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Fundamental Groups in Geometry

September 8th – September 14th, 2002

The workshop was organized by Fedor A. Bogomolov (New York), Jürgen Jost (Leipzig), Mina Teicher (Ramat-Gan) and Michael Zaidenberg (Saint-Martin-d'Heres) and attended by about forty participants from Europe, North America, China, Israel, Japan and Singapore.

The official program consisted of 18 lectures, the talks covered a wide range of new researchs of "Fundamental groups in Geometry". There were plenty of time for questions and many informal discussions among smaller groups.

The organizers and participants thank the "Mathematisches Forschungsinstitut Oberwolfach" to make the conference possible in the usual comfortable and inspiring setting.

Abstracts

Braid monodromy and topology of conjugate plane curves

Enrique Artal Bartolo

(joint work with J. Carmona and J.I. Cogolludo)

We are interesting in studying the spaces $\mathcal{M}$ which are obtained as the quotient by the projective group of the space of complex plane projective curves with the same combinatorial type (essentially, the degree of the irreducible components and topological type of singularities). The main problems are related with existence and connectedness of such spaces. The topology of the embedding of the curves in the projective plane $\mathbb{P}^2$ is an invariant of the connected components of $\mathcal{M}$.

We have found examples of discrete spaces $\mathcal{M}$ (for sextic curves) such that the representatives of the points in $\mathcal{M}$ have conjugate equations in some number field $\mathbb{K}$. In order to understand the embedding of this curves in $\mathbb{P}^2$ we have to find invariants which go beyond the algebraic structure. This invariant is braid monodromy of curves; we extend the classical definition of this invariant by allowing the projection point to be in the curve and we find that some special braid monodromy is different for two conjugate curves having equations in $\mathbb{Q}(\sqrt{2})$. Braid monodromy is an orbit in B_d^r by an action of $B_d \times B_r$ (by Hurwitz moves and conjugation) where B_n is the braid group on n strings; we find these braid monodromies to be different by means of a representation of B_d onto a finite group. Using a common result of the three authors we prove that, after adding some straight lines to the curves, there are conjugate curves in $\mathbb{Q}(\sqrt{2})$ of degree 12 having non-homeomorphic embeddings.

Fundamental groups of complements of plane curves and symplectic invariants

Denis Auroux

(joint work with S. Donaldson, L. Katzarkov and M. Yotov)

Given a compact symplectic manifold (X^{2n}, ω) for which $[\omega]$ is an integral cohomology class, its topology can be studied by means of the approximately holomorphic techniques introduced by Donaldson in the mid-90's: fixing an almost-complex structure, a complex line bundle L with $c_1(L) = [\omega]$ behaves like an ample line bundle, in the sense that suitable sections of $L^{\otimes k}$ for $k \gg 0$ can be used to define hyperplane sections, Lefschetz pencils, etc.

Three well-chosen sections of $L^{\otimes k}$ define a projection map to $\mathbb{CP}^2$ with generic local models; this construction is canonical up to isotopy for $k \gg 0$. In the case of a symplectic 4-manifold, we obtain a branched covering whose branch curve D is symplectic, with complex (2,3)-cusps and nodes of either orientation as only singularities. The curve D can be studied using the braid monodromy techniques introduced by Moishezon and Teicher in algebraic geometry.

Braid monodromy is a complete symplectic invariant: it determines D up to isotopy and, together with the monodromy morphism $\theta : \pi_1(\mathbb{CP}^2 - D) \to S_N$ of the covering, allows one to recover (X, ω) up to symplectomorphism. However there is no algorithm for comparing braid monodromies.

An easier invariant is $\pi_1(\mathbb{CP}^2 - D)$ itself. In the symplectic case, we need to allow for creations or cancellations of pairs of nodes, so the actual invariant is a certain quotient $G = \pi_1(\mathbb{CP}^2 - D)/K$, where K is generated by commutators $[\gamma, \gamma']$, where γ, γ' are conjugates of standard generators such that $\theta(\gamma)$ and $\theta(\gamma')$ are disjoint transpositions.

There is an exact sequence $1 \to G^0 \to G \to S_N \times \mathbb{Z}_d \to \mathbb{Z}_2 \to 1$, where the map $G \to S_N \times \mathbb{Z}_d$ is given by the monodromy θ and by the abelianization map $\delta : G \to \mathbb{Z}_d$ sending generators to 1.

In the case $\pi_1(X) = 1$, we have a structure theorem for the kernel G^0: there exists a natural surjective homomorphism $\phi : G^0 \to (\mathbb{Z}^2/\Lambda)^{N-1}$, where $\Lambda = \{(L^{\otimes k} \cdot C, K_X \cdot C),\ C \in H_2(X, \mathbb{Z})\}$.

Moreover, the available examples wherefore high degree projections ($k \gg 0$), i.e. $\mathbb{CP}^2$, $\mathbb{CP}^1 \times \mathbb{CP}^1$, some Del Pezzo or K3 complete intersections, Hirzebruch surfaces, and double covers of $\mathbb{CP}^1 \times \mathbb{CP}^1$ (by work of Moishezon, Teicher, Robb, A-D-K-Y, ...), suggest a much stronger conjecture when X is a simply connected complex surface and $k \gg 0$: namely, one expects that:

1) $K = \{1\}$ (i.e. $G = \pi_1$),
2) $\operatorname{Ker} \phi = [G^0, G^0]$ (i.e. $Ab(G^0) = (\mathbb{Z}^2/\Lambda)^{N-1}$),
3) $[G^0, G^0]$ is a quotient of $\mathbb{Z}_2 \times \mathbb{Z}_2$.

Equivariant refined Seiberg-Witten theory

Stefan Bauer

The monopole map can be defined for a K-oriented closed Riemannian 4-manifold. It is used to define (joint with M. Furuta) an element in some equivariant stable cohomotopy group, which is independent of the chosen metric. The Hurwitz homomorphism relates this element to the integer valued Seiberg Witten invariant.

