Mathematisches Forschungsinstitut Oberwolfach

Report No. 6/2003

Combinatorics, Convexity and Algebraic Geometry

January 26th – February 1st, 2003

The conference was organized by Victor V. Batyrev (Tübingen), Peter McMullen (London), Bernd Sturmfels Berkeley) and Bernard Teissier (Paris).

38 scientists participated. During the five days of the conference 18 main talks were given presenting recent developments in combinatorics, convexity and algebraic geometry. The productive interplay among these disciplines was emphasized, being especially evident in the theory of toric varieties.

The organizers and participants thank the 'Mathematisches Forschungsinstitut Oberwolfach' for making this conference possible.

The abstracts are listed in the order the talks were given.

Abstracts

Flag Vectors, Toric h-Vectors and Ordinary Polytopes

Margaret Bayer

The *toric h-vector* $h(P) = (h_0, h_1, \ldots, h_d)$ of a rational polytope is the sequence of (nonzero) middle perversity intersection homology Betti numbers of the toric variety associated to P. The h-vector depends only on the combinatorial data known as the flag vector of the polytope. The generalized Dehn-Sommerville equations (by Bayer and Billera) give all linear equations on the flag vectors; they imply $h_i = h_{d-i}$. The hard Lefschetz theorem on the toric variety gives the unimodality of the h-vector: $1 = h_0 \leq h_1 \leq \cdots \leq h_{\lfloor d/2 \rfloor}$. (Recently Kalle Karu extended this theorem from rational to real polytopes.)

In order even to conjecture further conditions on h-vectors or flag vectors of polytopes, we need to know a good variety of nonsimplicial polytopes and their vectors. These have been hard to find. I am studying *ordinary polytopes*, a class of nonsimplicial polytopes introduced by Bisztriczky. They are a natural generalization of cyclic polytopes, which played a central role in the study of face numbers of simplicial polytopes.

In even dimensions, all ordinary polytopes are cyclic. In odd dimensions, there is a unique combinatorial type $P^{d,k,n}$ of ordinary polytope for each triple n, k, d, with $n \geq k \geq d = 2m+1 \geq 5$. The h-vector has a surprisingly simple form:

$$h_i(P^{d,k,n}) = \binom{k-d+i}{i} + (n-k)\binom{k-d+i-1}{i-1}$$

for $0 \leq i \leq d/2$. The extreme cases are when $k = d$, in which case $P^{d,d,n}$ is a *multiplex* and has h-vector $(1, n-d+1, n-d+1, \ldots, n-d+1, 1)$; and when $k = n$, in which case $P^{d,n,n}$ is a cyclic polytope and has h-vector given by $h_i(P^{d,n,n}) = \binom{n-d+i}{i}$ for $0 \leq i \leq d/2$. The ordinary polytopes thus give a nice distribution of h-vectors between h-vectors representing the extremes of the simplicial case.

Lower Bounds for the Generalized h-Vectors of Centrally Symmetric Polytopes

Annette A'Campo-Neuen

In 1987, R. Stanley proved tight lower bounds for the coefficients of the h-vector of a simplicial centrally symmetric polytope that were previously conjectured by A. Björner.

In 1999, we obtained an analogous result for the generalized h-vector of a nonsimplicial but rational centrally symmetric polytope using equivariant intersection cohomology of the corresponding projective toric variety. Recently, we could show that the same lower bounds are valid for the generalized h-vector of an arbitrary centrally symmetric polytope even though we cannot associate a toric variety to it.

Our proof is based on the combinatorial intersection theory for arbitrary fans developed by G. Barthel, J.-P. Brasselet, K.-H. Fieseler and L. Kaup, and on the hard Lefschetz theorem in this context that was recently proved by K. Karu.

