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Arbeitsgemeinschaft mit aktuellem Thema: Convex Integration

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The Arbeitsgemeinschaft brought together experts from two fields, learning about the implementation and the applications of the convex integration method in differential geometry and in the calculus of variations.

These two aspects were not presented in two separated parts of the meeting but rather developed in parallel. In this way we could see how M. Gromov's approach, which was motivated by the famous J. Nash's flexibility theorem for C^1-isometric embeddings and which provided powerful tools for solving differential equations and inequalities arising in geometry, was interacting with more analytical ideas, in particular from L. Tartar's work, and recently led to a variety of new results – reaching from material science to surprising counterexamples in the theory of elliptic and parabolic PDEs.

Abstracts

Introduction to the h-principle: topological methods for solving differential equations and inequalities

Yasha Eliashberg

There exists a large class of differential equations and inequalities (relations) which have huge spaces of solutions. Amazing classical example of this kind is the famous C^1-isometric immersion theorem of J. Nash. In the talk we described the language of jets, necessary for the geometric description of this class of partial differential relations (PDR). We discussed the notion of formal and genuine solutions of the PDR, and introduced the notion of the h-principle, which is due to M. Gromov. One of the general methods for proving the h-principle, the convex integration was illustrated for the problem of constructing a closed k-form of norm 1.

Differential inclusions in the Calculus of Variations

László Székelyhidi

The talk gave an outline of convex integration in the context of multidimensional Calculus of Variations. To motivate, we showed that in many typical minimization problems the direct method fails. In particular for functionals

$$I[u] = \int_\Omega f(Du)\mathrm{d}x$$

where $f \geq 0$ and $K := \{f = 0\}$ is compact, global minimizers are Lipschitz mappings satisfying

$$Du(x) \in K \text{ a.e. in } \Omega,$$

and *may exist even if the direct method fails.* The idea is to approximate solutions from the class of piecewise affine mappings, obtaining members of the sequence from previous ones by local perturbation in a way that the gradients used get "closer" to K. In the method therefore we require that the original inclusion problem posed in a domain where some approximating mapping is affine with matching boundary data admits a solution. The significance of this observation is that it gives a global restriction on the class of functions from which we can hope to approximate the solution. We analysed this restriction, introducing approximate gradient distributions and generalized convexities.

On the other hand, using explicit constructions, iteration by local perturbation leads to a subset of approximate gradient distributions called *laminates*. An important point emphasized was that infinite iteration can lead to laminates which are supported on sets without rank-1 connections. This highlights the difficulty in *a priori* deciding whether a differential inclusion admits nontrivial Lipschitz solutions by convex integration or not.

Methods for proving the h-principle 1: Holonomic Approximation

Nikolay Mishachev

The *Holonomic Approximation Theorem* shows that in some sense there are unexpectedly many holonomic sections *near* a submanifold $A \subset V$ of positive codimension. The Holonomic Approximation Theorem significantly simplifies Gromov's *continuous sheaves method* for solving partial differential relations.

In order to understand the meaning of the theorem, it is useful to start with the following naive question: *Is it possible to approximate any section $F : V \to X^{(r)}$ by a holonomic section? In other words, given an r-jet section and an arbitrarily small neighbourhood of the image of this section in the jet space, can one find a holonomic section in this neighbourhood?* Though in general the answer is evidently negative, the Holonomic Approximation Theorem says that we *always can find* a holonomic approximation of a section $F : V \to X^{(r)}$ *near a slightly C^0-deformed* submanifold $\widetilde{A} \subset V$ if the original submanifold $A \subset V$ is of positive codimension.

Given an arbitrary submanifold $V_0 \subset V$ of positive codimension, the Holonomic Approximation Theorem allows us to solve any *open* differential relations $\mathcal{R}$ near a perturbed submanifold $\widetilde{V}_0 = h(V)$ where $h : V \to V$ is a C^0-small diffeomorphism. Gromov's h-principle for open Diff V-invariant differential relations on open manifolds, his directed embedding theorem, as well as some other results in the spirit of the h-principle are immediate corollaries of the Holonomic Approximation Theorem.

Methods for proving the h-principle 2: Removal of singularities

Bernhard Leeb

Following section 4.3 of the paper *Construction of nonsingular isoperimetrical films* by Eliashberg and Gromov (1971) we explained the method of removal of singularities for proving h-principles on the example of finding immersions of smooth manifolds into Euclidean space.

Convergence of Gradients I: Controlled L^∞-convergence

Pietro Celada

The talk describes the convex integration of Lipschitz partial differential relations (PDR) as exposed in S. Müller and V. Šverák's paper (*Attainment results for the two-well problem by convex integration* in Geometric analysis and the calculus of variations, J. Jost ed. , Cambridge MA 1996).

The problem consists of finding Lipschitz continuous solutions $u\colon \Omega \subset \mathbb{R}^N \to \mathbb{R}^N$ to the partial differential relation $\nabla u \in K$ having given affine boundary values on $\partial\Omega$. In the interesting cases, K is some compact set of $N \times N$ matrices with empty interior.