Some structural results were presented:

1. There is a connected sum theorem. In contrast to Seiberg-Witten theory or Donaldson theory the invariants of connected sums need not vanish, but are torsion elements.

2. There is a universal invariant defined by the parametrized monopole map over the space of all choices (metrics and connections). A fixed point map relates this universal element to the $Diff(X)$-equivariant stable cohomotopy Euler class of the $H^2_+(X; \mathbb{R})$-bundle over the space of metrics. This gives restrictions on the possible elements which may arise as (universal) invariants. It also leads to an understanding of the "chamber structure" phenomenon for the Seiberg-Witten invariants.

3. (Report on results of M. Szymik) If X comes with a free action of a finite group G, one gets a G-equivariant invariant. It contains all information on the (non-equivariant) invariants of quotients X/H for subgroups $H < G$. M. Szymik showed that already in the case of a group of prime order the comparison map is neither surjective nor injective. This leads to relations amongst the Seiberg-Witten invariants of the quotients X/H on the one hand and on the other hand (potentially) to an invariant of the action itself.

Special varieties, orbifolds, and classification theory

Frederic Campana

We describe two structure theorems in the birational geometry of complex projective manifolds, analogues of the structure theorems for Lie algebras, reducing these first to semi-simple and solvable ones, the solvable being iterated extensions of abelian ones. The roles of semi-simple, solvable and abelian are respectively played here by the orbifolds of general type, special manifolds, and third: manifolds either rationally connected or with $\kappa = 0$.

This decomposition is deeply linked with other aspects of classification theory: we indeed conjecture, and show in some few cases, that special manifolds have an almost abelian fundamental group, and are exactly the ones having zero Kobayashi metric, or a potentially dense set of rational points over any field of definition finitely generated over $\mathbb{Q}$. This immediately leads to a natural extension of Lang's conjectures to arbitrary X's (and even to orbifolds).

This decomposition gives a simple synthetic view of the structure of arbitrary X's, and indicates that the natural frame of classification theory is the category of orbifolds, to which our observations should be extended.

Orbifold fundamental groups

FABRIZIO CATANESE
(joint work with P. Frediani)

Scope of the lecture was to illustrate various applications of the notion of π_1^{orb}, (rest. of a fibration). For Y a normal $\mathbb{C}$-space, B closed analytic, with B_1,...,B_r the divisorial components of B, $\overrightarrow{m} = (m_1, \ldots, m_r) \in \mathbb{N}_+^r$ one defines

$$\pi_1^{orb}(Y, B, \overrightarrow{m}) := \pi_1(Y - B) / \ll \gamma_i \gg^{m_i}$$

γ_i being a geometric loop around B_i.

For $Y = X/G$ (X: manifold), we let B the branch locus and $\overrightarrow{m}$ the vector of multiplicities of inertia groups, thus we get

$$1 \to \pi_1(X) \to \pi_1^{orb}(Y, B, \overrightarrow{m}) \to G \to 1$$

The first application is for surfaces (varieties) etale quotients of products of curves, $S = C_1 \times C_2/G$. Here, $1 \to \pi_1(C_1) \times \pi_1(C_2) \to \pi_1(S) \to G \to 1$ and moding out by $\pi_1(C_2)$ we get the orbifold sequence of the quotient map $C_1 \to C_1/G$. This method plus the isotropic subspace method leads to the

Theorem. *Let $S = C_1 \times C_2/S$, S' with $\pi_1(S') \cong \pi_1(S)$, $e(S') = e(S)$ (e: Euler number). Then S' is diffeo to S and the moduli space is either irreducible, or it has 2 irreducible components $\mathcal{M}_1$, $\mathcal{M}_2$ with $\mathcal{M}_2 = \overline{\mathcal{M}_1}$, $\mathcal{M}_1 \cap \mathcal{M}_2 = \emptyset$. There are infinite examples of the second alternative.*

After discussing other counter-example to the Freidman-Morgan conjecture that S diffeo to S' implies S, S' belong to the same connected component of the moduli space, I introduced the orbifold fundamental group sequence of a fibration: $\pi_1(F) \to \pi_1(X^0) \to \pi_1^{orb}(Y^0) \to 1$ and explained an application

Theorem A (-,Keum,Oguiso). *Let Y be a normal elliptic $K3$ surface, Y^0 the smooth locus. Then either*

(1) $|\pi_1(Y^0)| < +\infty$ *or*
(2) *There exits $T \to Y$ etale in cod. 1, finite, T is torus.*

Without the assumption "Y elliptic" then is a conjecture of D.Q.Zhang. I also mentioned

Theorem B (-,Keum,Oguiso). *Let Y be as in Theorem A, S its minimal resolution, $Y^0 = S \cup_{i=1}^r E_i$. Then if $r \leq 15$ then $|\pi_1(Y^0)| < +\infty$.*

I finally mentioned that one can define the orbifold fundamental group sequence of a real variety (X, σ),

$$1 \to \pi_1(X) \to \pi_1^{orb}((X, \sigma)) \to \mathbb{Z}/2 \to 1$$

and that this notion has revealed itself as very useful.

Non-compact representations of surface groups

Oscar García-Prada
(joint work with Steven B. Bradlow and Peter B. Gothen)

Using the L^2 norm of the Higgs field as a Morse function, we study the moduli spaces of $U(p,q)$-Higgs bundles over a Riemann surface. We require that the genus of the surface be at least two, but place no constraints on (p,q). A key step is the identification of the function's local minima as moduli spaces of holomorphic triples. We prove that these moduli spaces of triples are non-empty and irreducible.

Because of the relation between flat bundles and fundamental group representations, we can interpret our conclusions as results about the number of connected components in the moduli space of semi-simple $PU(p,q)$-representations. The topological invariants of the flat bundle are used to label components. These invariants are bounded by a Milnor–Wood type inequality. For each allowed value of the invariants satisfying a certain coprimality condition, we prove that the corresponding component is non-empty and connected. If the coprimality condition does not hold, our results apply to the irreducible representations.

