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Functional Analytic and Complex Analytic Methods in the Theory of Linear PDE

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The meeting was organized by R. Meise (Düsseldorf), B.A. Taylor (Ann Arbor) and D. Vogt (Wuppertal). The 21 talks delivered during the meeting covered many different aspects of functional analysis and complex analysis as related to the theory of partial differential equations. The abstracts of the talks are presented in alphabetical order.

Abstracts

On a theorem of Lelong and Bremermann

Aydin Aytuna (Ankara)

The aim of this talk is to report on a particular result obtained in the ongoing joint research with V. Zahariuta.

Theorem. *Let M be a Stein manifold of dimension n and let p be a plurisubharmonic function on M. Then there is an l, $0 \leq l \leq 2n+2$, such that for every compact subset $K \subset M$ and every $\varepsilon > 0$ there exists a $C > 0$ and l analytic functions $f_1, \ldots, f_l$ on M such that*

$$p(z) - \varepsilon \leq \frac{1}{C} \max_{1 \leq i \leq l} \ln |f_i(z)| \leq p(z) + \varepsilon \ \ \forall z \in K.$$

I will say a few words about tyhe circle of ideas/problems that lead us to this result and consider some applications.

Evolution for overdetermined systems

Chiara Bioti (Ferrera)

We are concerned with the problem of evolution for (overdetermined systems of linear partial differential operators with constant coefficients; this means that we want to give necessary and/or sufficient conditions for the existence of solutions of the associated Cauchy problem. Hörmander gave a necessary and sufficient condition for evolution in the case of a scalar operator and Cauchy data on a hyperplane of $\mathbb{R}^N$ in the class of C^∞ functions. With Prof. Necinovich we generalized this condition to the case of (overdetermined) systems and Cauchy data on an affine subspace of $\mathbb{R}^N$ of arbitrary codimension, always in the C^∞ class. We obtain a necessary condition which naturally generalizes the one given by Hörmander, but we need a stronger condition for the sufficiency. These two conditions coincide in the case of Cauchy data given on a hyperplane, so that we find again that Hörmander original condition is necessary and sufficient for evolution. Then we generalized the above results to the case of (small) Gevrey functions obtaining a necessary condition and a stronger sufficient condition which coincide in the case of the hyperplane. Applying these results, for instance, to the classical heat operator

$$\partial_t - \Delta_x,$$

we find that the Cauchy problem admits at least a local solution in the C^∞ class and in the classes $\gamma^(\tau, s)$ of (small) Gevrey functions of order $s > 1$ in the space - variables $x \in \mathbb{R}^n$ and of order $\tau \geq 2s$ in the time - variable $t > 0$.

Weighted (LF) - spaces of holomorphic functions

José Bonet (Valencia)

We report on recent research about the projective description of weighted inductive limits of Fréchet spaces of holomorphic functions. In the first part, which is joint work with R. Meise, we denote by $\mathrm{VH} = ind_n \mathrm{HV_n}$ the (LF) -space of entire functions on $\mathbb{C}^n$ isomorphic, via the Fourier - Laplace transform, to the strong dual of the space of quasi - analytic functions of Roumieu type on $\mathbb{R}^n$. We show that VH coincides with its projective hull $\mathrm{H}\overline{\mathrm{V}}$

algebraically, but it has a strictly finer topology. Accordingly the topology of VH cannot be described by weighted sup - semi-norms. In the second part we report on joint work with K.D. Bierstedt. We show that certain weighted (LF) - spaces of holomorphic functions on the disc are a topological subspace of its projective hull if the weights in the original space are radial, vanish at the boundary and are of moderate growth. We use methods developed by Lusky.

The local Phragmén - Lindelöf principle and hyperbolicity conditions

Rüdiger W. Braun (Düsseldorf)

Reporting on joint work with R. Meise and B.A. Taylor, the local Phragmén - Lindelöf condition of Hörmander is characterized for analytic varieties in $\mathbb{C}^3$. The characterization is given in terms of a finite number of hyperbolicity conditions in conoids. A conoid is similar to a cone, but may shrink towards its tip at a faster than linear rate. Which conoids have to be considered is decided by looking at the singularities of certain limit varieties. Using a theorem of Hörmander, this result yields a characterization of the surjective constant coefficient partial differential operators

$$P(D) : \mathcal{A}(\mathbb{R}^+) \to \mathcal{A}(\mathbb{R}^+)$$

in geometric terms.

