Mathematisches Forschungsinstitut Oberwolfach

Report No. 40/2002

Mathematical Theory and Modelling in Atmosphere-Ocean-Science

August 18th – August 24th, 2002

Introduction

Atmospheric and oceanic motions are characterized by a very wide range of length and time scales, and by a rich collection of varying physical phenomena, see e.g., [1], [2]. The mathematical description of these motions reflects this multitude of scales and mechanisms in that it involves strong non-linearities and various scale-dependent singular limit regimes. Furthermore, the chaotic nature of atmosphere-ocean flows and the inherent uncertainties in available measured data necessitate the incorporation of stochastic and/or statistical components in comprehensive mathematical models.

Considerable progress has been made in recent years in the numerical simulation of geophysical flows. Examples include detailed computational studies of cloud formation processes, medium range weather forecasts, and long term predictions of El Niño climate phenomena (see, e.g., [3,4]).

These promising developments and the growing public awareness of potentially dangerous climate change impacts stimulate a staggering demand for increased precision and scope of computational predictions, and for improved estimates of the remaining uncertainties. Matching these expectations will require the systematic exploration of all the scientific potential available today. In particular, modern applied mathematics may be expected to contribute a number of important results and techniques from the areas of partial differential equations, stochastic analysis, statistics, optimization and control theory, numerical analysis, and scientific computing.

In turn, the complexity of the mathematical problems in atmosphere-ocean science makes this field a rich source of new, intriguing challenges in pure and applied mathematics.

This workshop brought together a group of mathematicians and theoretically oriented disciplinary researchers, and covered the following specific areas:

(1) *Stochastic modelling of under-resolved processes*
The scales of atmosphere-ocean flows cover a range from a few centimetres to thousands of kilometres. These scales cannot simultaneously be represented on any existing computer. Thus the net effects of unresolved scale processes must be modelled or "parametrized". Methods of stochastic analysis in combination with scaling

theories for small scale flow regimes may provide unconventional new approaches to the problem of subgrid scale modelling.

(2) *Development and rigorous justification of reduced asymptotic models*
Successful derivations of reduced models from more comprehensive equation systems, [2,5], enable scientific progress for (at least) two reasons. First, these reduced models identify critical physical interactions and allow one to study them in a simplified setting where rigorous mathematical analyses are still feasible. Secondly, reduced models often lead to more efficient computational representations of the associated flow regimes. The techniques of scale analyses traditionally used for such derivations in theoretical meteorology and oceanography are generally equivalent or closely related to systematic singular perturbation methods, such as matched asymptotic expansions, boundary layer techniques, etc.. However, the full potential of multiple space and multiple time scale analysis is yet to be explored in atmosphere-ocean science. In the context of assessing uncertainties of model predictions, an increasingly important role of mathematical theory will be the rigorous justification of reduced models and/or the suggestion of judicious modifications that will render them asymptotically correct.

(3) *Efficient and accurate numerical methods*
The computational simulation of atmosphere-ocean phenomena requires the numerical integration of complex nonlinear (stochastic) partial differential equations on the most advanced super-computers available, [6]. The complexity of the governing equations of atmosphere-ocean models require sophisticated solution techniques that must excel in the areas of both accuracy and computational efficiency. The transfer of modern numerical methods into computational codes for atmosphere-ocean simulations promises considerable progress, provided that particular attention is payed to a number of implicit conservation laws that play a key role for the large scale, long time dynamics in atmosphere and ocean.

(4) *Blends of 1.-3.*
The vast complexity of atmosphere-ocean systems makes it necessary to combine ideas from areas 1.-3. when addressing many, if not most, relevant scientific questions. For example,
   - *numerical representations* of atmosphere – ocean flows must account for the degeneration of the governing equations towards *reduced asymptotic models* in the various singular limit regimes mentioned earlier,
   - *reduced asymptotic models* for particular characteristic flow patterns may serve als key building blocks of *stochastic subgrid scale models*,
   - rigorous analyses for the validity of reduced models and for the influence of stochastic noise enable improved estimates of the uncertainties of computational predictions, and
   - the combination of classical *pde's for atmosphere-ocean flows* with *stochastic models* for the subgrid scale will necessitate *new computational strategies* that allow one to merge the stochastic and quasi-deterministic aspects of the resulting dynamics.

