Mathematisches Forschungsinstitut Oberwolfach

Report No. 27/2002

Geometric Analysis and Singular Spaces

June 2nd – June 8th, 2002

This meeting was organized by J.-M Bismut, J. Brüning, and R. Melrose; it was attended by 35 scientists from 8 countries. The conference must be considered very successful in view of the importance of the results communicated and the fruitfulness of the ensuing discussions, even though many interesting mathematicians were not able to come this time, for a variety of reasons.

As usual, the topics of discussion were manifold but, nevertheless, centering around a few major themes. A quite substantial number of talks dealt with *questions of spectral theory* and L^2-*invariants* on various singular spaces, like compact singular spaces (Grieser: compact semialgebraic sets, Shioya: Alexandrov spaces, Bär, Weingart: compact manifolds), complete noncompact Riemannian manifolds (Brüning: manifolds with cusps, Carron: manifolds with flat ends, Eyssidieux: Galois coverings of complex projective manifolds, Leichtnam: Etale groupoids, Müller: locally symmetric spaces with finite volume, Piazza: Galois coverings of compact manifolds), or smooth spaces with singular metrics (Callias).

Another focus of active interest was the *analytic torsion* which was addressed in the talks of Bismut (holomorphic and de Rham torsion forms), Goette (the family case), Köhler (quaternionic analytic torsion), Ma (analytic torsion on manifolds with boundary), and Yoshikawa (relation between equivariant analytic torsion and modular forms).

The importance of the notion of *gerbes* for the subject was demonstrated in talks by Bunke and Melrose.

Finally, there was some interest in *algebras of operators and functions* (Brasselet on function algebras with computable Hochschild homology, Schrohe on K-theory of the Boutet de Monvel algebra) and in various questions related to *geometric quantization* (Alexeev on group valued moment maps, Braverman on certain noncompact Kähler manifolds, Szenes on counting lattice points in polytopes).
The meeting showed clearly that geometric analysis on singular spaces is an important and still vigorously expanding subject which should be the object of future Oberwolfach meetings, too. The present team of organizers feels, however, that after conducting quite a few meetings it may be time to leave this task to others.

Abstracts

On the quantization of group valued moment maps

Anton Alekseev

Let g be a finite dimensional Lie algebra. The Duflo map $Sg \to Ug$ establishes an algebra isomorphism between the invariant polynomials $(Sg)^g$ and the center of the universal enveloping algebra $Z(Ug)$. Let $h \subset g$ be a Lie subalgebra. In general, the Duflo maps for g and h do not intertwine the natural embeddings $Sh \to Sg$ and $Uh \to Ug$.

Let g be a quadratic Lie algebra, that is, a Lie algebra with an ad-invariant scalar product B, and let h be a Lie subalgebra such that the restriction of B on h be nondegenerate. We show that there exists a natural extensions of the Duflo map $Q_g : Sg \otimes \wedge g \to Ug \otimes Cl(g)$, where $Cl(g)$ is the Clifford algebra corresponding to B, such that Q_g and Q_h are intertwined by the natural embeddings induced by $h \to g$.

We use this result to give an extension to quadratic Lie algebras of the recent result of Huang-Pandzic and of Kostant on the Vogan conjecture for Dirac cohomology.

Small Eigenvalues of the Yamabe Operator

Christian Bär

We define a differential topological invariant of compact manifolds by counting the small eigenvalues of the Yamabe operator for suitable Riemannian metrics. We bound this invariant from above and from below by the α-genus. For simply connected manifolds it turns out to be computable in terms of the α-genus and one sees that it can distinguish certain exotic spheres. Hence it is not a homeomorphism invariant.

As methods we use a spectral comparison principle due to Gallot and Meyer, refined Kato inequalities, a surgery result for the Yamabe spectrum, and bordism theory. As a geometric application we show that if a compact spin manifold has sufficiently large $\widehat{A}$-genus, then it has a “neck of bounded size” for all metrics. This is not true if one drops the assumption of the $\widehat{A}$-genus as one can e. g. see in the example of the torus.

Algebras of functions on singular spaces

Jean-Paul Brasselet

The Hochschild homology of the algebra $C^\infty(M)$ of smooth functions on a compact smooth manifold M has been proved to be isomorphic to the de Rham complex of differential forms on the manifold (A. Connes). In order to generalize such a result for singular varieties, one has to determine what are "good" functions and "good" forms on a singular variety. One idea comes from the concept of shadow forms, i.e. differential forms defined on the smooth part of the singular space X admitting poles of given order on the strata of a suitable stratification of X. The cohomology of the complex of shadow forms is isomorphic to intersection homology of X for a suitable perversity, related to the orders of the poles. Another idea comes from the Teleman's localization technique: "the Hochschild complex of $C^\infty(M)$, M smooth, is localized along the main diagonal". This procedure explains the local character of the Hochschild homology of the algebra $C^\infty(M)$ and is adapted to the computation of Hochschild homology of algebras which are significative for singular spaces:

- the algebra of smooth functions on $\mathbf{R}^n$ which are bounded at infinity as well as all their derivative,
- the algebra of functions with suitable controlled properties on cones over smooth manifolds,
- the algebra of Whitney functions on X closed in $\mathbf{R}^n$.

The results of this lecture come from joint works with A. Legrand, N. Teleman and M. Pflaum.

Background cohomology of a holomorphic vector bundle over a tamed Kähler manifold

Maxim Braverman

Let M be a complete Kähler manifold endowed with a circle group action. Let W be an equivariant holomorphic vector bundle over M. The manifold M is called *tamed* if it is endowed with an invariant proper function $\phi : M \to \mathbb{R}$ satisfying certain technical conditions.

We introduce an invariant of the triple (M, W, ϕ) called *the background cohomology*. By definition it is the cohomology of certain deformation of the Dolbeault complex of M with coefficients in W. We show that the background cohomology is rather stable with respect to ϕ and behaves very much like a cohomology of a holomorphic bundle over a compact manifold. In particular, we prove that it is semi-continuous in families. We also prove analogues of Kodaira and Andreotti-Grauert vanishing theorems. We discuss some application of these results to geometric quantization of non-compact Kähler manifolds.

