

Report No. 53/2002

## Combinatorial Optimization

November 24th – November 30th, 2002

This conference was organized by Rolf H. Möhring (Berlin), Thomas M. Liebling (Lausanne), and Uwe T. Zimmermann (Braunschweig), and was the latest in a series of Oberwolfach meetings on *combinatorial optimization*. The 43 participants came from Austria, Canada, France, Germany, Hungary, Italy, Japan, Switzerland, and the United States. There were 32 presentations in total, and each day featured two 50 minute survey talks. The attendees particularly appreciated that these were given by younger specialists in their fields. Topics covered a broad range of the discipline, like

- approximation algorithms
- online optimization
- scheduling
- graph and matroid theory
- submodular functions
- non-linear, stochastic, and integer programming
- various application areas.

On Thursday evening an *open problem session* took place, and one of the eleven problems was declared as solved the very same evening. Selected open problems are listed in this report. During the whole week participants seized the opportunity to informally discuss problems and jointly work together. One talk of the last day was a result of such work.

The organizers and all participants would like to thank the *Mathematisches Forschungsinstitut Oberwolfach* for its hospitality and the outstandingly stimulating atmosphere.

# Abstracts

Monday, November 25th, 2002

## Approximation Algorithms for the $k$ -Median Problem and Related Facility Location Problems

DAVID SHMOYS

In the  $k$ -median problem, we are given  $n$  points in a metric space (given by the distance between each pair of points), and we wish to select  $k$  of them (the "medians") so as to minimize the total distance of each point to its nearest selected median. An  $r$ -approximation algorithm for an optimization problem is a polynomial-time algorithm guaranteed to find a feasible solution of objective function value within a factor of  $r$  of optimal. We survey recent results on approximation algorithms for the  $k$ -median problem. We present results that highlight three approaches: LP rounding, primal-dual algorithms, and local search. In the first, we give an integer programming formulation and show that the optimal solution to the linear programming relaxation can be rounded to an integer one without increasing its cost by more than a particular constant factor. In the second, the linear program is used only implicitly to simultaneously construct a feasible integer primal solution and a feasible dual solution of objective function values that are within a constant factor of each other. In the last, we consider the paradigm in which there is a notion of a neighbouring feasible solution, and we iteratively check if any neighbour has better objective function value, until we converge at a so-called local optimum. The first constant performance guarantee for the  $k$ -median problem was obtained by LP rounding by Charikar, Guha, Tardos, and Shmoys. We present the primal-dual approach of Jain and Vazirani, in a variant derived by Mettu and Plaxton. Finally, we give a result of Arya, Garg, Khandekar, Meyerson, Munagala, and Pandit, that the process of swapping one median into the solution simultaneously with deleting one converges to a local optimum guaranteed to be within a factor of 5 of optimal.

## Cyclical Scheduling, Circular Ones Matrices, and the Minimax Problem

DORIT HOCHBAUM

(joint work with Paul Tucker)

The minimax problem is of the structure

$$\begin{aligned} \min \quad & \sum_{j=1}^n c_j x_j \\ & \max_{j=1 \dots n} a_{ij} x_j \geq b_i \\ & x_j \geq 0. \end{aligned}$$

Although the problem is stated in continuous variables, we demonstrate it is in fact a discrete problem equivalent to the set cover problem. Therefore it is  $\mathcal{NP}$ -hard.

When the matrix  $(a_{ij})$  is *bitonic* in columns (meaning one peak and one valley at most), the equivalent set cover is defined on a circular ones matrix. We provide a polynomial time algorithm for the problem, and discuss the more general multicover problem on circular ones matrices. When the matrix  $(a_{ij})$  is *bitonic* in columns (meaning one peak and one

valley at most), the equivalent set cover is defined on a circular ones matrix. We provide a polynomial time algorithm for the problem, and discuss the more general multicover problem on circular ones matrices. This problem models the important cyclical scheduling problem. We show it is equivalent to a network flow with side constraint problem. The side constraint is to require a subset of arcs to include at least a certain amount of flow. The complexity status of this problem is open. (In the bipartite case it is solved with a randomized polynomial algorithm of Mulmuley, Vazirani, and Vazirani.)

## Combinatorial Online Optimization in Practice: Models, Obstacles, Prospects

JÖRG RAMBAU

More and more optimization problems have to be solved online. That means, the input of the optimization problem is not given in advance but is revealed over time. An online algorithm has to make irrevocable decisions with incomplete knowledge about the future, and in practice the time for computations means a delay.

In this survey talk, we present three examples from practical projects: the online control of a cargo elevator system in a large distribution center, the dynamic configuration of optical telecommunication networks, and the online dispatching of service units for the ADAC (the german automobile association). Three important methods to evaluate the performance of an online algorithm are reviewed: dynamic systems (distributional analysis), competitive analysis (worst-case analysis), and queuing systems (asymptotic analysis). We show that all of them have their short-comings and can lead to computational problems or practically undesirable decisions. As an example for a new type of performance guarantee that overcomes some of the difficulties, we present the analysis under reasonable load.

In any case, additional simulation experiments on real data and an a-posteriori analysis are still required to convince people from practice of the value of an online algorithm.