Generalized triangle inequalities in symmetric spaces and buildings with applications to algebra

Misha Kapovich
(joint work with B.Leeb and J.Millson)

Everybody knows how to construct triangles with the prescribed side-lengths α_1, α_2, α_3 in the Euclidean plane: the necessary and sufficient conditions for this are the usual triangle inequalities $\alpha_i \leq \alpha_j + \alpha_k$. In this talk I will explain how to solve (in a unified fashion) the analogous problem for other geometries X: non-positively curved symmetric spaces (and their infinitesimal analogues) and Euclidean buildings. The notion of "side-length" in this generality becomes more subtle: *side-lengths* are elements of the appropriate Weyl cone Δ. One of the surprising results is that the "generalized triangle inequalities" for X determine a polyhedral cone $D_3(X) \subset \Delta^3$, which depends on X and on the type of geometry only weakly: $D_3(X)$ is completely determined by the finite Coxeter group W corresponding to X. (The polyhedra D_3 provide complete solutions to algebra problems **Q1, Q2** below, solutions to the algebra problems **Q3**, **Q4** are certain lattice points in D_3.) The linear inequalities describing X are determined by the "Schubert calculus" (computing the integer cohomology ring) in the associated generalized flag varieties. Our techniques are purely geometric (with a bit of dynamics). One relates and solves these problems using weighted configurations "at infinity" corresponding to the triangles.

Here are some algebra problems which one can solve (at least to some extent) using the geometric results about triangles. Recall that the *singular values* of an $m \times m$ matrix A are the square-roots of the eigenvalues of the matrix AA^*. For a matrix $A \in GL(m, \mathbb{Q}_p)$, the double coset

$$GL(m, Z_p) \cdot A \cdot GL(m, \mathbb{Q}_p) \subset GL(m, \mathbb{Z}_p)$$

is represented by a diagonal matrix $D = Diag(p^{e_1}, ..., p^{e_m})$. (Here $\mathbb{Q}_p$ are the p-adic numbers and $\mathbb{Z}_p$ are the p-adic integers.) The *invariant factors* of a matrix A are the integers e_i arranged in the decreasing order.

Let α, β and γ be m-tuples of real numbers arranged in decreasing order. In the Problem **P4** we will assume that α, β and γ are dominant weights of $GL(m, \mathbb{C})$ (i.e. they are vectors in $\mathbb{Z}^m$) and that V_α, V_β and V_γ are the irreducible representations of $GL(m, \mathbb{C})$ with these highest weights.

• **P1.** Give necessary and sufficient conditions on α, β and γ in order that there exist Hermitian matrices A, B and C such that the sets of eigenvalues of A, B and C are α, β and γ respectively, and

$$A + B + C = 0.$$

• **P2.** Give necessary and sufficient conditions on α, β and γ in order that there exist matrices A, B and C in $GL(m, \mathbb{C})$ the logarithms of whose singular values are α, β and γ, respectively, so that

$$ABC = 1.$$

• **P3.** Give necessary and sufficient conditions on the integer vectors α, β and γ in order that there exist matrices A, B and C in $GL(m, \mathbb{Q}_p)$ with invariant factors α, β and γ, respectively, so that

$$ABC = 1.$$

• **P4.** Give necessary and sufficient conditions on α, β and γ in order that

$$(V_\alpha \otimes V_\beta \otimes V_\gamma)^{GL(m,\mathbb{C})} \neq 0.$$

These problems have a long history and their complete solution and the relation between them were established only recently due to the efforts of several people: Klyachko, Tao and Knutson, et al., their proofs where based on algebraic geometry and combinatorics.

Our main contribution is to the extension of the above problems to other reductive groups. Let $\mathbb{F}$ be either the field $\mathbb{R}$ or $\mathbb{C}$; for simplicity, let G be a split reductive group over $\mathbb{Z}$ (think of something like $Sp(n)$) and let K be a maximal compact subgroup of $G(\mathbb{F})$. Instead of working with p-adics one can consider other fields with nonarchimedean valuations.

• **Q1.** Let $\mathfrak{g}$ be the Lie algebra of $G(\mathbb{F})$, and let $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ its Cartan decomposition. Give necessary and sufficient conditions on $\alpha, \beta, \gamma \in \mathfrak{p}/Ad(K)$ in order that there exist elements $A, B, C \in \mathfrak{p}$ whose projections to $\mathfrak{p}/Ad(K)$ are α, β and γ, respectively, so that

$$A + B + C = 0.$$

• **Q2.** Give necessary and sufficient conditions on $\alpha, \beta, \gamma \in K\backslash G(\mathbb{F})/K$ in order that there exist elements $A, B, C \in G(\mathbb{F})$ whose projections to $K\backslash G(\mathbb{F})/K$ are α, β and γ, respectively, so that

$$ABC = 1.$$

• **Q3.** Same as above for $A, B, C \in G(\mathbb{Q}_p)$ and $\alpha, \beta, \gamma \in G(\mathbb{Z}_p)\backslash G(\mathbb{Q}_p)/G(\mathbb{Z}_p)$.

• **Q4.** Let $G^\vee$ be the Langlands' dual group of G. Give necessary and sufficient conditions on highest weights α, β, γ of irreducible representations V_α, V_β, V_γ of $G^\vee(\mathbb{C})$ in order that

$$(V_\alpha \otimes V_\beta \otimes V_\gamma)^{G^\vee(\mathbb{C})} \neq 0.$$

Braid monodromy invariants of Hurwitz curves

Viktor S. Kulikov

(joint work with V. Kharlamov)

Let F_N be a relatively minimal ruled rational surface, $N \geq 1$, $p : F_N \to \mathbb{P}^1$ the ruling. R a fibre of p and E_N the exceptional section, $E_N^2 = -N$. By definition, the image $H = f(\Sigma) \subset F_N$ of a smooth map $f : \Sigma \to F_N$ of oriented closed real surface Σ is called a Hurwitz curve of degree m if

(i) f is an embedding except for a finite number of points $s_1, \dots, s_n \in \Sigma$;

(ii) for each s_i there is a neighbourhood $U_i \subset F_N$ of $f(s_i)$ and local complex analytic coordinates (x_i, y_i) s.t. $H \cap U_i$ is a germ of complex analytic curve and the complex orientation on $H \cap U_i \setminus \{f(s_i)\}$ coincides with the orientation transported from Σ by f;

(iii) for $s \neq s_i$, $i = 1, \dots, n$, H and the fibre $R_p(f(s))$ of p meet at $f(s)$ transversally with positive intersection number;

(iv) $H \cap \Sigma_N = \emptyset$ and the restriction of p to H is a finite map of degree m.

