Mathematisches Forschungsinstitut Oberwolfach

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Topologie

September 22nd – September 28th, 2002

The conference was organized by Cameron Gordon (Austin, Texas, USA), Wolfgang Lück (Münster, Germany), and Bob Oliver (Paris, France). About fifty mathematicians—from all over Europe, North America, and Australia—attended the meeting.

A highlight of the program was a series of three lectures delivered by Martin R. Bridson (London, UK) on geometric group theory. In line with the well-established tradition of the "Topologie Tagung", the other sixteen talks covered a wide variety of areas of current research in algebraic topology and related fields—such as stable homotopy theory, algebraic K- and L-theory, L^2-invariants, p-compact groups, three and four dimensional manifolds, and geometric group theory.

With an average of four one-hour lectures a day, the participants also had plenty of time for discussion and research. As usual the staff of the Mathematisches Forschungsinstitut Oberwolfach provided all the ideal conditions for a successful meeting.

Abstracts

Lectures on Geometric Group Theory

Martin R. Bridson

1. The Universe of Geometric Group Theory

What is geometric group theory: what are the mutual benefits of its interactions with other areas of mathematics; what are some of the main issues, and why; why is unconstrained group theory arbitrarily hard (in a quantifiable sense) and what are reasonable constraints that lead to a tractable, rich and effective theory; which classes of groups demand special attention, and why???

A brief review of why finitely presentations of groups are the natural objects of study. The heart of *combinatorial group theory*: the word, conjugacy and isomorphism problems and their geometric/topological origins (Dehn 1912). *Topological group theory*: topological models of groups – the standard 2-complex, its strengths and weaknesses, ambiguity, strategies for improvement; $K(\Gamma, 1)$, finiteness issues; manifold models, geometric properties (e.g. symplectic, complex, Kähler), aspherical models, uniqueness issues (Borel conjecture etc.).

Naturally arising classes of groups in the above context: 1-relator groups, 3-manifold groups, Kähler groups, Thompson's groups.

Geometric Group Theory: Strand 1 – illuminating groups and spaces by studying (and constructing) group actions (preferably by isometries). Complexes of groups. Examples of core algebraic problems solved by the Strand 1 approach.

Strand 2 – finitely generated groups as geometric objects (à la Gromov). Quasi-isometries and intrinsic geometry. Limits and ultralimits; the polynomial growth theorem (sketch of proof, emphasis on the synthesis of approaches [GH-limits, Montgomery-Zippin theorem, Jordan, Tits]). Quasi-isometric rigidity. Asymptotic cones of free groups (whence hyperbolic groups). Special classes emerging: nilpotent groups, hyperbolic groups.

Connecting the core of combinatorial group theory to geometry: the word problem as measured by Dehn functions; connection to isoperimetrical properties of manifolds via the Filling Theorem; the isoperimetrical spectrum; the clear demarcation of hyperbolic groups (again).

The universe of groups with two sides of simple development (the amenable side of the universe and the hyperbolic side), with wild interior ("here there are dragons").

The isolation of the class of hyperbolic groups: hyperbolic versus atoroidal, cf. 3-manifold theory. What other ideas enter from 3-manifold theory? JSJ decomposition, automorphisms of (hyperbolic) groups; the special role of mapping class groups and automorphism groups of free groups.

2. Decision Problems, Non-Positive Curvature and Subgroups

We investigate how complex the universe of groups becomes as one moves beyond hyperbolic groups in the direction of non-positive curvature. Moving in this direction, we focus on two fundamental questions:

* how can we determine when groups or spaces are isomorphic?

* what horrors can lurk among the subgroups of benign groups?

The many properties of hyperbolic groups; the hope that the next constraint on the word problem may have similar impact to the linear isoperimetrical inequality defining hyperbolic groups. Positive and negative results. Automatic groups versus non-positive curvature. CAT(0) spaces (à la Alexandrov), the group theoretic properties of their groups of isometries (no proofs). Proof that CAT(0) implies a quadratic isoperimetrical inequality. Quasification of CAT(0) – semihyperbolic ideas, combability.

