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Discontinuous Galerkin Methods: Analysis and Implementation

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Background

Over the last three decades mathematical research on classical Galerkin Finite Element Methods (CGFEM), based on the use of continuous piecewise polynomial approximations, has resulted in a coherent body of knowledge both in terms of theoretical foundations and with respect to efficient and robust implementations. This general class of methods is particularly well-suited for the numerical approximation of self-adjoint elliptic problems, but exhibits instabilities when applied to problems of hyperbolic or nearly hyperbolic character, such as transport (or transport-dominated) problems and problems of mixed or changing type (e.g. mixed elliptic-hyperbolic problems, degenerate elliptic problems, and the like). For such problems, Galerkin finite element methods based on the use of discontinuous piecewise approximations are much more promising.

This family of numerical techniques, generally referred to as Discontinuous Galerkin Finite Element Methods (DGFEM), has a long and distinguished history. Its roots can be traced back to the work of Pian and collaborators in the early 1960s on hybrid methods for elliptic problems (see also [?] for a historical survey); the mathematical analysis of hybrid methods was initiated by Babuška [?]. In 1971, J. Nitsche [?] considered an alternative scheme where to recover the optimal convergence rate. These ideas were further developed by Arnold [?], Wheeler [?], and more recently by Oden, Babuška and Baumann [?], Riviere, Wheeler and Girault, [?], Süli, Schwab and Houston [?], Arnold, Brezzi, Cockburn and Marini [?], and Hansbo and Larson [?]. In a different context, and completely independently, discontinuous Galerkin finite element methods were introduced by Reed and Hill [?], and Lesaint and Raviart [?] (and further improved analytically by Johnson, Nävert and J. Pitkäranta [?] and Johnson and Pitkäranta [?]), in order to overcome the stability limitations of conventional continuous finite element approximations

to first-order hyperbolic problems. Although subsequently much of the research in the field of numerical analysis of partial differential equations concentrated on the development and the analysis of CGFEM, in recent years there has been an upsurge of interest in discontinuous schemes. This paradigm shift was stimulated by several factors: the desire to handle, within the finite element framework, nonlinear hyperbolic problems (see [?] and [?]) which are known to exhibit discontinuous solutions even when the data are perfectly smooth; the need to treat convection-dominated diffusion problems without excessive numerical stabilization; the computational convenience of discontinuous finite element methods due to a large degree of locality; and the necessity to accommodate high-order hp-adaptive finite element discretizations in a flexible manner (see [?], [?]). The discontinuous Galerkin finite element method can also be thought of as the high-order generalization of the classical cell centre finite volume method – a popular discretization technique in the computational aerodynamics community. Finally, given that unlike their continuous counterparts DGFEMs work well both for elliptic and hyperbolic problems without excessive stabilization, such as SDFEM or the like, they lead to a unified framework of discretization methods for a large class of partial differential equations with nonnegative characteristic form, including (self-adjoint and non-self adjoint) elliptic problems, first-order hyperbolic problems and various unsteady problems.

The meeting

The Oberwolfach meeting aimed to explore recent mathematical advances in the analysis and implementation of discontinuous Galerkin finite element methods. This dual objective is reflected by the list of participants for the meeting which includes mathematicians who work on the analysis of these methods and engineers who use DGFEMs for large-scale simulations. The primary goal was to bridge the gap between the (lack of) mathematical understanding of the stability and accuracy properties of discontinuous Galerkin methods and their successful implementation in engineering computations. The meeting was only half-size, i.e. 25 participants were invited and a parallel meeting on dispersive wave equations also took place during this week. The meeting was successful in bridging the gap between the elliptic and the hyperbolic communities. The techniques in the elliptic case can be applied seamlessly also to hyperbolic problems, by merely taking into account the proper design of the numerical fluxes for either problem. 25 Researchers from 8 countries attended the meeting and presented 21 lectures on their work. The two groups of 'elliptic' and 'hyperbolic' researchers exchanged ideas and strongly interacted. A strong point of the meeting was the prominent presence of young researchers on the postdoctoral and assistant professorial level. Their lively interaction and eagerness to discuss and to tackle new problems will definitely continue beyond this Oberwolfach meeting. In addition, lively exchanges of ideas on how to build good software based on discontinuous Galerkin discretizations resulted as well. Highlights of the meeting included the presentation of Professor Franco Brezzi on DGFEM for Reissner–Mindlin plates. Professor Brezzi showed new results (joint work with Professors Donatella Marini and Douglas Arnold) which indicated that there are DG-discretizations which perform as well as the best known mixed Finite Element Methods for the numerical solution of plate models with shear. In related work, Professor Peter Hansbo showed compelling numerical evidence that suitable DG - discretizations of elasticity problems are free of volume locking. In linearly elastic problems, this is also mathematically understood; Professor Hansbo's numerical results strongly indicated that this is also the case in problems of nonlinear elasticity at large strains.
The presentation by Professor Chi-Wang Shu on DGFEM for higher order evolution problems showed once again the versatility of the method and in particular its applicability to dispersive wave equations. We also mention the survey lecture by Professor Klainerman to participants of both meetings on recent developments in the mathematical analysis of the Einstein equations.