A Hurwitz curve H is called cuspidal if in (ii) the intersection $H \cap U_i$ is given by $y_i^2 = x_i^k$ for some $k \geq 1$. A Hurwitz curve H is called almost algebraic if there is a disc $D \subset \mathbb{P}^1$ containing the images of $f(s_i)$ and s.t. $H \cap (p^{-1}(D) \setminus E_N)$ can be given by $P(z, w) = 0$, where $P \in \mathbb{C}[z, w]$, $\deg_w(P) = m$. For any Hurwitz curve $H \subset F_N$, $\deg H = m$, one can associate a factorization $\Delta_m^{2n} = b_1 \dots b_n$, where $\Delta_m \in B_m$ is so called Garside's element in the braid group B_m, and each b_i is a braid of a germ of polynomial of degree m over 0. Such a factorization is called braid monodromy factorization (bmf). The group $B_n \times B_m$ acts on the set of the factorizations of Δ_m^{2n} of length n and the orbits under this actions are called braid monodromy types (bmt). We proved the following results.

Theorem 1. *Two cuspidal Hurwitz curves H_1 and $H_2 \subset F_N$ are H-isotopic iff $bmt(H_1) = bmt(H_2)$.*

Theorem 2. *For any cuspidal braid monodromy factorization β of Δ_m^{2n} there is an almost algebraic Hurwitz curve $H \subset F_N$ s.t. $bmf(H) = \beta$.*

Similarly, if $S \subset \mathbb{CP}^2$ is a singular symplectic surface, i.e., S is a J-holomorphic curve for some almost complex structure J on $\mathbb{CP}^2$, one can define $bmf(S)$ w.r.t. a generic pencil of J-lines.

Theorem 3. *Two symplectic cuspidal surfaces $S_1, S_2 \in \mathbb{CP}^2$ are symplectically isotopic iff $bmt(S_1) = bmt(S_2)$.*

Holomorphic mappings of certain configuration spaces

Vladimir Lin

The n^{th} configuration space $\mathcal{C}^n(X)$ of a space X consists of all n point subsets $Q = \{q_1, ..., q_n\} \subseteq X$. It may be viewed as the *regular orbit space* $\mathcal{C}^n(X) = \mathcal{E}^n(X)/\mathbf{S}(n)$, $\mathcal{E}^n(X) \stackrel{\text{def}}{=} \{q = (q_1, ..., q_n) \in X^n \mid q_i \neq q_j \ \forall\, i \neq j\}$ and $\mathbf{S}(n)$ is the symmetric group. $\mathcal{C}^n(X)$ is a complex manifold if X is so.

How to construct holomorphic self-maps of $\mathcal{C}^n(X)$? What can one say of the automorphism group $\operatorname{Aut}\mathcal{C}^n(X)$ and its orbit space $\mathcal{C}^n(X)/\operatorname{Aut}\mathcal{C}^n(X)$?

The $\operatorname{Aut} X$ action in X induces the diagonal $\operatorname{Aut} X$ action in X^n; thus, any $A \in \operatorname{Aut} X$ produces the automorphism $\{q_1, ..., q_n\} \mapsto \{Aq_1, ..., Aq_n\}$ of $\mathcal{C}^n(X)$. Moreover, for any map

$T\colon \mathcal{C}^n(X) \to \operatorname{Aut} X$ one may define the map

$$f_T\colon \mathcal{C}^n(X) \to \mathcal{C}^n(X)\,, \quad \text{where } f_T(Q) = T(Q)Q \stackrel{\text{def}}{=\!=} \{T(Q)q_1, ..., T(Q)q_n\}$$

for all $Q = \{q_1, ..., q_n\} \subset X$. If $\operatorname{Aut} X$ is a complex Lie group[1] and T is holomorphic, f_T is holomorphic, too; such a map $f = f_T$ is called *tame.*[2]

For T as above, pick up a point $Q^* = \{q_1^*, ..., q_n^*\} \in \mathcal{C}^n(X)$ and define

$$f_{T,Q^*}\colon \mathcal{C}^n(X) \to \mathcal{C}^n(X)\,, \quad f_{T,Q^*}(Q) = T(Q)Q^* \stackrel{\text{def}}{=\!=} \{T(Q)q_1^*, ..., T(Q)q_n^*\}$$

for all $Q = \{q_1, ..., q_n\} \subset X$; such a map $f = f_{T,Q^*}$ is called *orbit like.* Its image is contained in one ($\operatorname{Aut} X$)-orbit; if the stabilizer of Q^* in $\operatorname{Aut} X$ is trivial (or at least connected), then the image of the induced endomorphism of the fundamental group[3] $(f_{T,O^*})_*\colon \pi_1(\mathcal{C}^n(X)) \to \pi_1(\mathcal{C}^n(X))$ is abelian.