What does it mean for a problem to be undecidable? Recursive and r.e. sets of integers, encodings into groups, the Higman embedding theorem. Proof (given the Higman theorem) of the existence of finitely presented groups with unsolvable word problem. Promoting this to the undecidability of the isomorphism problems for groups (proof) and manifolds of dimension 4 and above (sketch).

Explanation of the conjectures for 3-manifolds, namely that the word, conjugacy and isomorphism problems should be solvable in the class of 3-manifold groups, and that the homeomorphism problem should be solvable for compact 3-manifolds. Note that all of these would follow (in a highly non-trivial manner) from a solution to Thurston's Geometrization Conjecture.

Theorem (Farrell-Jones; Sela). *The homeomorphism problem is solvable in the class of fundamental groups of closed negatively curved manifolds of dimension* ≥ 5.

Conjecture. *The homeomorphism problem for closed non-positively curved manifolds is unsolvable in dimensions 4 and above.*

Theorem (MRB). *The isomorphism problem is unsolvable in the class of combable groups.*

Sketch of construction and proof.

Examples and theorems to illustrate the delicacy of deciding which subgroups of semihyperbolic groups are themselves semihyperbolic. Positive results in low dimensions, tameness and wildness results in higher dimensions.

3. Some Gems of the Theory, and Aspects of Rigidity

Mostow rigidity, automorphisms of hyperbolic groups, the theorems of Rips-Sela and Paulin. The special role of mapping class groups and (outer) automorphism groups of free groups.

The various manifestations of $\mathrm{SL}(2,\mathbb{Z})$ and the natural families of generalization to which they lead: Thompson's groups; $\mathrm{SL}(n,\mathbb{Z})$ (arithmetic lattices,...); mapping class groups; (outer) automorphism groups of free groups.

A detailed list of some of the many levels to the analogies between lattices (particularly $\mathrm{SL}(n,\mathbb{Z})$), mapping class groups, and (outer) automorphism groups of free groups – sketch ideas of proofs where possible.

Symmetric space / Teichmüller space / Outer space. The isometry-rigidity theorems of Tits, Royden, N. Ivanov, and Bridson-Vogtmann.

Rigidity manifests in the absence of outer automorphisms: the theorems of Bridson-Vogtmann and Ivanov, analogous to Mostow's work concerning lattices. Commensurators.

Super-rigidity: rigidity for maps from lattices to mapping class groups and automorphism groups of free groups (Kaimanovich, Masur, Farb; Bestvina, Fujiwara; Bridson, Farb) and between the latter classes of groups (Bridson-Vogtmann). Zimmer programme, actions on the circle; quasi-isometric rigidity; property (T), etc.

p-compact groups as framed manifolds

Tilman Bauer

Every compact Lie group G comes with a canonical framing of its tangent bundle given by left translation of a basis of its Lie algebra. Hence it gives rise to an element $[G]$ in the stable homotopy groups of spheres by means of the Thom-Pontryagin construction. It is possible to construct this maps less geometrically using a stable transfer map

$$BG^{\mathfrak{g}} \to BH^{\mathfrak{h}}$$

which is defined whenever H is a closed subgroup of G. The element $[G]$ is then given as the composite

$$\mathbf{S}^d \to BG^{\mathfrak{g}} \to B\{1\}_+ = \mathbf{S}^0.$$

This construction can be extended to the class of p-compact groups. Indeed, following ideas of J. Klein, for any connected p-compact group G, we have

$$S_G := (\Sigma_+^\infty G)^{hG} \simeq \mathbf{S}^d$$

where the G-action is given by right multiplication and d is the mod p homological dimension of G. Everything is understood to be formed in the p-complete category. This spectrum still has a left action of G which behaves like the adjoint action of a Lie group on the one-point compactification of its Lie algebra. A suitable analog of $BG^{\mathfrak{g}}$ should therefore be defined as the homotopy orbit spectrum of S_G under this action, and a transfer map is constructed as the G-homotopy orbits of the map

$$S_G \to (\Sigma_+^\infty G)^{hH} \simeq G \wedge_H S_H.$$

Homotopy classes arising in this way from p-compact groups include α_1 at any prime and β_1 at $p = 3, 5$.