## Cyclic Timetabling and Cyle Bases of Graphs

CHRISTIAN LIEBCHEN

(joint work with Leon Peeters)

We consider the problem of constructing cyclic timetables for public transportation companies. This is usually modelled by the Periodic Event Scheduling Problem (PESP), in that we are given a period time  $T$  for the traffic network, and where we have to find a node potential  $\pi$  that periodically fulfills constraints of the form  $l_{ij} \leq \pi_j - \pi_i + pT \leq u_{ij}$ , where  $p$  is required to be integer.

The PESP is  $\mathcal{NP}$ -complete. Moreover, in practice it is already very difficult to optimize a linear objective over only a few hundreds of constraints. Hence, we are seeking for an advantageous formulation of the problem.

To this aim, we generalize the usual notion of tensions to periodic tensions. Further, we conclude that an arc mapping is precisely a periodic tension, if a certain cycle periodicity property holds for every cycle. But the latter already is implicitly true by ensuring the cycle periodicity property only for the  $m - n + 1$  elements of an integral cycle basis, which means that we can formulate the problem with  $n - 1$  integer variables less than in the PESP formulation.

The cycle inequalities introduced by Odijk (1996) yield box constraints for the integer variables. We get an approximation for the size of a cycle basis by weighting cycles only by

a linear cost function over their arcs. Whereas recognition of integral cycle basis is trivial by thinking of them in terms of integral lattices, constructing a minimal integral cycle basis does not seem to be easy. Nor does constructing a minimal (generalized) fundamental cycle basis, in the sense of Whitney (1935). Restricting to strictly fundamental basis, which stem from spanning trees, as proposed by Nachtigall (1998), does not help either, because minimization is an  $\mathcal{NP}$ -hard task there.

## The Bundle Method in Combinatorial Optimization

FRANZ RENDL

(joint work with I. Fischer, G. Gruber, R. Sotirov, and A. Wiegele)

The Bundle method is a standard tool from nonsmooth optimization to minimize convex functions. In the context of combinatorial optimization it can be used to get approximate solutions to optimization problems with a huge number of linear (in)equalities. These are treated only indirectly through Lagrangian relaxation.

Computational results applied to semidefinite relaxations of Max-Cut in combination with the triangle inequalities are discussed. Finally, lower bounds for the Quadratic Assignment Problem, based on semidefinite programming, in combination with combinatorial (in)equalities are presented, which have the potential to improve current Branch and Bound methods to solve this problem to optimality.

## Chromatic Properties of Balanced $0, \pm 1$ Matrices

DOMINIQUE DE WERRA

In order to generalize bipartite graphs,  $0, 1$ -balanced matrices have been introduced by Berge, who characterized them by a simple bicolouring property.

This class has been extended to  $0, \pm 1$ -matrices; Conforti and Cornuejols gave a corresponding characterization in terms of bicolouring of the columns of the matrix.

In addition, Berge showed that the balanced  $0, 1$ -matrices have for any  $k \geq 2$  a so-called *good*  $k$ -colouring (each row contains the largest possible number of colours, i.e.  $\min\{k, \sum_j a_{ij}\}$ ).

The concept of good  $k$ -colouring can be extended in several ways to  $0, \pm 1$ -matrices. We give some extensions and show that such generalized  $k$ -colourings exist in balanced  $0, \pm 1$ -matrices for any  $k \geq 2$ .

These variations suggest to explore some extensions of balanced matrices which may admit some relaxations of the concept of good  $k$ -colourings.

## Symmetry and Integer Linear Programs

FRANÇOIS MARGOT

Components of a branch-and-cut algorithm for solving integer linear programs with a large symmetry group are presented. The pruning algorithm, cut generation procedures, and algorithms for setting variables are based on the symmetry group of the problem. Pruning and orbit computations are performed by backtracking procedures using a Schreier-Sims table for representing the symmetry group. Applications to hard set covering problems, generation of covering designs and error correcting codes are given.

## Difficult Problems in Propositional Logic

KLAUS TRUEMPER

The extraction of logic from data, as part of data mining approaches, is becoming increasingly important. We describe a method that reliably and without tuning or parameter selection produces the desired logic functions plus probability distributions of their accuracy.

When learned logic functions replace normative logic formulations, certain problems that for normative formulations reside at the second level of the polynomial hierarchy become the logic minimization problem MINSAT. This fact could be viewed as an explanation for the curious fact that problems at the second level of the polynomial hierarchy are very hard for computers, yet are solved by the human brain with ease. Indeed, if one postulates that the human brains learns logic from data, then problems that seemingly reside at the second level of the polynomial hierarchy become MINSAT cases, which the brain is quite good at.

We discuss some applications in intelligent systems such as medical diagnosis, natural language processing, and computer vision.

Tuesday, November 26th, 2002

## Linear Programming over Symmetric Cones

CHRISTOPH HELMBERG

With the development of interior point methods the task of optimizing a linear cost function over the intersection of an affine subspace with a symmetric cone (a convex cone that is self-dual and homogeneous) came into the realm of tractability. This includes linear programming (LP), second order cone programming (SOCP), and semidefinite Programming (SDP). We present a survey on duality and complexity results, discuss useful modeling concepts in formulating optimization problems as linear programs over symmetric cones, review some major applications (robust linear programming, chance constraints, robust control, global optimization over polynomials, sum of squares decomposition) and algorithms (primal-dual interior point methods and large scale approaches via nonconvex quadratic programming as well as bundle methods employing specialized cutting models).

