Mathematisches Forschungsinstitut Oberwolfach

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# Modulformen

September 15th – September 21st, 2002

The conference was organized by S.Böcherer (Mannheim), T.Ibukiyama (Osaka) and W.Kohnen (Heidelberg). 23 talks were given, covering a wide range of topics. One speaker (Prof.Ikeda) was asked to give two talks on his results.
The theory of modular forms has many branches and it was one of the main purposes of the conference to bring together experts from various corners of the field.

Topics covered include

- $L$-functions and zeta-functions (not only of Langlands-type)
- Dimension formulas for spaces of modular forms
- explicit structure of rings of modular forms
- Applications of trace formulas
- Liftings of modular forms
- Local representation theory
- congruences for modular forms
- Relations with geometry

We hope that the stimulating atmosphere of the conference helped to create new collaborations among the participants.

# Abstracts

### A representation theoretic Kohnen-Zagier formula

M.Baruch

The Kohnen-Zagier formula relates the Fourier coefficient of a half-integral weight form in the Kohnen +space to a twisted central value of an $L$-function of an integral weight form. A formula of this type was first proved by Waldspurger. We prove a generalized formula of this type in a representation theoretic setting and show that it implies the Kohnen-Zagier formula.

### Eisenstein Series and Moments of Zeta

D.Bump
(joint work with J.Beinecke)

It is shown that if the parameters of an Eisenstein series on $GL(2k)$ are chosen so that its (integrated) L-function is the $2k$-th moment of the Riemann zeta function, then the $\binom{2k}{k}$ terms in its constant term agree with the $\binom{2k}{k}$ factors appearing in a conjectural formula for the 2k-th moment by Conrey, Farmer, Keating, Rubinstein and Snaith. A method of eliminating the problematic "arithmetic part" is shown for the 6-th moment. Furthermore a method of Sarnak is worked out using Eisenstein series on $GL(2)$ to estimate the 4-th moment of $\zeta$.

### Asymptotics of class numbers

A.Deitmar

For an order $\mathcal{O}$ in a number field let $h(\mathcal{O})$ be its class number, $R(\mathcal{O})$ its regulator and $D(\mathcal{O})$ its discriminant. In 1983 Peter Sarnak proved that

$$\sum_{\mathcal{O}:e^{2R(\mathcal{O})}\leq x} h(\mathcal{O})\,R(\mathcal{O}) \;\sim\; \frac{x}{\log x},$$

as $x \to \infty$. The main result of my talk is a generalization of Sarnak's result to number fields of prime degree. Let $d$ be a prime number $\geq 3$. Let $r, s \geq 0$ be integers with $d = r + 2s$. A number field $F$ is said to be *of type* $(r, s)$ if $F$ has $r$ real and $2s$ complex embeddings. Let $S$ be a finite set of primes with $|S| \geq 2$. Let $C_{r,s}(S)$ be the set of all number fields $F$ of type $(r, s)$ with the property $p \in S \;\Rightarrow\; p$ is non-decomposed in $F$. Let $O_{r,s}(S)$ denote the set of all orders $\mathcal{O}$ in number fields $F \in C_{r,s}(S)$ which are maximal at each $p \in S$. For such an order $\mathcal{O}$ let $h(\mathcal{O})$ be its class number, $R(\mathcal{O})$ its regulator and $\lambda_S(\mathcal{O}) = \prod_{p\in S} f_p$, where $f_p$ is the inertia degree of $p$ in $F = \mathcal{O} \otimes \mathbf{Q}$. Then $f_p \in \{1, d\}$ for every $p \in S$.

For $\lambda \in \mathcal{O}^\times$ let $\rho_1, \ldots, \rho_r$ denote the real embeddings of $F$ ordered in a way that $|\rho_k(\lambda)| \geq |\rho_{k+1}(\lambda)|$ holds for $k = 1, \ldots, r-1$. For the same $\lambda$ let $\sigma_1 \ldots \sigma_s$ be pairwise non conjugate complex embeddings ordered in a way that $|\sigma_k(\lambda)| \geq |\sigma_{k+1}(\lambda)|$ holds for $k = 1, \ldots, s-1$.