Zariski-Riemann Spaces of Real Fans

Masanori Ishida

The theory of toric varieties says that there exists a natural one-to-one correspondence between the set of toric varieties with the torus $(\mathbb{C}^*)^r$ and the set of (rational) fans in the real space $\mathbb{R}^r$ with the lattice $\mathbb{Z}^r$. A finite fan Δ is said to be complete, if the support $|\Delta| = \bigcup_{\sigma\in\Delta} \sigma$, i.e., the union of all cones in Δ, is equal to $\mathbb{R}^n$. Then the corresponding toric variety is a complete algebraic variety. For a given finite fan Δ, it is a non-trivial problem to find a complete fan $\overline{\Delta}$ which contains Δ as a subfan. It used to be done as follows. Let $Z(\Delta)$ be the corresponding toric variety. Then by applying Sumihiro's equivariant completion theorem, we can find a complete toric variety Z' which contains $Z(\Delta)$ as an open subvariety. Since Z' is equal to $Z(\overline{\Delta})$ for a complete fan $\overline{\Delta}$, this $\overline{\Delta}$ is the required fan. Sumihiro's theorem based on Nagata's compactification theorem of algebraic varieties. Nagata proved this theorem by using the compactness of the Zariski-Riemann space, i.e., the set of all valuation rings of the function field.

In January 2001, G. Ewald informed me that a combinatorial proof of the existence of $\overline{\Delta}$ is possible, and his method is valid also for not necessarily rational fans.

In my talk, I defined the Zariski-Riemann space for fans as the set of additive preorders of the dual $\mathbb{Z}$-module $M \cong \mathbb{Z}^r$ of the lattice. I can translate the proof of Nagata's theorem written in the language of algebraic varieties into that of fans. The Zariski-Riemann space for not necessarily rational fans is also defined as the set of $\mathbb{R}$-additive preorders of $M_{\mathbb{R}} \cong \mathbb{R}^r$. Then the existence of the completion $\overline{\Delta}$ follows from this translation in case of both rational and real fans. In order to translate a difficult part of Nagata's proof, we interpret the notion of the blowing-up of a variety at a subvariety as well as at a fractional ideal into that of a fan by defining ideals and fractional ideals for fans.

Rational Hypergeometric Functions

Alicia Dickenstein

Multivariate A-hypergeometric functions associated with toric varieties were introduced by Gel'fand, Kapranov and Zelevinsky. Singularities of such functions are discriminants, that is, divisors projectively dual to torus orbit closures. In joint work with Eduardo Cattani and Bernd Sturmfels, we show that the existence of rational hypergeometric functions with poles along most of these potential denominators imposes strong combinatorial restrictions on the configuration A. We conjecture that the denominator of any rational hypergeometric function is a product of resultants, that is, a product of special discriminants arising from Cayley configurations, and that in this case, all such functions can be described in terms of toric residues. These results are proved for toric hypersurfaces, for toric varieties of dimension at most three, for toric fourfolds in $\mathbb{P}^6$ and for Lawrence configurations.

Combinatorial Constructions of Real Algebraic Varieties

Ilia Itenberg

Real algebraic varieties seem to be quite distant from combinatorial geometry. However, it is possible to construct real algebraic varieties in a completely combinatorial fashion: one can patchwork them from pieces which essentially are hyperplanes. This procedure is called the *combinatorial patchworking*. It is a particular case of the Viro method and is directly related to Maslov's dequantization of positive real numbers. We present one of the applications of the combinatorial patchworking: a construction of maximal (in the sense of the Smith-Thom inequality) hypersurfaces and, more generally, maximal complete intersections in projective spaces (joint work with O. Viro). The construction of maximal hypersurfaces leads to a nice description of the Hodge numbers of an algebraic hypersurface in $\mathbb{CP}^n$ in terms of the numbers of certain simplices in a primitive triangulation of the corresponding Newton simplex.