Evening discussion

On Tuesday, 23rd April, in the evening, a plenary discussion was held. The focus of the discussion were adaptive DGFEM for hyperbolic and convection-dominated problems. During the discussion, two approaches were identified: (a) residual-based a-posteriori error estimation (e.g. L^1-norm based) without reference to the dual problem, and (b) "dual problem"-based approach to error estimation and algorithm steering. Some contributions to the discussion are summarized below.

Professor Brezzi asked, regarding the duality-based approach for a-posteriori error estimation, if there is a systematic way to identify "admissible" target functionals with respect to which one should adapt meshes and/or orders. It was agreed that the "residual" based approach gives in general less precisely tailored, adapted meshes than the approach based on the numerical solution of the dual problem. It was commented that the main problem with the duality-based approach is the specification of appropriate functionals. For example, maximum pointwise errors are at present too difficult to control with the goal-oriented, duality-based approach, since the maximum may either not exist (e.g maximum pointwise stresses) or one might not be able to specify it.

Another important issue identified in the discussion concerns the error control for target functionals in time-dependent, transient problems. The need to store numerical solutions over all time levels in the duality-based approach was considered too expensive in 3-d. Professor Rolf Rannacher suggested to borrow techniques from computational optimal control theory in order to avoid the storage of numerical solutions over a large number of time levels. Professor Joe Flaherty commented that in 3-d transient problems memory is a major issue. His group overcame this difficulty by transporting their DG algorithms scalably to massively parallel hardware.

Professor Rannacher criticized the discussion as being overly narrow, i.e., centered on CFD and aircraft design problems only. He insisted that goal-oriented adaptivity for DGFEM should also be investigated for general elliptic/hyperbolic problems, time-dependent or not. It was remarked by several participants that the highly structured meshes with goal-oriented adaptivity will in general only yield a good approximation for a single objective. Professor M. Feistauer insisted that engineers usually look for a number of quantities in any simulation - the topic of goal-oriented adaptivity with respect to a *group* of target functionals was addressed as an open issue. Professor Rannacher replied that the savings in goal-oriented adaptive calculations are so large that there one could even afford parallel numerical adaptivity with respect to several goals. Another open problem was the issue of the quality of the computed dual solution. If the dual solution is available explicitly, the duality based approach to adaptivity is rigorous and, in a sense, the best possible one. Usually the dual solution can only be computed on the mesh optimized on the primal calculation, i.e. the mesh which is geared towards the objective functional in the primal problem. Several people mentioned that it is practically impossible to construct an example for which the dual solution is so bad that adaptivity in the primal variable runs off a near optimal refinement trajectory. The problem of making this apparent robustness of the duality-based approach mathematically rigorous is at present open.

All participants greatly regretted that one of the organizers, Endre Süli, was absent due to the death of a close member of his family just prior to the meeting.

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Abstracts

Kinetic Entropies, Hyperbolic Systems and the Discontinuous Galerkin Method

Tim Barth, NASA Ames R.C., USA

Hyperbolic moment systems derived from the kinetic Boltzmann equation with Levermore closure are considered within the framework of the discontinuous Galerkin method. Included in these moment systems are the familiar Euler equations of gas dynamics. A striking attribute of the Boltzmann moment viewpoint is the simplicity by which certain otherwise tedious theoretical results are obtained. Using Levermore's exponential conjugate entropy, a new class of computable monotone flux functions for systems are shown to be globally energy-stable when combined with Gauß-Lobatto state space integration. Numerical results are presented to verify the theory.