For $k = 1, \dots s-1$ let $\alpha_k(\lambda) := 2k(d-2k)\,\log\left(\frac{|\sigma_k(\lambda)|}{|\sigma_{k+1}(\lambda)|}\right)$.
If $s > 0$ let

$$\alpha_s(\lambda) := 2rs\,\log\left(\frac{|\sigma_s(\lambda)|}{|\rho_1(\lambda)|}\right).$$

For $k = s+1, \dots, r+s-1$ let

$$\alpha_k(\lambda) := (k+s)(r+s-k)(\,\log\left(\frac{|\rho_{k-s}(\lambda)|}{|\rho_{k-s+1}(\lambda)|}\right).$$

For $T_1, \dots, T_{r+s-1} > 0$ set

$$v_{\mathcal{O}}(T_1, \dots T_{r+s-1}) := \#\{\lambda \in \mathcal{O}^\times / \pm 1 \mid 0 < \alpha_k(\lambda) \le T_k,\ k = 1, \dots, r+s-1\}.$$

Let

$$c = (\sqrt{2})^{1-r-s}\left(\prod_{k=1}^{s-1}(4k(d-2k)\right)4rs\left(\prod_{k=s+1}^{r+s-1} 2(k+s)(r+s-k)\right),$$

where the factor $4rs$ only occurs if $rs \neq 0$. The main result is
**Theorem** *With*

$$\vartheta_S(T) := \sum_{\mathcal{O}\in\mathcal{O}(S)} v_{\mathcal{O}}(T)\,R(\mathcal{O})\,h(\mathcal{O})\,\lambda_S(\mathcal{O})$$

*we have, as* $T_1, \dots, T_{r+s-1} \to \infty$,

$$\vartheta(T_1, \dots, T_{r+s-1}) \ \sim\ \frac{c}{\sqrt{r+s}}\,T_1 \cdots T_{r+s-1}.$$

The proof involves a new Lefschetz formula for higher rank symmetric spaces.

## On two geometric theta lifts

J.Funke

The theory of theta lifts has been a major tool in the investigation of the geometry (of cycles) of locally symmetric spaces of orthogonal type. Here the singular theta lift introduced by Borcherds and extended by Brunier and the Kudla-Millson theta lift have been of particular interest.
In this talk, we show that in the Hermitian case (i.e. for $O(p,2)$) the Kudla-Millson lift and $dd^c$(extended Borcherds lift) are adjoint maps. We also introduce a suitable Borcherds lift for general signature and obtain a similar relation to the Kudla-Millson lift. Moreover, we show that this generalized Borcherds lift gives rise to “differential characters” in the sense of Cheeger and Simons for certain special cycles of codimension q.

## Siegel 3-folds of geometric genus one

V.Gritsenko

Let $\mathcal{A}_{s,t}$ be the moduli space of $(s,t)$-polarized abelian surfaces:

$$\mathcal{A}_{s,t} = \mathbf{H}_2/\Gamma_{s,t}\,,$$

where $\Gamma_{s,t}$ is the paramodular group of type $(s,t)$. We prove the following result:

**Theorem:** $h^{3,0}(\mathcal{A}_{1,t}) = \dim S_3(\Gamma_{1,t}) = 1$ *if t=13, 17, 19*

We formulate the **Conjecture:**

$$h^{3,0}(\mathcal{A}_{1,t}) = 1 \iff t = 13, 17, 19, 21, 22, 23, 25, 27, 28, 32, 35, 40, 42, 48, 60.$$

We sketch the proof of the theorem for $t = 13$, which goes through the explicit construction of certain Jacobi cusp forms of weight 3.

## Shintani Cocyycles on $GL_n(\mathbf{Q})$

R.Hill

In the case $n = 2$, D.Solomon constructed a 1-cocycle on $GL_2(\mathbf{Q})$ with values in the Shintani functions corresponding to cones in $\mathbf{Q}^2$. Essentially, to two $2\times 2$-matrices $\alpha$, $\beta$ the cocycle assigns the Shintani function of the cone generated by the first column of $\alpha$ and the first column of $\beta$.
The aim of the talk is to generalize this construction to give an (n-1)-cocycle on $GL_n(\mathbf{Q})$. The difficulties arise when the cones arising degenerate or when one needs to decide whether a boundary component of the cone should be included or not. The difficulties are overcome by passing to a field extension $F = \mathbf{Q}(\epsilon_1, \ldots, \epsilon_n)$ of $\mathbf{Q}$, in which the $\epsilon_i$ are regarded as infinitesimally small. The boundaries of the cones obtained in $F^n$ only intersect $\mathbf{Q}^n$ at the origin and the cones are never degenerate.