Combinatorial Patchworking of Pseudo-Holomorphic Curves in Toric Surfaces

Eugenii Shustin

Let Δ be a lattice convex polygon in the positive quadrant, $\mathbb{C}\Delta = \Delta \times (S^1)^2 \subset \mathbb{C}^2$. Define the extended moment map $\mathbb{C}\mu_\Delta : (\mathbb{C}^*)^2 = {\mathbb{R}_+}^2 \times (S^1)^2 \overset{(\mu_\Delta, \mathrm{Id})}{\longrightarrow} \mathbb{C}\Delta$ with $\mu_\Delta : {\mathbb{R}_+}^2 \to \Delta$ being the usual moment map. For any curve C in the toric surface $T(\Delta)$ associated with Δ, define the chart $\mathrm{Ch}(C)$ as the closure $\overline{\mathbb{C}\mu_\Delta(C \cap (\mathbb{C}^*)^2)}$. For a subdivision $\Delta = \Delta_1 \cup \ldots \cap \Delta_N$ into convex lattice polygons, and a set

$$A = \{a_{ij} \in \mathbb{C} \mid (i,j) \in \Delta,\ a_{ij} \neq 0 \text{ as } (i,j) \text{ is a vertex of some of } \Delta_1, \ldots, \Delta_N\},$$

define $f_k = \sum_{(i,j)\in\Delta_k} a_{ij}x^iy^j$, and the C-curve $\bigcup_{k=1}^{N} \mathrm{Ch}(f_k = 0)$.

Theorem: (Itenberg, Shustin) If $T(\Delta)$ is the projective plane or a Hirzebruch surface, and all the curves $(f_k = 0)$ are real reduced and generic at infinity, then the corresponding C-curve is equivariantly isotopic in $\mathbb{C}\Delta$ to the chart of some real pseudo-holomorphic curve.

Example: For any $d > 5$, there exist real pseudo-holomorphic plane curves with any number of real tacnodes (i.e., of type A_3) from 0 to $d^2/5 + O(d)$.

We conjecture that among these real singular pseudo-holomorphic curves there are those which are not isotopic to any algebraic curve of the same degree.

Toric HyperKähler Manifolds

Hiroshi Konno

The real torus T^N acts on the quaternionic vector space $\mathbb{H}^N$, preserving its hyperKähler structure. This induces the action of a subtorus $K \subset T^N$ on $\mathbb{H}^N$, which admits a hyperKähler moment map $\mu\colon \mathbb{H}^N \to k^* \otimes \mathbb{R}^3 \cong k^* \times k^*_{\mathbb{C}}$. If K acts freely on $\mu^{-1}(\alpha,\beta)$, then we have a smooth hyperKähler quotient $X(\alpha,\beta) = \mu^{-1}(\alpha,\beta)/K$. It has the natural hyperKähler structure (g, I_1, I_2, I_3), which is preserved by the action of the quotient torus $T^n = T^N/K$. Bielawski and Dancer called $X(\alpha,\beta)$ a *toric hyperKähler manifold*. It is not a toric manifold in the usual sense, but a corresponding object in hyperKähler geometry.

In the talk, we discussed the topology and geometry of toric hyperKähler manifolds. First we computed the cohomology ring of $X(\alpha, \beta)$. Under the natural isomorphism $\kappa\colon k^* \to H^2(X(\alpha,\beta);\mathbb{R})$, the parameters α, β correspond to the de Rham cohomology classes represented by the Kähler form and the holomorphic symplectic form on $(X(\alpha,\beta), I_1)$ respectively.

Next we study the variation of the complex structure of the holomorphic symplectic manifold $(X(\alpha,\beta), I_1)$ according to the parameter (α,β) as follows. If we fix an arbitrary $\beta \in k^*_{\mathbb{C}}$, then we have a chamber structure in k^*.
(a) If α' is in the same chamber $\mathcal{C}$ as α, then $(X(\alpha',\beta), I_1)$ is canonically isomorphic to $(X(\alpha,\beta), I_1)$ as a complex manifold. Moreover, the Kähler cone of this complex manifold is $\kappa(\mathcal{C}) \subset H^2(X(\alpha,\beta);\mathbb{R})$.
(b) If α' is in the next chamber to α, then $(X(\alpha',\beta), I_1)$ is obtained from $(X(\alpha,\beta), I_1)$ by applying Mukai's elementary transformation, which is a special birational transformation between holomorphic symplectic manifolds.