A Glaeser type theorem for the space of real analytic functions

Pawel Domański (Poznań)

Let $\varphi : \Omega_1 \to \Omega_2$ be a real analytic map between open subsets $\Omega_1 \subseteq \mathbb{R}^{d_1}$, $\Omega_2 \subseteq \mathbb{R}^{d_2}$, we define $C_\varphi(f) := f \circ \varphi$. We give a short survey of results of Glaeser, Bierstone, Milman and Pawlucki on closeness of the range of $C_\varphi : C^\infty(\Omega_2) \to C^\infty(\Omega_1)$.
Then, in a joint work with M. Langenbruch (Oldenburg), we analyse the analogous problem for $C_\varphi : \mathcal{A}(\Omega_2) \to \mathcal{A}(\Omega_1)$, where $\mathcal{A}(\Omega)$ denotes the space of real analytic functions on an open domain in $\mathbb{R}^d$. We find necessary conditions for C_φ to have closed range or to be open onto a closed subspace of $\mathcal{A}(\Omega_1)$. We show that even injective C_φ with closed range need not be open onto its image. Finally, we give a characterization of those injective C_φ which are open onto their images.

Applications of extension operators to solution operators for PDE

Leonhard Frerick (Wuppertal)

For a closed set $S \subseteq \mathbb{R}^n$ let $\mathcal{D}_S$ denote the ideal of those test functions on $\mathbb{R}^n$ which are flat on S. Then every constant coefficient linear PDO P(-D) acts surjectively on $\mathcal{D}'_S$. Concerning right inverses for P(-D) we give the following result:

Theorem. *Let $S \subset \mathbb{R}^n$ be compact. Then $P(-D) : \mathcal{D}'_S \to \mathcal{D}'_S$ has a continuous linear right inverse if and only if*

i) *$P(-D) : \mathcal{D}' \to \mathcal{D}'$ has a continuous linear right inverse and*
ii) *The space $\mathcal{E}_p(S)$ of those Whitney jets f on S which solve the equation $P(D)f = 0$ has the property (DN).*

From this we obtain the following:

Corollary. *Let $K \subseteq \mathbb{R}^n$ be compact, $P \in \mathbb{C}[x_1, \ldots, x_{n+1}]$, $\hat{K} := \mathbb{R}^n \times \{0\}$. Then $P(-D) : \mathcal{D}'_{\hat{K}} \to \mathcal{D}'_{\hat{K}}$ has a continuous linear right inverse if and only if*

i) *$P(-D) : \mathcal{D}'(\mathbb{R}^{n+1}) \to \mathcal{D}'(\mathbb{R}^{n+1})$ has a continuous linear right inverse and*

ii) *P is partially hypoelliptic with respect to $\mathbb{R}^n \times \{0\}$.*

iii) *The space $\mathcal{E}(K)$ of all Whitney jets on K has the property (DN).*

Morera theorems on manifolds

Eric Grinberg (Temple University)

This is joint work with Todd Quinto, Tufts University.
The classical Morera theorem states that a continuous function of one complex variable is analytic in a region if it has vanishing contour integrals over "all simple closed curves" in that region. Many variations, extensions and generalizations of this result exist. For instance, one can consider only "large" curves, or even a family of curves obtained by rotating and translating a fixed curve Gamma. We consider extensions of the classical Morera theorem to functions on complex Riemannian manifolds. The tests of holomorphy obtained involve contour integrals over geodesic spheres. Both families of spheres with fixed radius and with variable radius may be used. The proofs are local and depend on microlocal propagation of singularities of a Radon transform related to the Morera integral and on a theorem of Hörmander, Kawai and Kashiwara regarding wave front sets.

Generalized functions and operational calculus discussed by Fourier and Heaviside

H. Komatsu (Tokyo)

As L. Schwartz says in his book "Théorie des distributions", it is generally believed that Dirac's δ - function and Hadamard's finite parts of divergent integrals are first examples of generalized functions and that Heaviside invented "operational calculus". In this talk I will show that Fourier introduced δ - functions much earlier than Dirac and that he represented solutions of partial differential equations by operational calculus more audaciously than Heaviside.

Splitting of mixed Fréchet - Schwartz and (DF) - Schwartz power series spaces

D. Kunkle (Wuppertal)

We consider the splitting problem for power series spaces that combine a structure of projective and inductive kind, which makes them reduces projective limits of a sequence of duals of Fréchet - Schwartz spaces. We give conditions for their splitting, obtaining in particular splitting of pairs of Fréchet - Schwartz and (DF) - Schwartz power series spaces.