Dirac systems

Jochen Brüning

(joint work with Werner Ballmann and Gilles Carron)

The spectral theory of geometric operators on noncompact manifolds differs drastically from the compact case, mainly through the possible presence of essential spectrum. To achieve significant results, the geometric singularity has to be translated into a functional analytic model of the operators involved which allows a detailed study. In this talk, we describe such a model for singularities of the type $U := (0, \varepsilon_0) \times N)$, where $0 < \varepsilon_0 \leq \infty$ and N is compact, equipped with a metric of the form $g := dx^2 \oplus g_N(x)$, with a smooth family, g_N, of metrics on N; we will apply the model to complete manifolds with finite volume and pinched negative curvature near infinity.

The model we propose consists of the following data:

1)we are given a C^1-Hilbert bundle, $\pi : \mathcal{H} \to (0, \varepsilon_0)$, together with a continuous metric connection ∂;

2) in addition, there is a family, $A(x)$, of self-adjoint operators in $\mathcal{H}_x := \pi^{-1}(x)$ with domain $\mathcal{H}^1_x$, such that $\mathcal{H}^1$ is a C^1-Hilbert bundle, too, and both A and the natural embedding become C^1-morphisms;

3) there is a C^1-field of endomorphisms, γ, with the properties

$$\gamma^* = \gamma^{-1} = -\gamma,$$
$$[\partial, \gamma] = 0,$$
$$\gamma A + A\gamma = 0.$$

Then it is readily seen that all first order geometric differential operators, D, can be written in the form

$$D = \gamma(\partial + A),$$

on $L^2(\mathcal{H})$ for a suitably chosen Hilbert bundle with the above properties.

If one studies simple examples of ends in complete Riemannian manifolds like cylinders or hyperbolic cusps, then one is lead to conjecture that the coefficient A in the corresponding Dirac systems converges at infinity, in a suitable sense, modulo a finite dimensional perturbation. It is rather difficult to make this intuition precise in some generality but we can do it in the case of manifolds with cusps. This leads to several abstract structural assumptions on the operator coefficient which imply precise descriptions of the essential spectrum and convenient index formulas in the Fredholm case. The operators under consideration will not be Fredholm in general, though, but they will be extended Fredholm operators in the sense of Carron. Correspondingly, we can derive formulas for the extended index under the additional structural assumptions.

Families with corners, eta forms, and Deligne cohomology

Ulrich Bunke

Given a geometric family, then under certain conditions the K-theoretic index of the associated family of Dirac operators can be refined to a Deligne cohomology class. Low dimensional examples of these refinements are the eta invariant, the determinant line bundle and the index gerbe. The refinement on level k exists iff the index of the family is trivial on the $k-1$-skeleton of the parameter space. The refinement still depends on choices, but if the parameter space is compact, then the set of all possible Deligne cohomology valued refinements is finite.

Deligne cohomology as well as geometric families admit the notion of transgression. Compatibility of the refinement with transgression was shown for the index gerbe, i.e. in level 2. This generalizes the result in level 1, i.e. the holonomy formula for the determinant line bundle.

Spectral invariants in the presence of singularities

Constantine Callias

As a model of a differential operator with singularities on an algebraic set, let H be the differential operator $-\Delta_n + \kappa(x)/|p(x)|^\alpha$ on $\mathbb{R}^n$, where p is a homogeneous polynomial, κ is a positive smooth function of compact support and α is an integer > 2. We prove the existence of an asymptotic expansion of the distributional trace of the heat operator, $\operatorname{Tr}\phi e^{-tH}$ for $\phi \in C_0^\infty(\mathbb{R}^n)$, as $t \to 0+$, with respect to the "power - logarithm" asymptotic forms $t^k \log^j$, $k \in \mathbb{C}$, $j \in \mathbb{Z}_+$. This extends the results on operators with irregular singularities that were the object of the article in Math. Res. Lett. **2**, 129-146 (1995), whose methods are applied to this case as well. The proof relies on an analysis of classical heat expansions as we approach the singularities together with a differential calculus of functions with "power - logarithm" asymptotic expansions in several variables. Explicit computations that demonstrate the power of this calculus are presented. Applications to calculations of indices, determinants and spectral invariants that provide crucial information for inverse spectral problems are also discussed. In particular, the hamiltonian of a many-body system of particles interacting via a two-body singular interaction is a special case of the model operators above and the trace of the heat operator (in a finite volume) is the partition function of such as system. Guided by the examples of two-body systems and weak interactions we conjecture that the high - temperature asymptotic expansion of the partition function of the many-body system determines the short - distance asymptotic expansion of a spherically symmetric interaction.

L^2 cohomology of manifold with flats ends

Gilles Carron

Let (M^n, g) be a complete Riemannian manifold. We denote by $\mathcal{H}^k(M, g)$ or $\mathcal{H}^k(M)$ its space of L^2-harmonic k-forms, that is to say the space of L^2 $k-$forms which are closed and coclosed:

$$\mathcal{H}^k(M) = \{\alpha \in L^2(\Lambda^k T^* M),\ d\alpha = \delta\alpha = 0\},$$

where

$$d\ :\ C_0^\infty(\Lambda^k T^* M) \longrightarrow C_0^\infty(\Lambda^{k+1} T^* M)$$

is the exterior differentiation operator and

$$\delta\ :\ C_0^\infty(\Lambda^{k+1} T^* M) \longrightarrow C_0^\infty(\Lambda^k T^* M)$$

its formal adjoint. The operator $(d+\delta)$ is elliptic hence the elements of $\mathcal{H}^k(M)$ are smooth and the L^2 condition is only a decay condition at infinity.