Polytopes, Algorithms and Enumeration

Maximilian Kreuzer
(joint work with Harald Skarke)

Toric Calabi–Yau varieties play an important role in mirror symmetry and other string dualities. Their construction is based on reflexive polytopes (i.e., lattice polytopes with an interior point whose dual vertices belong to the dual lattice). An algorithm that was used to complete the enumeration of these polytopes in 3 and 4 dimensions was inspired by the reflexivity of Newton polytopes for transversal polynomials of the appropriate degree in weighted projective spaces. A constructive proof of the finiteness of the relevant weight systems for fixed dimension and the combinatorics of minimal lattice polytopes with interior point provide us with a set of maximal reflexive polytopes that contain all others as subpolytopes, possibly on a coarser lattice.

A crucial ingredient for an implementation of this algorithm was the definition of a normal form, whose evaluation also gives us the symmetries of the polytope as the stabilizer of the group of vertex permutations. By keeping track of the posed structure of all reflexive polytopes with respect to inclusion (up to lattice automorphisms) we established the connectedness of the corresponding toric hypersurfaces via singular transitions. Assuming an approximately random generation of mirror pairs we got a surprisingly good prediction for the final result already at an early stage of the computation.

The results are available from our web page and the programs became part of a package called PALP (math.NA/0204356]), which can be used to reconstruct all our results and to generate large numbers of polytopes in higher dimensions. It also has tools for analyzing lattice quotients, NEF partitions and fibration structures and it computes string theoretical Hodge numbers of hypersurfaces and complete intersections.

The Toric Mirror Residue Conjecture of Batyrev-Materov

András Szenes
(joint work with Michèle Vergne)

In this talk I prove the conjecture of Batyrev and Materov on the equality of two seemingly unrelated expressions: that of a generating function of intersection numbers on moduli spaces of maps from $\mathbb{P}^1$ into a toric variety and that of the corresponding toric residue on the Batyrev dual variety. The main ingredient is the careful study of the map $(p^{\lambda_1}, \ldots, p^{\lambda_n})$ where λ_i are the generators of the Mori cone and $p^\lambda = \prod_{i=1}^{n} \alpha_i{}^{\langle \alpha_i, \lambda \rangle}$, where α_i are the Gale dual vectors to the edges of the fan of the original toric variety.

Counting Curves Tropically

Grigory Mikhalkin

The talk presented a new formula for the Gromov-Witten invariants of arbitrary genus in the projective plane as well as for the related enumerative invariants in other toric surfaces. The answer is given in terms of certain lattice paths in the relevant Newton polygon. The length of the paths is responsible for the genus of the holomorphic curves in the count. The formula is obtained by working in terms of the so-called tropical algebraic geometry. This version of algebraic geometry is simpler than its classical counterpart in many aspects. In particular, complex algebraic varieties themselves become piecewise-linear objects in the real space. The transition from the classical geometry is provided by consideration of the "large complex limit" (which is also known as "dequantization" or "patchworking" in some other areas of mathematics).

Toric Hilbert Schemes

Diane Maclagan
(joint work with Rekha R. Thomas)

The toric Hilbert scheme of a lattice $\mathcal{L} \subseteq \mathbb{Z}^n$ parameterizes all ideals with the same $\mathbb{Z}^n/\mathcal{L}$-graded Hilbert series as the lattice ideal $I_{\mathcal{L}}$. It is a multigraded Hilbert scheme in the sense of Haiman and Sturmfels. In the case where $\mathcal{L} = \ker(A)$ for A a $d \times n$ integer matrix, this was first developed by Peeva and Stillman, based on work by Sturmfels. When $\dim \mathcal{L} = n$, the toric Hilbert scheme is Namakamura's G-Hilbert scheme, which arose in the study of the McKay correspondence.

