Mathematisches Forschungsinstitut Oberwolfach

Report No. 37/2002

Dynamical System Methods in Fluid Mechanics

July 28th – August 3rd, 2002

Traditional methods in fluid mechanics continue to be invigorated with the infusion of ideas from dynamical systems, analysis of PDE's, geometric methods, and new developments in scientific computation. This meeting was organized around the mixture of these general themes.

The specific themes of the conference were as follows:

- **Stability, Bifurcation and Normal Forms.**
  These techniques continue to be of great interest in fluid mechanics and this remains an active area of research. These areas include stability problems for liquid drops and free boundary problems, bifurcations in rotating flows, etc.
- **Nonlinear Waves and Pattern Formation.**
  There have been interesting developments linking some of the traditional topics (such as the Benjamin-Feir instability) with geometric (multisymplectic) structures and further links with pattern formation and symmetry can be expected.
- **Averaged Euler and Navier Stokes Equations.**
  These new models for fluids have remarkable geometric structure (connections to diffeomorphism groups) and computational properties. Current investigations that link this approach with LES (large eddy simulation) and boundary layer models (based on transient growth rates and uncertainty modelling) will be discussed.
- **Control Theory.**
  Many of the interesting recent developments are motivated and enriched by control theory. This includes boundary layer control and the locomotion of underwater vehicles as well as fish, both robotic and biological.
- **Analytical and Numerical Aspects.**
  Recent formulations of fluids as discrete (in the Veselov sense), multisymplectic systems and their numerical implementation should be an exciting emerging area. Several groups worldwide are investigating these aspects. In addition progress has been continuing in traditional topics such as the analytical aspects as well as accurate computation of eigenvalues and singular values for the linearized Navier-Stokes equations, in for example, planar Couette and pipe flow.
- **Vortex Dynamics.**
  Progress relating the averaged Euler equations and vortex blobs is also an exciting area. Also, it has been recently shown how to compute the dynamic equations for vortices interacting (dynamically) with solid and elastic bodies. This is important

for underwater vehicles as well as for propulsion of fish etc. Again, several groups worldwide are investigating these issues and getting them together would be most profitable.

The conference aimed for, and largely succeeded in having a healthy mix of senior researchers, postdoctoral fellows and graduate students. Our philosophy, consistent with the general approach advocated by Oberwolfach was to select about 20–23 people to give a lecture. We suggested to the Oberwolfach administration names of students, postdoctoral fellows and junior faculty to the meeting. We limited the number of talks so there will be ample time for discussion, but give everyone a chance to participate through two poster sessions. The posters were mounted on the first day and remained for most of the week, with hours in the schedule specifically for the viewing of the posters.

Conference Schedule

Monday, July 29

- 9:00-9:15: **WELCOME**
  Gert-Martin Greuel, Director of the Institute
  Jerrold Marsden and Jürgen Scheurle, Organizers
- 9:15-10:00: Paul Newton, University of Southern California,
  "Vortex crystals, quasi-crystals, and beyond"
- 10:05-10:50: Guido Schneider, Karlsruhe
  "Transient self similar decay in Poiseuille flow at criticality for exponentially long times"
- 10:55-11:30: **PHOTO AND COFFEE BREAK**
- 11:30-12:15: Robert McLachlan, Massey
  "Some topics in the time integration of fluid and wave equations"
- 12:30: **LUNCH**
- 1:30-3:30: **INFORMAL DISCUSSIONS**
  AFTERNOON TEA AND COFFEE, 2:30-3:30
- 3:30-4:15: Mark Roberts, Surrey
  "Stability Criteria for Hamiltonian Relative Equilibria"
- 4:20-5:00: **BREAK**
- 5:00-5:45: Bernold Fiedler, Berlin
  "Plane Kolmogorov Flows"
- 6:30-7:30: **DINNER**
- **EVENING FREE**