Moreover, every p-compact groups satisfies a self-duality equivalence

$$\Sigma_+^\infty G \simeq S_G \wedge DG_+$$

and, for a monomorphism $H < G$,

$$G \wedge_H S_H \simeq D(G/H)_+ \wedge S_G.$$

This last identity, for $H = \mathbf{S}^1$, is the missing piece of information for proving the following, using surgery theory and some structure theory of p-compact groups:

Theorem (Kitchloo, Notbohm, Pedersen, B.)**.** *Every quasifinite loop space is homotopy equivalent to a closed, compact, smooth, parallelizable manifold.*

The hyperbolicity of the curve complex

Brian Bowditch

The complex of curves is a simplicial complex, $\mathcal{C}$, associated to a compact surface. Its vertex set is the set of homotopy classes of simple closed curves, and a (finite) subset of vertices is deemed to span a simplex if the corresponding curves can be realised disjointly. This complex was originally introduced by Harvey, and has been studied by many authors since. For example it has been used by Harer to investigate the cohomology of the mapping class group, M, and by Ivanov to study the outer automorphism group of M. More recently the large scale geometry of $\mathcal{C}$ has been exploited. In particular, Masur and Minsky have shown that (except in certain trivial cases) it is hyperbolic in the sense of Gromov. In this talk we outline a simplified proof of this result. The arguments are more combinatorial in nature, and are based on an analysis of intersection numbers. As a consequence one can obtain certain refinements. For example, the hyperbolicity constant is bounded above by a logarithmic function of the complexity (the genus plus the number of punctures). One can also obtain a fairly explicit description of quasigeodesics in terms of intersection numbers. It is hoped that these methods might lead to further understanding of this complex, for example, its hierarchical finiteness properties as laid out in the second of Masur and Minsky's papers.

Homotopy self-equivalences of 4-manifolds

IAN HAMBLETON
(joint work with Matthias Kreck)

Let M be a closed, oriented topological 4-manifold. We fix a base-point x_0 in M and consider the group $Aut(M)$ of homotopy classes of orientation-preserving homotopy self-equivalences $f\colon M \to M$ which fix the base-point. The group $Aut(M)$ is a natural invariant, but is not well understood except in the simplest cases.

Techniques from surgery theory can be used to give a conceptual description of $Aut(M)$, under fairly general assumptions, in terms of bordism groups and the base-point preserving automorphisms $Aut(B)$ of the 2-type of M. The answer is expressed in a commutative braid diagram.

If M is a spin manifold and $\pi_1(M, x_0)$ is a finite group of odd order, the braid diagram can be computed to give a formula:

$$Aut(M) = H_2(M; \mathbb{Z}/2) \rtimes Isom(\pi_1(M), \pi_2(M), k_M, s_M)$$

as a semi-direct product of $H_2(M; \mathbb{Z}/2)$, and the isometries of the intersection form s_M on $\pi_2(M)$ respecting the action of $\pi_1(M, x_0)$ and the k-invariant k_M. For simply-connected 4-manifolds the first complete proof of this formula was obtained by Cochran-Habegger using homotopy-theoretic methods. The non-simply-connected results are new. Our approach is based on studying the group of h-cobordisms between M and itself, and the results for this part also apply to smooth 4-manifolds.

The Scott-Wiegold conjecture and applications to Dehn surgery

JAMES HOWIE

Theorem. *Let p, q, r be integers greater than 1, and $W = W(x, y, z)$ an element of the free group $\langle x, y, z\rangle$. Then the group*

$$G = \langle x, y, z | x^p = y^q = z^r = W(x, y, z) = 1\rangle$$

is nontrivial.