DG Methods for Reissner-Mindlin Plates

F. Brezzi, Pavia - joint work with D.N. Arnold and L.D. Marini

We considered three families of discontinuous F.E. approximations of RM plate equations; each family could be seen as a generalisation of the Arnold-Falk element. For each family of polynomial spaces, we considered several options, related to various continuity requirements (on ϑ and on w), to the presence of a consistency term for w, and to the choice of the weight in the penalty/super penalty term. We found in particular that using a continuous ϑ requires the addition of a certain number of suitable bubble-functions to the space θ_h of discrete rotations, while this is not necessary when we use totally discontinuous rotations.

Discontinuous Galerkin Methods for Incompressible Navier-Stokes Equations

B. Cockburn

It is shown that a rewriting of the Navier-Stokes equations is necessary in order to obtain a system to which the discontinuous Galerkin method can be applied successfully. In particular, it is shown that the pressure "p" has to be replaced by "$p - \frac{1}{2}\,\bar{u}\cdot\bar{u}$ ", where $\bar{u}$ is the velocity.

Discontinuous Galerkin Methods for Porous Media Flow and Shallow Water Equations

Clint Dawson, Texas University at Austin

In the first part of the talk, we discuss a new local discontinuous Galerkin Method for elliptic flow problems which allows for piecewise constant approximations. Theoretical and numerical results are given, extensions to higher order are also discussed. In the second part, we discuss DG methods for shallow water flow. These include fully DG formulations and combined DG - continuous Galerkin methods.

On Some Aspects of the DGFEM for the Solution of Nonlinear Conservation Laws and Compressible Flow)

Miloslav Feistauer, Charles University Prague - joint work with Vit Dolejši, Charles University and Christoph Schwab, ETH Zürich

The subject of the paper is the numerical solution of compressible flow. The goal is to develop a sufficiently accurate and robust method. We present the application of the DGFEM combined with finite volume techniques. This method is theoretically analysed on a model nonlinear scalar conservation law equation with a diffusion term. Namely, error estimates are investigated for one version of the DGFEM combined with a finite volume approach to the discretization of convection terms. Attention is also paid to the choice of a new limiting of the order of the method in the vicinity of discontinuities in order to avoid spatial oscillations. The approximation of a curved boundary is also discussed. Some results of numerical simulation of compressible flows are presented.

Adaptive and Parallel Discontinuous Galerkin Methods for Hyperbolic Systems

J. E. Flaherty, Rensselaer Polytechnic Institute, NY

We address solution techniques for hyperbolic systems using a discontinuous Galerkin strategy. We present several aspects of the method including (i) local time stepping, (ii) flux evaluation, (iii) solution limiting, (iv) parallel strategies, (v) and a posteriori error estimation. We show that DG methods with piecewise-polynomials of degree p exhibit superconvergence at Radau points of degree $p+1$. The solution at the downwind ends of elements exhibits a strong superconvergence and converges as $\mathcal{O}(h^{2p+1})$. These results hold in multiple dimensions relative to a set of orthogonal polynomials that may be considered as extension of the Radau polynomials.

Discontinuous Galerkin Methods for Plasticity Problems Related to Granular Flows

Pierre Gremaud, North Carolina State University

Several computational challenges related to the modelling of granular flows are described. The general structure is that of PDAEs (Partial Differential Algebraic Equations), the algebraic constraint corresponding to the yield condition being satisfied. DG results are presented.

In the second part of the talk, other aspects are discussed, in particular, the appearance of secondary circulation.

Nonconforming Elements and the Discontinuous Galerkin Method

Peter Hansbo, Chalmers University of Technology, Göteborg, Sweden - joint work with M.G. Larson

Three different problem classes - incompressible elasticity, Kirchoff plates, and Reissner-Mindlin plates were considered. We showed how to construct low order, non-locking, optimally convergent discontinuous Galerkin elements for these problems. Furthermore, we showed how some classical non-conforming elements (Crouzeix-Raviart and Morley) naturally come out of the framework if one desires to obtain independence of the penalty parameters. In this context, a stable version of the Crouzeix-Raviart element for elasticity was obtained.