## Spherical functions on $Sp_2$ as a spherical homogeneous $Sp_2 \times Sp_1^2$-space

Y.Hironaka

Let $\mathbf{G} = Sp_2 \times Sp_1^2$, where $Sp_1^2$ is considered as a subgroup of $Sp_2$ in the usual way (diagonally). Then $\mathbf{X} = Sp_2$ is a spherical homogeneous $\mathbf{G}$-space with $\mathbf{G}$-action defined by $(g_1, g_2)\cdot x = g_1 x g_2^t$.
Let $k$ be a local field with odd residual characteristic. We put $G = \mathbf{G}(k)$, $X = \mathbf{X}(k)$ and $K = \mathbf{G}(O_k)$. We study spherical functions on $X$. Among other things, we obtain

- a complete set of representatives of $K$-orbits in $X$
- employing spherical functions as kernel functions, we obtain an $\mathcal{H}(G,K)$-isomorphism

$$\mathcal{S}(K\backslash X) \simeq \left(\mathbf{C}[q^{\pm z_1}, \ldots, q^{\pm z_4}]^W \oplus \prod_{i=1}^{4}(q^{\frac{z_i}{2}} + q^{-\frac{z_i}{2}} \cdot \mathbf{C}[q^{\pm z_1}, \ldots, q^{\pm z_4}]^W\right)^2;$$

  therefore $\mathcal{S}(K\backslash X)$ is a free $\mathcal{H}(G,K)$-module of rank 4 and a basis is given explicitly.
- Eigenvalues for spherical functions are parametrized by $\mathbf{C}/W$, where $W$ is the Weyl group of G. The space of spherical functions on $X$ corresponding to $z \in \mathbf{C}^4/W$ has dimension 4 and a basis is given explicitly.

## Multiple Dirirchlet series: What one can prove and what one can't

J.Hoffstein

Consider the $L$-series $L(s,f,\chi_d^{(n)})$, where $f$ is an automorphic form on $GL(r)$ and $\chi_d^{(n)} = \left(\frac{*}{d}\right)_n$. We assume that the ground field contains the $n$-th roots of unity.
We define the "imperfect" double Dirichlet series

$$Z(s,w) = \sum_{d \text{ squarefree}} \frac{L(s,f,\chi_d^{(n)})}{(Nd)^w}$$

and obtain the meromorphic continuation as a function of two variables on a convex subset of $\mathbf{C}^2$. This continuation is sufficient to establish the existence of a pole in $w$ with non-zero residue for any $s$ with $1-\frac{1}{n+1} < \Re(s) < 1$ (for $n=2$ there is a stronger result). This implies that for any $s$ in this range, there exist infinitely many $d$ with $d$ squarefree such that $L(s,f,\chi_d^{(n)}) \neq 0$. We also sketch lot of the cases where $L(s,f,\chi_d^{(n)})$ can be extended to all of $d$ (i.e. not just squarefree $d$). In some cases, the corresponding "perfect" double Dirichlet series can then be extended to all of $\mathbf{C}^2$.

## Liftings of Modular Forms

T.Ikeda

The Fourier coefficients $A_{k+n}^{(2n)}$ of the Siegel Eisenstein series $E_{k+n}^{(2n)}$ are given as follows

$$C \cdot A_{k+n}^{(2n)}(B) = L(1-k,\chi_B) f_B^{k-\frac{1}{2}} \prod_{p|f_B} \tilde{F}_p(B, p^{-k+\frac{1}{2}})$$