(5) *Interdisciplinary discourse*

Of utmost importance in the present context is an interdisciplinary discourse. When addressing problems of practical interest, a judicious choice of scientific foci is needed to guide the mathematical research. The necessary experience and expertise can only be found in the disciplines of theoretical meteorology, oceanography, and climate research. In turn, by incorporating the fundamentals of applied mathematics new research directions, which would otherwise not have been thought of, may develop.

In summary, the projected workshop on mathematical aspects of atmosphere-ocean modelling stimulated the development of novel research ideas in applied and computational mathematics drawing from a truly cross-disciplinary exchange.

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References

[1] A. E. Gill, (1982), *Atmosphere-Ocean Dynamics*. International Geophysical Series, **30**, Academic Press, London

[2] J. Pedlosky, (1987), *Geophysical Fluid Dynamics*, Springer, 2nd edition

[3] K. A. Emanuel, (1994), *Atmospheric Convection*, Oxford University Press.

[4] M. Beniston, (1998), *From Turbulence to Climate*, Springer

[5] R. K. Zeytounian, (1991), *Meteorological Fluid Dynamics*, Springer

[6] D. R. Durran, (1999), *Numerical Methods for Wave Equations in Geophysical Fluid Dynamics*, Springer

Abstracts

Primitive-equation-based low-order models with seasonal cycle: Model construction and application to non-linearity and complexity of large-scale atmosphere dynamics

ULRICH ACHATZ

In a continuation of previous investigations on deterministic reduced atmosphere models with compact state space representation two main modifications we introduced: First, primitive equation dynamics is used to describe the nonlinear interaction between resolved scales. Secondly, the seasonal cycle in its main aspects is incorporated. Stability considerations lead to a grid-point formulation of the basic equations in the dynamical case. A total matrix consistent with the equations can be derived, provided surface pressure is treated as constant in time. Using this matrix, a reduction in the number of degrees of freedom is achieved by a projection onto three-dimensional EOFs, each of them encompassing simultaneously all prognostic variables (winds and temperatures). The impact of unresolved scales and not explicitly described physical processes is incorporated via an empirical linear parameterisation. The basis patterns having been determined from a GCM data set, it is found that, in spite of the presence of a seasonal cycle, at most 500 modes are needed for describing 40% of the variance produced by the GCM. If compared to various low-order models with quasi-geostrophic dynamics our reduced models exhibit, at these resolutions, a considerably enhanced capability to predict GCM tendencies. Fairly different results are obtained with respect to the dependence of short-time predictability as climate simulations on the number of employed degrees of freedom. Models using 500 basis patterns are significantly better in short-term predictions than smaller counterparts. Meaningful predictions of the first 500 EOFs are possible for 4-5 days, while the mean anomaly correlation for the leading 30 EOFs stays above 0.6 for nine days. In a 30-EOF model this is only six days. A striking feature is found when it comes to simulations of the monthly mean states and transient fluxes: the 30-EOF model is performing just as well as the 500-EOF model. Since similar behaviour is also found in the reproduction of the number and shape of the three significant cluster centroids in the January data of the GCM, one can speculate on a characteristic dimension in the range of a few tens for the large-scale dynamics of the climate attractor. Finally, no support is found for the classic hypothesis that the observed cluster centroids, indicating multi-modality in the climate statistics, can be interpreted as quasi-steady states of the GCM's low-frequency dynamics.

Recent simulations of stratified turbulence without rotation

PETER BARTELLO

Simulations of stratified turbulence are presented. The focus was on contrasting the statistical features of the modes possessing potential vorticity (PV) from those that do not. It was found that when forcing is applied to the PV modes the vertical energy spectra of both types of modes were almost flat in the buoyancy range. By contrast, when high-frequency waves were forced, the wave energy spectrum decayed approximately as k_z^{-3}, consistent with the notion of a "saturated gravity wave spectrum". However, the vortical spectrum of the PV modes remained relatively flat. Horizontal spectra consistently showed a progressive steepening as stratification increased. During all of these simulations the energy of modes with zero horizontal wave number increased very slowly.