Following Gromov's ideas, Müller and Šverák proved that, under suitable convexity assumptions involving the notion of *lamination convexity*, a sequence of approximated problems can be recursively solved. Whilst the approximating gradients ∇u_k change faster and faster to satisfy the PDR, the speed of L^∞ convergence of the functions u_k to the limit function u can be chosen so as to enforce strong convergence of ∇u_k to ∇u, thus proving that u is a solution to the PDR.

Convergence of Gradients II: The Baire category

MARIANNA CSÖRNYEI

I gave a survey talk on the Baire-category theorem and the basic definitions and properties of Baire-1 functions, focusing on their continuity: a function f is *Baire-1* if and only if $f|_K$ admits a continuity point for every compact set K.

It is immediate to check that the gradient as a mapping from an L^∞-closed subspace of Lipschitz functions to L^p is Baire-1 for every $p < \infty$. As a corollary, one can always find a Lipschitz function f so that $f_n \xrightarrow{L_\infty} f$ implies $\nabla f_n \xrightarrow{L_1} \nabla f$. I showed how this simple observation can be used to solve the finite gradient problem (B. Kirchheim: Deformations with finitely many gradients and stability of quasiconvex hulls, C.R. Acad. Sci. paris Sér I Math. 332/3 (2001) pp. 289-294.) I also showed how the Baire-category theorem can be applied to solve certain quasiconvex Hamilton-Jacobi equations (B. Dacorogna, P. Marcellini: General existence theorems for Hamilton-Jacobi Equations in the scalar and vectorial cases, Acta Mathematica, 178/1 (1997), pp. 1-37.)

Method of convex integration for solving partial differential inequalities 1.

DARKO MILINKOVIC

If $\mathcal{R}$ is an open partial differential relation *ample* in coordinate directions, then it satisfies all forms of the h-principle. The idea of the proof is finding a C^0-approximation of the map f with derivative $\frac{\mathrm{d}f}{\mathrm{d}t}$ in Conv(A) by a map g with derivative $\frac{\mathrm{d}g}{\mathrm{d}s}$ in A.

Modification of gradients I: Existence of functions with prescribed jacobian determinant

SERGIO CONTI

Typical applications to materials science require considering partial differential inclusions

$$\nabla u \in K \quad \text{where } u : \Omega \subset \mathbb{R}^n \mapsto \mathbb{R}^m$$

and

$$K \subset \Sigma := \{F \in \mathbb{R}^{m\times n} : M(F) = t\}$$

where $t \in \mathbb{R}$ and M is a given minor. In this case no open set can be generated by K.

This talk discussed the approach by S. Müller and V. Šverák, who showed that the concept of in-approximation and hence the method of convex integration can be generalized to *relatively* open subsets of Σ.

The basic approximation step is based on the existence of diffeomorphisms with given determinant, as proved by B. Dacorogna and J. Moser.

Modification of gradients II: convex integration in the rank-1 convex hull

Andrew Lorent

We present some of the results proved in the paper *Convex integration with constraints and applications to phase transitions and partial differential equations* (Müller, S., Šverák. J. Eur. Math. Soc. (JEMS) 1 (1999) 393-422).

This paper generalises previously known results in the following two ways. Firstly it provides a proof of the existence of differential inclusions $u : \Omega \to \mathbb{R}^2$ satisfying

$$\begin{cases} Du \in U & \Omega \\ u = F \text{ on } \partial\Omega \end{cases}$$

where F is in the rank-1 convex hull of the open (or open inside the surface of matrices satisfying a minor constraint) set of matrices U. Previously this was known for functions satisfying the affine boundary condition with respect to the the much smaller *lamination convex hull* .

Secondly it provides the existence of differential inclusions satisfying a minor constraint of the form $M(Du) := \det\left(\left(\frac{\partial u_i}{\partial x_j}\right)_{i,j=1}^{k}\right) = t$ where $t \neq 0$.

We present a proof of the first of these generalizations.

Method of convex integration for solving partial differential inequalities 2.

Hansjörg Geiges

Explicit examples are given of differential relations in jet bundles for which the so-called ampleness criterion holds or does not hold. Ample differential relations satisfy the h-principle, as demonstrated in the companion talk. Specifically, it is shown that the immersion relation in positive codimension is ample, whereas the submersion relation is not (none the less, the latter also satisfies the h-principle, as can be shown using the holonomic approximation method). Then an ampleness criterion for directed immersions is proved, which allows to deduce the h-principle for totally real immersions and, with a little more care, totally real embeddings into almost complex manifolds. Finally, as an appetiser for applications of convex integration to symplectic and contact geometry, it is shown by explicit computation of the corresponding differential relation that any nowhere zero differential 2-form on a 3-manifold can be homotoped through nowhere zero 2-forms to a closed 2-form representing any given cohomology class.

Modification of Gradients III: Other constraints

Dragomir Dragnev

We present a technique for modifying an affine function whilst keeping its boundary values unchanged and having its gradient up to an arbitrary small error distributed on two desired rank-one connected matrices. Next we show that this construction can be extended to the situation of affine functions with symmetric gradients and that it is possible to approximate any C^1 function with symmetric gradient by piecewise affine C^1 functions with symmetric gradients on domains in $\mathbb{R}^n$.