Tuesday, July 30

- 9:00-9:45: Klaus Kirchgässner, Stuttgart
  "Dispersive dynamics in Euler-systems and applications"
- 9:50-11:00: Poster Presenters
  Each person can present a 2 minute summary of their poster using at most two transparencies
- 11:00-12:30: **COFFEE BREAK AND POSTER SESSION**
- 12:30: **LUNCH**
- 1:30-3:30: **INFORMAL DISCUSSIONS**
  AFTERNOON TEA AND COFFEE, 2:30-3:30
- 3:30-4:15: Alexander Mielke, Stuttgart
  "Energetic stability of solitary water waves"
- 4:20-5:00: **BREAK**
- 5:00-5:45: Pascal Chossat, Valbonne
  "Dynamo dynamics in a rotating spherical shell"
- 6:30-7:30: **DINNER**
- **EVENING FREE**

Wednesday, July 31

- 9:00-9:45: Marcel Oliver, Tübingen,
  "Smoothing properties of hierarchies of
  geophysical balance models"
- 9:45-10:00: **BREAK**

- 10:00-10:45: Steve Shkoller, Davis
  "Turbulent channel flow in weighted Sobolev spaces"
- 10:45-11:00: **BREAK**
- 11:00-11:45: Igor Mezic, Santa Barbara
  "Adiabatic control of vortex elements"
- 12:00: **LUNCH**
- 1:00-6:00: **HIKE**
- 6:30-7:30: **DINNER**
- 8:00-8:45: Gerard Iooss
  "Elementary bifurcations of reversible systems with an essential spectrum"

Thursday, August 1

- 9:00-9:45: Jerry Marsden, Pasadena
  "Variational integrators for the dynamics of point vortices"
- 9:45-10:00: **BREAK**
- 10:00-10:45: Sebastian Reich, London
  "Multisymplectic geometry and numerical methods
  for fluid mechanics"
- 10:45-11:30: **BREAK**
- 11:30-12:15: Claudia Wulff, Berlin
  "Approximate momentum conservation for spatial
  semi-discretizations of nonlinear wave equations"
- 12:30: **LUNCH**
- 1:30-4:00: **INFORMAL DISCUSSIONS**
  AFTERNOON TEA AND COFFEE, 2:30-3:30
- 4:00-4:45: Jochen Denzler, Knoxville
  "Spectrum of the Orr-Sommerfeld equation
  for plane Couette flow"
- 5:00-6:30: **POSTER SESSION**
- 6:30-7:30: **DINNER**
- 8:00-8:45: Tudor Ratiu, Lausanne
  "Poisson brackets for complex fluids"

Friday, August 2

- 9:00-9:45: Genevieve Raugel
  "Generic transversality property for the damped wave equation"
- 9:45-10:00: **BREAK**
- 10:00-10:45: Mariana Haragus, Talence
  "Three dimensional traveling water waves"
- 10:45-11:30: **BREAK**
- 11:30-12:15: Darryl Holm, Los Alamos
  "Introduction to the Lagrangian-averaged
  Navier-Stokes model of turbulence"
- 12:30: **LUNCH**
- 1:30-6:30 **INFORMAL DISCUSSIONS**
  AFTERNOON TEA AND COFFEE, 2:30-3:30
- 6:30-7:30: **DINNER**

Conference Participants

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Abstracts

Dynamo Dynamics in a Rotating Spherical Shell...

Pascal Chossat, INLN (UMR 6618 du CNRS) and Ambassador of France in New Delhi, India

We show that convective states in a slowly rotating spherical shell can undergo a "dynamo" bifurcation, which leads to intermittent dynamics with a magnetic dipole showing random reversals. The analysis exploits the symmetries of the system to show the existence of robust heteroclinic cycles in the purely convective regime undergoing, though the dynamo bifurcation, an instability which leads to a new heteroclinic cycle which now involves a self-sustained negative field exhibiting the intermittent-like behavior.