On climate stability, climate sensitivity and the dynamics of the enhanced greenhouse effect

I. Ray Bates

The dynamics of the enhanced greenhouse effect resulting from a CO_2 increase are studied using a simple two-zone aquaplanet model in which the earth's rotation as manifested through atmospheric angular momentum (AM) transport from the tropics to the extratropics plays a central role. The model's sensitivity to forcing is viewed against the background of its stability to free perturbations about equilibrium. Free perturbation in sea surface temperature (SST) are subject to a destabilising influence from the effects of the water vapour infrared radiative (WVIR) feedback and are stabilized by evaporation which results in moist convection and precipitation that deposits the latest heat removed from the surface above the level of the main water vapour absorbers, where it is radiated to space. The rate of evaporation depends on the surface wind strength, which is a function of the AM transport. The model's equilibrium sensitivity to forcing is found to be strongly related to the degree of stability of its slow normal mode, which is sensitively dependent on the strength of the WVIR feedback in the extratropics. The model's sensitivity is also strongly dependent on the latitudinal distribution of the forcing, which determines the extent to which the wind sensitivity acts as a positive or a negative feedback. Using estimates of the parameters derived from observation and detailed radiative model calculations, the model gives a temperature increase for a on doubling CO_2 that lies in the range of that given by general circulation models (GCMs). Extratropical amplification of the temperature response, which is also seen in GCMs, is found to be a feature of the model's sensitivity even if the forcing is uniform. Dynamical reasons for the extratropical amplification are seen from the model and these involve the AM transport in an important way.

The Nonlinear Interaction of Barotropic and Equatorial Baroclinic Rossby Waves

Joseph A. Biello, Andrew J. Majda

Simplified asymptotic equations are developed for the nonlinear interaction of long wavelength equatorial Rossby waves and barotropic Rossby waves with a significant midlatitude projection in the presence of suitable horizontally and vertically sheared zonal mean flows. The simplified equations allow for nonlinear energy exchange between the barotropic Rossby waves and the baroclinic equatorial waves for non-zero zonal mean vertical shear through wave-wave interactions. Idealised examples in the model demonstrate that midlatitude Rossby wave trains in a baroclinic mean shear can transfer their energy to localized equatorially trapped baroclinic Rossby waves through a nonlinear “westerly wind burst” mechanism. Conversely, equatorially trapped baroclinic Rossby wave trains in the idealised model can transfer substantial energy to the midlatitude barotropic Rossby waves.

Local equilibrium balanced approximations

Nicola Botta

In the computation of nearly balanced motions, particular care has to be taken in constructing discrete approximations. In fact, standard discrete methods generate, on realistic finite grids, unbalanced truncation errors which are much larger than the ”true” signal. We analyse the origin of this problem and discuss a general and simple approach for constructing balanced approximations.

Statistical mechanics of strong and weak vortices

Oliver Bühler

The motion of one-hundred point vortices in a circular cylinder is simulated numerically and compared with theoretical predictions based on statistical mechanics. The novel aspect considered here is that the vortices have greatly different circulation strengths. Specifically, there are four strong vortices and ninety-six weak vortices, the net circulation in either group is zero, and the strong circulations are five times larger than the weak circulations. As envisaged by Onsager [Nuovo Cimento **6** (suppl.), 279 (1949)], such an arrangement leads to a substantial amplification of statistical trends such as the preferred clustering of the strong vortices in either same-signed or oppositely-signed pairs, depending on the overall energy level. It is found that the statistical mechanics predictions compare remarkably well with the numerical results,including a prediction of vortex accumulation at the cylinder wall for low energies.

New results on the semi-geostrophic model of large-scale atmosphere/ocean flows

Mike Cullen

The semigeostrophic (SF) model is an accurate model of large scale atmosphere/ocean flows with errors of order $R_0 : (L_R/L)^{1/2}$, where R_0 is the Rossby number (U/fL, f the Coriolis parameter) and L_R the deformation radius (NH/F, N the buoyancy frequency). It predicts flows with very stationary statistics, with only weak cascades to small or large scales. It can be proved that weak solutions of SF exist in Cartesian geometry with constant F for shallow water, 3D Boussinesq, and 3D compressible flows. Some of the key steps in extending the results to spherical geometry have also been proved, though no full result is available. The latter case is important as the condition $L > L_R$ implies that the variation of f is important.

Attractors of atmospheric models: stability and sensitivity to small perturbation of external forcing

V. Dymnikov

The theorems of existence of global attractors, necessary and sufficient conditions of attractor stability (as sets), and symmetry properties of Lyapunov exponents are given for some atmospheric models. We define the conditions of "quasi regular behaviour" of atmospheric dynamics and applicability of fluctuation-dissipation theorems for construction of response operator to small perturbations of external forcing. The applicability of the above approach is verified numerically for atmospheric models, of different complexity - two-dimensional barotropic model, two-layer quasi geostrophic model, and GCM of mid resolution.