The first result of this talk was that the toric Hilbert scheme is smooth and irreducible when $\dim \mathcal{L} = 2$. This generalizes previous versions of this result for the two special cases $\mathcal{L} = \ker(A)$ and $\mathbb{Z}^n/\mathcal{L}$ finite. We then discussed the connectedness of the scheme, relating this to the connectedness of an associated graph of bistellar flips of triangulations. This leads to Santos' example of a disconnected toric Hilbert scheme.

Rings of Differential Operators on Affine Toric Varieties

Mutsumi Saito
(joint work with William N. Traves)

Let $A := \{ \mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n \}$ be a finite set of integral vectors. We denote by $\mathbb{N}A$, $\mathbb{Z}A$, and $\mathbb{R}_{\geq 0}A$ the monoid, the abelian group, and the cone generated by A, respectively.

Let $R_A := \mathbb{C}[\mathbb{N}A]$ denote the semigroup algebra of $\mathbb{N}A$. We consider two rings: the ring $D(R_A)$ of differential operators on the affine toric variety $\mathrm{Spec}(R_A)$ and its graded ring $\mathrm{Gr}(D(R_A))$ with respect to the order filtration.

As a starting point for the study of $D(R_A)$, we consider the finite generations of $D(R_A)$ and $\mathrm{Gr}(D(R_A))$. While studying the finite generation of $\mathrm{Gr}(D(R_A))$, we defined the notion of a scored semigroup; a semigroup $\mathbb{N}A$ is *scored* if the difference $(\mathbb{R}_{\geq 0}A \cap \mathbb{Z}A) \setminus \mathbb{N}A$ consists of a finite union of hyperplane sections of $\mathbb{R}_{\geq 0}A \cap \mathbb{Z}A$ parallel to facets of the cone $\mathbb{R}_{\geq 0}A$.

We have proved the following:

Theorem:

(1) $\mathrm{Gr}(D(R_A))$ is finitely generated if and only if R_A is a scored semigroup algebra.
(2) $D(R_A)$ is finitely generated for all semigroup algebras R_A.

Scoredness is somehow mysterious to us. We can easily check that scoredness implies Serre's condition (S_2). However there exist a non-scored Cohen-Macaulay example and a non-Cohen-Macaulay scored example.

Flag Invariants of Polytopes

Louis J. Billera

We consider the flag f-vector of a convex polytope P, which counts all flags of faces of P according to the sets of dimensions involved. We discuss what is known about the behaviour of this and two related invariants, the flag h-vector and the **cd**-index.

In particular,

(1) the flag h-vector of a polytope is the flag f-vector of a coloured (balanced) simplicial complex,
(2) the **cd**-index is minimized (termwise) over all n-dimensional polytopes by the n-simplex,
(3) the **cd**-index is minimized over all n-dimensional zonotopes by the n-cube (equivalently, over all essential central hyperplane arrangements in $\mathbf{R}^n$ by the coordinate arrangement),
(4) for n-dimensional polytopes with v vertices, the **cd**-index is maximized by the cyclic n-polytope with v vertices.

In addition, there are known relations between various **cd**-coefficients for any polytope.

Finally, we discuss the state of knowledge about flag f-vectors of graded posets, Eulerian posets and coloured simplicial complexes.

Incidence Combinatorics of Resolutions

EVA MARIA FEICHTNER

We provide a combinatorial framework that is designed to describe the incidence change in stratifications throughout a resolution process, moreover even to *pre*scribe the resolution by intrinsic combinatorial data of the space we are starting with.

Inspired by the combinatorics that is involved in the construction of DeConcini-Procesi "wonderful" models for arrangement complements, we propose notions of combinatorial building sets, nested sets and blowups on a purely order-theoretic level. Our notions describe the incidence combinatorics of stratifications through every step of the DeConcini-Procesi model construction, but also serve in other contexts, e.g., resolutions of toric varieties (joint work with D. Kozlov).