## Local Branching

MATTEO FISCHETTI

(joint work with Andrea Lodi)

The availability of effective exact or heuristic solution methods for general Mixed-Integer Programs (MIPs) is of paramount importance for practical applications. In the talk we investigate the use of a generic MIP solver as a black-box “tactical” tool to explore effectively suitable solution subspaces defined and controlled at a “strategic” level by a simple external branching framework. The procedure is in the spirit of well-known local search metaheuristics, but the neighborhoods are obtained through the introduction in the MIP model of completely general linear inequalities called *local branching* cuts.

The new solution strategy is exact in nature, though it is designed to improve the heuristic behaviour of the MIP solver at hand. It alternates high-level strategic branchings to define the solution neighborhoods, and low-level tactical branchings to explore them. The result is a completely general scheme aimed at favoring early updatings of the incumbent solution, hence producing high-quality solutions at early stages of the computation.

The method is analyzed computationally on a large class of very difficult MIP problems by using the state-of-the-art commercial software ILOG-Cplex 7.0 as the black-box tactical MIP solver. For these instances, most of which cannot be solved to proven optimality in a reasonable time, the new method exhibits consistently an improved heuristic performance: in 20 out of 24 cases, the MIP solver produced significantly better incumbent solutions when driven by the local branching paradigm.

# VLSI Design Today: Minimizing Cycle Time and Power Consumption

JENS VYGEN

Maximizing the clock frequencies, or equivalently minimizing cycle times/meeting timing constraints, has become the most important objective in VLSI design. Power consumption is also becoming more and more important and may become the dominating objective soon.

However, the timing analysis methodology currently used in industry is not accurate enough for reasonable power minimization under tight timing constraints. We point out severe weaknesses of the classical static timing analysis model and propose an extension that does not have any of these drawbacks. For the first time, meaningful slacks can be computed efficiently, and this even works for non-acyclic timing graphs.

In addition to presenting this new timing model, we survey several applications and other recent theoretical advances that lead to better algorithms for faster chips consuming less power. These are the basis of the so-called Bonn tools, which are being used for the design of many of the most complex chips.

## A Duality Result for 0/1 Programs

ROBERT WEISMANTEL

(joint work with Dimitris Bertsimas)

Motivated by results from algebraic geometry, we develop a Farkas type Lemma for binary programs. We also give elementary proofs that every binary programming problem has an explicit dual linear program with exponentially many variables and constraints for which weak duality, strong duality and complementary slackness hold. This dual coincides with the dual linear program that one obtains from lifting the original program into exponential space by multiplication by products of variables as suggest by Balas, Serali and Adams, Lovasz and Schrijver, Balas, Ceria, Cornuejols. We prove that under modest assumptions the optimal solutions of a partial dual – a subset of the dual variables is included – define a nested family of tight constraints. This result gives rise to a couple of algorithmic ideas.

## Minimum Turn Cycle Covers in Grid Graphs

MARCO E. LÜBBECKE

(joint work with Gunnar W. Klau)

A grid graph  $G$  is a node induced subgraph of the integer lattice. We seek a collection of cycles which cover each of  $G$ 's vertices at least once. Each turn of a cycle by 90 degree counts as one turn. The objective is to minimize the total number of turns. The complexity status of this minimum turn cycle cover problem is open. We present a natural integer programming formulation. The linear programming relaxation gives empirically excellent lower bounds on the integer optimum. We outline a possible approach to proving that it in fact gives raise to a 2-approximation. For the case that every vertex of  $G$  has to be covered exactly once we prove that the optimal face of the polytope associated with the linear programming relaxation is integer. Thus, this special case is solvable in polynomial time.

## Inverse 1-Median Problems

RAINER E. BURKARD

(joint work with C. Pleschiutchnig and J. Zhang)

The inverse 1-median problem can be stated as follows: Let  $n$  points with weights  $w_i > 0$  be given. Let  $d(i, j)$  denote the distance between the points  $i$  and  $j$ . The problem consists in adjusting the weights at minimum cost such that a prespecified point  $s$  becomes a 1-median. If the points correspond to vertices of a graph  $G$  and  $d(i, j)$  is the shortest path length between vertices  $i$  and  $j$ , then we speak of a discrete problem. It is easy to see that a discrete inverse 1-median problem can be formulated as a linear program. Therefore it is solvable in linear time. If  $G$  is a tree we know according to a poem of Hua et al. that  $s$  is optimum, if all subtrees rooted in  $s$  have a weight less or equal one half of the total weight. This property can be used to design a greedy algorithm for the inverse 1-median problem on a tree. If the points lie in the plane and the distances are measured in  $l_1$ -norm, then a similar optimality condition holds as in a tree. Therefore also in this case a greedy-type algorithm can be designed which takes care that the weights in both directions  $x$  and  $y$  are suitably balanced.

## Polynomials and Polyhedra

MARTIN GRÖTSCHEL

A beautiful result of Bröcker and Scheiderer on the stability index of basic closed semi-algebraic sets implies, as a very special case, that every  $d$ -dimensional polyhedron admits a representation as the set of solutions of at most  $d(d+1)/2$  polynomial inequalities. Even in this polyhedral case, however, no constructive proof is known, even if the quadratic upper bound is replaced by any bound depending only on the dimension. In a joint paper with Martin Henk, we give, for simple polytopes, an explicit construction of polynomials describing such a polytope. The number of used polynomials is exponential in the dimension, but in the 2- and 3-dimensional case we get the expected number  $d(d+1)/2$ . The talk relates these results to polyhedral combinatorics and speculates about possible uses in combinatorial optimization.