$\mathcal{C}^n(X)$ may admit "sporadic" holomorphic self-maps, which are neither tame nor orbit like. But I believe that in "simple" cases a holomorphic map $\mathcal{C}^n(X) \to \mathcal{C}^n(X)$ must be tame if *its topology is sufficiently complicated.*

One form of the latter requirement involves the fundamental groups. A continuous map $f\colon Y \to Z$ of arcwise connected spaces is called *non-abelian* if the image of the induced homomorphism $f_*\colon \pi_1(Y) \to \pi_1(Z)$ is a non-abelian group.[4] It was proven in the 1970's that *if $n > 4$ and $X = \mathbb{C}$ or $X = \mathbb{C}^*$ then every non-abelian holomorphic map $f\colon \mathcal{C}^n(X) \to \mathcal{C}^n(X)$ is tame.*[5] Recently I proved the following theorem:

Theorem. *Let $X = \mathbb{CP}^1$ and $n > 4$. Then every non-abelian holomorphic map $f\colon \mathcal{C}^n(X) \to \mathcal{C}^n(X)$ is tame.*

My student Yoel Feler proved that *for $n > 4$ and any torus $X = \mathbb{C}/(a\mathbb{Z} + b\mathbb{Z})$ $(a, b \in \mathbb{C}^*$, $\operatorname{Im}(a/b) \neq 0)$ every automorphism of $\mathcal{C}^n(X)$ is tame.*

These results imply:

Corollary 1. *Let X be an irreducible smooth non-hyperbolic algebraic curve and $n > 4$. Then every automorphism of $\mathcal{C}^n(X)$ is tame. The orbits of the natural $\operatorname{Aut}\mathcal{C}^n(X)$ action in $\mathcal{C}^n(X)$ coincide with the orbits of the diagonal $\operatorname{Aut} X$ action, and the orbit space $\mathcal{C}^n(X)/\operatorname{Aut}(\mathcal{C}^n(X))$ coincides with the orbit space $\mathcal{C}^n(X)/\operatorname{Aut} X$. In particular, for $X = \mathbb{CP}^1$ the orbit space $\mathcal{C}^n(X)/\operatorname{Aut}(\mathcal{C}^n(X))$ may be identified with the moduli space $M(0, n)$ of the Riemann sphere with n punctures.*

Corollary 2. *Let $n > 4$ and X be either $\mathbb{C}$ or $\mathbb{C}^*$ or $\mathbb{CP}^1$. Then the homotopy classification of non-abelian holomorphic mappings $\mathcal{C}^n(X) \to \mathcal{C}^n(X)$, up to a homotopy in the class of holomorphic mappings, coincides with the homotopy classification of holomorphic mappings $\mathcal{C}^n(X) \to \operatorname{Aut} X$. Moreover, according to the classical Grauert theorem, the latter coincides with the homotopy classification of all continuous mappings $\mathcal{C}^n(X) \to \operatorname{Aut} X$.*

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[1] By the classical Bochner-Montgomery theorem, this is certainly the case whenever X is a compact complex manifold ($\operatorname{Aut} X$ may have infinite number of connected components or/and be discrete).

[2] This construction may be slightly generalized, by considering a map T of $\mathcal{C}^n(X)$ to the space of all maps $X \to X$ that satisfies the following condition: $\#[T(Q)Q] = n$ for each $Q \in \mathcal{C}^n(X)$ and the map $\mathcal{C}^n(X) \ni Q \mapsto T(Q)Q \in \mathcal{C}^n(X)$ is holomorphic.

[3] From now on, we suppose that X is arcwise connected.

[4] Every *automorphism* of $\mathcal{C}^n(X)$ is non-abelian if $\dim_{\mathbb{C}} X > 0$ and $n \geq 3$.

[5] I proved this for $X = \mathbb{C}$ in 1972; for $X = \mathbb{C}^*$ it was proven by V. Zinde in 1977.

Corollary 3. *For $n > 4$ the orbit space $\mathcal{C}^n(\mathbb{CP}^1)/\operatorname{Aut}\mathcal{C}^n(\mathbb{CP}^1)$ is isomorphic to the orbit space $\mathcal{C}^n(\mathbb{CP}^1)/\mathbf{PSL}(2, \mathbb{C}) \cong M(0, n)$, where $M(0, n)$ is the moduli space of the n-punctured Riemann sphere.*

To prove the theorem, we establish certain algebraic properties of the braid groups B_n and $B_n(S^2)$, which provide us with an equivariant lifting of a non-cyclic self-mapping of the space $\mathcal{C}^n(X)$ to its Galois $\mathbf{S}(n)$ covering $\mathcal{E}^n(X)$, and then apply the appropriate analytic techniques to study the equivariant holomorphic self-mappings of $\mathcal{E}^n(X)$.

The birational geometry of complex orbifold

Steven Shin-Yi Lu

A fibration $f : X \to Y$ naturally imposes an orbifold structure on Y whose divisorial part has the form $D(f) = \sum_i (1 - 1/m_i)D_i$ on Y. Here, m_i is the minimum of the multiplicities of the components of $f^*(D_i)$ that dominates D_i as opposed to the classical gcd definition of the multiplicity. This choice of multiplicity is naturally imposed on us by our considerations in holomorphic geometry and by the canonically associated Bogomolov sheaf of the fibration. This latter sheaf, defined as the saturation L_f of f^*K_Y in Ω^p_X, is a natural birational invariant of f at least in the sense that $\kappa(Y, f) := \kappa(L_f)$ is. This then allows us to define the same for any meromorphic fibration or even map. By the flattening theorem, one can always arrange, replacing f by a birationally equivalent one, to have $\kappa(Y, f) := \kappa(L_f)$. After Campana, f is said to be of general type if $\kappa(Y, f) = p := dimY$, X special if X has no meromorphic map of general type. By solving an orbifold version of the theorems of Kawamata and Viehweg on the additivity of Kodaira dimension, we show that any compact complex manifold X has a meromorphic fibration of general type that is proper and holomorphic on an open subset and whose general fibres are special. It is defined by the Iitaka fibration of the highest Bogomolov sheaf, the one with the highest p which is necessarily unique by this theorem. Using this, we show that the core c_X constructed by Campana is the same fibration and hence resolves his conjecture that c_X is of general type. This also resolves in a weak sense a problem that is central to Mori's classification program. Furthermore, we work out the above in the very general setting of orbifolds with arbitrary rational multiplicities and in the compact complex category.

Weak Lefschetz Theorems

George Marinescu

Napier and Ramachandran generalized the weak Lefschetz theorems of Nori to the case of higher codimensional subvarieties with positive normal bundle. Their method works even when the subvariety doesn't move in the ambient manifold. (In the case when it moves, the weak Lefschetz theorem was proved by Campana and Kollàr). We propose here the following differential-geometric variant.