Our framework provides the outset to go even beyond the geometric context of resolutions and yet return to geometry in a somewhat unexpected way: starting with an arbitrary lattice and a combinatorial building set we define an algebra which, in particular cases, is known to be the cohomology algebra of DeConcini-Procesi compactifications of hyperplane arrangements. There is yet another geometric interpretation: for an arbitrary atomic lattice and a combinatorial building set, we construct a toric variety whose Chow ring is isomorphic to the proposed algebra (joint work with S. Yuzvinsky).

The Nash Problem on Arc Families of Singularities

SHIHOKO ISHII
(joint work with János Kollár)

In 1968 John F. Nash introduced the arc spaces and the jet schemes for algebraic and analytic varieties in his preprint which is published as "Arc structures of singularities" in Duke Math. J. in 1995. The idea of these spaces led to many works by Hickel, Lejeune-Jalabert, Reguera-López, Gonzalez-Sprinberg and so on. The study of these spaces has been developed by M. Kontsevich, J. Denef and F. Loeser as the theory of motivic integration.

However the main subject of Nash's paper is the map from the set of the families of arcs at singular points to the set of essential components of the resolutions of the singularities. He proved that this map is injective and posed a question if this is always bijective. Reguera-López prove it for a minimal singularity of dimension 2.

In this talk we show the affirmative answer to this problem for a toric singularity and the negative answer in general by giving a counterexample.

The affirmative answer for a toric singularity is proved by showing that the following diagram for $X = \mathrm{U}_\sigma$ $(\sigma \subset N_{\mathbb{R}})$ gives just the identity map:

$$\{\text{minimal element of } (\bigcup_{\substack{\tau<\sigma \\ \text{singular}}} \tau^\circ) \cap N\} \hookrightarrow \{\text{good component of } \Pi^{-1}(\operatorname{Sing} X)\} \overset{\text{Nash map}}{\hookrightarrow}$$

$$\{\text{essential component}\} = \{\text{essential divisor over } X\} \hookrightarrow \{\text{divisorially essential divisor}\} \hookrightarrow \{\text{toric divisorially essential divisor}\} \hookrightarrow \{\text{minimal element of } (\bigcup_{\substack{\tau<\sigma \\ \text{singular}}} \tau^\circ) \cap N\}.$$

Counterexample: $\{{x_1}^3 + {x_2}^3 + {x_3}^3 + {x_4}^3 + {x_5}^6 = 0\} \subset k^5$, $\operatorname{char} k \neq 2, 3$, has two essential components, but only one good component (= family of arcs passing through the singularity).

Multidegrees and Positive Combinatorial Formulae

Allen Knutson

(joint work with Ezra Miller)

We defined the multidegree, i.e., a multigraded generalization of the degree of a projective variety, and proved the coefficients of this polynomial are positive. We show that Schubert polynomials are the multidegrees of matrix Schubert varieties, and thus can use a Gröbner degeneration to compute them in a positive way. Our degeneration is reduced and Cohen-Macaulay, and the components are easily enumerated.

What is the Right "Polytopal" Generalization of Toric Varieties to the Noncommutative Case?

Valery Alexeev

(joint work with Michel Brion)

Suppose Q is a lattice polytope, symmetric with respect to a reflection group W, or, more generally, $\Delta = \{Q_i\}$ is a W-invariant complex of polytopes. To this combinatorial data, we associate a projective variety which we call *reductive* - it is irreducible, normal, and comes with a double action of a reductive group G, whose dimension is generally higher than $\dim \Delta$. These are remarkably simple varieties whose properties generalize those of toric varieties in a very intuitively clear way. We illustrate this by describing their structure, degenerations, moment map and the moduli.