If M is compact without boundary, then these spaces have finite dimension, and we have the theorem of Hodge-de Rham : the spaces $\mathcal{H}^k(M)$ are isomorphic to the real cohomology groups of M. For noncompact manifolds, there is no such general interpretation. In (1982, [D]), J. Dodziuk asked the following question: according to Vesentini ([V]), if M is flat outside a compact set, the spaces $\mathcal{H}^k(M)$ are finite dimensional. Do they admit a topological interpretation ?

We give the following complete answer to this question. Let (M^n, g) be a complete Riemannian Manifold with one flat end E. Then

(1) If the volume growth of geodesic balls is at most quadratic, i.e. if

$$\lim_{r\to\infty} \frac{\text{vol } B_x(r)}{r^2} < \infty,$$

then we have

$$\mathcal{H}^k(M, g) \simeq \text{Im}\left(H_c^k(M) \longrightarrow H^k(M)\right).$$

(2) If $\lim_{r\to\infty} \frac{\text{vol } B_x(r)}{r^2} = \infty$, then the boundary of E has a finite covering diffeomorphic to the product $\mathbf{S}^{\nu-1} \times \mathbf{T}$, where $\mathbf{T}$ is a flat $(n-\nu)$-torus ; let $\pi\ :\ \mathbf{T} \longrightarrow \partial E$ be the induced immersion, then

$$\mathcal{H}^k(M, g) \simeq H^k(M \setminus E, \ker \pi^*),$$

where $H^k(M \setminus E, \ker \pi^*)$ is the cohomology associated to the complex of differential forms on $M \setminus E$ which are zero when pulled back to $\mathbf{T}$:

$$H^k(M \setminus E, \ker \pi^*) =$$

$$\{\alpha \in C^\infty(\Lambda^k T^*(M\setminus E)),\ d\alpha = 0,\ \ pi^*\alpha = 0\} / \{d\alpha, \alpha \in C^\infty(\Lambda^{k-1} T^*(M \setminus E)), \pi^*\alpha = 0\}.\ :$$

This theorem was already known for asymptotically euclidean manifolds, i.e. when each end is simply connected ([C1, M]).

This theorem is obtained as an application of the analysis we have developped in ([C2]) and of the work of Eschenburg and Schroeder which describes the endstructure of flat manifolds ([E-S], see also [G-P-Z]). The preprint is available at: www.math.sciences.univ-nantes.fr/carron

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L_2-invariants for coherent analytic sheaves

Philippe Eyssidieux

After the pioneering work of Gromov (J.Diff. Geom 1992), several algebraic geometers (most notably Campana and Kollár) recognized the fruitfulness of the following problem:

Let X be a complex projective manifold, $\pi : \tilde{X} \to X$ be an infinite Galois covering of X and L be a holomorphic line bundle on X. Study the Hilbert space of L_2 holomorphic sections of $\pi^* L$ (L_2 with respect to $Gal(\tilde{X}/X)$-equivariant metrics).

Cohomological techniques for coherent analytic sheaves are well known from the classical work by Oka, Cartan and Serre in the 1950s to provide efficient techniques to study linear series. It is therefore very natural to construct a cohomological formalism for studying these L_2- linear series.

Let $(\tilde{X}, O)$ be a complex analytic space endowed with a properly discontinuous action of a discrete group Γ. Let $C_\Gamma(\tilde{X})$ be the category of Γ-equivariant coherent analytic sheaves on $\tilde{X}$. Let A be the von Neumann algebra which is the commutant of the right regular representation of Γ. Let $\pi : \tilde{X} \to \tilde{X}/\Gamma := X$ be the quotient map. For any $F \in Ob(C_\Gamma(\tilde{X}))$ we construct a sheaf of A-modules on X $l_2\pi_* F \subset \pi_* F$, requiring that $s \in F(\pi^{-1}(U)$ should be 'L_2 along the fibers '. The functor $F \to l_2\pi_* F$ is then exact. Setting $H^q_{(2)}(\tilde{X}, F) := H^q(X, l_2\pi_* F)*$ gives rise to a cohomological δ-functor from $C_\Gamma(\tilde{X})$ to (A-modules).

The category of hilbertian A-modules defined by Murray-von Neumann (on which a real valued dimension function is defined) can be embedded in an abelian category $E(A)$, as shown by Farber. This category is a subcategory of (A-modules). Denote by $C_\Gamma(\tilde{X})_c$ the full subcategory of $C_\Gamma(\tilde{X})$ consisting of Γ-cocompactly supported sheaves. We construct a lift $R\Gamma_{(2)} : D^b(C_\Gamma(\tilde{X})_c) \to D^b(E(A))$ of the preceding δ-functor, hereby proving an analogue of Cartan's theorem A.

Consider an equivariant proper morphism $f : \tilde{Z} \to \tilde{X}$, then we have $R\Gamma_{(2)}(\tilde{Z}, .) = R\Gamma_{(2)}(\tilde{X}, Rf_*.)$ (Leray spectral sequence). We also have an analogue of Atiyah's L_2-index theorem which states that $\sum_q (-1)^q$
$\dim_\Gamma H^q_{(2)}(\tilde{X}, F)) = \sum_q \dim H^q_\Gamma(\tilde{X}, F)$ (here H^q_Γ is the equivariant cohomology).

This work was published in Math. Ann. (2000);simultaneously, independent work on this topic was done by Campana and Demailly.

Higher torsion invariants

Sebastian Goette

We compare two generalisations of Franz-Reidemeister torsion to the family case of families.

The higher analytic torsion form of Bismut and Lott arises in a Grothendieck-Riemann-Roch theorem for flat bundles. Let $F \to M$ be a flat vector bundle over the total space of a proper submersion $p\colon M \to B$, then the fibrewise cohomology $H = H^*(M/B;F) \to B$ is again a flat vector bundle. The analytic torsion $\mathcal{T}$ compares two natural characteristic forms associated with F and H. If both bundles carry parallel metrics, this form gives rise to a cohomology class $\mathcal{T}(M/B;F) \in H^{\mathrm{even},\geq 2}(B;\mathbb{R})$, the *higher analytic torsion.*

On the other hand, if $\dim M > 2\dim B$, there exists a function $h\colon M \to \mathbb{R}$ whose fibrewise singularities are either of Morse type, or cubical and unfolded over B, and whose unstable tangent bundle is trivialised. If F admits a parallel metric and is fibrewise acyclic, Igusa's *higher Franz-Reidemeister torsion* $\tau(M/B;F)$ measures the variation of the fibrewise Thom-Smale complex over generic points in B.