Stochastic Analysis of the Global Axial Angular Momentum Budget

J. Egger

Given the budget equations for the global axial angular momentum M, the related covariance equations are derived. These equations allow one to study the response of M to and its impact on the mountain and friction torques in a stochastic framework. Data are used to evaluate these terms and to assess their relative importance.

All covariance equations are satisfied quite well by the data. The mountain torque is found to be generally more important in these covariance equations than the friction torque. The case of the equatorial components is considered as well where the torque by the Earth's bulge is dominating by far.

A circulation model for teaching and research

Klaus Fraedrich

A portable, modular general circulation model is presented and numerical experiments with it. Experimental design, results and physical interpretation is given for two examples. Low frequency variability and long-term memory induced by two storm tracks show properties of the teleconnection patterns observed in the atmosphere.

Statistical predictions and dynamical simulations for models of open-ocean deep convection

Marcus I. Grote

Within basins that exhibit open-ocean deep convection, convectively mixed fluids are often observed in regions of upward-doming isopycnal surfaces. Consistent with observations, a recent equilibrium statistical theory (DiBattista, Majda, Marshall 2001) applied to a simplified two-layer QG model predicts the concentration of convective overturning about the peaks of upwelling isopycnals induced either by a cyclonic background gyre or bottom typography. Here we compare the predictions of statistical mechanics with a DNS of the simplified model to study the dynamic relaxation to statistical equilibrium.

Mesoscale heat transport due to baroclinic vortices in singular representation

Vladimir Gryanik

A problem of heat transport of turbulent isolated deep-convection is considered. Ballistic rather than diffusion approach is introduced and studied. Ballistic stirring gives the upper bound for heat transport at mesoscales in the ocean $R^2 \sim C^2 t^2$.

How can informational entropy be used in the atmospheric climate theory?

Michael V. Kurgansky

This tentative talk is aimed to discuss some aspects of equilibrium statistical fluid mechanics and its possible implications to the atmospheric climate theory. A principle of maximum of informational entropy is applied to introduce: (i) a steady spectral distribution of kinetic energy on a barotropic atmosphere and (ii) steady statistical distribution of properly modified Ertel's potential vorticity in a hemispheric atmosphere.

Prediction of ENSO using cluster-weighted modelling

FRANK KWASNIOK

Predictions of ENSO (El Niño Southern Oscillation) using the methodology of cluster-weighted modelling (also referred to as probabilistic or Bayesian network) are presented. The predictand is the Niño3.4 sea surface temperature anomaly index. Sea surface temperature and sea level pressure in the tropical Pacific area are used as predictors. The predictor-predictand relationship is modelled by a mixture of local linear models. Each model is associated with two Gaussian probability distributions, one describing its domain of influence in phase space and one describing its prediction output. The actual prediction is then a convex combination of the individual probability distributions. Hence the cluster-weighted modelling approach provides a non-linear, non-Gaussian probabilistic prediction scheme. The locations in phase space, the prediction coefficients and the prediction uncertainties of the models are determined from observational data according to maximum likelihood using the expectation-maximization algorithm. The method is evaluated both as a deterministic and a categorical probabilistic prediction scheme. A cross-validated prediction skill is estimated and compared to that of more conventional methods. Implications of the results for nonlinearity and stochasticity of ENSO are discussed.

Characterization of shear-stratified turbulence for a model tropospheric jet

ALEX MAHALOV

Large-scale (1024x512x512) direct numerical simulations are performed to study an atmosphere jet at the tropopause. The basic state is characterized by a jet centred at the tropopause and a non-uniform vertically variable background stratification. The turbulent field is characterized by the presence of two asymmetric shear layers above and below the jet stream core and layered structure. Vertical variability of the turbulence outer scales (Ozmidov, Tatariskii, Ellison, buoyancy, shear and others) are investigated. The ratios of the Ellison to buoyancy scales are much smaller than unity in mixed regions and approach unity at the edges, confirming that mechanical turbulence prevails on the core whereas nonlinear wave interactions are significant at the edges. With decreasing background stratification, a clear separation was observed between the altitudes where enhanced nonlinear wave interactions occur and where the shear production is peaked.