Theorem A. *Let (X, ω) a complete hermitian manifold, such that the torsion operator of ω is bounded on X. Let $E \longrightarrow X$ be a line bundle which is uniformly positive outside a proper compact set. Assume moreover that*

$$\int_{X(\leqslant 1)} \Theta(E)^n > 0$$

where $\Theta(E)$ is the curvature of E and $X(\leqslant 1)$ is the open set where $\Theta(E)$ is non–degenerate and has at most 1 negative eigenvalue. Let $i : Y \hookrightarrow X$ be a compact complex space, with *a fundamental system of neighbourhoods* $\{V\}$, *such that* $\dim H^0(V, E^k) < \infty$ *for* $k >> 1$. *Assume moreover that* $i_*\pi_1(Y) \subset \pi_1(X)$ *is a normal subgroup. Then the index of* $i_*\pi_1(Y)$ *in* $\pi_1(X)$ *is finite.*

The theorem can be applied to the case of Zariski open sets in Moishezon manifolds as well as for some q-concave projective manifolds. A space Y satisfies the property in Theorem A if, for example, $\Theta(N_Y)$ has at least one positive eigenvalue (in case $\operatorname{codim} Y = 1$), or $\Theta(N_Y)$ is positive in the sense of Griffiths (for general codimension).

For the proof of Theorem A we cannot use the solution of the $\overline{\partial}$-equation as in Napier and Ramachandran, since ω is not Kähler. We resort in turn to the following variant of the asymptotic Morse inequalities of Demailly for covering manifolds. We denote the von Neumann dimension of a Γ-module by $\dim_\Gamma$.

Theorem B. *Let* $p : \widetilde{X} \longrightarrow X$ *be a Galois covering of* X *of group* Γ *and let* $\widetilde{E} = p^*E$. *Denote by* $H^0_{(2)}(\widetilde{X}, \widetilde{E}^k)$ *the space of holomorphic sections of* $\widetilde{E}^k$ *which are* L^2 *with respect to the pull-back metrics on* $\widetilde{E}^k$ *and* $\widetilde{X}$. *Set* $n = \dim X$. *Then,*

$$\dim_\Gamma H^0_{(2)}(\widetilde{X}, \widetilde{E}^k) \geqslant k^n \int_{X(\leqslant 1)} \Theta(E)^n + o(k^n),\ k \longrightarrow \infty.$$

The proof of Theorem is based on the spectral analysis of the laplacian on the covering $\widetilde{X}$, and uses technical elements borrowed from the proof of the Novikov -Shubin inequalities (usual Morse inequalities for coverings). The results of this talk were obtained jointly with R. Todor and I. Chiose in a paper from Nagoya Math. J., **163**(2001), 145-165.

Symplectic structures of moduli space of Higgs bundles over a curve and Hilbert scheme of points on the canonical bundle

Avijit Mukherjee

(joint work with I.Biswas)

The moduli space of triples of the form (E, θ, s) are considered, where (E, θ) is a Higgs bundle on a fixed hyperbolic Riemann surface X, and s is a (non-zero) holomorphic section of E. Such a moduli space admits a natural map to the moduli space of Higgs bundles simply by forgetting s. If (Y, L) is the spectral data for the Higgs bundle (E, θ), then s defines a section of the line bundle L over Y. The divisor of this section gives a point of a Hilbert scheme parametrizing 0-dimensional subschemes of the total space of the canonical bundle K_X, since Y is a curve on K_X. The main result of this work says that the pullback of the symplectic form on the moduli space of Higgs bundles to the moduli space of triples coincides with the pullback of the natural symplectic form on the Hilbert scheme $\mathrm{Hilb}^l(K_X)$, using the map that sends any triple (E, θ, s) to the divisor of the corresponding section of the line bundle on the spectral curve.

Seiberg-Witten invariants and normal surface singularities

Andras Nemethi

(joint work with Liviu Nicolaescu)

In the talk I presented a very general conjecture formulated by Liviu Nicolaescu and me which relates the analytical invariants of a normal surface singularity to the Seiberg-Witten invariants of the link of the singularity, provided that the link is a rational homology sphere. The talk contained a historical presentation of the background as well.

The conjecture can be formulated as follows. First we define in a topological way a "canonical" $spin^c$ structure of the link. The first part of the conjecture provides a topological upper bound (expressed in terms of the Seiberg-Witten invariant of the "canonical" $spin^c$ structure) for the geometric genus of the singularity. The second part states that for $\mathbb{Q}$-Gorenstein singularities this upper bound is optimal; in particular, it gives a topological description of the geometric genus in terms of the link for these singularities. Finally, for a smoothing of a Gorenstein singularity, the last part gives a topological characterization of some smoothing invariants, like the signature and the Euler characteristic of the Milnor fibre.

As supporting evidence for this conjecture, I discussed some cases when the validity was verified: singularities with good $\mathbb{C}^*$ actions, suspension hypersurface singularities, and some rational and minimally elliptic singularities.

These results extend previous work of Artin, Laufer and S. S.-T. Yau, respectively of Fintushel-Stern and Neumann-Wahl.

Alexander polynomials and Zariski pairs of sextic curves

Pho Duc Tai

(joint work with S.Kaplan, H.Maakestad and M.Teicher)

Following Artal Bartolo, we recall that a pair of irreducible plane curves (C_1, C_2) is a Zariski pair if they have the same degree and there is a 1-1 correspondence between singular points of C_1 and C_2 preserving topological types but $\mathbb{CP}^2 \setminus C_1$ is not homeomorphic to $\mathbb{CP}^2 \setminus C_2$.