Spectrum of the Orr-Sommerfeld Equation for Plane Couette Flow

Jochen Denzler, Knoxville

The spectrum $\{\lambda\}$ for the Orr–Sommerfeld equation for plane Couette flow, i.e., for the equation

$$\left(R^{-1}\left(\partial_y^2-\alpha^2\right)-i\alpha y-\lambda\right)\left(\partial_y^2-\alpha^2\right)w(y)=0$$

with

$$w(0)=0=w(1), \qquad w'(0)=0=w'(1),$$

where R is the Reynolds number and α is the wave number, is determined completely and rigorously, as a function of the parameters $R\in[0,\infty]$, $\alpha>0$, with the limiting cases $R=0$, $R=\infty$ interpreted appropriately.

The limiting case $R=0$ can be dealt with by classical complex variable methods. Branches of the spectrum (any $R>0$) can be numbered according to their limiting behaviour as $t:=\frac{1}{2}(\alpha R)^{1/3}\to 0$. Even though this limiting behaviour is quantitatively different for the cases $R=0$ and $0<R\leq\infty$, the ordering of the branches is not affected by a jump discontinuity as $R\to 0$. Continuation methods permit one to follow the branches thus identified along increasing t all the way into a critical region in which they connect to each other, each such connection spawning a continuation branch with t increasing to ∞ along this branch.

Two possible behaviours occur: for small R, neighbouring branches coming from $t\approx 0+$ are connected pairwise in the critical region, whereas for $R=\infty$, they are connected in groups of four. The transitions ("mode crossings") accumulate (only) as $R\to\infty$, in an intriguing way: for low branch number, only a single mode crossing takes place. Higher numbered branches undergo mode crossings that are subsequently undone and then give way to a new mode crossing with a different (adjacent) pairing. This pattern follows from a combined asymptotics in the critical region, where $\arg\lambda\approx\pm\frac{2\pi}{3}$, α bounded (or only logarithmically growing with R), and $R\to\infty$.

Plane Kolmogorov Flows

Bernold Fiedler, FU Berlin
(joint work with Andrei Afandikov and Stefan Liebscher)

We consider the Kolmogorov problem of a viscous incompressible fluid flow in a plane channel. Stationary solutions in a vicinity of the critical Reynolds number are studied by a Kirchgässner spatial dynamics reduction to a six-dimensional centre manifold. Further reduction yields a three-dimensional reversible system with a line of equilibria. This line of equilibria is not induced by symmetries or first integrals and is not normally hyperbolic. We explore the resulting "bifurcation without parameters" by blow-up and averaging methods. The set of small bounded solutions consists entirely of lots of heteroclinic orbits of various geometric types and of the trivial equilibria. The heteroclinics, alias (multi-pulse) front waves, form a quite complicated but non-recurrent set.

Introduction to the Lagrangian-averaged Navier-Stokes alpha (LANS-alpha) model of turbulence

Darryl D. Holm, Los Alamos National Laboratory

We sketch some of the properties of the LANS-α model and it's predictions for two fundamental experimental laws of fully developed turbulence. These laws are: (1) the $k^{-5/3}$ law for the kinetic energy spectrum; (2) the $r^{2/3}$ law for average squared increments in velocity parallel to a displacement vector $\mathbf{r}$. Lagrange averaging causes the roll–offs $k^{-5/3} \to k^{-3}$ and $r^{2/3} \to r^2$ at length scales smaller than the correlation length α of the Lagrangian fluid trajectories. These two roll–offs are proved via the Kármán- Howarth theorem for velocity autocorrelation dynamics of LANS-α. These roll–offs establish that energy transport balances with dissipation at lower wavenumbers than for NS and, thus, allows LANS-α to compute in two decades of resolution what would cost three decades of resolution for Navier-Stokes.

Three-dimensional travelling water waves

Mariana Haragus (Bordeaux
(joint work with Mark Groves (Loughborough))

We present a rigorous existence theory for small-amplitude 3-dimensional travelling water waves. The hydrodynamic problem is formulated as an infinite-dimensional Hamiltonian system in which an arbitrary horizontal spatial direction is the time-like variable. Wave motions which are periodic in a second, different horizontal direction are detected using a centre-manifold reduction technique by which the problem is reduced to a locally equivalent Hamiltonian system with a finite of degrees of freedom. A catalogue of bifurcation scenarios is compiled by means of a geometric argument based upon the classical dispersion relation for travelling water waves. Nonlinear bifurcation theory is carried out for a representative selection of bifurcation scenarios. We find truly 3-dimensional waves which have periodic, solitary-wave or generalized solitary-wave profiles in one direction and are periodic in another.