Surjective partial differential operators on spaces of real analytic functions

M. Langenbruch (Oldenburg)

The lecture is concerned with the basic question when

(1) $$P(D) : \mathcal{A}(\Omega) \to \mathcal{A}(\Omega)$$

is surjective. Here P(D) is a partial differential operator with constant coefficients, $\Omega \subseteq \mathbb{R}^n$ is open and $\mathcal{A}(\Omega)$ is the space of real analytic functions on Ω. Hörmander (1973) showed that for convex open sets Ω, (1) is equivalent to a certain Phragmen-Lindelöf condition valid on the complex variety of the principle part P_m of P. Sufficient conditions for (1) have been given by Kawai (1972) and Kaneko (1985) using (Fourier) hyperfunctions. In the lecture, we will present a new characterization of (1) for general open sets Ω by means of shifted elementary solutions which are real analytic on an arbitrary relatively compact open subset of Ω. This contains and extends results of Andersson (1972), Kawai (1972), Zampieri (1984) and Kaneko (1985). The characterization also implies that in many situations, (1) implies that P_m is locally hyperbolic and the localizations of P_m at ∞ are also hyperbolic.

Entire solutions of certain partial differential equations

B. Q. Li (Miami)

We consider entire solutions of partial differential equations defined by Fermat varieties and give characterizations of solutions of the equations.

Algebraic varieties on which the classical Phragmén - Lindelöf theorem holds

R. Meise (Düsseldorf)

(Joint work with R.W. Braun and B.A. Taylor).
An algebraic variety V in $\mathbb{C}^n$ has the property (SPL) if the following holds: There exists $A \geq 1$ such that each plurisubharmonic function u on V which satisfies

$$u(z) \leq |z| + O(|z|),\ z \in V$$

and

$$u(z) \leq 0,\ z \in V \cap \mathbb{R}^n$$

already satisfies

$$u(z) \leq A|\mathrm{Im} z|,\ z \in V.$$

The classical Phragmén - Lindelöf Theorem states that $\mathbb{C}^n$ satisfies (SPL) for $A = 1$. If $P_m \in \mathbb{C}[z_1, \ldots, z_n]$ is homogeneous of degree $m \geq 2$ and $P \in \mathbb{C}[z_1, \ldots, z_{n+1}]$ is defined as $P(z', z_{n+1}) := P_m(z') - z_{n+1}$, then $P(D) : \mathcal{D}'(\mathbb{R}^{n+1}) \to \mathcal{D}'(\mathbb{R}^{n+1})$ admits a continuous linear right inverse iff $V(P)$ satisfies (SPL). The aim of the talk was to explain the following result:

Theorem. *An algebraic surface V in $\mathbb{C}^n$ satisfies (SPL) iff the following conditions hold:*

1. *V(P) satisfies the local Phragmén Lindelöf condition $PL_{loc}(\xi)$ at each $\xi \in V(P) \cap \mathbb{R}^n$.*
2. *For each real simple curve γ in $\mathbb{R}^n$ and $d \leq 1$ the limit variety $T_{\gamma,d}V$ satisfies $PL_{loc}(\xi)$ at each of its real points.*
3. *For each real principle curve γ in $\mathbb{R}^n$, $d \leq 1$, and each $\xi \in (T_{\gamma,d}V)_{reg} \cap \mathbb{R}^n$, V is (γ, d) - hyperbolic at ξ.*

The conditions (1) - (3) are necessary for each pure k - dimensional algebraic variety V in $\mathbb{C}^n$ to satisfy (SPL).

Landau - Pollak dimension theorem and Gabor analysis

V. P. Palamodov (Tel Aviv)

An arbitrary square integrable function can not be expanded in a series of Gabor functions (Weyl-Heisenberg orbit of the Gaussian), if the cell area of the lattice is equal 1 (critical density). However it can be done for functions f satisfying mild smoothness condition and the lattice of critical density is extended by means of "sharp" elements. The coefficients of this expansion can be estimated in terms of scalar products of f with the Gabor functions. It implies that a function f is properly approximated by a linear combination of the elements of the Gabor series supported by the points of the lattice in a domain D of the phase space, provided the scalar products of f are small in the complement to D. This is a generalization of the Landau-Pollak dimension-theorem for several variables and domains D of arbitrary shape, whereas the original result concerns only bandlimited functions and domains that are products of time and frequency intervals.