## Stable Sets in Bipartite Graphs

ULRICH FAIGLE

(joint work with Gereon Frahling)

Computing a maximum weighted stable set in a bipartite graph is considered well-solved and usually approached with preflow-push, Ford-Fulkerson or network simplex algorithms. However, a combinatorial algorithm for this problem can be designed that is not based on flows but on relaxations to spanning subtrees and iterative removal of infeasibilities in the tentative solutions. Numerical tests suggest that this algorithm outperforms flow based algorithms in practice by a considerable margin especially in the the case of dense graphs.

Wednesday, November 27th, 2002

## Improved Approximation Algorithms for the Minimum-Space Advertisement Scheduling Problem

MICHEL X. GOEMANS

(joint work with Brian Dean)

We study a scheduling problem involving the placement of advertisements on a webpage over time. One variant of the problem can be formulated as an extension of  $P||C_{max}$  in which every job  $j$  has a multiplicity  $c_j$  and we need to assign  $c_j$  copies of each job on different machines so as to minimize the maximum total processing time assigned to any machine. Our main result is a proof that the natural generalization of Graham's algorithm also yields a  $4/3$ -approximation to this minimum-space advertisement scheduling problem. Previously, this algorithm was only known to give an approximation ratio of 2, and the best known approximation ratio for any algorithm for the minimum-space ad scheduling problem was  $3/2$ . Graham's proof does not extend to our situation, and our analysis uses a novel lower bounding approach for the problem which can be formulated as a linear program. We also provide a pseudo-polynomial algorithm (its running time is polynomial in the number of jobs and the number of machines) with approximation ratio of  $(1 + \epsilon)$  for any constant  $\epsilon > 0$ .

## A Set-Covering Based Heuristic Approach for Bin-Packing Problems

PAOLO TOTH

(joint work with Michele Monaci)

Several combinatorial optimization problems can be formulated as large size Set-Covering Problems. This is the case of many problems in the Cutting & Packing area, as well as of other well-known and relevant problems in combinatorial optimization, such as the Vehicle Routing Problem, the Graph Colouring Problem or the Crew Scheduling Problem.

In this work, we use the *Set-Covering formulation* to obtain a general heuristic algorithm for this type of problems, and describe our implementation of the algorithm for solving two variants of the well-known (One-Dimensional) Bin Packing Problem: the *Two-Constraint Bin Packing Problem (2CBP)*, in which we are required to pack a set of items (each having a positive weight and a positive volume) into the minimum number of identical bins having a given weight and volume capacity, and the basic version of the *Two-Dimensional Bin Packing Problem (2DBP)*, in which we have to pack a given set of rectangular items (each having a positive width and a positive height) into the minimum number of identical rectangular bins having a given width and height. For the considered problems, each *column* of the associated Set-Covering formulation corresponds to a feasible (and inclusion maximal) item set (i.e., to a feasible "filling" of a bin).

In our approach, both the "column-generation" and the "column-optimization" phases are heuristically performed. In particular, in the first phase, we do not generate the entire set of columns, but only a small subset of it, by using greedy procedures and fast constructive heuristic algorithms from the literature. In the second phase, we heuristically solve the associated Set-Covering instance by means of a Lagrangian-based heuristic algorithm.

Extensive computational results on a large set of instances from the literature show that, for the two considered problems, this approach is competitive, with respect to both the quality of the solution and the computing time, with the best heuristic and metaheuristic algorithms proposed so far.

## Pursuit-Evasion and Buffer Minimization in Online Scheduling

GÜNTER ROTE

This is a case study on the interplay between combinatorial optimization and geometry, and on the application of geometric ideas to combinatorial optimization problems.

We consider a game between two persons where one person (the “sheriff”) tries to catch the other person (the “thief”), but the sheriff only knows an approximation of the true position of the thief. The two players have identical constraints on their speed. It turns out that the thief can increase his distance from the sheriff beyond any limit. However, when the speed constraints are given by a polyhedral metric, the sheriff can always remain within a constant distance of the thief. These results hold in any dimension.

We apply this problem to buffer minimization in an on-line scheduling problem with conflicts that was first studied by Chrobak, Csirik, Imreh, Noga, Sgall, and Woeginger. The on-line player is modeled by the sheriff and the accumulated target workload that arrives over time is modeled by the apparent position of the thief. The true position of the thief represents the off-line player.

The speed constraint in this case is determined by the stable set polytope of the conflict graph. For bipartite conflict graphs, the structure of this polytope is particularly simple. In this case, our approach leads to new approximation bounds.

## Dynamic Flow Models for Traffic Networks

EKKEHARD KÖHLER

(joint work with Katharina Langkau and Martin Skutella)

Traffic in road networks serves as a standard example for the application of static network flow algorithms. While for restricted situations like rush-hour traffic this is indeed a suitable model, it is not appropriate for a more general setting. The reason is, that it does not capture two main features of road traffic. On the one hand such a model should be time-dependent, i.e., flow unit should move through the network over time. On the other hand transit times on the arcs of such a network should not be constant but rather dependent on the current flow situation on the particular arc.