This answers in the affirmative a question of Jim Wiegold (Kourovka Notebook, Problem 5.53 (1976), attributed to Peter Scott). The problem had also been raised by Fintushel and Stern (1980) and Gordon (1983) in connection with Dehn Surgery on knots in S^3. In this connection the following conjecture is of interest.

Conjecture (Cabling conjecture, Gonzalez-Acuña and Short, 1986)**.** *If Dehn surgery on a knot in S^3 yields a non-prime 3-manifold, then the knot is a cable knot and the surgery slope is that of the cabling annulus.*

In the special situation described in the conjecture, it is well known that there are precisely two prime factors, (at least) one of which is a lens space. So the conjectured upper bound on the number of prime factors is 2.

An easy consequence of the theorem is the following:

Corollary. *Let $M = M_1 \# \cdots \# M_n$ be a 3-manifold obtained by Dehn surgery on a knot in S^3. Then at least $n-2$ of the M_i are homology spheres.*

Combining this with a result of Valdez Sánchez and Sayari that says at least $n-1$ of the M_i are lens spaces, we get an upper bound of 3 prime factors. More precisely:

Corollary. *Let M be as above. Then $n \leq 3$. Moreover, if $n = 3$ then, up to renumbering, M_1 is a homology sphere, M_2 and M_3 are lens spaces, and $|\pi_1 M_2|$ is coprime to $|\pi_1 M_3|$.*

Sketch proof of Theorem. It is easy to reduce to the case where p, q, r are distinct primes, and the exponent-sums of x, y, z in W are all equal to 1.

In this case, we prove nontriviality of G by producing a representation

$$\rho : G \to SO(3) = SU(2)/\{\pm I\} = S^3/\{\pm 1\}$$

such that the images of x, y, z have orders p, q, r respectively.

Assuming there is no such representation, we can construct a smooth, $SO(3)$-equivariant map

$$\psi : S^2 \times S^2 \times S^2 \to S^2$$

as follows. Allow each of the variables X, Y, Z to vary inside an appropriate conjugacy class ($\cong S^2$) in S^3. Define $\psi(X, Y, Z)$ to be the S^2-coordinate of

$$W(X, Y, Z) \in S^3 \setminus \{\pm 1\} \cong S^2 \times \mathbb{R}.$$

Alternatively, we may think of ψ as a 2-parameter family of smooth maps

$$\psi_{X,Y} : S^2 \to S^2,$$

with parameters $X, Y \in S^2$.

The degrees of the maps in this family are clearly independent of the choice of parameters X, Y. If $X = \pm Y$, then the map $\psi_{X,Y}$ is S^1-equivariant with respect to the rotation action of S^1 on S^2 with fixed points $\pm X$. The degree of an S^1-equivariant map $S^2 \to S^2$ is determined by the images of the two fixed-points, so can be easily calculated.

A final observation is that we can set everything up in such a way that the degrees of the two equivariant maps corresponding to $X = \pm Y$ are -1 and $+1$ respectively, contradicting invariance of degree. □

Heegaard splittings, the virtually Haken conjecture and property (τ)

MARC LACKENBY

This talk explained the interaction of the seemingly unrelated areas appearing in the title. Its focus was the following three important conjectures in 3-manifold theory: the virtually Haken conjecture, the positive virtual b_1 conjecture and the virtually fibred conjecture. The main result was:

Theorem. *Let M be a compact orientable irreducible 3-manifold, with boundary a (possibly empty) collection of tori. Suppose that*
(1) $\pi_1(M)$ fails to have Property (τ), and

(2) M has non-zero strong Heegaard gradient.
Then M is virtually Haken.