Elementary Bifurcations of Reversible Systems with an Essential Spectrum Filling the Real Line

Gérard Iooss, Nice

We consider reversible systems having a one parameter family of equilibria, such that the linearized operator has an essential spectrum filling the entire real axis, in addition to a zero eigenvalue, and a pair of simple imaginary eigenvalues diving into the essential spectrum at 0 when the parameter ϵ crosses 0 (other eigenvalues lie far from the imaginary axis). We give a characterization of such systems with examples coming from travelling water wave problems, in the case when one layer of fluid is infinitely deep. We show under what general conditions haw a bifurcation occurs to an homoclinic solution (with algebraic decay) of Benjamin-Ono type.

Variational Integrators and Vortex Dynamics

Jerrold E. Marsden, CDS, Caltech
(joint work with Clancy Rowley and Paul Newton)

We review some of the features of discrete mechanics, which is based on a discrete variational principle and the associated variational integrators. Features of the algorithms include their symplectic nature, their conservation properties, as well as generalizations to the asynchronous context for spatially extended systems. We illustrate these properties using the dynamics of point vortices in the plane as well as on the sphere.

Dispersive Dynamics in Eulerian Systems and Applications...

Klaus Kirchgässner, Stuttgart

A new method is given for analyzing the motion of an inviscid fluid-layer with free boundary under the action of gravity; an observer is moving with constant speed c and the mean depth is h. We treat the supercritical case $\mu^2 = gh/c^2$ less than one. The method is based on the discussion of the dispersion relation $(s+\sigma)^2 = \mu\sigma$ in $\mathbb{C}^2$ and the use of Paley-Wiener. In the appropriate Hardy-Lebesgue-spaces, we can formulate the full problem as a spatially dynamical system parametrized by the complex Laplace-variable s. The following results are given:

(1) For small initial values there exists a unique solution globally in time.
(2) The quiescent state is Liapunov-stable.
(3) The solitary waves existing in the given μ^c-interval are bi-orbital stable, i.e., perturbed initial values converge towards a solitary wave appropriately shifted in phase and speed.

A Comparison of Some Geometric Numerical Integrators for the KdV Equation

ROBERT MCLACHLAN, MASSEY UNIVERSITY, NEW ZEALAND

This study arose from a question of Uri Ascher: is there a good explicit or semi-explicit scheme for the Korteweg-de Vries equation? If so, why would one want to use an implicit scheme? In addition, KdV is a good equation on which to probe any differences between symplectic and multisymplectic (variational) integrators. Four geometric integrators are constructed and compared: (i) an unconditionally stable scheme which preserves $|u|^2$; a standard conservative Hamiltonian finite difference scheme, which is then integrated either by (ii) an explicit symplectic splitting method or by (iii) the symplectic implicit midpoint rule; and (iv) a multisymplectic integrator, which is equivalent to the 3-time level, 12-point Preissman scheme proposed by Qin and Zhou. We find that although (i,ii) are much faster, the implicit methods (iii,iv) are stable for much larger time steps. The standard conservative finite difference (ii, iii) appears to generate small sawtooth waves, which can provoke instabilities during the time integration. The multisymplectic discretization does not do this and is also stable on much coarser spatial grids for which the solution is poorly resolved.

The Algebraic Entropy of Classical Mechanics

ROBERT MCLACHLAN, MASSEY UNIVERSITY, NEW ZEELAND

I describe be the ''Lie algebra of classical mechanics", modelled on the Lie algebra generated by kinetic and potential energy of a simple mechanical system with respect to the canonical Poisson bracket. It is a polynomially graded Lie algebra, a class we introduce. We describe these Lie algebras, give an algorithm to calculate the dimensions c_n of the homogeneous subspaces of the Lie algebra of classical mechanics, and determine the value of its entropy $\lim_{n\to\infty} c_n^{1/n}$. It is $1.82542377420108\ldots$, a fundamental constant associated to classical mechanics.