Hypoellipticity and local solvability in Gevrey classes

L. Rodino (Torino)

We present the following result, obtained in collaboration with A. Albanese and A. Corli. If P, a linear partial differential operator with analytic coefficients in an open set $\Omega \subseteq \mathbb{R}^n$, is hypoelliptic with respect to the Gevrey classes of order s, then the transposed operator P^t is locally solvable in the same Gevrey classes at any point of Ω. The proof, in terms of abstract functional analysis, extends to more general topological vector spaces. Applications are given to certain classes of PDE.

Existence of continuous linear extension maps for spaces of Whitney jets

J. Schmets (Liege)

The material of the talk comes from a long standing joint work with Manuel Valdivia. If Ω is a proper open subset of $\mathbb{R}^n$ (resp. $\mathbb{C}^n$), $BC^\infty(\Omega)$ (resp. $\mathcal{H}_\infty(\Omega)$) designates the Fréchet space of the C^∞ - functions (resp. holomorphic functions) on Ω which are bounded on Ω as well as all their derivatives, endowed with its canonical topology. Given $\Omega \subset \mathbb{R}^n$, it is possible to construct an open subset D_Ω of $\mathbb{C}^n$ such that $D_\Omega \cap \mathbb{R}^n = \Omega$ and $(u + i\,v \in D_\Omega \Rightarrow u \in \Omega)$ such that the following key result holds:

Theorem. *There is a continuous linear map $T_\Omega : BC^\infty(\Omega) \to \mathcal{H}_\infty(\Omega)$ such that for every $f \in BC^\infty(\Omega)$, $s \in \mathbb{N}$ and $\varepsilon > 0$, there is a compact subset K of Ω such that*

$$|D^\alpha(T_\Omega f)(u + i\,v) - D^\alpha f(u)| \leq \varepsilon$$

for every $u + i\,v \in D_\Omega$ and $\alpha \in \mathbb{N}_0^n$ verifying $u \in \Omega \setminus K$ and $|\alpha| \leq s$.

This key result leads to results mentioned in the title; e.g.

Theorem. *If the closed subset F of $\mathbb{R}^n$ is compact or such that $\mathbb{R}^n \setminus F = \Omega$ is bounded, then the existence of a continuous linear extension map from the space $\mathcal{E}(F)$ of the Whitney jets of F into $C^\infty(\mathbb{R}^n)$ implies the existence of such a map from $\mathcal{E}(F)$ into $\mathcal{H}_\infty C^\infty(\Omega)$.*

In this statement $\mathcal{H}_\infty C^\infty(\Omega)$ is the Fréchet space of the functions f defined on $\mathbb{R}^n \cup D_\Omega$ verifying $f|_{\mathbb{R}^n} \in C^\infty(\mathbb{R}^n)$, $f|_{D_\Omega} \in \mathcal{H}_\infty(D_\Omega)$ and

$$\lim_{z\to x} D^\alpha(f|_{D_\Omega})(z) = D^\alpha(f|_{\mathbb{R}^n})(x) \quad \forall\alpha \in \mathbb{N}_0^n, \quad \forall x \in \partial_{\mathbb{R}^n} F$$

endowed with the seminorms

$$\sup_{|\alpha|\le m} \|D^\alpha f\|_{b_m \cup D_\Omega}.$$

The extension to the case of closed subsets of $\mathbb{R}^n$ holds if and only if F satisfies the Frerick - Vogt condition. The extension of these results to the spaces of the ultra-differentiable jets/functions has been indicated.

Elliptic differential operators on manifolds with conical singularities

E. Schrohe (Potsdam)

In my talk I tried to illustrate how methods from singular pseudodifferential analysis and functional analysis can be brought together to obtain new results for parabolic equations.

Blowing up near the tip, a manifold with a conical singularity becomes a manifold with boundary, say $\mathbb{B}$, the boundary $\partial\mathbb{B} = X$ being the cross-section of the cone. Instead of differential operators on the singular manifold we then study Fuchs type degenerate operators, i.e. operators which in a neighbourhood of the boundary have the form

$$A = t^{-\mu} \sum_{j=0}^{\mu} a_j(t; x, D_x)(-t\partial_t)^j.$$

Here we have chosen coordinates (t, x) on $[0, 1) \times X$, and a_j is smooth in t, taking values in differential operators of order $\mu - j$ on X.