Property (τ) is a concept from geometric group theory, introduced by Lubotzky and Zimmer. It has many equivalent definitions, in terms of differential geometry, graph theory and representation theory. Lubotzky and Sarnak proposed the following:

Conjecture. *The fundamental group of a closed hyperbolic 3-manifold fails to have Property (τ).*

The *strong Heegaard gradient* is

$$\lim_i \inf\{\chi_-^{sh}(M_i)/d_i : M_i \to M \text{ is a degree } d_i \text{ cover}\}$$

where

$$\chi_-^{sh}(M_i) = \min\{-\chi(F) : F \text{ is a strongly irreducible Heegaard surface for } M_i\}.$$

This is at least the *Heegaard gradient* which is

$$\inf_i \{\chi_-^h(M_i)/d_i : M_i \to M \text{ is a degree } d_i \text{ cover}\}$$

where

$$\chi_-^h(M_i) = \min\{-\chi(F) : F \text{ is a Heegaard surface for } M_i\}.$$

Conjecture. *The strong Heegaard gradient of a closed hyperbolic 3-manifold is non-zero.*

Conjecture. *The Heegaard gradient of a closed hyperbolic 3-manifold is zero if and only if it is virtually fibred.*

Thus, either of the above conjectures, together with the Lubotzky-Sarnak conjecture, would imply the virtually Haken conjecture for hyperbolic 3-manifolds.

I then gave a result, establishing geometric conditions on a finitely presented group that are equivalent to having positive virtual b_1. These are very similar to the equivalent definitions of 'not (τ)'. This has applications in 3-manifold theory, including:

Theorem. *Let M be a closed orientable 3-manifold with a negatively curved Riemannian metric. Suppose that, for some collection of finite regular covers $\{M_i \to M\}$ with degree d_i, $\inf_i \chi_-^h(M_i)/\sqrt{d_i} = 0$. Then M satisfies the positive virtual b_1 conjecture.*

The Morava K-theory of spaces related to BO

Gerd Laures

(joint work with Nitu Kitchloo and W. Stephen Wilson)

We consider the Morava K-homology Hopf algebras of the connective covers $BO\langle k\rangle$ of BO. For $k = 1$ or 2 they have been computed in the 80's by Wilson. The cases $BSpin$ and $BO\langle 8\rangle$ have been described for Morava $K(1)$ and $K(2)$ by Ando, Hopkins and Strickland in terms of the universal rings of real k-structures. A result of Kitchloo and myself shows that the concept of real k-structures does not apply to describe all Morava $K(n)$ of connective covers.

Especially the computation of the higher Morava K-homologies of $BO\langle 8\rangle$ turns out to be a hard problem. In our work we look at all maps and spaces of all connective covers of the infinite loop spaces in the spectrum of KO. Our main tool is the bar spectral sequence to compute these maps. It turns out that once we look at a connective cover E of a sufficiently large connective cover B then the map into the Eilenberg-MacLane spaces become trivial in homology and we obtain a short exact sequence of Hopf algebras ($K = K(n)$)

$$K_* \longrightarrow K_*F \longrightarrow K_*E \longrightarrow K_*B \longrightarrow K_*.$$

Since the fibre F has a finite Postnikov system its homology is known from results of Ravenel and Wilson. This result leaves us with only a few spaces to compute. We use the fibration of infinite loop spaces associated to the fibration

$$\Sigma bo \xrightarrow{\eta} bo \longrightarrow bu$$

to get our hands on the spectral sequences. It turns out that we had to compute almost all spaces in order to get to the desired space $BO\langle 8\rangle$. Our main theorem is ($p = 2$)

Theorem. *The maps*

$$K(\mathbb{F}_2, 2) \longrightarrow K(\mathbb{Z}, 3) \longrightarrow BO\langle 8\rangle \longrightarrow BSpin \longrightarrow K(\mathbb{Z}, 4) \xrightarrow{2} K(\mathbb{Z}, 4)$$

give rise to an exact sequence of Hopf algebras in Morava K-homology. As an algebra, $K_*BO\langle 8\rangle$ is polynomial tensor with $K_*K(\mathbb{Z}, 3)$ and concentrated in even degrees.