Fibre Tilings

Peter McMullen

Generalizing the earlier idea of secondary polytopes, Billera and Sturmfels introduced fibre polytopes, and showed how they were related to certain kinds of subdivision induced by projections of polytopes onto other polytopes. Here, this concept is extended to possibly unbounded polyhedra, by making the definition a combinatorial one. Applying the notion to the epigraphs of convex functions which lift finite tilings having strong (orthogonal) duals then permits a definition of fibre tilings. Various useful properties of such tilings under sections and projection are consequences.

Edited by Benjamin Nill

Participants

Dr. Annette A'Campo-Neuen
acampo@mathematik.uni-mainz.de
annette.acampo@math-lab.unibas.ch
Fachbereich Mathematik und
Statistik
Universität Konstanz
D–78457 Konstanz

Prof. Dr. Valery Alexeev
valery@math.uga.edu
Dept. of Mathematics
University of Georgia
Athens, GA 30602 - USA

Prof. Dr. Klaus Altmann
altmann@math.fu-berlin.de
Institut für Mathematik II (WE2)
Freie Universität Berlin
Arnimallee 3
D–14195 Berlin

Prof. Dr. Gottfried Barthel
Gottfried.Barthel@uni-konstanz.de
Fakultät für Mathematik
und Statistik
Universität Konstanz
Fach D 203
D–78457 Konstanz

Prof. Dr. Victor V. Batyrev
batyrev@bastau.mathematik.uni-tuebingen.de
victor.batyrev@uni-tuebingen.de
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10
D–72076 Tübingen

Prof. Dr. Margaret M. Bayer
bayer@math.ukans.edu
Department of Mathematics
University of Kansas
405 Snow Hall
Lawrence, KS 66045-7567 - USA

Prof. Dr. Louis J. Billera
billera@math.cornell.edu
Department of Mathematics
Cornell University
310 Malott Hall
Ithaca NY 14853-4201 - USA

Prof. Dr. Anders Björner
bjorner@math.kth.se
Dept. of Mathematics
Royal Institute of Technology
S-100 44 Stockholm

Prof. Dr. Alicia Dickenstein
alidick@dm.uba.ar
Dto. de Matematica - FCEYN
Universidad de Buenos Aires
Ciudad Universitaria
Pabellon 1
1428 Buenos Aires - Argentina

Prof. Dr. Günter Ewald
ewaldfamily@t-online.de
Fakultät für Mathematik
Ruhr-Universität Bochum
Universitätsstr. 150
D–44801 Bochum

Prof. Dr. Eva Maria Feichtner
feichtne@math.ethz.ch
Departement Mathematik
ETH-Zentrum
Rämistr. 101
CH-8092 Zürich

Prof. Dr. Yi Hu
yhu@math.arizona.edu
Dept. of Mathematics
University of Arizona
Tucson, AZ 85721 - USA

Prof. Dr. Masanori Ishida
ishida@math.tohoku.ac.jp
Mathematical Institute
Graduate School of Science
Tohoku University
Aoba
Sendai 980-8578 - Japan

Prof. Dr. Shihoko Ishii
shihoko@math.titech.ac.jp
Department of Mathematics
Graduate School of Sciences
Tokyo Institute of Technology
Tokyo 152-8551 - Japan

Prof. Dr. Ilia Itenberg
itenberg@univ-rennes1.fr
itenberg@maths.univ-rennes1.fr
I.R.M.A.R.
Universite de Rennes I
Campus de Beaulieu
F-35042 Rennes Cedex

Prof. Dr. Allen Knutson
allenk@math.berkeley.edu
Department of Mathematics
University of California at Berkeley
Berkeley, CA 94720-3840 - USA

Prof. Dr. Hiroshi Konno
konno@ms.u-tokyo.ac.jp
Graduate School of
Mathematical Sciences
University of Tokyo
3-8-1 Komaba, Meguro-ku
Tokyo 153-8914 - Japan

Prof. Dr. Maximilian Kreuzer
kreuzer@hep.itp.tuwien.ac.at
Institut für Theoretische Physik
Technische Universität Wien
Wiedner Hauptstr. 8-10/136
A-1040 Vienna

