)) - \int_0^t \int_E F(X_s, y) N(X_s, dy) dH_s\right).$$

M_t is a nonnegative local martingale. M_t is called a pure jump Girsanov transform of X. M_t is obviously a nonnegative supermartingale multiplicative functional of X. Let Y be the strong Markov process obtained from X via the Girsanov transform M. One of the main results of this paper is that the Green function of Y is comparable to that of X. We also have a generalization of this result for the mixture of pure jump Girsanov transform and Feynman-Kac transform.

For $\alpha \in (0,2)$, a relativistic α-stable process $Y = (Y_t, Q^x)$ in $\mathbf{R}^n$ is a Lévy process whose characteristic function is given by

$$e^{-t\left(|\xi|^2 + m^{2/\alpha})^{\alpha/2} - m\right)}, \quad \xi \in \mathbf{R}^n,$$

where $m > 0$ is a constant. By regarding the relativistic α-stable process Y as a perturbation of the symmetric α-stable process X and applying our general results, we get the Green function of the killed relativistic stable process on a bounded $C^{1,1}$ domain is comparable to that of the killed symmetric stable process on the same domain.

Integration with respect to fractional motions

Jerzy Szulga (Auburn)

Using the spectral analysis, we extend the rudimentary integral, defined first for simple functions with respect to a process with stationary increments $W_t = \int_0^\infty (\exp\{itw\} - 1) dZ_w$, to a maximal domain. Integrands form a Hilbert space of certain tempered distributions but they still could be seen as functions under mild conditions imposed on the spectral measure.

In particular, we describe completely the integrands related to the fractional Brownian motion. The integration is equivalent to the classical integration with respect to Brownian motion via a certain isomorphism playing the role of the chain rule. The obtained integral is not Riemannian, for an integrable function may be not integrable on a sub-interval. Extension to double or multiple integrals follows.

Almost verbatim procedure applies to a fractional Lévy stable motion (the spectral random measure dZ_w is a Lévy stable motion with a power weight).

Linearly additive random fields with independent increments on time-like curves

Shigeo Takenaka (Okayama)

Let V be a convex cone in R^n. A curve $\{\ell(t); t \in R_+\}$ in R^n is called a time-like curve if it holds $\{\ell(t); t \ge s\} \subset \ell(s) + V$, $\forall s$. A random fields $\{X(\mathbf{t}); \mathbf{t} \in V\}$ is called a multiparameter additive process if the restriction $X|_\ell(t) = X(\ell(t))$ is an additive process for any time like curve ℓ.

Consider an $S\alpha S$ linearly additive process $\{X(\mathbf{t}); \mathbf{t} \in R^n\}$ that is the restriction $X|_L$ on any strait line L is an $S\alpha S$ motion.

The result is: If there exists a convex cone V such that the parameter restriction $\{X(\mathbf{t}); \mathbf{t} \in V\}$ becomes a multiparameter additive process, then there uniquely exists a measure μ on the dual cone $V^* = \{u; u \cdot v \leq 0, \ \forall v \in V\}$ with which the process $\{X\}$ has representation

$$X(\mathbf{t}) = Y(S_{\mathbf{t}}),$$

where $\mathbf{Y} = \{Y(B); B \subset R^n\}$ is the $S\alpha S$-random measure controlled by μ and $S_{\mathbf{t}} = \{x; x \cdot \mathbf{t} \leq -1\}$. The converse is also true.

The structure of self-similar stable mixed moving averages

Murad S. Taqqu (Boston)

Let X be a symmetric α–stable process with stationary increments given by a mixed moving average representation and assume in addition, that X is self-similar. We obtain a decomposition of the process X, unique in distribution, into three independent components, which we characterize and associate with flows. The first component is associated with a dissipative flow. Examples include the limit of renewal reward processes, the so-called "random wavelet expansion" and Takenaka processes. The second component is associated with a conservative identity flow. Particular cases include linear fractional stable motions, log-fractional stable motion and the stable Levy motion. We provide an example of the third component, one which is associated with a conservative cyclic flow.

Edited by Werner Linde

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