Techniques have been developed to take information of the Morava K-theory of a space and use it to compute its BP cohomology. This done in the last part of the talk.

On the geometrization of 3-dimensional orbifolds

Bernhard Leeb

(joint work with Michel Boileau and Joan Porti)

We discuss joint work with Boileau and Porti regarding the geometrization of compact connected orientable 3-orbifolds. This is an aspect of Thurston's Geometrization Program, and geometrization here refers to locally homogeneous Riemannian metrics.

Our main result is that a small orbifold with non-empty singular locus is geometric. Together with known results this implies that irreducible atoroidal orbifolds with non-empty singular locus are geometric, i.e. the so-called Orbifold Theorem. Thurston had formulated it in 1982 and outlined a strategy for its proof which had remained basic for all later work on the topic.

Applications include (i) the Generalized Smith Conjecture, that non-free actions of finite groups on the 3-sphere by orientation preserving diffeomorphisms are conjugate to orthogonal actions, and (ii) the geometrization of such actions on hyperbolic manifolds.

Our argument involves a study of representation varieties resp. deformation spaces of cone structures and methods from the geometry of metric spaces with curvature bounded below. A basis for the whole approach is the Hyperbolization Theorem for Haken manifolds. The role of the Haken condition is played in the orbifold case by the non-emptiness of the singular locus. One starts by putting a complete hyperbolic structure on the smooth part of the orbifold and deforms it to a geometric structure on the orbifold.

p-local groups: the homotopy of p-fusion systems

RAN LEVI

(joint work with Carles Broto and Bob Oliver)

In this talk we give a preliminary report on a joint project with Broto and Oliver on the theory of p-local compact groups. These are algebraically defined objects which admit a "classifying space". The family of spaces thus arising are expected to have homotopy theoretic properties which resemble those shared by p-completed classifying spaces of compact Lie groups and p-compact groups. The motivation for the definition comes from the finite analogue – p-local finite groups. The main change is replacing finite p-groups in the definition of the finite case by discrete p-toral groups, i.e., extension of a discrete p-torus by a finite p-group. As one might expect though, giving up finiteness introduces an array of problems which we will address. The main result we will present is that all compact Lie groups as well as all p-compact groups give rise to p-local compact groups whose classifying spaces coincide up to homotopy with the p-completed classifying space of the original object. We will also give an example of a family of exotic 2-local finite groups arising from the family of fusion systems introduced by Benson. These 2-local groups can be organized in a sequence whose "colimit" is a 2-local compact group which we identify as the Dwyer-Wilkerson space BDI(4) – the space realizing the rank 4 Dickson algebra at the prime 2.

A finiteness result for Heegaard splittings of 3-manifolds

MARTIN LUSTIG

(joint work with Yoav Moriah)

We show that for a given 3-manifold and a given Heegaard splitting there are finitely many preferred decomposing systems of $3g - 3$ disjoint essential disks. These are characterized by a combinatorial criterion which is a slight strengthening of Casson-Gordon's rectangle condition. This is in contrast to the fact that in general there can exist infinitely many such systems of disks which satisfy just the Casson-Gordon rectangle condition.

Recent progress on the Bass conjecture

GUIDO MISLIN

(joint work with Jon Berrick and Indira Chatterji)

For each finitely generated projective (left) $\mathbb{Z}G$-module P, there exists an idempotent matrix $(m_{ij}) = M \in M_n(\mathbb{Z}G)$ such that P is isomorphic to the image under right multiplication $\mathbb{Z}G^n \to \mathbb{Z}G^n$ by M. Writing $[\mathbb{Z}G, \mathbb{Z}G]$ for the additive subgroup of $\mathbb{Z}G$ generated by the elements $gh - hg$ $(g, h \in G)$, we identify $\mathbb{Z}G/[\mathbb{Z}G, \mathbb{Z}G]$ with $\bigoplus_{[s]\in[G]} \mathbb{Z} \cdot [s]$, where $[G]$ is the set of conjugacy classes of elements of G. The *Hattori-Stallings rank* r_P is then defined by

$$r_P = \sum_{i=1}^{n} m_{ii} + [\mathbb{Z}G, \mathbb{Z}G] = \sum_{s\in[G]} r_P(s)[s] \in \bigoplus_{[s]\in[G]} \mathbb{Z} \cdot [s].$$

In 1976, H. Bass made the following conjecture.