We define and compare two different models that try to cope with the requirements of traffic in road networks. First we define a load-dependent model for flows over time. In this model the speed of the flow units on an arc depend at every point in time on the load on that arc, where the transit time function is convex. The second model determines the transit time of a unit of flow in an arc using the flow rate at the time of entering this arc. For both models we consider the quickest flow problem and give approximation algorithms. Furthermore we compare the quality of these models.

## Open Problem Session

### Minimize number of consecutive-1 matrices in a decomposition of integer matrices

HORST W. HAMACHER

Given an integer matrix  $I$ , we want to find a representation of  $I$  as linear combination of matrices with the strict consecutive-1 property, such that the number of matrices is as small as possible.

### Problem: Contractions to $C_4$

GERHARD WOEGINGER

Let  $G$  be a simple, connected, undirected input graph. What is the complexity of the following problem: Does there exist a sequence of edge-contractions that transform  $G$  into  $C_4$ , the cycle on four vertices? For instance, it is easy to see that any chordal graph  $G$  cannot be contracted to  $C_4$ . Or for instance, a graph  $G$  with a dominating vertex (a vertex that is adjacent to all other vertices) cannot be contracted to  $C_4$ .

I conjecture that this problem can be solved in polynomial time. The related question about contractions to  $C_3$  is trivial, and the related question about contractions to  $C_5$  (or about contractions to any fixed  $C_k$  with  $k \geq 5$ ) is NP-complete.

### Realizability of final football tables

ANDRÁS FRANK

We are given  $n$  football teams playing in  $\binom{n}{2}$  matches against each other. The team that wins a match obtains three points, on a draw, both teams get one point.

What is the complexity of deciding whether a given final table is realizable?

### Extending bipartite graphs to permit a unique perfect matching

GÜNTER ROTE

We are given a bipartite graph  $G = (V, E)$ . What is the complexity of deciding whether one can add edges  $E'$ , such that  $G = (V, E \cup E')$  is still bipartite, but has a unique perfect matching?

Recognizing bipartite graphs that have a unique perfect matching is polynomial, as it is equivalent to matrix upper triangulation.

Thursday, November 28th, 2002

## Connected Rigidity Matroids and Uniquely Realizable Graphs

TIBOR JORDÁN

A framework  $(G, p)$  is a pair, where  $G$  is a graph with  $n$  vertices and  $p = (p_1, p_2, \dots, p_n)$  is a corresponding set of  $n$  points in  $R^d$  ( $p$  is called a configuration). Two frameworks  $(G, p)$  and  $(G, q)$  are equivalent if corresponding edges of the two frameworks have the same length. A framework  $(G, p)$  is a unique realization of  $G$  if all equivalent realizations of  $G$  can be obtained from  $(G, p)$  by a rigid congruence of  $R^d$ . The unique realization problem is to decide whether a given realization is unique, or whether all realizations of a given graph  $G$  are unique. We shall consider generic realizations only, i.e. we shall assume that all coordinates of the configurations are algebraically independent over the rationals.

Bruce Hendrickson conjectured in '88 that all realizations of a given graph  $G$  are unique in  $R^2$  if and only if  $G$  is 3-connected and redundantly rigid. Motivated by this problem, Bob Connelly conjectured in '89 that every 3-connected circuit of the two-dimensional rigidity matroid can be obtained from  $K_4$  by a sequence of 'degree-three extensions'. With Alex Berg (Aarhus) we proved Connelly's conjecture and with Bill Jackson (London) we showed that Hendrickson's conjecture is also true.

## Mixed Integer Models for the Optimization of Gas Networks

ALEXANDER MARTIN

A gas network basically consists of a set of compressors and valves that are connected by pipes. The task of the transient technical optimization is to optimize the drives of the gas and to set in the compressors cost-efficiently such that the required demands are satisfied. This problem leads to a complex mixed integer nonlinear optimization problem. We approach it by approximating the non-linearities by piece-wise linear functions leading to a huge mixed integer program. We study the polyhedral consequences of this model and present a polynomial separation algorithm for certain substructures. Our preliminary computational results show the benefits when incorporating this separation algorithm into a general mixed integer programming solver.

## Submodular Function Minimization—Theory and Practice

SATORU IWATA

Recently, combinatorial strongly polynomial algorithms for minimizing submodular functions have been developed by Iwata, Fleischer, and Fujishige (IFF) and by Schrijver. All the previously known polynomial-time algorithms for this problem are based on the ellipsoid method, which is not efficient in practice. This talk provides a survey on these and subsequent combinatorial algorithms and reports some results of preliminary computational experiments.

## Approximation Algorithms for the General Covering Problem

KLAUS JANSEN

We generalize a method by Grigoriadis et al. to compute an approximate solution of the fractional covering (and max-min resource sharing) problem with  $M$  nonnegative linear (or concave) constraints on a convex set  $B$  to the case with general approximate block solvers (i.e. with only constant, logarithmic, or even worse approximation ratios). The algorithm is based on a Lagrangian decomposition which uses a modified logarithmic potential function and on several other ideas (scaling phase strategy, two stopping rules in a phase, eliminating functions larger than a threshold value  $T$ , reducing the step length and taking a convex combination among different iterates in a phase). We show that the algorithm runs in iterations (or block optimization steps), a data and approximation ratio independent bound which is optimal up to poly-logarithmic factors for any fixed relative accuracy  $\epsilon \in (0, 1)$ . Furthermore, we show how to apply this method for the fractional weighted graph colouring problem.