The operator A naturally acts on scales of L_p-Mellin-Sobolev spaces, $\mathcal{H}_p^{s,\gamma}(\mathbb{B})$, $s, \gamma \in \mathbb{R}$, $1 < p < \infty$. Here, however, we consider it as an unbounded operator, initially defined on $C_0^\infty(\operatorname{int}\mathbb{B})$, and study its closed extensions in $\mathcal{H}_p^{0,\gamma}(\mathbb{B})$, which is a weighted L_p-space on $\mathbb{B}$. Under certain ellipticity conditions (i.e. invertibility assumptions for both the principal symbol and the conormal symbol) one can describe all these extensions.

We next address the question of maximal regularity, which is important for many applications. According to a theorem of Dore and Venni, maximal regularity follows from the boundedness of the purely imaginary powers, i.e. an estimate $\|A^{is}\|_{\mathcal{H}_p^{0,\gamma}(\mathbb{B})} \le C_p e^{\theta|s|}$, $s \in \mathbb{R}$.

Using an adapted pseudodifferential calculus for parameter-dependent operators, we find conditions that allow us to construct a parametrix to $A - \lambda$ which coincides with the resolvent and gives us very precise information on its structure – indeed sufficient to check Dore and Venni's criterion. As an example, we can treat the case where A is the Laplace-Beltrami operator for a conical metric on $\mathbb{B}$, provided $\dim\mathbb{B} > 4$.

In a next step, we see how this can be combined with an abstract result of Clément and Li in order to obtain solvability of certain quasilinear evolution equations (joint work with S. Coriasco (Torino) and J. Seiler (Potsdam)).

Higher order tangents to analytic varieties along curves

B. A. Taylor (Ann Arbor)

(Reporting on joint work with R. Meise and R. Braun).
Let V be an analytic variety in an open set in $\mathbb{C}^n$ containing the origin and with $0 \in V$. We suppose V is purely d-dimensional. For a curve

$$\gamma(t) = \{t + \xi_2 t^{d_2} + \ldots, d_j = \frac{j}{q}, j \geq 1, |\xi| = 1\}$$

define

$$V_t = \{w : \gamma(t) + t^d w \in V\}.$$

Then the currents defined by V_t converge to a limit current $T_{\gamma,d}V$ as $t \to 0$. $T_{\gamma,d}V$ is either zero or (supported by) an algebraic variety in $\mathbb{C}^n$ of pure dimension k. Properties of such limit currents and examples are given. The results are applied in other places to derive necessary conditions for varieties to satisfy the local Phragmen - Lindelöf condition.

The bounded factorization property for Fréchet spaces

T. Terzioglu (Tuzla/Istanbul)

The aim of this talk is to report on results obtained in ongoing research done with V. P. Zahariuta and M. Yurdakul about Fréchet spaces E,F,G such that each continuous linear operator $T : E \to F$ which can be expressed as $T = R \circ S$ with $S \in L(E,G)$, $R \in L(G,F)$ is bounded. If this happens, we write $(E,G,F) \in \mathcal{BF}$. We give a characterization of the relation $\mathcal{BF}$ for triples of Fréchet spaces following the approach used by Vogt in characterizing pairs $(E,F) \in \mathcal{B}$. In case $(E,\lambda(B)) \in \mathcal{B}$ and $(\lambda(C),F) \in \mathcal{B}$ where $\lambda(B)$ and $\lambda(C)$ are nuclear, we prove $(E,\lambda(B)\hat{\otimes}_\pi\lambda(C),F) \in \mathcal{BF}$. With the help of this result we can really differentiate $\mathcal{BF}$ and $\mathcal{B}$. In case E, G and F are all Köthe spaces we show that if $(E,G,F) \notin \mathcal{BF}$ then there are quasidiagonal unbounded Operators $S : E \to G$, $R : G \to F$ such that $T = R \circ S$ is also quasidiagonal and unbounded. This extends a recent result of Djakov and Ramanujan.

Fréchet quotients of $\mathcal{A}(\mathbb{R}^d)$

D. Vogt (Wuppertal)

Report on joint work with P. Domanski and L.Frerick.
The investigation of the Fréchet structure of the space $\mathcal{A}(\Omega)$ of real analytic functions on an open set $\Omega \subseteq \mathbb{R}^d$ turned out to be the basis for various interesting results. The Fréchet subspaces were characterized by Domanski and Langenbruch. Domanski and the speaker proved that all complemented Fréchet subspaces of $\mathcal{A}(\Omega)$ are finite dimensional (basic for their proof that $\mathcal{A}(\Omega)$ admits no Schauder basis). For the Fréchet quotients it was only known that they fulfill the very restrictive condition $(\bar{\bar{\Omega}})$ and this was an ingredient for the previously mentioned result. It was proved only very recently that there exist nontrivial Fréchet quotients. We give the following characterization:

Theorem. *A Fréchet space E is isomorphic to a quotient of $\mathcal{A}(\Omega)$ if and only if $E \in (\bar{\bar{\Omega}})$ and is n^j - nuclear.*

A principal tool which is of independent interest is:

Theorem. *For a Fr'echet space E we have* $\mathrm{Ext}^1(\mathcal{A}(\Omega), E) = 0$ *if and only if $E \in (\bar{\bar{\Omega}})$.*

The derived functors of Hom in the category of locally convex spaces

J. Wengenroth (Trier)

We consider the derivatives $\mathrm{Ext}^k(E,\cdot)$ of the functor $L(E,\cdot)$ (for a fixed locally convex space E) acting on the category of l.c.s. After explaining the relevance of Ext^1 for splitting, extension and lifting problems and the role of Ext^k for the stability of $\mathrm{Ext}^{k-1}(E,\cdot)=0$ we briefly recall the behaviour of $\mathrm{Ext}^k(E,X)$ for Fréchet spaces X and the connection to the countable projective limit functor. The main result deals with a conjecture of V.I. Palamodov:

$\mathrm{Ext}^k(E,X)=0$ for all $k\in\mathbb{N}$ if E is a metrizable l.c.s., X is a complete (DF) - space and one of them is nuclear.

Using properties of the projective limit functor for uncountable spectra we obtain answers to Palamodov's question at least if we assume the continuum hypothesis: For each infinite dimensional nuclear (LB)-space X we then have

(1) $\mathrm{Ext}^k(E,X)=0$ holds for $k\geq 3$ and each l.c.s. E.
(2) There is a normed space E with $\mathrm{Ext}^1(E,X)\neq 0$.
(3) $\mathrm{Ext}^1(\mathbb{K}^{\mathbb{N}},X)=0$ and $\mathrm{Ext}^2(\mathbb{K}^{\mathbb{N}},X)\neq 0$.

On some linear topological invariants

V. Zahariuta (Metu, Ankara and Sabanci University, Istanbul

Joint work with T. Terzioglu and M. Yurdakul.

Let E be a locally convex space, $\mathcal{U}(E)$ $(\mathcal{B}(E))$ be the set of all neighbourhoods of zero (absolutely convex bounded sets). If $F:\mathbb{R}_+\to\mathbb{R}_+$, $\varphi(t)\to\infty$, then (Ω_φ), (DN_φ) are Vogt's interpolation classes. We denote by (QN) and (AN) the classes of all quasinormable (Grothendieck) and, respectively, asymptotically normable (Terzioglu - Vogt) spaces. We say that E satisfies strict Ω_φ $(E\in(s\Omega_\varphi))$ if

$$\exists M\in\mathcal{B}(E)\forall U\in\mathcal{U}(E)\exists V\in\mathcal{U}(E)\exists C: V\subset C\varphi(t)M+\frac{1}{t}U,\quad t>0;$$

We write $\left(E\in(\overline{s\Omega_\varphi})\right)$ if $E=F'$ and M is supposed to be equicontinuous.

Proposition. *Let φ satisfy the condition:*

$$\exists\alpha>1:\frac{\varphi(t^\alpha)}{\varphi(t)}\to\infty.\quad(*)$$

Let E be a Fréchet Schwartz space. Then $E\in(\Omega_\varphi)$ iff $E\in(s\Omega_\varphi)$.

Theorem. *$E \in (DN)_\varphi$ implies $E' \in \left(\overline{s\Omega_\varphi}\right)$.*

Theorem. *Let φ satisfy (*) and E be a Montel Köthe space admitting a continuous norm. Then $E \in (\Omega)_\varphi$ iff $E' \in (DN)_\varphi$.*

We say that E is strictly quasinormable ($E \in (sQN)$) if

$$\exists M \in \mathcal{B}(E) \forall U \in \mathcal{U}(E) \exists V \in \mathcal{U}(E) \forall \delta \exists \Delta : V \subseteq \Delta M + \delta U;$$

if $E = F'$ and M is equicontinuous here, then we write $E \in \left(\overline{sQN}\right)$. We define the class $(\overline{AN})$ as the union of all classes $(DN)_\varphi$. For Fréchet spaces $(QN) = (sQN)$ (Grothendieck) and $(AN) = (\overline{AN})$ (Terzioglu - Vogt).

Theorem. *$E \in (\overline{AN})$ implies $E' \in (\overline{sQN})$.*

Edited by Daren Kunkle

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