Conjecture (Classical Bass Conjecture)**.** *For any finitely generated projective $\mathbb{Z}G$-module P, the values $r_P(s) \in \mathbb{Z}$ of the Hattori-Stallings rank r_P are zero for $s \in G \setminus \{1\}$.*

We show that groups that satisfy the *Bost conjecture* satisfy the classical Bass conjecture too, and indeed a more general versions thereof that we call the ℓ^1 Bass conjecture. The proof is obtained via a chain of deductions, describing a tour from geometric functional analysis, through operator algebra K-theory, algebraic topology and combinatorial group theory (we refer to Lafforgue's work for the definition of the Bost assembly map).

Theorem. *Let G be a countable discrete group for which the Bost assembly map*

$$\beta_*^G : K_*^G(\underline{E}G) \to K_*^{\mathrm{top}}(\ell^1(G))$$

is rationally an epimorphism in degree 0. Then the ℓ^1 Bass conjecture holds for G.

Known information on the Bost conjecture implies for instance the following.

Corollary. *Amenable groups satisfy the classical Bass conjecture.*

The proof of the theorem involves a natural embedding of G in an acyclic group $A(G)$ that is injective on conjugacy classes, with the centralizer of any finitely generated abelian subgroup of $A(G)$ acyclic as well. This allows us to control the image of the universal trace

$$T^1 : K_0(\ell^1(G)) \to HH_0(\ell^1(G)).$$

Some applications of noncommutative localization in topology

Andrew Ranicki

The (Cohn) localization of a noncommutative ring A inverting a set Σ of morphisms of f.g. projective A-modules is a ring $\Sigma^{-1}A$ with a universally Σ-inverting morphism $A \to \Sigma^{-1}A$. Amalgamated free products and HNN extensions are special cases of noncommutative localization. The applications of noncommutative localization to topology involve an analogy between the geometric properties of fundamental domains of covers and the algebraic properties of modules and quadratic forms over triangular matrix rings.

The 1980's results of Schofield and Vogel on the algebraic K- and L-theory of $\Sigma^{-1}A$ have been recently generalized by Neeman and myself, to obtain localization exact sequences of the type

$$\cdots \to K_i(A) \to K_i(\Sigma^{-1}A) \to K_i(A,\Sigma) \to K_{i-1}(A) \to \dots\ ,$$

$$\cdots \to L_i(A) \to L_i(\Sigma^{-1}A) \to L_i(A,\Sigma) \to L_{i-1}(A) \to \dots$$

(`http://arXiv.org/abs/math.RA/0109118`) in the case of injective $A \to \Sigma^{-1}A$ which are 'stably flat' :

$$\mathrm{Tor}_*^A(\Sigma^{-1}A, \Sigma^{-1}A) \ = \ 0\ .$$

The relative groups are the algebraic K- and L-groups of homological dimension 1 Σ-torsion A-modules. The localization exact sequences give new proofs of the 1970's results of Waldhausen and Cappell on the algebraic K- and L-groups of generalized free products, with applications to codimension 1 splitting properties of manifolds.

Algebraic K-theory of group rings and topological cyclic homology

Holger Reich

(joint work with Wolfgang Lück, John Rognes and Marco Varisco)

We use topological cyclic homology and the cyclotomic trace to detect elements in $K_n(\mathbb{Z}[G]) \otimes_{\mathbb{Z}} \mathbb{Q}$, the rationalized higher algebraic K-theory groups of integral group rings. Modulo a certain conjecture in number theory and under mild homological finiteness conditions on the group G, we prove that the algebraic K-theory assembly map for the family of finite subgroups is rationally injective. This vastly generalizes a result of Bökstedt, Hsiang and Madsen, and leads to a concrete description of a large direct summand inside $K_n(\mathbb{Z}[G]) \otimes_{\mathbb{Z}} \mathbb{Q}$ in terms of group homology. Along the way we also prove integral splitting results for THH and TC assembly maps with arbitrary coefficients.