Extending previous joint work with Bismut, we show that if h has no cubical singularities, then $\mathcal{T}(M/B;F)$ can be expressed in terms of a torsion class $T(M/B;F,h)$ similar to Igusa's, and a characteristic class ${}^0J$ of the vertical tangent bundle along the fibrewise singularities. This leads us to conjecture a general relation between the torsion classes of Igusa and Bismut-Lott.

As an application, we use the higher analytic torsion to detect infinite families of smooth bundles $M_j \to B$ with diffeomorphic fibres that are homeomorphic but not diffeomorphic as bundles.

Weyl's law for semialgebraic sets

Daniel Grieser

A semialgebraic set is a subset of $\mathbb{R}^N$ which can be described by a finite number of polynomial equations and inequalities. Let X be a compact semi-algebraic set. Such a set is stratifyable. We assume that X has an open dense stratum Ω of dimension n. We equip Ω with the Riemannian metric induced by restriction from the ambient space. We prove Weyl's law for Ω, i.e.

$$N(\lambda) = c_n \mathrm{vol}_n(\Omega)\lambda^{n/2} + O(\lambda^{n/2-\epsilon_0}),$$

where c_n is the 'usual' constant, that is $(2\pi)^{-n}$ times the volume of the unit ball in $\mathbb{R}^n$, and ϵ_0 is some positive number. Here, $N(\lambda)$ is the number of eigenvalues of the Laplacian on Ω, with Dirichlet or Neumann boundary conditions. That is,

$$N(\lambda) = \sup\{\dim V : \int_\Omega u^2 \leq \lambda \int_\Omega |\nabla u|^2\},$$

where V varies over subspaces of $H_0^1(\Omega)$ or $H^1(\Omega)$, respectively. Similar but weaker results have been obtained previously by various authors, among them Nagase, Pati, Li-Tian, and Gromov.

For the proof we use the classical domain decomposition method, which relies on the minimax principle for $N(\lambda)$. In order to make this work, we divide Ω into one large piece, a smooth manifold with boundary of volume close to $\mathrm{vol}_n(\Omega)$, and many small pieces of diameter roughly $\lambda^{-1/2}$ which may be singular. The Weyl asymptotics then follows from a Poincaré inequality for the small pieces. This inequality can be proved for 'standard cusps', i.e. cusps defined inductively (over dimension) using monomials. By a theorem of A. Parusiński X has a decomposition into such standard cusps.

Quaternion analytic torsion

Kai Köhler

(joint work with Gregor Weingart)

Analytic torsions were introduced by Ray and Singer as real numbers constructed using certain $\mathbf{Z}$–graduated complexes of elliptic differential operators acting on forms with coefficients in vector bundles on compact manifolds. The real analytic torsion was defined for the de Rham-operator associated to flat Hermitian vector bundles on Riemannian manifolds. The complex Ray-Singer torsion was defined for the Dolbeault-operator acting on anti-holomorphic differential forms with coefficients in a holomorphic Hermitian vector bundle on a compact complex manifold.

Thus it seems natural to investigate torsions for other $\mathbf{Z}$–graded complexes occurring in geometry. We give a definition of an (equivariant) quaternionic torsion for quaternionic Kähler manifolds M, with coefficients in the antiselfdual vector bundles $\mathcal{W}$. This is done by decomposing the action of a natural Dirac operator on Salamon's complex on these manifolds

$$\begin{array}{ccccc} 0 & \longrightarrow & \mathrm{Sym}^k H \otimes \mathcal{W} & \longrightarrow & \mathrm{Sym}^{k+1} H \otimes Ł^{1,0} E^* \otimes \mathcal{W} \\ & & \longrightarrow \quad \ldots & \longrightarrow & \mathrm{Sym}^{2n+k} H \otimes Ł^{2n,0} E^* \otimes \mathcal{W} \quad \longrightarrow \quad 0 \end{array}$$

for a parameter $k \in \mathbf{N}_0$ even and $TM \otimes_{\mathbf{R}} \mathbf{C} \cong H \otimes E$. The Laplace operator defining the torsion is the square of this Dirac operator. Also we compute the equivariant quaternionic torsion for all known quaternionic Kähler manifolds of positive curvature, i.e. for the quaternionic symmetric spaces of the compact type, with respect to the action of any element of the associated Lie group and any equivariant antiselfdual vector bundle.

Finally we comment briefly on the special case of hyperkähler manifolds, in which the quaternionic torsion can be expressed in terms of a Dolbeault-operator.

APS Index theory for etale groupoids

Eric Leichtnam

(joint work with Paolo Piazza)

Connes has proved a cohomological form of the index theorem for leafwise elliptic operators on a closed foliated manifold $(X, \mathcal{F})$. He considered the etale groupoid G_T^T associated with a complete transversal T where G_T^T denotes the set of paths of the holonomy groupoid of $(X, \mathcal{F})$ whose extremeties lie on T.

Gorokhosvky and Lott have given a heat superconnection proof of Connes' theorem by combining techniques of Bismut (the family index theorem) and Lott (the superconnection proof of the Connes-Moscovici higher index theorem for coverings). The advantage of the approach of Gorokhosvky and Lott lies in the fact that it provides an explicit representative of the Chern character of the index.

In this work, we use the constructions of Gorokhosvky and Lott to state and prove an Atiyah-Patodi-Singer type index theorem for a class of etale groupoids G and proper

$G-$manifolds with boundary. We assume that the boundary Dirac operator is invertible and that the etale groupoid is (in some sense) of polynomial growth so that we are able to prove the convergence of the higher eta invariant. We also give an application to the foliation case.