### Consecutive-1 Decomposition of Integer Matrices and Applications.

HORST W. HAMACHER

In this talk we consider the following problem: Given an integer matrix  $I = (I_{ij})$ , we want to find a decomposition of  $I$  as linear combination of matrices with the consecutive-1 property (C1P), i.e.  $I = \sum_{t \in T} \alpha_t Y_t$ . Here,  $T$  is the set of all C1P matrices and the C1P is meant in the strict way for the rows, such that the matrices  $Y_t$  satisfy the property  $y_{ij_1} = y_{ij_2}$  for  $j_1 < j_2$  implies  $y_{ij} = 1, \forall j_1 \leq j \leq j_2$ .

We first consider the problem of minimizing the decomposition time  $\sum_{t \in T} \alpha_t$ . This problem is, for instance, relevant in the implementation of radiation during a cancer therapy using so-called multi-leaf collimators (MLC). While this problem is trivially solvable if all C1P matrices are allowed, it becomes more involved if only specific subsets of  $T$ , are allowed. We show how to reduce the problem to network flow problems in two suitably chosen networks. The second version of the transformation makes it also possible to show that an optimal solution exists, which has the integrality property. In this way, the applied problem is solved with a small number of C1P matrices, a property which is particularly important in the application context.

### Node-Capacitated Ring Routing

ANDRÁS FRANK

This is an account on a research whose results are published in two works:

1. A. Frank, Z. Király, and B. Kotnyek, *Routing in ring networks*, in preparation.
2. A. Frank, B. Shepherd, V. Tandon, and Z. Végh, *Node-capacitated ring routing*, *Mathematics of Operations Research*, 27, 2. (2002) pp. 372-383.

We consider the node-capacitated routing problem in an undirected ring network along with its fractional relaxation, the node-capacitated multicommodity flow problem. For the feasibility problem, Farkas' lemma provides a characterization for general undirected graphs asserting roughly that there exists such a flow if and only if the so-called distance inequality holds for every choice of distance functions arising from non-negative node-weights. For rings this (straightforward) result will be improved in two ways. We prove

that, independent of the integrality of node-capacities, it suffices to require the distance inequality only for distances arising from (0-1-2)-valued node-weights, a requirement which will be called the double-cut condition. Moreover, for integer-valued node-capacities, the double-cut condition implies the existence of a half-integral multicommodity flow. In this case even an integer-valued multicommodity flow exists which may though be infeasible but not much: it exceeds every node-capacity by at most one.

Our approach gives rise to a fully combinatorial, strongly polynomial algorithm to compute either a violating double-cut or a node-capacitated multicommodity flow. The approach is then used to compute a node-capacitated routing, if one exists. A relation of the problem to its edge-capacitated counterpart will also be explained.

### **New Maximum Flow Algorithms by MA Orderings and Scaling**

SATORU FUJISHIGE

(joint work with Shiguo Isotani)

Maximum adjacency (MA) ordering has effectively been applied to graph connectivity problems by Nagamochi Ibaraki. We show a nice application of MA ordering to the maximum flow problem with integral capacities to get a new polynomial-time algorithm and propose its scaling version which requires  $\mathcal{O}(mn \log U)$  running time, where  $m$  is the number of arcs,  $n$  the number of vertices,  $U$  the maximum capacity. We also show some computational results to examine behaviours of our algorithms.

### **The 2-Edge-Connected Subgraph with Bounded Rings Problem**

A. RIDHA MAHJOUR

(joint work with B. Fortz, S.T. McCormick, and P. Pesneau)

The 2-edge-connected subgraph with bounded rings problem in a graph consists in determining a minimum cost 2-edge-connected subgraph such that the shortest cycle to which each edge belongs (a “ring”) does not exceed a given length  $K$ . We present here a formulation of that problem in the space of the natural design variables and derive facet results for different classes of valid inequalities. We study the separation problems associated to these inequalities and their integration in a Branch-and-Cut algorithm, and provide extensive computational results.

**Friday, November 29th, 2002**

## **The Structure of Group Relaxations**

REKHA THOMAS

(joint work with Serkan Hosten)

In this talk I will survey recent results on the structure of all the minimal group relaxations that are needed to solve all integer programs in a family with fixed coefficient matrix and cost vector but varying right hand sides. These results come from the algebraic approach to integer programming using Groebner bases but have a self contained explanation in math programming language.

## **Strongly Polynomial Algorithms for Submodular and Bisubmodular Function Minimization**

S. THOMAS MCCORMICK

(joint work with Satoru Fujishige)

Recently Fujishige and Iwata showed how to extend the IFF algorithm for submodular function minimization (SFM) to bisubmodular function minimization (BSFM). However, they were able to extend only the weakly polynomial version of IFF to BSFM. Here we investigate the difficulty that prevented them from also extending the strongly polynomial version of IFF to BSFM, and we show a way around the difficulty. This new method gives a somewhat simpler strongly polynomial SFM algorithm, as well as the first combinatorial strongly polynomial algorithm for BSFM.