Here,

| | |
|---|---|
| $B = B^t > 0$ | : half integral symmetric matrix |
| C | : some constant depending only on $n$ and $k$ |
| $D_B$ | $= (-1)^n det(2B)$ |
| $\mathcal{D}_B$ | $=\mid Disc(\mathbf{Q}(\sqrt{D_B})) \mid$ |
| $\mid D_B \mid$ | $= \mathcal{D}_B \cdot f_B^2$ , $(f_b > 0)$ |
| $\chi_B$ | :Dirichlet character corresponding to $\mathbf{Q}(\sqrt{D_B})$ |
| $\tilde{F}_p(B,X)$ | : some Laurent polynomial |

Let $f(\tau) = \sum_N a(N) q^N \in S_{2k}(SL_2(\mathbf{Z}))$ be a normalized Hecke eigenform. Put

$$A(B) = C(\mathcal{D}_B) f_B^{k-\frac{1}{2}} \prod_{p|f_B} \tilde{F}_p(B,\alpha_p)$$

Here, $1-a(p)X+p^{2k-1}X^2 = (1-p^{k-\frac{1}{2}}\alpha_p X)(1-p^{k-\frac{1}{2}}\alpha_p^{-1}X)$ and $C(\mathcal{D}_B)$ denotes the Fourier coefficient of the corresponding modular form of half-integral weight. Then we have
**Theorem:** Put $F(Z) = \sum_B A(B) exp(2\pi i tr(BZ))$ with $Z \in \mathbf{H}_{2n}$. Then $F \in S_{k+n}(Sp_{2n}(\mathbf{Z}))$; it is a Hecke eigenform and

$$L(s,F,stand) = \zeta(s) \prod_{i=1}^{2n} L(s+k+n-i, f)$$

We can also construct a hermitian cusp form from the Fourier coefficient formula for Hermitian Eisenstein series.

## On the conjecture of Miyawaki

T.Ikeda

Let $f = \sum_N a(N)q^N \in S_{2k}(SL_2(\mathbf{Z}))$ be a normalized Hecke eigenform, whose Satake parameters are $\alpha_p^{\pm 1}$. For non-negative integers $n, r$ such that $n + r \equiv k \bmod 2$, there is a Hecke eigenform $F \in S_{k+n+r}(Sp_{2n+2r}(\mathbf{Z}))$ such that

$$L(s, F, st) = \zeta(s) \prod_{i=1}^{2n+2r} L(s + k + n + r - i, f).$$

Given $g \in S_{k+n+r}(Sp_r(\mathbf{Z}))$, we put

$$\mathcal{F}_{f,g}(Z) = \int_{Sp_r(\mathbf{Z}\backslash \mathbf{H}_r} F(\begin{pmatrix} z & 0 \\ 0 & z' \end{pmatrix})\overline{g(z')}(\det \Im(Z'))^{k+n-1} dz'$$

Then $\mathcal{F}_{f,g} \in S_{k+n+r}(Sp_{2n+r}(\mathbf{Z}))$.

**Theorem:** Assume that $g$ is a Hecke eigenform and that $\mathcal{F}_{f,g} \neq 0$. Then $\mathcal{F}_{f,g}$ is a Hecke eigenform and

$$L(s, \mathcal{F}_{f,g}, st) = L(s, g, st) \cdot \prod_{i=1}^{2r} L(s + k + n - i, f).$$

As for the non-vanishing, we describe a conjecture for the special case $n = 0$, giving a conjectural formula for the double scalar product

$$\langle F_{|\mathbf{H}_r \times \mathbf{H}_r}, g \times g \rangle$$

in terms of values of some $L-$function.

## CAP Automorphic Representations of $U_{E/F}(4)$

T.Konno

(joint work with K.Konno)

The term CAP is the short hand for "cuspidal but associated to parabolic", which was first used by I.Piatetski-Shapiro. Thus a CAP-form is a cuspidal automorphic form which shares almost all local components with some residual discrete automorphic representation. The image of the Saito-Kurokawa-lifting and Ikeda's lifting are examples of such forms.
In this talk, we consider the quasisplit unitary group $U_{E/F}(4)$ in 4 variables, for which all the residual discrete forms are obtained. We construct the candidates of all CAP-forms expected by Arthur's conjectures on the automorphic spectrum. In particular, we show:
**Theorem:** *The CAP-forms are obtained as $\theta$-lifts of some discrete automorphic representations of unitary groups in two variables.*
Looking at the precise description of the local and global $\theta$-correspondence, we also show that the occurrence of these CAP forms agrees with the multiplicity formula conjectured by Arthur.