Analytic torsion for manifolds with boundary

XIAONAN MA
(joint work with Jochen Brün ing)

It is interesting in itself to understand various index theorems from the point of view of local index theory, and it also helps us to understand other geometric invariants such as the η-invariant or the analytic torsion. Let us briefly recall the definition of the Ray-Singer metric: Let F be a flat vector bundle on a compact manifold, and let $\det H(X, F)$ be the determinant of the (absolute) cohomology of F. The Ray-Singer metric on the complex line $\det H(X, F)$ is the product of the standard L_2-metric on $\det H(X, F)$ and the Ray-Singer analytic torsion.

In this talk, we prove at first a local index version of the theorem of Gauss-Bonnet-Chern for manifolds with boundary, equipped with a general Riemannian metric (which needn't have a product structure near its boundary).

As a natural continuation of the above local index theorem, we establish the anomaly formula for Ray-Singer metrics associated with a flat vector bundle on a compact manifold with boundary. We do not assume that metrics on the flat vector bundle are flat nor that the Riemannian metrics have product structure near the boundary. Thus we generalize the corresponding result of Bismut-Zhang to manifolds with boundary.

Pseudodifferential algebras and index on manifolds with boundary

SERGIU MOROIANU
(joint work with Robert Lauter)

Let M^N be a compact manifold with boundary together with a boundary fibration $\partial X \to Y$ of closed manifolds with fiber F. Let $x : X \to [0, \infty)$ be a boundary-defining function, y and z local coordinates on Y, respectively on F. The double-edge and the fibered cusp Lie algebras are the Lie sub-algebras of $\mathcal{V}(X)$ spanned over $C^\infty(X)$ by $x^2\partial_x, x^2\partial_y, x\partial_z$, respectively by $x^2\partial_x, x\partial_y, \partial_z$. They induce algebras of differential operators ${}^{de}\mathrm{Diff}(X)$, ${}^{\Phi}\mathrm{Diff}(X)$. There exist algebras of pseudodifferential operators $\Psi_{de}(X)$, $\Psi_\Phi(X)$ containing ${}^{de}\mathrm{Diff}(X)$ and ${}^{\Phi}\mathrm{Diff}(X)$, respectively, as all the differential operators. We compute the Hochschild homologies of these algebras and of some ideals and quotients. The results are certain geometric cohomology spaces, different in the double-edge and fibered-cusp cases. As a corollary we deduce the existence of unique traces on these algebras. The traces can be identified in terms of double zeta-functions in the spirit of the Wodzicki residue. In this setting we give an index formula for fully elliptic operators:

$$\mathrm{Index}(A) = \overline{AS}(A) + \hat{\mathrm{Tr}}_\partial([\log x, B]]A),$$

where the first term is a local expression in the symbol of A vanishing rapidly at the boundary, while the second is concentrated on the boundary.

We specialize to $A \in {}^{\Phi}\mathrm{Diff}^1(X, E \oplus E)$ elliptic, $Y = S^1$, D a family of invertible operators on $\partial X \to S^1$ with values in E and A near ∂X of the form $A = x^2 \partial_x I_2 + \delta_x$, where

$$\delta_x := \begin{bmatrix} -ix\tilde{\nabla}_{\partial_\sigma} & D^* \\ D & ix\tilde{\nabla}_{\partial_\sigma} \end{bmatrix},$$

∂_σ is the unit vector filed along S^1 and $\tilde{\nabla}_{\partial_\sigma} = \nabla_{\partial_\sigma} + \frac{1}{4}\mathrm{Tr}(L_{\tilde{\partial}_\sigma} g^F)$ is a correction to the connection on E.

Theorem. Let $\lim_a \eta(\delta_x)$ be the limit as $x \to 0$ of the eta invariant of δ_x (the adiabatic limit). Then

$$\mathrm{Index}(A) = \overline{AS}(A) - \frac{1}{2}\lim_a \eta(\delta_x).$$

On the discrete spectrum of the Laplacian on locally symmetric spaces of finite volume

Werner Müller

Let $X = G/K$ be a Riemannian symmetric space of non-positive curvature and let $\Gamma \subset G$ be a discrete subgroup with $\mathrm{vol}(\Gamma\backslash G) < \infty$. Let Δ be the Laplacian of X. Regarded as operator $\Delta : C_c^\infty(\Gamma\backslash X) \to L^2(\Gamma\backslash X)$, Δ is essentially selfadjoint. It follows from the work of Langlands on Eisenstein series that the spectrum of Δ consists of a pure point spectrum $\mathrm{Spec}_{pp}(\Delta)$ and an absolutely continuous spectrum $\mathrm{Spec}_{ac}(\Delta)$. For arithmetic quotients $\Gamma\backslash X$, the point spectrum has deep connections with number theory. There are a number of conjectures centered around the intrinsic and fine structure of the point spectrum. Due to the presence of a large continuous spectrum, eigenvalues tend to be highly unstable and, therefore, are difficult to study. We are concerned with the existence problem. Let $0 = \lambda_0 < \lambda_1 \le \lambda_2 \le \cdots$ be the possible eigenvalues of Δ and let

$$N_\Gamma(\lambda) = \#\{i \mid \lambda_i \le \lambda\}$$

the eigenvalue counting function. The following conjecture is due to Sarnak and the author.
Conjecture: If $\mathrm{rk}\, G > 1$, then $N_\Gamma(\lambda)$ satisfies Weyl's law, i.e.,

$$N_\Gamma(\lambda) \sim \frac{\mathrm{vol}(\Gamma\backslash X)}{\Gamma(n/2+1)(4\pi)^{n/2}} \lambda^{n/2}$$

as $\lambda \to \infty$. Here $n = \dim X$.
Using the trace formula, Selberg has shown that this holds for the principal congruence subgroups $\Gamma(N)$ of $\mathrm{SL}(2,\mathbb{Z})$. Efrat has established the conjecture for the Hilbert modular groups. St. Miller proved that it holds for $\mathrm{SL}(3,\mathbb{Z})$. Our main result is the following theorem.