## **Mean-Risk Models in Stochastic Integer Programming**

RÜDIGER SCHULTZ

We consider mixed-integer linear programs under uncertainty where data information is revealed stepwise and decisions have to be taken accordingly, based on the information available. Optimization then has to deal with implicitly given random objective functions that arise from the scheme of alternating decision and observation. Traditional stochastic programming models aim at optimizing the expectation of these functions. When addressing risk aversion one arrives at mean-risk models. This requires the selection of proper risk measures. In the talk we introduce different risk measures and discuss consequences of their inclusion into purely expectation-based stochastic integer programs. We study consistency of the risk measures with ordering principles from stochastics, and we investigate well-posedness of the resulting mean-risk models. For discrete probability distributions, the mean-risk models are equivalent to large-scale, block-structured, mixed-integer linear programs, the block structure of which depends on the risk measure employed. We identify risk measures that lead to decomposable block structures, and we outline resulting decomposition strategies.

## **Progress in Linear and Integer Programming**

BOB BIXBY

A quick review was given of computational progress in linear programming (a combined algorithm + machine factor nearing 2,000,000) and integer programming (with an emphasis on finding good feasible solutions).

*Edited by Christian Liebchen and Marco Lübbecke*

## Participants

**Prof. Dr. Robert E. Bixby**

bixby@caam.rice.edu  
Jesse H. Jones Graduate School of  
Management  
Rice University  
6100 Main Street  
Houston TX 77005 - USA

**Prof. Dr. Rainer E. Burkard**

burkard@opt.math.tu-graz.ac.at  
Institut für Mathematik  
Technische Universität Graz  
Steyrergasse 30  
A-8010 Graz

**Prof. Dr. Ulrich Faigle**

U.Faigle@math.utwente.nl  
faigle@zpr.uni-koeln.de  
faigle@octopussy.mi.uni-koel  
Mathematisches Institut  
Universität zu Köln  
D-50923 Köln

**Prof. Dr. Sandor P. Fekete**

s.fekete@tu-bs.de  
Abteilung für Mathematische  
Optimierung  
TU Braunschweig  
Pockelsstr. 14  
D-38106 Braunschweig

**Prof. Dr. Matteo Fischetti**

fisch@dei.unipd.it  
Dipartimento di Elettronica e  
Informatica  
Università di Padova  
Via Gradenigo 6/A  
I-35131 Padova

**Prof. Dr. Andras Frank**

frank@cs.elte.hu  
Department of Operations Research  
Eötvös Lorand University  
ELTE TTK  
Pazmany Peter setany 1/C  
H-1117 Budapest

**Prof. Dr. Satoru Fujishige**

fujishig@sys.es.osaka-u.ac.jp  
Dept. of Systems and Human Science  
Osaka University  
Toyonaka  
Osaka 560-8531 – Japan

**Prof. Dr. Michel Goemans**

goemans@math.mit.edu  
Department of Mathematics  
Massachusetts Institute of  
Technology  
Cambridge, MA 02139-4307 – USA

**Prof. Dr. Martin Grötschel**

groetschel@zib.de  
Konrad-Zuse-Zentrum für  
Informationstechnik Berlin (ZIB)  
Takustr. 7  
D-14195 Berlin

**Prof. Dr. Horst W. Hamacher**

hamacher@mathematik.uni-kl.de  
Fachbereich Mathematik  
Universität Kaiserslautern  
Kurt-Schumacher-Str. 26  
D-67663 Kaiserslautern

**Prof. Dr. Christoph Helmberg**

helmberg@mathematik.tu-chemnitz.de  
Fakultät für Mathematik  
Technische Universität Chemnitz  
D-09107 Chemnitz

**Prof. Dr. Dorit S. Hochbaum**  
dorit@hochbaum.Berkeley.Edu  
Department of Industrial  
Engineering and Operations Research  
University of California  
Etcheverry Hall  
Berkeley, CA 94720-1777 – USA

**Prof. Dr. Klaus Jansen**  
kj@informatik.uni-kiel.de  
Institut für Informatik und  
Praktische Mathematik  
Universität Kiel  
Olshausenstr. 40  
D-24118 Kiel

**Prof. Dr. Tibor Jordan**  
jordan@cs.elte.hu  
Department of Operations Research  
Eötvös Lorand University  
ELTE TTK  
Pazmany Peter setany 1/C  
H-1117 Budapest

**Prof. Dr. Michael Jünger**  
mjuenger@informatik.uni-koeln.de  
Institut für Informatik  
Universität zu Köln  
Pohligstr. 1  
D-50969 Köln

**Dr. Ekkehard Köhler**  
ekoehler@math.tu-berlin.de  
Fakultät II-Institut f. Mathematik  
Technische Universität Berlin  
Skr. MA 8-3  
Straße des 17. Juni 136  
D-10623 Berlin

**Prof. Dr.Dr.h.c. Bernhard Korte**  
dm@or.uni-bonn.de  
Forschungsinstitut für  
Diskrete Mathematik  
Universität Bonn  
Lennestr. 2  
D-53113 Bonn

**Christian Liebchen**  
liebchen@math.tu-berlin.de  
Institut für Mathematik  
Technische Universität Berlin  
Straße des 17. Juni 136  
D-10623 Berlin

**Prof. Dr. Thomas M. Liebling**  
thomas.liebling@epfl.ch  
Institut de Mathématiques  
Ecole Polytechnique Fédérale  
de Lausanne  
MA-Ecublens  
CH-1015 Lausanne

**Dr. Marco E. Lübbecke**  
m.luebbecke@tu-bs.de  
Abteilung für Mathematische  
Optimierung  
TU Braunschweig  
Pockelsstr. 14  
D-38106 Braunschweig