## Some vanishing theorems for the cohomology of arithmetic manifolds

J.-S.Li

(joint work with J.Schwermer)

Suppose that $X = G/K$ is a Riemannian symmetric space, $\Gamma \subset G$ is a lattice in $G$ and $E$ is a finite dimensional representation of $G$. The Matsushima formula, when combined with the Vogan-Zuckermann classification of unitary representations with non-zero cohomology, gives the first basic vanishing results for $H^*(\Gamma, E)$ (or at least the cuspidal cohomology). There is much evidence showing that this vanishing result cannot be improved without posing further conditions on $G$ and $\Gamma$. In this talk we describe our joint work with Schwermer, where we prove that $H^j(\Gamma, E) = 0$ for all $j < q_0(G)$, if $E$ has regular highest weight. Here

$$q_0(G) = \frac{1}{2}\left(\dim(X) - rk(G) + rk(K)\right)$$

## On a Rankin-Selberg Convolution of n Variables for Siegel Modular Forms

Y.Martin

(joint work with O.Imamoglu)

Let $F(Z) = \sum_T a(T)e(TZ)$ and $G(Z) = \sum_T b(T)e(TZ)$ be two Siegel cusp forms of weight $k$ for $Sp(n, \mathbf{Z})$. We study the Dirichlet series

$$D(F, G, \omega) = \sum_{T/\sim} \frac{a(T)\overline{b(T)}}{\xi_T} E(T, \omega),$$

where $E(Y, \omega)$ is the Selberg Eisenstein series for $GL(n)$, which depends on $n$ complex variables $\omega = (\omega_1, \ldots, \omega_n)$.
One can prove that

$$\Lambda(F, G, \omega) = (\text{suitable gamma factors}) \times D(F, G, \omega)$$

- has a meromorphic continuation to $\mathbf{C}^n$
- satisfies $2^n n!$ functional equations
- at a particular point $\omega_0 \in \mathbf{C}^n$ we have $\Lambda(F, G, \omega_0) = \text{constant} \times \langle F, G\rangle$
- Particular specializations of $\Lambda(F, G, \omega)$ yield the one variable Rankin-Selberg convolutions of Kalinin and Yamazaki.

## Modular Form Congruences and Selmer Groups

W.McGraw

(joint work with K.Ono)

A famous question of Hida asks: When do integral weight congruences between modular forms descend, via the Shimura correspondence, to congruences between half-integral weight forms ? Serre shows that there are congruences between forms in $S_2(\Gamma_o(p))$ and forms in $S_{p+1}$. We show that the pre-images of these forms in $S^+_{\frac{3}{2}}(\Gamma_0(4p))$ and $S^+_{\frac{p+2}{2}}(\Gamma_0(4))$are congruent after acting on the first form by the $U(p)$-operator. Combining this result with work of Dummigan, we give a large class of examples for which the existence of elements in certain Selmer groups associated to these integral weight forms agrees with famous conjectures on special values of $L$-functions for these forms.

## Spherical functions of real reductive groups of low rank

T.Oda

The problem of generalized spherical functions of a reductive group over a local field $k$ is formulated as follows: Given a reductive group $G$ over $k$, and an admissible representation $\pi$ of $G$ which is irreducible; given a closed subgroup $R$ and an irreducible unitary representation $(\eta, V)$ of $R$, we can construct the (say, smooth) induced $G$-module $Ind_R^G(\eta)$. The following problem is fundamental in the local theory of automorphic forms:

**Problem:** (i) Finds triads so that the intertwining space $Hom_G(\pi, Ind_R^G(\eta))$ is of finite dimension.

(ii) For each non-zero element $I \in Hom_G(\pi, Ind_R^G(\eta))$, describe the image $Im(I) \hookrightarrow Ind_R^G(\eta)$.