Theorem *Let $\Gamma \subset \mathrm{SL}(n,\mathbb{R})$ be a congruence subgroup. Then Weyl's law holds for Γ.*

More generally, we may take any number field F and the algebraic group G over $\mathbb{Q}$ obtained from SL_n by restriction of scalars from F to $\mathbb{Q}$. Then Weyl's law holds for every congruence subgroup of $G(\mathbb{Q})$.

The proof of this theorem relies on the following results:

(1) The Arthur trace formula,
(2) the theory of Eisenstein series,
(3) the theory of Rankin-Selberg L-functions,
(4) the description of the residual spectrum for GL_n by Moeglin and Waldspurger.

On the cut and paste invariance of Novikov's higher signatures.

Paolo Piazza
(joint work with Eric Leichtnam)

Let M be a compact orientable manifold (without boundary) and let $L(M)$ the Hirzebruch L-genus of M. It is well known that the integer $\sigma(M) :=< L(M), [M] >$ satisfies the following properties:

(i) it is a *homotopy invariant.*

(ii) it is a *cut-and-paste invariant.*

By cut-and-paste invariance is meant the following: if

$$M_1 = M_+ \cup_{(F,\phi_1)} M_- \,, \quad M_2 = M_+ \cup_{(F,\phi_2)} M_- \,,$$

with $\partial M_+ = F = -\partial M_-$, $\phi_j \in \text{Diffeo}(F)$, then $\sigma(M_1) = \sigma(M_2)$.

Let Γ be a finitely generated discrete group. One might wonder whether these two properties are still true for Novikov's higher signatures $\sigma((M,r),[c])$ associated to a pair $(M, r : M \to B\Gamma)$ and to the cohomology classes $[c]$ in $H^*(B\Gamma, \mathbf{R})$:

$$\sigma((M,r),[c]) :=< L(M) \cup r^*[c]; [M] > \ .$$

The first property is known as the Novikov conjecture and is still open for an arbitrary Γ as above (although established for several classes of groups); the second property is in general false. Nevertheless, one can give sufficient conditions on Γ and on F ensuring that the cut-and-paste invariance property does hold. In order to understand how it is possible to do so we work with the index class associated to the signature operator on the covering defined by $(M, r : M \to B\Gamma)$. This is a class $\text{Ind}(\mathcal{D}_{(M,r)}) \in K_*(C^*_r\Gamma)$. We show that if $(M_1, r_1 : M \to B\Gamma)$ is cut-and-paste equivalent to $(M_2, r_2 : M \to B\Gamma)$ then

$$\text{Ind}(\mathcal{D}_{(M_1,r_1)}) - \text{Ind}(\mathcal{D}_{(M_2,r_2)}) = \text{hsf}(\mathcal{D}_F(\theta)) \quad \text{in} \quad K_*(C^*_r\Gamma).$$

On the right hand side, the higher noncommutative spectral flow of a suitable S^1-family of operators on F appears. Let $m = [(\text{dim}M + 1)/2]$ and assume that the $m-th$ Novikov-Shubin invariant of $(F, r_1|_F \to B\Gamma)$ is equal to ∞^+; under this assumption on F we show that $\text{hsf}(\mathcal{D}_F(\theta)) = 0$ in $K_*(C^*_r\Gamma) \otimes \mathbb{Q}$. Thus, in this case, $\text{Ind}(\mathcal{D}_{(M_1,r_1)}) = \text{Ind}(\mathcal{D}_{(M_2,r_2)})$ for two cut-and-paste equivalent pairs.

If in addition Γ is such that the Baum-Connes map $K_*(B\Gamma) \to K_*(C^*_r(\Gamma))$ is rationally injective then from the equality of the two index classes we get the equality of the higher signatures. Summarizing: $\sigma((M_1,r_1),[c]) = \sigma((M_2,r_2),[c])$ for two cut-and-paste equivalent pairs satisfying the two assumptions above.

This result was first proved by Leichtnam-Lott-Piazza for Gromov-hyperbolic groups by working in the noncommutative de Rham homology of the Connes-Moscovici algebra. It was then proved in the above more general form by Leichtnam-Lück-Kreck using algebraic surgery. The proof presented in this talk is a purely analytic treatment of the latter result.

A geometric description of equivariant K-homology

THOMAS SCHICK
(joint work with Paul Baum and Nigel Higson)

Equivariant homology theories play a more and more important role in modern geometric topology. In particular, they feature in various isomorphism conjectures. We study equivariant K-homology, the left hand side of the Baum-Connes conjecture.

K-theory has a very geometric description in terms of vector bundles (under appropriate conditions also its equivariant version). Contrarily, definitions of equivariant homology are given in terms of Kasparov's KK-theory, or using homotopy theory (done by Lück-Oliver), and are not very intuitive.

After a motivational introduction to the Baum-Connes conjecture, a very geometric and concrete definition of equivariant K-homology for proper actions of discrete groups is the main part of the talk. The cycles are simply (certain) differential operators on equivariant manifolds. We sketch the problems and the solution to prove that this coincides with the previous definition. This implies certain finiteness properties for the K-theory of the C^*-algebra of groups for which the Baum-Connes conjecture is true.

One application is a geometric description of an equivariant Chern character, computing (complexified) equivariant K-homology in terms of equivariant homology.

K-theory of Boutet de Monvel's algebra

ELMAR SCHROHE
(joint work with S. Melo and R. Nest)

We considered the algebra A of all operators of order and class zero in Boutet de Monvel's calculus on a compact manifold X with boundary. A is known to be a Fréchet-$*$-subalgebra of $\mathcal{L}(H)$, where H is the Hilbert space $L^2(X) \oplus H^{-1/2}(\partial X)$. Its closure $\mathcal{A}$ therefore is a C^*-algebra. We studied its K-theory.