Let us denote $\mathcal{Z}(n)$ the set of all Zariski pairs of degree n, $\mathcal{Z}(n, \pi_1)$ (resp. $\mathcal{Z}(n, \Delta_1)$) the set of all Zariski pairs (C_1, C_2) of degree n such that $\pi_1(\mathbb{CP}^2 \setminus C_1) \ncong \pi_1(\mathbb{CP}^2 \setminus C_2)$ (resp. $\Delta_1(C_1) \neq \Delta_1(C_2)$). Thus $\mathcal{Z}(n) \subset \mathcal{Z}(n, \pi_1) \subset \mathcal{Z}(n, \Delta_1)$.

The equisingular families of conics, cubics, quartics and quintics are irreducible, i.e. $\mathcal{Z}(n) = \mathcal{Z}(n, \pi_1) = \mathcal{Z}(n, \Delta_1) = \emptyset$ for $n < 6$. For degree ≥ 6 this is not true, the first example, is a pair of sextics (with 6 cusps) was given by Zariski.

Using results of Oka on the computation of Alexander polynomials of sextics (math.AG-0205092), we describe the method to list up all of the Zariski pairs of degree 6 which can be distinguished by their Alexander polynomials, i.e. the set $\mathcal{Z}(6, \Delta_1)$. We prove that for any $(C_1, C_2) \in \mathcal{Z}(6, \Delta_1)$, one of them is of torus type and the other is of non-torus type, and the Alexander polynomial are $\Delta_1(t) = t^2 - t + 1$ (for sextic of torus type) and $\Delta_1(t) = 1$ (for sextic of non-torus type).

A characterization of Shimura curves in moduli stacks of abelian varieties and Calabi-Yau manifolds

Eckart Viehweg

(joint work with Kang Zuo)

Let $f : X \to Y$ be a semi-stable family of complex abelian varieties over a curve Y of genus q, and smooth over the complement of s points. If $F^{1,0}$ denotes the non-flat $1,0$ part of the corresponding variation of Hodge structures, the Arakelov inequalities say that

$$2\deg(F^{1,0}) \leq \operatorname{rank}(F^{1,0})(2q - 2 + s).$$

We study families for which this inequality becomes an equality, or equivalently families whose Higgs field

$$\theta_{1,0} : F^{1,0} \to F^{0,1} \otimes \Omega^1_Y(\log S)$$

is an isomorphism. As it turns out, this property is reflected in the existence of "too many" Hodge cycles of a general fibre of f, and it forces Y to be a Shimura curve. As a byproduct one obtains an explicit description of all possible examples.

For other semistable families of n dimensional varieties one considers the induced variation of Hodge structures of weight n and the corresponding Higgs bundle (E, θ) on Y. Then θ is maximal, if (roughly speaking) (E, θ) is the direct sum of sub Higgs bundles (F_i, θ_i) of length i, for which

$$\theta_i^{p,q} : F_i^{p,q} \to F_i^{p-1,q+1} \otimes \Omega_Y^1(\log S)$$

are isomorphisms, as soon as both sheaves are non zero. This definition is not the most general one. Contrary to the case of abelian varieties, we exclude here the existence of unitary parts. The maximality of the Higgs field implies that the families are rigid, and that the special Mumford Tate group of a general fibre F is the smallest algebraic subgroup of $Sl(H^n(F, \mathbb{Q}))$ which contains the image of the monodromy representation.

For $K3$ surfaces, X. Sun, S.L. Tan and K. Zuo have shown, that the maximality of the Higgs field implies that the Picard number of a general fibre is 19, and that the family is constructed from a product of modular families of elliptic curves.

The latter also seems to be true for families of Calabi-Yau threefolds.

Artin groups and geometric monodromy

Bronislaw Wajnryb

Let $f(x, y) = 0$ be a polynomial equation which defines an algebraic curve in a neighbourhood of $(0, 0)$ in $\mathbb{C}^2$, with an isolated singular point at $(0, 0)$. A versal deformation of this singularity induces a fibration $V \to B(\epsilon)$ which is locally trivial over the complement $U = B(\epsilon) - \Sigma$ of the singular set Σ (the discriminant) and whose fibre is a compact orientable surface S with a boundary. The fibration induces the geometric monodromy representation $\mu : \pi_1(U) \to M(S)$, where $M(S)$ is the mapping class group of S, the group of the isotopy classes of the orientation preserving diffeomorphisms of S pointwise fixed on the boundary. Dennis Sullivan asked around 1975 whether μ is always injective. For simple singularities A_n, D_n, E_6, E_7, E_8 the group $\pi_1(U)$ is isomorphic to Artin group of type A_n, D_n, E_6, E_7, E_8 respectively and μ is a geometric homomorphism, it takes standard generators of Artin group onto Dehn twists in $M(S)$. Any Artin group corresponding to Coxeter matrix with entries 2 and 3 only has an essentially unique geometric homomorphism ϕ into the suitable mapping class group which coincides with μ for the groups A_n, D_n, E_6, E_7, E_8. In 1992 Perron and Vannier proved that ϕ is injective for the groups A_n and D_n. In 1997 Labruere proved that ϕ is not injective for any Artin group corresponding to Dynkin diagram which is not a tree or a Dynkin diagram which is a tree with more than 3 ends. The groups E_6, E_7, E_8 belong to the missing cases. In this work we show that ϕ is not injective for all other Artin groups. In particular μ is not injective for singularities E_6, E_7, E_8 so the question of Sullivan has the negative answer already for simple singularities.