Usually we assume the existence of a double coset decomposition

$$G = RA_RK$$

with a subgroup of the split component of a maximally split Cartan subgroup $A$ of $G$. Then choosing a $K$-type $\tau \overset{i}{\hookrightarrow} \pi$ of $\pi$ we have the restriction homomorphism

$$\begin{array}{rcl} Hom_G(\pi, Ind_R^G(\eta)) & \to & Hom_K(\tau, Ind_R^G(\eta)) \\ & \cong & \{\tau^* \otimes Ind_R^G(\eta))\}^K \\ & \hookrightarrow & \tau^* \otimes C^\infty(A_R) \end{array}$$

The determination of the image of the composition map $r_\tau$ is solved by considering the holonomic system, i.e. a maximally overdetermined system of partial differential equations, when $G = Sp(2, \mathbf{R})$ and $G = SU(2,2)$ for many $R$.

The description of the maximal compact $K$'s representations is quite important. To settle those $K \hookrightarrow U(1)^l \times SU(2)^m \times SU(3)^n$, the projector of a certain tensor product of the representations of $\mathbf{gl}_3 \cong Lie(U(3)) \otimes \mathbf{C}$ in terms of the canonical basis was discussed, which is joint work with M.Hirano.

## Arithmetic Differential Operators on nearly holomorphic Siegel Modular Forms

A.Panciskin

Nearly holomorphic Siegel modular forms over a ring $A$ are certain formal expansions

$$f = \sum_{\xi, \mathbf{n}} a(\xi) R^{\mathbf{n}} q^\xi$$

where $\xi$ runs over all half-integral positive semidefinte symmetric matrices of size $m$, $q^\xi = exp(2\pi i tr(\xi z)$, $z \in \mathbf{H}_m$, $R = (R_{ij}) = (4\pi \Im(Z))^{-1}$, $\mathbf{n} = (n_{ij})$ and $R^{\mathbf{n}} = \prod_{i,j} R_{ij}^{n_{ij}}$.

We describe the action of the arithmetic differential operators of Maaß and Shimura (Theorem 1). This gives a method of proving congruences between nearly holomorphic Siegel modular forms (Theorem 2) and Kummer type congruences for certain L-values attached to Siegel modular forms (Theorem 3).

## $L$ and $\epsilon$ -Factors of some Representations of $GSp(4)$

B.ROBERTS

In our proof of an analogue for $GSp(4)$ of the dihedral case of the Langlands-Tunnel theorem we defined some $L$-packets for $GSp(4)$. In this talk we describe the $L$- and $\epsilon$-factors associated to the generic elements of these local $L$-packets by the Novodvorsky integral representation in the nonarchimedean case. As pointed out by Takloo-Bighash, the theory also has implications for archimedean zeta integrals. We also discuss the problem of finding canonical vectors in such generic representations which represent their $L$-factors.

## The Burkhardt Group and Modular Forms

R.SALVATI-MANNI

(joint work with E.Freitag)

We investigate the ring of Siegel modular forms of genus 2 and level 3. We determine the structure of this ring. It is generated by 10 modular forms (5 of weight 1 and 5 of weight 3) and there are 20 relations. The proof consists of two steps: In a first step we prove that the projective variety associated to this ring of modular forms is the normalization of the dual of Burkhardt's quartic. The second step consists in the normalization of the Burkhardt dual. Several complicated polynomial identities will occur. We first construct an element of the normalization, then using the action of Burkhardt's group we obtain the other elements; at the end we obtain a ring contained in the same ring of fractions that satisfies Serre's criterion of normality. This is the ring of modular forms.

## An integral Representation of the Singular Series and its Applications

F.SATO

The (global) singular series $b(T,w)$ is defined (for $\Re(w) > \frac{n+1}{2}$) by

$$b(T,w) = \sum_{R \in Sym_n(\mathbf{Q}/\mathbf{Z})} \nu(R)^{-w} \, e^{2\pi i \, tr(TR)}$$

where $T$ is a half-integral symmetric matrix of size $n$ and $\nu(R)$ is the product of the denominators of the elementary divisors of $R$.