Assuming that all connected components of X have nonempty boundary, we show that $K_1(\mathcal{A}) \simeq K_1(C(X)) \oplus \ker\chi$, where $\chi : K_0(C_0(T^*\dot{X})) \to \mathbb{Z}$ is the topological index and $T^*\dot{X}$ denotes the cotangent bundle of the interior. Also $K_0(\mathcal{A})$ is topologically determined. In case the boundary has torsion free K-theory, we get $K_0(\mathcal{A}) \simeq K_0(C(X)) \oplus K_1(C_0(T^*\dot{X}))$.

Geometry and analysis on Alexandrov spaces

TAKASHI SHIOYA

It is interesting to study the spectral properties of Alexandrov spaces under perturbations with respect to the Gromov-Hausdorff topology. We have a natural C^0 Riemannian metric defined outside the measure zero singular set of an Alexandrov space, and this induces the $(1, 2)$-Sobolev space. The Laplacian is defined as the generator of the energy form. A remarkable property is that the Poincaré inequality holds on each metric ball, where the Poincaré constant is estimated independently of the space and depends only on the dimension and the lower curvature and volume bounds. This is essential to obtain the continuity of the spectral structure of an Alexandrov space with respect to the Gromov-Hausdorff topology under lower curvature and volume bound.

Trigonometric partial fraction decompositions and applications

Andras Szenes
(joint work with Michèle Vergne)

Questions of classical enumerative geometry may be analyzed using localization techniques. One of the first such methods was Bott's residue formula for characteristic numbers of complex manifolds. Recently, starting with the work of Witten on nonabelian localization, and then from the papers of Jeffrey and Kirwan a new principle emerged: the manifold is obtained as a quotient by a group action, but the characteristic numbers are still obtained from contributions at the fixed points of this action. In the simplest case of a torus action, this principle gives a new and efficient way to compute volumes of convex polytopes. We describe joint work with Michèle Vergne, which contains an extension of this method, allowing one to compute the number of lattice points in polytopes. The result is based on a refined partial fractions decomposition for trigonometric rational functions. This type of decomposition can be successfully applied to other problems, for example to the computation of the coefficients of Verlinde polynomials.

Adibatic Limit and Szegő projections

Grigore Raul Tataru

Given a Riemannian manifold (X, g), compact without boundary, an adapted complex structure can be introduced on T^*X near the 0-section such that the coball bundles $\mathbb{B}^*_\epsilon X = \{g(x,\xi) \leq \epsilon^2\}$ are strictly pseudoconvex for ϵ small.

Our objective is to prove that the fibre integration map:

$$\mathcal{H}(\mathbb{B}^*_\epsilon X) \cong S(C^\infty(\mathbb{S}^*_\epsilon X)) \ni u \longrightarrow Tu(x) = \int_{\mathbb{S}^*_{\epsilon x}X} u(x,\xi)d\xi \in C^\infty(X)$$

is an isomorphism for ϵ small enough; above $\mathcal{H}(\mathbb{B}^*_\epsilon X)$ stands for the space of holomorphic functions on the interior of $\mathbb{B}^*_\epsilon X$ smooth up to the boundary identified with the range of the Szegő projection on $\mathbb{S}^*_\epsilon X$. T is known to be an isomorphism for $X = \mathbb{R}^n$ (Hőrmander) and $X = \mathbb{S}^n$ (Lebeau) and Fredholm in general (Boutet de Monvel-Guillemin); also a variant of the map, integration over the balls $\mathbb{B}^*_{\epsilon x}X$ instead of the spheres $\mathbb{S}^*_{\epsilon x}X$, was proved to be an isomorphism for ϵ small (Epstein-Melrose).

The proof is based on studying the behavior of the Szegő projections S_ϵ on $\mathbb{S}^*_\epsilon X$ as $\epsilon \searrow 0$; it is shown that they can be understood as an element of an algebra Ψ^*_{aH} of operators on $S^*X \times [0, \epsilon_0)$. A general element of this algebra, $A \in \Psi^*_{aH}(\mathbb{S}^*X)$, 'restricts' for each $\epsilon > 0$ to be a standard Heisenberg operator on $\mathbb{S}^*X$; its limit at $\epsilon = 0$ is a family $N(A)$ of translation invariant Heisenberg operators $N(A)_x$ on $T_xX \times \mathbb{S}^*_xX$, $x \in X$.

Since the limit at $\epsilon = 0$ is just the $\mathbb{R}^n$ case and T is an isomorphism there, the same holds for small ϵ.

Calculation of the Heat Kernel Coefficients

Gregor Weingart

By the work of Minakshisundaram–Pleijel the heat kernel of a selfadjoint differential operator L of Laplace type on sections of a vector bundle E has an asymptotic expansion

$$k_t^L(x,y) \quad \sim \quad \frac{1}{\sqrt{4\pi t}^n} e^{-\frac{\mathrm{dist}^2(x,y)}{4t}} \sum_{\mu \geq 0} t^\mu \, a_\mu(x,y)$$

with coefficients $a_\mu(x,y)$ which carry important geometric information about the operator L. If we trivialize the bundle E using radial parallel transport we can compare L with the standard euclidian Laplacian Δ and get as a more or less formal consequence of the above asymptotic expansion the intertwining property of the heat kernel coefficients

$$\frac{(-1)^k}{k!} \Big[\, L^k \, \psi \, \Big](x) \quad = \quad \sum_{\mu=0}^{k} \frac{(-1)^\mu}{\mu!} \Big[\, \Delta^\mu \left(\widehat{a}_{k-\mu}(\,\cdot\,, y) \, \psi(y) \right) \Big](x)$$

where the $\widehat{a}_{k-\mu}(x,y) \; := \; a_{k-\mu}(x,y) j(x,y)$ are the coefficients rescaled by the Jacobian $j(x,y)$ of the exponential map in x with respect to the Riemannian metric defined by the principal symbol of L.