Fluid Particle Motion Control and Anti-KAM Theory

IGOR MEZIC, UNIVERSITY OF CALIFORNIA, SANTA BARBARA

We consider questions related to control of motion of fluid particles in incompressible fluids, or a subset of related questions for volume-preserving and Hamiltonian systems in arbitrary dimensions. These are in the integrable case foliated by group, so a natural problem to study is controllability of group translations. The controllability question is the following: given $g_1, g_2 \in G$, where G is a topological group, is there a sequence $u_0, \ldots, u_n$ such that $g_2 = g_1 \cdot u_0 \cdot u_1 \cdot \ldots \cdot u_n$? The answer is positive if U is of positive Haar measure and contains an ergodic element u_e (i.e. translation by u_e on G is ergodic). A more complicated problem is that of controllability of near-integrable twist maps, of the form $x' = x + y + \epsilon u f(x, y), y' = y + \epsilon u g(x, y)$ where we show that, under weak conditions on f, g and $u \in [0, 1]$ with arbitrary small ϵ, there is a sequence of u' s such that the system transitions from arbitrary initial to arbitrary final conditions. (This is joint work with U. Vasdya). In addition to having consequences for control of fluid particle motion, these results show non-robustness of KAM- type theorems to bounded noise perturbations that are arbitrarily small.

On the Energetic Stability of Solitary Water Waves

Alexander Mielke, Universität Stuttgart

We study solutions of the water wave problem for a fluid layer of finite depth in the presence of gravity and surface tension. We use the canonical Hamiltonian formulation of Zakharov in terms of the surface elevation and the trace of the velocity potential on the surface. With a new continuity result for the Dirichlet-Neumann operator in terms of the surface as a function in $H^1(\mathbb{R})$, we show conditional energetic stability of the trivial solution in certain regions of the parameter space. In the same region we obtain stability of solitary waves under the additional assumption that the second variation of the energy has only one negative eigenvalue. The latter assumptions is shown to be fulfilled for the small amplitude solitary waves constructed first by Amick and Kirchgässner.

Vortex Crystals, Quasi-crystals, and Beyond

Paul K. Newton, University of Southern California

This talk will present an overview of our current knowledge of relative equilibrium patterns of point vortices in the plane and on the sphere. Recent experiments on vortex formation in Bose-Einstein condensates by Ketterle et al (2002) offer some of the cleanest realizations of the complex lattice structures that collections of rotating identical vortices can display (up to 160 vortices in a 1mm diameter circle). Since the circulation of each vortex is quantized, and since the rotation frequency is a known function of the geometric properties of the lattice and the circulation, these experiments present the intriguing possibility of measuring microscopic quantities such as Planck's constant by careful measurements of macroscopic properties of the lattice, such as angular frequency and lattice spacing. On the sphere, there is a direct connection to the large and growing literature on the 'charge-on-a-sphere' problem, where one considers the equilibrium configurations of N point charges repelled by their mutual Coulomb interactions. Also called the 'dual problem for stable molecules', the most celebrated configuration is the stable carbon-60 molecule, or buckminster fullerine, with atoms arranged in a soccer-ball pattern. We will describe Kelvin's variational principle for vortex equilibria, the connection between the spacing of N vortices on a line so that the configuration rotates like a rigid body and the zeros of a well known family of orthogonal polynomials. On the sphere, we will describe equilibria generated from the Platonic solids and an interesting class of periodic and quasi-periodic orbits which bifurcate from them and have intriguing Floquet multiplier structure. We will also describe the use of Monte Carlo algorithms to locate ground states and ring equilibria of identical vortices on the sphere, reminiscent of the planar catalogue of two-dimensional vortex patterns of Campbell and Ziff (1978). In the end, however, I hope to convince you how little is known, particularly when it comes to asymmetric patterns, patterns with defects, vortex crystals made up of point vortices of different strengths, and stability theory.