Prof. Dr. Carl W. Lee
lee@ms.uky.edu
Dept. of Mathematics
University of Kentucky
715 Patterson Office Tower
Lexington, KY 40506-0027 - USA

Prof. Dr. Diane Maclagan
maclagan@math.stanford.edu
Department of Mathematics
Stanford University
Stanford, CA 94305-2125 - USA

Dr. Evgeny Materov
materov@atomlink.ru
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10
D–72076 Tübingen

Prof. Dr. Peter McMullen
p.mcmullen@ucl.ac.uk
pmm@math.ucl.ac.uk
Department of Mathematics
University College London
Gower Street
GB-London, WC1E 6BT

Prof. Dr. Grigory Mikhalkin
mikhalkin@math.harvard.edu
mikhalkin@math.utah.edu
Dept. of Mathematics
University of Utah
1155 S.1400 E. JWB 233
Salt Lake City, UT 84112-0090 - USA

Benjamin Nill
benjamin@nillweb.de
nill@poincare.mathematik.uni-tuebingen.de
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10
D–72076 Tübingen

Prof. Dr. Tadao Oda
odatadao@math.tohoku.ac.jp
Mathematical Institute
Graduate School of Science
Tohoku University
Aoba
Sendai 980-8578 - Japan

Prof. Dr. James Pommersheim
jpommersheim@pomona.edu
Dept. of Mathematics
University of California-San Diego
La Jolla, CA 92093-0112 - USA

Prof. Dr. Mutsumi Saito
saito@math.sci.hokudai.ac.jp
Dept. of Mathematics
Hokkaido University
Kita-ku
Sapporo 060-0810 - Japan

Dr. Eugenii Shustin
shustin@post.tau.ac.il
Department of Mathematics
School of Mathematical Sciences
Tel Aviv University
Ramat Aviv, P.O. Box 39040
Tel Aviv 69978 - Israel

Prof. Dr. Bernd Sturmfels
bernd@math.berkeley.edu
Department of Mathematics
University of California at Berkeley
Berkeley, CA 94720-3840 - USA

Prof. Dr. Andras Szenes
szenes@math.bme.hu
BME Institute of Mathematics
Geometria Tsz, H ep.
Egry Jozsef u. 1
H-1111 Budapest

Prof. Dr. Bernard Teissier
teissier@math.jussieu.fr
Equipe "Géometrie et dynamique"
Institut Mathématique de Jussieu
175 rue du Chevaleret
F-75013 Paris

Dr. Rekha Thomas
thomas@math.washington.edu
Dept. of Mathematics
University of Washington
Box 354350
Seattle, WA 98195-4350 - USA

Dr. Ilya Tyomkin
tyomkin@post.tau.ac.il
School of Mathematical Sciences
Tel Aviv University
Ramat Aviv
Tel Aviv 69978 - Israel

Prof. Dr. Michele Vergne
vergne@math.polytechnique.fr
vergne@daphne.math.polytechnique.fr
Centre de Mathématiques
Ecole Polytechnique
Plateau de Palaiseau
F-91128 Palaiseau Cedex

Prof. Dr. Volkmar Welker
welker@mathematik.uni-marburg.de
FB Mathematik und Informatik
Universität Marburg
Hans-Meerwein-Strasse (Lahnbg)
D–35032 Marburg

Prof. Dr. Jaroslaw Wlodarczyk
wlodar@math.purdue.edu
Dept. of Mathematics
Purdue University
West Lafayette, IN 47907-1395 - USA

Prof. Dr. Christopher Woodward
ctw@math.rutgers.edu
Department of Mathematics
Rutgers University
Hill Center, Bush Campus
110 Frelinghuysen Road
Piscataway, NJ 08854-8019 - USA

Prof. Dr. Günter M. Ziegler
ziegler@math.tu-berlin.de
Institut für Mathematik
MA 6-2
Technische Universität Berlin
Straße des 17. Juni 136
D–10623 Berlin