**Prof. Dr. Ali Ridha Mahjoub**  
Ridha.Mahjoub@math.univ-bpclermont.fr  
Laboratoire LIMOS  
Université de Clermont II-Blaise  
Pascal, Complexe Scientifique des  
Cézeaux  
F-63177 Aubiere Cedex

**Dr. Francois Margot**  
fmargot@ms.uky.edu  
Dept. of Mathematics  
University of Kentucky  
715 Patterson Office Tower  
Lexington, KY 40506-0027 - USA

**Prof. Dr. Alexander Martin**  
martin@mathematik.tu-darmstadt.de  
FB Mathematik, AG 7  
TU Darmstadt  
Schlossgartenstr. 7  
D-64289 Darmstadt

**Prof. Dr. S. Thomas McCormick**

stmv@adk.commerce.ubc.ca  
Faculty of Commerce and Business  
Administration  
University of British Columbia  
463 Henry Angus Building  
Vancouver V6T 1Z2 - Canada

**Prof. Dr. Rolf Möhring**

moehring@math.tu-berlin.de  
Institut für Mathematik - Fak. II  
Technische Universität Berlin  
Skr. MA 6-1  
Straße des 17. Juni 136  
D-10623 Berlin

**Prof. Dr. Alain Prodon**

alain.prodon@epfl.ch  
Institut de Mathématiques  
Ecole Polytechnique Fédérale  
de Lausanne  
MA-Ecublens  
CH-1015 Lausanne

**Dr. Jörg Rambau**

rambau@zib.de  
Konrad-Zuse-Zentrum für  
Informationstechnik Berlin (ZIB)  
Takustr. 7  
D-14195 Berlin

**Prof. Dr. Gerhard Reinelt**

Gerhard.Reinelt@Informatik.Uni-Heidelberg.de  
Institut für Informatik  
Ruprecht-Karls-Universität  
Heidelberg  
Im Neuenheimer Feld 368  
D-69120 Heidelberg

**Prof. Dr. Franz Rendl**

franz.rendl@uni-klu.ac.at  
Institut für Mathematik  
Universität Klagenfurt  
Universitätsstr. 65-67  
A-9020 Klagenfurt

**Dr. Giovanni Rinaldi**

rinaldi@iasi.rm.cnr.it  
Istituto di Analisi dei Sistemi ed  
Informatica  
CNR  
Viale Manzoni 30  
I-00185 Roma

**Dr. Günter Rote**

rote@inf.fu-berlin.de  
Institut für Informatik  
Freie Universität Berlin  
Takustr. 9  
D-14195 Berlin

**Prof. Dr. Iwata Satoru**

iwata@sr3.t.u-tokyo.ac.jp  
Dept. of Math. Eng. and Inf. Physic  
Grad. School of Engineering  
University of Tokyo  
7-31, Hongo, Bunkyo-ku  
Tokyo 113-8656 – JAPAN

**Prof. Dr. Rainer Schrader**

schrader@zpr.uni-koeln.de  
Institut für Informatik/ZAIK  
Universität zu Köln  
Weyertal 80  
D-50931 Köln

**Prof. Dr. Alexander Schrijver**

lex@cw.nl  
CWI  
Postbus 94079  
NL-1090 GB Amsterdam

**Prof. Dr. Rüdiger Schultz**

schultz@math.uni-duisburg.de  
Institut für Mathematik  
Gerhard-Mercator-Universität  
Duisburg  
Lotharstr. 65  
D-47048 Duisburg

**Prof. Dr. David Shmoys**  
shmoys@orie.cornell.edu  
shmoys@cs.cornell.edu  
Department of Computer Science  
Cornell University  
232 Frank H. T. Rhodes Hall  
Ithaca NY 14853 - USA

**Dr. Rekha Thomas**  
thomas@math.washington.edu  
Dept. of Mathematics  
Box 354350  
University of Washington  
Seattle, WA 98195-4350 - USA

**Prof. Dr. Paolo Toth**  
ptoth@deis.unibo.it  
DEIS  
Universita di Bologna  
Viale Risorgimento, 2  
I-40136 Bologna

**Prof. Dr. Klaus Truemper**  
klaus@utdallas.edu  
Dept. of Computer Science EC 31  
The University of Texas at Dallas  
Box 83 06 88  
Richardson, TX 75083-0688 – USA

**Jens Vygen**  
vygen@or.uni-bonn.de  
Forschungsinstitut für  
Diskrete Mathematik  
Universität Bonn  
Lennestr. 2  
D-53113 Bonn

**Prof. Dr. Robert Weismantel**  
weismantel@imo.math.uni-magdeburg.de  
Fakultät für Mathematik  
Otto-von-Guericke-Universität  
Magdeburg  
Universitätsplatz 2  
D-39106 Magdeburg

**Prof. Dr. Dominique de Werra**  
dewerra.ima@epfl.ch  
Institut de Mathématiques  
Ecole Polytechnique Fédérale  
de Lausanne  
MA-Ecublens  
CH-1015 Lausanne

**Prof. Dr. Gerhard Woeginger**  
g.j.woeginger@math.utwente.nl  
Department of Mathematics  
University Twente  
PO Box 217  
NL-7500 AE Enschede

**Prof. Dr. Uwe Zimmermann**  
u.zimmermann@tu-bs.de  
Abteilung für Mathematische  
Optimierung  
TU Braunschweig  
Pockelsstr. 14  
D-38106 Braunschweig