Cohomology of variations of Hodge structures over quasi-compact Kähler manifolds and applications to algebraic geometry

Yihu Yang
(joint work with J. Jost and K. Zuo)

Let $U = Y \setminus S$, smooth quasi-compact Kähler surface, S is a normal crossing divisor in Y; let $\mathbb{V}$ be a polarized variation of Hodge structure defined over $\mathbb{R}$ over U, with unipotent local monodromies around S. Let

$$E = \bigoplus_{p+q=m} E^{p,q},\ \theta^{p,q} : E^{p,q} \to E^{p-1,q+1} \otimes \Omega^1_Y(\log S)$$

denote the Higgs bundle corresponding to $\mathbb{V}$ (afterwards, we briefly write $E^{p,q}$ by E^p). Using the Pioncaré-like metric on U and the Hodge metric on $\mathbb{V}$, we can define an L^2-subcomplex of the above complex by taking the sheaves of local sections satisfying the L^2-integrable condition:

$$E_{(2)} \xrightarrow{\theta} (E \otimes \Omega^1_Y(\log S))_{(2)} \xrightarrow{\theta} (E \otimes \Omega^2_Y(\log S))_{(2)} \cdots .$$

Then, we can prove the following

Main Theorem *There exists a natural isomorphism*

$$H^*_{\mathrm{DR}}(\Gamma([Gr^*_F A^0(E)]_{(2)}) \xrightarrow{D''} \Gamma([Gr^*_F A^1(E)]_{(2)}) \cdots)$$
$$\simeq \mathbb{H}^*(E_{(2)} \xrightarrow{\theta} (E \otimes \Omega^1_Y(\log S))_{(2)} \cdots).$$

In another direction, one also has the theorem duo to E. Cattani, A. Kaplan and W. Schmid (for this, see Inventiones Math., **87**, 1987, 217-252).

The Main Theorem together with the Cattani-Kaplan-Schmid's theorem and the Kähler identity of the Laplacians for the situation of VHS gives rise to

Corollary 2. *There exists a natural isomorphism*

$$H^*_{\mathrm{int}}(Y, \mathbb{V}) \simeq \mathbb{H}^*(E_{(2)} \xrightarrow{\theta} (E \otimes \Omega^1_Y(\log S))_{(2)} \cdots).$$

In this talk, we also give some applications to algebraic geometry.

Log Terminal Algebraic Varieties and the Fundamental Groups of Their Smooth Loci

De-Qi Zhang

We work over $\mathbb{C}$. We are interested in algebraic varieties X with log terminal singularities, especially the topological fundamental group $\pi_1(X^0)$, where for variety V, $V^0 := V - SingV$.

From the minimal model program we know that a minimal model will inevitably contain some terminal singularities. Also a degenerate fibre of a family of varieties will have some singularities. So we can not help but considering varieties with some mild singularities.

Motivation: If $\pi_1(V^0)$ has an index-m normal subgroup, then we have a corresponding Galois $\mathbb{Z}/(m)$-cover $U \to V$ unramified over V^0. So the study of V may be reduced to that of U whose singularities should be better. See also [Keum- Zhang, Proc. Alg. Geom. in East Asia, Kyoto, 2001, A. Ohbuchi (ed.)]

Below, V is $\mathbb{Q}$-Fano (resp. weak $\mathbb{Q}$-Fano) if the anti-canonical divisor $-K_V$ is $\mathbb{Q}$-ample (resp. nef and big). According to the min.model program (completed in dim ≤ 3), every proj.variety is birational to either a min.terminal variety or a Fano fibration. This is the reason why we consider Fano varieties.

Conjecture A. Let V be a log terminal $\mathbb{Q}$-Fano variety. Then the topological fundamental group $\pi_1(V^0)$ of the smooth locus V^0 of V is finite. (see results below to support it).

Theorem B [Gurjar-Zhang (Tokyo 1994-95), Zhang (Osaka 1995), Fujiki-Kobayashi-Lu, Keel-McKernan]. *Conjecture A is true if either* $\dim V \leq 2$ *or the Fano index* $r(X) > \dim X - 2$.

Theorem C [Takayama]. *Suppose that* V *is a log terminal weak* $\mathbf{Q}$*-Fano variety. Then* $\pi_1(X) = (1)$.

Remark D. "Log terminal" in Conj.A can not be weakened to "log canonical" [Zhang,Trans A.M.S.(1996)].

According to the Iitaka fibration theorem, every proj. variety is birational to a fibration where a general fibre is of Kodaira dim. 0 and the base variety has dim. equal to the Kodaira dim. of the source variety. This is a motivation for us to consider varieties V of Kodaira dim. 0. If further, V is minimal and assume the abundance conjecture (proved when dim ≤ 3) then $mK_V \sim 0$ for some $m \geq 1$.

Definition E [Zhang, Kyoto, 1991-93]. A log terminal proj. surface Y is *log Enriques* if $mK_Y \sim 0$ for some $m \geq 1$ and if $H^1(Y, \mathcal{O}_Y) = 0$. The $I = I(X) := \min\{m | m \geq 1, mK_X \sim 0\}$ is called the index of Y.

The one below was formulated, when X is Du Val $K3$, in [Catanese-Keum-Oguiso, Math. Ann. 2002?].

Conjecture F. Let Y be a log Enriques surface. Then either $\pi_1(Y^0)$ is finite, or there is a quasi-etale (= etale in co-dim 1) morphism $X \to Y$ with X an abelian surface.

Let $\widetilde{Y} \to Y$ the min.resolution, $D = \sum D_i$ the exceptional divisor and $\#D$ the number of irred.comp. of D.

Theorem G (1) [Shimada-Zhang, Nagoya 2001] *When* Y *is Du Val* $K3$*, we have* $\pi_1(Y^0) = (1)$ *if the lattice* $\mathbf{Z}[\cup_i D_i]$ *is primitive in* $H^2(\widetilde{Y}, \mathbf{Z})$*, if* $\#D \leq 18$ *and if the discriminant group* $(\mathbf{Z}[\cup_i D_i])^\vee / (\mathbf{Z}[\cup_i D_i])$ *is generated by no more than* $\min\{\#D, 20 - \#D\}$ *elements (the last two conditions due to Nikulin are to guarantee the uniqueness of a primitive lattice embedding).*
(2) [Keum-Zhang JPAA 2002] *When* Y *is either Du Val* $K3$ *or Du Val Enriques, Conjecture* F *is true if* Y *has a few singularities of type* A_{p-1} *and no others, where* p *is a prime number.*
(3) [Catanese-Keum-Oguiso] *When* Y *is Du Val* $K3$*, Conjecture* F *is true if either* Y *has an elliptic fibration, or the exceptional divisor of the minimal resolution of* Y *has at most* 15 *components.*

Edited by Pho Duc Tai

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