**Theorem:** *Put* $H_n = \frac{1}{2}\begin{pmatrix} 0_n & 1_n \\ 1_n & 0_n \end{pmatrix}$ *and* $X(T) = \{x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in M_{2n,n}(\mathbf{Q}_p) \mid H_n[x] = T\}$. *Let* $\omega_T$ *be the gauge form on* $X(T)$ *given by* $\frac{dx}{d(H_n[x])}$. *Then*

$$\int_{X(T)} \mid det(x_2) \mid_p^{w-n} \omega_T = b_p(T,w) \prod_{i=1}^{n} \frac{1-p^{-i}}{1-p^{-(w-i+1)}}$$

**Application 1:** The local singular series $b_p(T,w)$ can be written as a finite linear combination of spherical functions on the p-adic symmetric space $SO(n,n)/S(O(n) \times O(n))$. This gives an explanation of the functional equation satisfied by $b_p(T,w)$ (joint work with Y.Hironaka)

**Application 2:** The Koecher-Maass series of the non-holomorphic Siegel Eisenstein series can be identified with the zeta function associated to a certain prehomogeneous vector space (joint work with T.Ueno).

**On Hecke eigenvalues of primitive forms and the analogue of Linnik's problem in the weight aspect**

J.Sengupta

Let $f$ and $g$ be two primitive cusp forms for $\Gamma_0(N)$ having distinct weights $k_1$ and $k_2$. Let

$$f(z) = \sum n^{\frac{k_1-1}{2}} \lambda_f(n) e^{2\pi i n z} \qquad g(z) = \sum n^{\frac{k_2-1}{2}} \lambda_g(n) e^{2\pi i n z}$$

be their respective Fourier expansions; $\lambda_f(n)$ is the normalized eigenvalue for the n-th Hecke operator. The strong multiplicity one theorem for $GL(2)$ says that there are infinitely many primes $p$ such that $\lambda_f(p) \neq \lambda_g(p)$. We are interested in the "smallest" such prime $p$. We show that given $\epsilon > 0$ there exists a prime $p$ as above such that $p = \mathcal{O}(k^{2+\epsilon})$ where $k = \max(k_1, k_2)$ and the implied constant depends only on $\epsilon$ and $N$. This is an improvement of the earlier result of Moreno.
In the second part of the talk we discuss an analogue of Linnik's classical problem in the context of primitive cusp forms of varying weight.

**Green currents for modular cycles in arithmetic quotients of complex hyperballs**

M.Tsuzuki

(joint work with T.Oda)

Let $X$ be a complex manifold and $Y$ an analytic subvariety of codimension $v$. In such a situation a Green current for $Y$ can be defined.
We want to construct a Green current for $Y$ analytically using the techniques of harmonic analysis on Lie groups, when $X$ is the quotient of a Hermitian symmetric domain $G/K$ by an arithmetic lattice $\Gamma$ and $Y$ is a modular cycle coming from a modular embedding $H/H \cap K \hookrightarrow G/K$. The focus of the talk was on the case when $G/K$ is a complex hyperball of dimension $n$ and $H/H \cap K$ is also a complex hyperball (of dimension $n-v$). In the case $v = 1$ (as was predicted by T.Oda) we have already succeeded to construct a Green current for a modular divisor $Y$ by taking the constant term at a suitable point $s_0$ of the meromorphic continuation of the Poincare series $\sum_{\gamma \in (\Gamma \cap H)\backslash \Gamma} \phi_s^{(2)}(\gamma g)$ with the "secondary spherical function" $\phi_s^{(2)}$.
So it is quite natural to ask whether the same method works when $v > 1$. In the talk we reported on the following points:

- To make precise the notion of vector-valued secondary spherical function and to show the existence of such functions $\phi_s^{(2)}$.
- To show the $L^1$-convergence of the series $\sum_\gamma \phi_s^{(2)} = \tilde{G}_s$ with $s \in \mathbf{C}$ lying in a certain nonempty domain.
- The differential equation of the current defined by $\tilde{G}_s$

We expect that the function $s \longmapsto \tilde{G}_s$ has a meromorphic continuation so that $s = n-2v+2$ is (at most) a simple pole and that the constant term of $\tilde{G}_s$ at $s = n-2v+2$ is closely related to a green current of $Y$.