'Vortex Crystals', H. Aref, P.K. Newton, M.A. Stremler, D.L. Vainchtein, to appear in *Advances in Applied Mechanics*, 2002.

Smoothing Properties for Hierarchies of Models for Shallow Water Wave Geostrophy

Marcel Oliver, Universität Tübingen and Bremen

Large scale flow in the atmosphere and in the ocean at mid-latitudes can often be considered as nearly geostrophic, i.e. the pressure gradient force is almost balanced by the Coriolis force, while inertia is a lower-order effect. It is therefore desirable to find reduced models that represent large scale flows near geostrophy while being computationally less expensive than the parent model. One important consideration is the need to remove inertia-gravity waves from the model, because such waves do not interact strongly with large scale circulation, yet force severe step-size constraints upon direct numerical simulations.

There are a number of well known models for geostrophic flow which differ in the assumed parent model, in the precise scaling assumptions, and the choice of coordinate system. It is often not well understood how these different models relate to each other.

In this talk I will demonstrate an ansatz based on variational asymptotics—involving an infinitesimal change of coordinates in the low-Rossby number expansion of the variational principle—which can reproduce a number of known models as well as new ones. This formulation makes it easy distinguish models based on their analytical properties (their well-posedness and regularity) and choose those most consistent with the model setting: Vortical motion at large scales.

Approximate Momentum Conservation for Spatial Semidiscretizations of Nonlinear Wave Equations

Marcel Oliver, Universität Tübingen; Matt West, California Institute of Technology; Claudia Wulff, University of Warwick

We prove that a standard second order finite difference uniform space discretization of the nonlinear wave equation with periodic boundary conditions, analytic nonlinearity, and analytic initial data conserves the momentum up to an error which is exponentially small in the stepsize. Our estimates are valid for as long as the trajectories of the full nonlinear wave equation remain real analytic. The method of proof is that of backward error analysis. We construct a modified equation which interpolates the discretized system, remains exponentially close to the semi-discrete system, but also possesses the full continuous translation symmetry, is Hamiltonian, and therefore conserves momentum. These properties directly imply approximate momentum conservation for the semidiscrete system. We also consider discretizations that are not variational as well as non-uniform grids. Through numerical examples as well as arguments from geometric mechanics and perturbation theory, we show that such methods generically do not approximately preserve momentum.

Poisson Brackets for Spin Glasses and it's Construction by Cocycles

Tudor Ratiu, Ecole polytechnique fédérale de Lausanne

Holm and Kupershmidt have given the Poisson bracket for spin glasses in 1988. The formula for this bracket consists of a relatively complicated Lie algebra, augmented by a two–cocycle. A general construction a for brackets of this type is given. It is based on a theorem that guarantees the compatibility of two Poisson brackets. The geometric interpretation of the cocycle in also presented.

Generic Transversality Property for the Damped Wave Equation

Genevieve Raugel, CNRS at Université de Paris-Sud (Orsay)
(joint work with P. Brunovsky (Bratislava)

Genevieve Raugel, CNRS at Université de Paris-Sud (Orsay); joint work with P. Brunovsky (Bratislava)

Let $\gamma > 0$, Ω be a bounded regular domain in $\mathbb{R}^n$, $n = 1, 2$ or 3. We consider the damped wave equation:

$$u_{tt} + \gamma u_t - \Delta u + u = f(x, u)$$
$$\left.\frac{\partial u}{\partial n}\right|_{\partial\Omega} = 0$$
$$(u(t, x), u_t(t, x))\,|_{t=0} = (u_0, v_0)$$

where $f : (x, y) \in \bar{\Omega} \times \mathbb{R} \mapsto f(x, y) \in \mathbb{R}$ is a C^k function for some $k \geq 2$.

In this talk we describe a generalization of an earlier generic transversality result of Burnovsky and Polacik (1997) for parabolic equations to the context of the above wave equation. For the case $n = 2, 3$, one can show that the transversality is not generic if f does not depend on x.