Without loss of generality we can simplify the problem notationally and consider a trivial vector bundle $V \times E$ on a euclidian vector space V and a differential operator L of Laplace type acting on sections of $V \times E$ besides the euclidian Laplacian Δ. If we assume in addition that the exponential map in the origin for the symbol metric of L is the identity map of V we can use the intertwining property of the heat kernel coefficients $\widehat{a}_\mu$ to get the formula

Theorem: (Polterovich's inversion formula)

$$(\, \widehat{a}_k \psi \,)(0) = \sum_{l=0}^{r} \left(-\frac{1}{4} \right)^l \binom{r + \frac{n}{2}}{r - l} \frac{(-1)^{k+l}}{(k+l)!} \, L^{k+l} \, (\frac{1}{l!} |x|^{2l} \psi \,)(0) \quad r \geq k \geq 0$$

where $|x|^2$ is the distance function to the origin with respect to both the euclidian and the symbol metric. Note that the coefficients $\widehat{a}_\mu$ are not well–defined but in a neighborhood of the origin and in fact the intertwining property is strong enough to fix their infinite order jets in 0 completely. These jets are elements of (the completion of) $\mathrm{Sym}\, V^* \otimes E$ and the corresponding formal power series $\mathrm{jet}^\infty \, \widehat{a}(z) \; := \; \sum_{\mu \geq 0} \mathrm{jet}^\infty \widehat{a}_\mu \, z^\mu$ is given by:

Theorem: (Inversion formula)

$$\mathrm{jet}^\infty \, \widehat{a}(z) \quad = \quad e^{z\Delta} \, (2z)^{-N} \, \sigma_{total}(\, e^{-zL} \, e^{z\Delta} \,)^\sharp$$

where $\sigma_{total}(\, e^{-zL} \, e^{z\Delta} \,) \in \mathrm{Sym}\, V \otimes \mathrm{End}\, E$ is the total symbol of the differential operator $e^{-zL} \, e^{z\Delta}$ in the origin, $\sharp$ is the musical isomorphism and N is the number operator of $\mathrm{Sym}\, V^*$. Looking closely at this formula we note that quite remarkably the coefficient of z^k in the formal power series $e^{-zL} e^{z\Delta}$ must be a differential operator of order $\leq k$ in the origin, because otherwise $(2z)^{-N}$ is undefined and the inversion formula makes no sense. However the origin is a rather singular point for this operator as its principal symbol of order $2k$ vanishes exactly at the points where the symbol metric agrees with the euclidian metric.

$K3$ Surfaces with Involution, Equivariant Analytic Torsion, and Automorphic Forms on the Moduli Space

Ken-Ichi Yoshikawa

Let $\mathbf{L}_{\mathrm{K3}}$ be the the even unimodular lattice of signature $(3, 19)$. Then, it is isometric to the 2nd integral cohomology lattice of a $K3$ surface. Let $M \subset \mathbf{L}_{\mathrm{K3}}$ be a primitive 2-elementary hyperbolic sublattice with rank $r(M)$.

Let (X, ι) be a $K3$ surface with involution. Then, the pair (X, ι) is called a 2-elementary $K3$ surface of type M if the following conditions are satisfied:

(1) ι is anti-symplectic, i.e., $\forall\, \eta \in H^0(X, \Omega^2_X)$, $\iota^*\eta = -\eta$.

(2) The invariant part of $H^2(X, \mathbf{Z})$ w.r.t. the ι action is isometric to M.

Let (X, ι) be a 2-elementary $K3$ surface of type M. Let $\mathbf{Z}_2 \subset \mathrm{Aut(X)}$ be the subgroup generated by ι. Let X^ι be the fixed point set of ι. Then, X^ι is either empty or the disjoint union of compact Riemann surfaces. Let $X^\iota = \sum_i C_i$ be the irreducible decomposition.

Let γ be an ι-invariant Kähler metric on X. Let $\tau_{\mathbf{Z}_2}(X, \gamma)(\iota)$ be the equivariant analytic torsion of (X, γ). Let $\tau(C_i, \gamma|_{C_i})$ be the analytic torsion of $(C_i, \gamma|_{C_i})$. For the triplet (X, ι, γ), we introduce the quantity:

$$\begin{aligned}\tau_M(X, \iota, \gamma) &= \mathrm{Vol}(\mathrm{X}, \frac{\gamma}{2\pi})^{\frac{14-r(M)}{4}} \tau_{\mathbf{Z}_2}(X, \gamma)(\iota) \prod_i \mathrm{Vol}(C_i, \frac{\gamma}{2\pi}|_{C_i}) \tau(C_i, \gamma|_{C_i}) \\ &\quad \times \exp\left[\frac{1}{8}\int_{C_i} \log\left(\frac{\eta\wedge\bar{\eta}}{\gamma^2/2} \cdot \frac{\mathrm{Vol}(\mathrm{X}, \frac{\gamma}{2\pi})}{\|\eta\|^2_{L^2}}\right)\Bigg|_{C_i} c_1(C_i, \gamma|_{C_i})\right].\end{aligned}$$

Then, $\tau_M(X, \iota, \gamma)$ is independent of the choice of γ. Regard τ_M as a function on the moduli space of 2-elementary $K3$ surfaces of type M, denoted by $\mathcal{M}^0_M$.

Since $\mathcal{M}^0_M$ is an arithmetic quotient of a symmetric bounded domain of type IV with a divisor $\mathcal{D}_M$ removed, one can consider automorphic forms on $\mathcal{M}_M := \mathcal{M}^0_M \cup \mathcal{D}_M$. In fact, we consider automorphic forms on $\mathcal{M}_M$ with values in some line bundle λ^q_M on $\mathcal{M}_M$. An automorphic form of weight p with values in λ^q_M is called an automorphic form of weight (p, q). The Petersson norm on λ^q_M is denoted by $\|\cdot\|$.

Theorem For some $m \in \mathbf{N}$, there exists an automorphic form Φ^m_M of weight $((r(M) - 6)m, 4m)$ with zero divisor $m\mathcal{D}_M$ such that $\tau_M = \|\Phi^m_M\|^{-\frac{1}{2m}}$.

Edited by Jochen Brüning

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