Adaptive Discontinuous Galerkin FE Methods for the 2D Compressible Euler Equations

Ralf Hartmann, Heidelberg University - joint work with Paul Houston, Leicester

Based on the DG FEM we develop an adaptive algorithm for the efficient computation of physically relevant quantities $J(\cdot)$ like drag and lift coefficients of airfoils immersed in an inviscid fluid. In particular, by employing a duality argument we derive the error representation formula

$$J(u) - J(u_h) = \sum_{\kappa \in T_h} \widetilde{\eta}_k \tag{1}$$

where $\widetilde{\eta}_\kappa$ on each element κ of the triangulation consists of the FE residuals multiplied by local weighting terms involving the solution z of a certain dual problem. We shall show in a variety of numerical examples that the approximate error representation

$$\eta = \sum_{\kappa \in T_h} \widetilde{\eta}_\kappa$$

originating from (1) by replacing the exact dual solution z by a numerical approximation is very close to the true error in the target quantity $J(\cdot)$. Furthermore, we show that any bounding of (1) from above, like applying triangle inequality and further bounding in order to derive so-called Type II error estimates will result in possibly very large overestimation of the true error.

Finally, we employ the computed local indicators $\widetilde{\eta}_k$, also referred to as 'weighted indicators', for adaptive mesh refinement resulting in meshes that are specifically tailored to the cost-efficient computation of the quantity of interest. We compare the efficiency of these meshes with meshes produced by so called 'ad hoc indicators' that simply rely on the residual or smoothness information of the solution. Furthermore, we compare with meshes that are designed by hand for the efficient computation of the quantity of interest.

We illustrate this approach by several different problems in combination with various different target quantities, including density or pressure point values or drag and lift coefficients of airfoils in subsonic, transonic and supersonic flows.

Adaptivity for High-Order/Spectral Finite Element Methods for Second-Order PDEs with Nonnegative Characteristic Form

Paul Houston, Leicester University - joint work with Endre Süli and Kathryn Harriman, Oxford, and Bill Senior, Leicester

The aim of this talk is to consider the a-posteriori error analysis of the *hp*-version of the discontinuous Galerkin finite element method for approximating second-order PDEs with nonnegative characteristic form. In particular, we consider the derivation of computable error bounds for certain target functionals of the solution of practical interest; relevant examples include the mean value of field or its flux through the outflow boundary of the computational domain.

By employing a duality argument we derive so-called weighted or Type I a posteriori estimates which bound the error between the true value of the prescribed functional, and the actual computed value. In these error estimates, the element residuals of the computed numerical solution are multiplied by local weights making the solution of a certain dual or adjoint problem. On the basis of the resulting a posteriori error bound, we design and implement an adaptive, finite element algorithm which incorporates both local h- and p-refinement. The performance of the proposed *hp*-refinement algorithm is demonstrated through a series of numerical experiments.

Preconditioning Discontinuous Galerkin Methods for Elliptic Problems

Guido Kanschat, Heidelberg University

The analysis of a multi-level preconditioner for the interior penalty method is presented. We investigate its performance with respect to the penalty parameter. It is shown to be robust with respect to the polynomial degree, if a block-smoother is used, while point smoothers deteriorate fast. Then, it is shown that downwind ordered Gauß-Seidel yields an optimal preconditioner for advection-diffusion problems, independent of the Peclet number. The same preconditioner is then applied successfully to the Schur complement of the LDG method. Finally, this preconditioner will be used to construct a block preconditioner for the saddle point problems arising when discretizing Poisson and Stokes equations with the LDG method.

Comparison of Finite Volume and Discontinuous Galerkin Methods for MHD

Dietmar Kröner, University of Freiburg

The main disadvantage of finite volume schemes of higher order is that the stencil for the discretization will strongly increase with the order of the scheme and the scheme becomes very expensive. For discontinuous Galerkin methods one can use locally higher order polynomials and the discretization is very local. Nevertheless it is not clear if discontinuous Galerkin methods are more efficient. We have studied several numerical experiments for the Euler and MHD equations with smooth as well as discontinuous solutions. It turned out that the DG methods are of higher accuracy but for discontinuous solutions they need more CPU time.