Geometric Integration Methods for Geophysical Fluid Dynamics

Sebastian Reich, Imperial College, London...

Large scale geophysical flows in the atmosphere and ocean are nearly incompressible and stratified into two-dimensional layers. As a result, vorticity and circulation play a central role in geographical fluid dynamics. One might therefore expect that a Lagrangian particle based approach would lead to good vorticity conservation also in a computational setting. In the talk, I developed a particle mesh based method that is Hamiltonian and exactly conserves circulation. To maintain solution regularity as well as balance in a conservative setting, we first reformulated the equations using a smoothing operator for the layer-depth. This model regularization is similar but different to α-Euler regularizations for incompressible fluids. Numerical results were presented.

Stability Criteria for Hamiltonian Relative Equilibria

Mark Roberts, Surrey

Several different types of stability can be consider for relative equilibria of left or right invariant geodesic flows on a Lie group G. Criteria for leaf-wise stability (i.e., stability of the reduced equilibrium point μ^c for the flow on its coadjoint orbit in $\mathfrak{g}^*$) are obtained from the Arnold form. Linear and nonlinear Eulerian stability are determined by considering the flow of the Euler equation on the whole of $\mathfrak{g}^*$. Finally linear Lagrangian stability on the whole phase space T^*G is determined by the Jacobi equation.

If the "velocity" of the relative equilibrium is $\xi \in \mathfrak{g}$, the "difference" between spectral leafwise stability and spectral Eulerian stability is given by the spectrum of $\left(\mathrm{ad}_\xi\,|_{\mathfrak{g}_\mu}\right)^*$. Similarly, the difference between spectral Eulerian stability and spectral Lagrangian stability is given by the spectrum of ad_ξ. Examples of these spectrum have been computed for semi-simple Lie groups and (formally) the Bott-Virasoro group.

Nonlinear Eulerian stability is typically determined using the Energy-Casimir method. This has been extensively generalized, and is foundation explored, in recent work of Patrick, Roberts and Wulff (preprint 2002). Their results also suggest a possible approach to nonlinear Lagrangian stability, using an extension of the Energy-Momentum method.

Almost Global Existence and Transient Self Similar Decay for Poiseuille Flow at Criticality

Guido Schneider, Karlsruhe
(joint work with Hannes Uecker)

We consider nonlinear diffusion equations as

$$\partial_t u = \partial_x^2 u + u^3,$$

with $x \in \mathbb{R}$ for small initial data in $L^1 \cap L^\infty$. It is well known that all solutions of this system explode in finite time. However, we make the observation that in terms of the norm of the initial conditions, it takes an exponentially long time. Moreover, before explosion the L^∞-norm of such solutions becomes exponentially small, which makes it almost impossible to observe the instability in experiments. As an application, we consider the long time transient self-similar decay to unstable Poiseuille flow at criticality for exponentially long times. This, together with subcritical bifurcation and short time transient amplification, is a principal obstruction in all attempts to measure the critical Reynolds number for this experiment more and more precisely.

Turbulent Channel Flow in Weighted Sobolev Spaces

Steve Shkoller, University of California Davis

Modeling the mean characteristics of turbulent channel flow has been one of the long-standing problems in fluid dynamics. While a great number of mathematical models have been proposed for isotropic turbulence, there are relatively few, if any, turbulence models in the anisotropic well-bounded regime which hold throughout the *entire* channel. Recently, the anisotropic Lagrangian averaged Navier- Stokes (LANS-α) equations have been derived. This talk is devoted to the analysis of this coupled system of nonlinear PDEs for the mean velocity and covariance tensor in the channel geometry. The vanishing of the covariance along the walls induces certain degenerate elliptic operators into the model, which require weighted Sobolev spaces to study. We prove that when the no-slip boundary conditions are prescribed for the mean velocity, the LANS-α equations possess unique global weak solutions which converge as time tends to infinity, towards the unique stationary solutions. Qualitative properties of the stationary solutions are also established. (Joint with D. Coutand)

Edited by Jerrold E. Marsden

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