Internal and inertial wave attractors

LEO MAAS

Continuously (here: uniformly) stratified fluids and rotating homogeneous fluids support linear waves that differ from surface waves. In contrast with the usual case one now has monochromatic waves whose spatial structure is determined by $\psi_{xx} - \psi_{zz} = 0$ (2D, linearized, inviscid, Boussinesq fluid). Inertial waves satisfy this equation in long channels, far from end walls (in general, they satisfy Poincaré's-eqn. $P_{xx} + P_{yy} - P_{zz} = 0$). In 2D, exact solutions with y=0 at the boundary follow by method of characteristics for arbitrary domains. Typically, when there is at least one side not parallel or perpendicular to gravity, focussing zones place onto wave attractors (limit cycle). The reflection point of the attractor from the boundary leads to mixing hotspots. In the rotating case this induces mixing of angular momentum and cyclonic mean flows. In a radially-stratified, rotating spherical shell focussing is on either equatorial wave attractors feeding a mean flow of the counter and undercurrent type observed near the equator. At the same time waves are also attracted to "their" inertial latitude, near the bottom, perhaps explaining ubiquitousness appearance of inertial oscillations in ocean. Laboratory experiments confirm the appearance of attractors.

Resonances between gravity waves and topography

Paul A. Milewski

We present the reduced dynamics for periodic shallow water waves propagating over periodic topography. The dynamics is governed by coupled Korteweg - de Vries equations where the coupling is a convolute of the waveform and the topography. This can be viewed as a resonant form of Bragg scattering. We show two types of simulations from the equations. Time-dependent solutions whose initial data is a nonlinear solution to the uncoupled KdV, and periodic solutions, satisfying no flow conditions at the boundary, corresponding to "sloshing" waves in an enclosed tank.

Multiscale organisation of tropical convection

Mitchell W. Moncrieff

The intra seasonal oscillation, or Madden-Julian oscillation (MJO), is a fundamental mode of tropical variability in which the interaction of convection with the tropical dynamics is important. However, climate models and NWP models have poor skill in the prediction of MJOs. This is likely due to deficiencies in convective parameterisations. The hypothesis explored herein is that convection organized on scales 100-1000 km (mesoscales) is a key process. This hypothesis is explored using two approaches. Firstly, cloud-resolving models are used in place of conventional convection parameterisation (super-parameterisation). This approach successfully predicts MJOs and gets the correct propagation speed, and overall structure. Secondly, an analytic model is derived to explain the results of the super-parameterisation. This model represents the MJO as a non-scale system - a large-scale circulation and organized convection - that is shown to be dynamically equivalent though a mapping of the characteristic nondimensional numbers (i.e., convective Froud number and Rossby number). This simple model explains the simulated MJO as well as the super-rotation that it generates. These results confirm the importance of organized convection on mesoscales to the MJO dynamics.

Atmospheric heat transport by synoptic-scale eddy ensembles in SDCMs: methods of description

Vladimir K. Petukhov

An overview of zonally averaged statistical-dynamical models is given. Some conventional theoretical models of heat transfer of synoptic eddy ensembles are analysed and important, unresolved problems are outlined. A horizontal non-stationary model of synoptic scale vortices ensemble is presented and the main results of the model are surveyed.

Particle methods in geophysical fluid dynamics

Sebastian Reich

The rotating shallow water equations are a simple model for geophysical fluid dynamics. The long term dynamics is strongly affected by conservation laws of mass and vorticity as well as geostrophic balance. The talk reviewed a class of Hamiltonian particle methods. These methods conserve mass, circulation and PV. This makes them ideally suited for long term simulations and statistical mechanics type investigations.

Baroclinic Eddy Fluxes and the Thermal Stratification of the Extratropical Atmosphere

Tapio Schneider

Baroclinic eddy fluxes dominate the transport of entropy in the extratropical atmosphere and, presumably, set the tropopause height and thermal stratification. A dynamical constraint on the extratropical tropopause height and thermal stratification is derived from a relation between entropy fluxes (isentropic mass fluxes) and eddy fluxes of potential vorticity and surface potential temperature. Based on diffusive eddy flux closures, the dynamical constraint relates the tropopause potential temperature to the surface potential temperature and its gradient. Implications of the dynamical constraint for the existence of a turbulent inverse energy cascade in the atmosphere are discussed.