Second Order Central Schemes on General Adaptive Unstructured Grids

Mario Ohlberger, University of Freiburg - joint work with Marc Küther

We give a reinterpretation of first order staggered schemes as a finite volume scheme with upwind flux on the intersection grid followed by an averaging step. Within this framework a posteriori and a priori error estimates are derived in the L^1-norm for scalar nonlinear hyperbolic conservation laws in arbitrary space dimension.

In a second step we then derive a second order central scheme based on piecewise linear reconstruction operators.

We use the rigorous error estimate for the first order method to derive local error indicators for an adaptive algorithm which is based on an equal distribution strategy. Finally numerical experiments demonstrate the efficiency of the adaptive second order scheme.

Discontinuous Galerkin Methods for Time-Harmonic Maxwell's equations

Ilaria Perugia, University of Pavia

We present discontinuous Galerkin methods for time-harmonic Maxwell's equations in low and high-frequency regimes.
The operators involved are discretized in discontinuous finite element spaces using suitable variants of IP and LDG techniques. Heterogeneous materials will be considered, in the low-frequency case, by incorporating a divergence free constraint either by a regularization approach or by Lagrange multiplier techniques. These approaches will then be extended to the high-frequency case.

A Discontinuous Galerkin Method with Non-Overlapping Domain

Béatrice Riviére - joint work with V. Girault and M.F. Wheeler,
University of Texas

We formulate and analyze a family of discontinuous Galerkin finite element methods for Stokes and Navier-Stokes problems. In each triangle the finite elements discretizing the velocity are polynomials of degree k with no continuity requirement between triangles and the finite elements discretizing the pressure are polynomials of degree $k-1$, also totally discontinuous. An inf-sup condition is established as well as optimal energy estimates for the velocity and L^2 estimates for the pressure. In addition, it is shown that the method can treat a finite number of non-overlapping domains with non matching grids at interfaces.

The Local Discontinuous Galerkin Method for the Oseen Equations

Dominik Schötzau, University of Basel

We introduce and analyze LDG methods for the Oseen equations of incompressible fluid flow. We derive optimal a-priori estimates for the errors in the velocity and the pressure. Numerical experiments are presented that show that the methods perform well for a wide range of Reynolds numbers.

Local Discontinuous Galerkin Method for Higher Order PDEs

Chi-Wang Shu, Brown University, USA

We discuss local discontinuous Galerkin method for solving KdV-type equations involving 3 spatial derivations; time-dependent biharmonic equations involving 4 spatial derivatives and PDEs involving 5th derivatives. Suitable numerical fluxes are defined so that the methods can be proven L_2 stable for quite general nonlinear cases. Numerical results are shown to demonstrate the accuracy and efficiency of the method especially for the "convection dominated" case, namely when the higher order derivatives have small coefficients.

A Finite Element Method for Domain Decomposition with Non-Matching Grid

Rolf Stenberg, Helsinki University of Technology

We review joint work with P. Hansbo (Chalmers) and R. Becker (Heidelberg) where we prepare the use of Nitsche's method in domain decomposition. It allows the use of different finite element grids on different subdomains. A-priori and a-posteriori error estimators are given together with numerical results. Finally, we show how Nitsche's method should be applied to Robin boundary conditions.

Discontinuous Galerkin for Flow Problems

Andrea Toselli, ETH Zürich

We present an *hp*-finite-element approximation on some matching grids for a scalar advection-diffusion-reaction problem. A-priori
error estimates are obtained which are optimal in h and slightly suboptimal in p. In the second part of our talk, we present a
discontinuous Galerkin method for the Stokes problem. It presents better stability properties than the corresponding *hp*
conforming method employing pressure spaces of the same degree.

The Symmetric and Antisymmetric Formulation of the DGFEM for Diffusion Problems

Thomas P. Wihler, Seminar for Applied Mathematics, ETH Zürich

It is well known from regularity theory that the exact solution of a diffusion problem may exhibit singularities in polygons. In this talk it is shown how these singularities may be resolved. Optimal order convergence results for the h-version DG are proved and experimental convergence results for the *hp*-version DG are presented.

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