A Quillen adjunction close to differential graded commutative algebras

Birgit Richter

Starting from the well-known Dold-Kan correspondence between simplicial modules and chain complexes, we show that the inverse of the normalization functor, D, from chain complexes to simplicial modules sends differential graded commutative algebras to simplicial E_∞-algebras. We prove this by an explicit construction of an E_∞-operad, which acts on $D(X)$ for all differential graded commutative algebras X. This operad is a slight generalization of an endomorphism operad and we prove that this operad is acyclic.

Differential graded commutative algebras do not possess any good homotopy category in characteristic different from zero. So the natural idea is to pass to a homotopy invariant analog of commutative algebras, namely differential graded E_∞-algebras.

With the help of the concept of parametrized operads we are able to show that the functor D maps differential graded E_∞-algebras to simplicial E_∞-algebras. Here the E_∞-operad on the simplicial side is constructed as an amalgamation of the E_∞-operad for the functor D and the operad which acts on the differential graded input.

The functor which assigns an Eilenberg-MacLane spectrum to a simplicial module prolongs the functor D and allows us to pass from differential graded commutative algebras (resp. E_∞-algebras) to the category of E_∞ ring spectra.

It is easy to see that the functor D possesses a left adjoint on the level of E_∞-algebras and this adjoint functor pair is in fact a Quillen adjunction, i.e., it induces an adjunction on the corresponding homotopy categories. These homotopy categories can be obtained with the help of Markus Spitzweck's concept of semi model categories.

L^2-invariants and geometric group theory

Roman Sauer

By analytic means D. Gaboriau extended the notion of L^2-Betti numbers of groups to discrete measured groupoids. Each measure preserving action of a countable group on a finite measure space gives rise to such a groupoid.

We provide an alternative definition of L^2-Betti numbers of discrete measured groupoids by means of homological algebra which still has the good properties of Gaboriau's definition.

Using that, we can give an algebraic proof of Gaboriau's theorem which says that L^2-Betti numbers of measure equivalent groups coincide up to a constant factor. Here measure equivalence is a measure-theoretic analog of quasi-isometry, and it can be expressed in terms of isomorphisms of certain groupoids. An important example of measure equivalent groups is given by discrete subgroups with finite co-volumes in a locally compact group.

The methods in the new proof of Gaboriau's theorem can also be applied to Novikov-Shubin invariants. They turn out not to be invariant under measure equivalence but we can show the following theorem.

Theorem. *The Novikov-Shubin invariants of quasi-isometric amenable groups which admit a finite model for the classifying space of proper actions coincide.*

Galois symmetries of 4-manifolds and equivariant stable homotopy theory

Markus Szymik

An equivariant extension of the Bauer-Furuta invariants of closed smooth 4-manifolds X has been discussed. Whenever a compact Lie group G acts on X preserving a complex spin structure σ_X on X, the automorphism group of (X, σ_X) contains an extension $\mathbb{G}$ of G by the circle group $\mathbb{T}$. The equivariant invariant is the $\mathbb{G}$-equivariant stable homotopy class of the Seiberg-Witten monopole map associated to (X, σ_X).

If G is finite and acts freely on X, the invariants of the quotient 4-manifolds X/H for subgroups $H \leqslant G$ are contained in the equivariant invariants of X. A comparison map sends the latter to all the former. This map is an isomorphism away from the order of G. Computations show that it is neither surjective nor injective in general. On the one hand, this implies relations among the non-equivariant invariants. On the other hand, it shows the potential of the equivariant invariants to contain more information.

Edited by Marco Varisco

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