## The twisted topological trace formula and liftings of automorphic representations

U.Weselmann

For a reductive connected group $G$ we consider the $G(\mathbf{A}_f)$-modules

$$H^i(G,E_\lambda) = \lim_{\rightarrow} H^i(G(\mathbf{Q})\backslash G(\mathbf{A})/K_\infty A_G(\mathbf{R})^o K_f, \tilde{E}_\lambda)$$

where $\tilde{E}_\lambda$ is the coefficient system attached to the representation with highest weight $\lambda \in X^*(T)$. If $\eta \in Aut(G)$ fixes some splitting, we may consider the $\eta$-action on $H^i(G,E_\lambda)$ too and construct the stable endoscopic group $G_1$ such that $\hat{G}_1 = \left(\hat{G}^{\hat{\eta}}\right)^o$, e.g.

| | | | | | |
|---|---|---|---|---|---|
| a) | $G = GL_{2n} \times GL_1$ | $\eta(A,a) = (J^t A^{-1} J^{-1}, det(A)\cdot a)$ | $G_1$ | $=$ | $GSpin_{2n+1}$ |
| b) | $G = PGL_{2n+1}$ | $\eta(A) = J^t A^{-1} J^{-1}$ | $G_1$ | $=$ | $Sp_{2n}$ |
| c) | $G = SO_{2n+2}$ | $\eta \in O_{2n+2} \setminus SO_{2n+2}$ | $G_1$ | $=$ | $Sp_{2n}$ |

To compute Hecke operators, we developed a topological trace formula

$$tr(\eta\circ h_f, H^*(G,E_\lambda)) = \sum_{I\subset\Delta, I^\eta = I} (-1)^{\sharp((\Delta\backslash I)/\eta)} \sum_{\substack{\gamma \in P_I(\mathbf{Q}) \\ \text{mod st.conj.}}} \alpha(\gamma_\infty)\cdot tr(\eta\gamma \mid E_\lambda)\cdot O^{st}(\eta\gamma, h_f)$$

By a comparison of two trace formulas one gets that $H^*(G,E_\lambda)$ is the lift of $H^*(G_1,E_\lambda)$ up to representations of the form $Ind_{G(\mathbf{A}_f)}^{G(\mathbf{A}_f)\times\eta}$ in the Grothendieck group of admissible $G(\mathbf{A}_f)\times\langle\eta\rangle$-modules in the situations a) and b) for $n=2$. Here we use a fundamental lemma due to Flicker and the equivalence of the fundamental lemmas in the situations a), b), c). From these identities one develops character identities between local representations and deduces for all representations $\pi_f$ of $G(\mathbf{A}_f)$ that contribute via the discrete spectrum to $H^3(GSp_4,E_\lambda)$ but are neither endoscopic lifts from $GL_2\times GL_2/GL_1$ nor CAP representations w.r.t. the Siegel parabolic:

- each $\tilde{\pi}_f$ in the packet of $\pi_f$ occurs in $H^3(G,E_\lambda)$ with multiplicity 4
- there exists $\hat{\pi}_f$ in the packet of $\pi_f$ such that $\pi_\infty^W\times\hat{\pi}_f$ is globally generic
- all automorphic representations $\pi_\infty^W\times\hat{\pi}_f$ and $\pi_\infty^H\times\hat{\pi}_f$ occur with multiplicity one in the cuspidal spectrum of $GSp_4$ for all $\hat{\pi}_f$ in the packet of $\pi_f$.

## Computation of Spaces of Siegel Modular Cusp Forms

David S.Yuen

(joint work with C.Poor)

We survey the known dimensions of $S_n^k$, the space of Siegel modular cusp forms of weight $k$ and degree $n$. We obtain new results for degrees $4,5,6$ by combining a Vanishing Theorem and a restriction technique. For fixed $n,k$, the Vanishing Theorem gives an explicit set of Fourier coefficients which determine $S_n^k$. The restriction of Siegel modular forms to elliptic modular forms reveal linear relations among these Fourier coefficients. Sometimes we produce enough relations to determine $S_n^k$. We discuss conjectures to the effect that this method always computes the dimension of $S_n^k$.

*Edited by Siegfried Böcherer*

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