Stability of Rossby waves in the B-plane approximation

Leslie Smith

The motivation is to understand the mechanisms for energy transfer from small-scale turbulence to anisotropic large-scale flows in various models of geophysical phenomena. To this end, Floquet theory is used to describe the unstable spectrum at large scales of the B-plane equation linearized about Rossby waves. Base flows of one to three Rossby waves are considered analytically using continued fractions and the method of multiple scales, while base flows with more than three Rossby waves are studied numerically. It is demonstrated that the mechanism for instability changes from inflectional to triad resonance at an 0(1) Rhines number, independent of Reynolds number. For a single Rossby wave base flow, the critical Reynolds number for instability is found in various limits, and resonant triads are shown to decrease the critical Reynolds number below the large Rhines number value. For more isotropic base flows consisting of many (up to forty) Rossby waves, the most unstable mode is purely zonal for Rhines numbers above two, and nearly zonal for Rhines number one-half, where the transition Rhines number is again 0(1).

Math modelling of the ocean's well-mixed layer

Esteban G. Tabak

A series of mathematical models is presented to shed light on various aspects of the dynamics of the wind-stirred upper ocean layers: resonant energy transfer from the wind into surface waves and from these into internal modes supported at the base of the mixed layer, entraining of waters from the ocean interior by breaking waves, and turbulent diffusion. It is envisioned that all these processes play a significant role in establishing the depth and main properties of the well-mixed layer.

Global regularity for a dissipative planetary geostrophic model

Edriss S. Titi

We consider a viscous thermocline geostrophic planetary model. In the conservation of momentum equation the velocity vector field is linearly related to the temperature. Since one can control the LP norms for the temperature one can in turn show that the velocity field remains bound in the L^6 norm. This is in turn sufficient to show that the temperature gradient remains finite for all time. As a result one can boot-strap to show global regularity of this three-dimensional geophysical system. In addition, one can show that the dynamical system induced by the solution operator is dissipative and possesses a finite dimensional global attractor. This was a joint work with Chang-Sheng Cao from the CNLS at Los Alamos National Laboratory.

Atmospheric turbulence, theory and models

Ka-Kit Tung

Atmospheric large-scale motions satisfying Quasi-Geostrophic scaling conserve pseudo-potential vorticity as well as total energy. It was thought that this twin conservation prohibits net downscale energy fluxes in QG turbulence, akin to the situation in 2D turbulence. We show that such "proofs" must be wrong because conservation laws should not by themselves determine the direction of energy flow. Instead the flux of energy should be directed from scales of injection to the scales of dissipation, whichever these may be. Observational evidence is presented to show that there is no inverse energy cascade inertial subrange in the atmosphere. The only possible inertial subrange over the subsynoptic and mesoscales has downscale energy fluxes. The observed energy spectrum in the atmosphere from tens of thousands of kilometres and tens of kilometres can be simulated in a model with energy and enstrophy injection in the synoptic scales (5,000 km - 2,500 km), Ekman damping at the planetary scales (>5,000 km) and hyperdiffusion at the smallest scales resolved (100 km). Power-law behaviour is found with k^{-3} behaviour between (2,500 km - 750 km) and k−5/3 behaviour over the mesoscales (<2,500 km). Both enstrophy and energy flux are downscale in this region.

Nonlinear geostrophic adjustment

Vladimir Zeitlin

Cauchy problem for a single-scaled localized initial perturbation is solved perturbatively in Rossby number in a series of models for rotating stratified fluid, namely in rotating shallow water model, two-layer rotating shallow water model and continuously stratified hydrostatic primitive equation models. The calculations performed in first three orders of perturbation theory in both geostrophic (Burger number of order unity) and frontal geostrophic (small Burger number) regimes show decoupling of fast (inertia-gravity waves) and slow (vorticity) component. The evolution of the slow component follows from elimination of resonances in fast dynamics. It is shown that there is no wave-drag in slow dynamics. However, in the presence of stratification (2-layer rotating shallow water, and primitive equations) the time scales of the evolution of the slow component and that of modulation of the fast component are the same in the frontal regime, i.e., time splitting is incomplete. Both evolution equations and well-defined initialisation procedure for slow and fast component of motion are obtained. The results prove that slow, balanced equations known previously in geophysical literature are consistent.

Edited by Rupert Klein

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