Further we discuss the existence of piecewise affine solutions of partial differential inclusions of the form:

$$\nabla f \in \mathcal{K} \subset M^{m\times n} \text{ a. e. in } \Omega \subset \mathbb{R}^n$$
$$f(x) = Ax \text{ on } \partial\Omega.$$

for boundary data A in some open set $\mathcal{U} \subset M^{m\times n}$, provided that the set $\mathcal{K}$ can be reached from the interior of $\mathcal{U}$ by rank-one segments. As an application, the existence of piecewise affine solutions of the $SO(2)$ two-well problem is established.

Convex integration as a method of finding C^1-solutions of first order PDE's

Bernd Mümken

In this lecture we considered a typical example for the $C^\perp$-dense h-principle for a closed differential relation $\mathcal{R} \subset X^{(r)}$. We illustrated the general idea of replacing $\mathcal{R}$ by a nearby system of open metaneighbourhoods $\mathrm{Met}_\epsilon\mathcal{R}$ to construct solutions $f_\epsilon \in \Gamma(X)$ of $\mathrm{Met}_\epsilon\mathcal{R}$, which converge to a solution of $\mathcal{R}$.

Failure of regularity for elliptic systems

Jan Kristensen

In this talk the counterexample to partial regularity of extremals for strongly quasiconvex integrals discovered by S. Müller and V. Šverák is presented. After a brief description of regularity results for global minimizers I describe how to rewrite the second order Euler-Lagrange equation as a first order differential inclusion. I then explain how the stable embedding of the T_4-configuration is performed by a genericity argument, and how this allows one to construct an in-approximation.

Failure of regularity II

Miroslav Chlebík

There have been a number of recent successes (by Müller and Šverák) of using Gromov's method of convex integration and its variants and extensions to construct elliptic and parabolic 2×2 systems which admit weak solutions that are Lipschitz but nowhere C^1. These examples at the same time show failure of (even partial) regularity, the lack of the uniqueness, and (in the parabolic case) the failure of the energy identities and inequalities for Lipschitz (weak) solutions.

The talk is concentrated on those recent results and on present limits of the methods to construct (or to exclude the existence of) 'wild' solutions to partial differential inclusions that come from strongly elliptic (or parabolic) systems. The methods are limited by the fact that we are still lacking a fundamental understanding of quasiconvexity, an efficiently manageable characterization of quasiconvex functions. Further progress will require a deeper understanding of the geometry of rank-one convexity as well. Many open problems with implications to regularity/failure of regularity for PDE systems are discussed.

Nash-Kuiper C^1-isometric embedding theorem

Peter Albers and Kai Zehmisch

In these two lectures a proof, employing convex integration, of the famous Nash–Kuiper Isometric C^1–Embedding Theorem was presented. The theorem states:

Theorem 1. (Nash '54: $n \leq q-2$, Kuiper '55: $n = q-1$) *Any strictly short C^1–immersion $f : (V^n, g) \longrightarrow (\mathbb{R}^q, h_{\mathrm{std}})$, $n < q$, can be C^0–approximated by isometric C^1–immersions. Moreover, if f is an embedding, it can be C^0–approximated by isometric C^1–embeddings.*

A C^1–map $f : (V^n, g) \longrightarrow (W^q, h)$ between (smooth) Riemannian manifolds (V^n, g) and (W^q, h) is by definition *strictly short*, if $f^*h < g$. The above theorem is the essential step to establish (various) h–principles for C^1–immersions. In the course of the proof, first the theorem is modified to hold for *ε–isometric* approximating maps $\tilde{f}$, i.e. maps satisfying $(1-\varepsilon)g < \tilde{f}^*h < (1+\varepsilon)g$, by using parametric one–dimensional convex integration iteratively. Thereby, C^1–control of $\tilde{f}$ is established. Using this C^1–control the ε–isometric approximation can be found to C^1–converge in the limit $\varepsilon \to 0$ to an honest C^1–isometry still C^0–close to the given strictly short map, constituting the desired approximation. As a corollary of the Nash–Kuiper theorem the existence of a C^1–smooth isometric embedding of the standard sphere S^2 or the standard disk D^2, respectively, into an arbitrarily small ball in $\mathbb{R}^3$ follows. In contrast to the theorem the following holds: any C^2–embedding of S^2 into $\mathbb{R}^3$ is conjugate to the standard one.

Applications to material science

Georg Dolzmann

This lecture discusses connections between minimizers for variational problems describing solid-solid phase transformations and solutions of partial differential inclusions (PDIs). A typical model for a cubic to tetragonal transformation is given by: Minimize

$$\int_\Omega W(Du)\mathrm{d}x$$

in $\mathcal{A}_F = \{u \in W^{1,\infty}(\Omega; \mathbf{R}^n),\, u(x) = Fx \text{ on } \partial\Omega\}$ where $W \geq 0$ is the free energy density which satisfies

$$K = \{W = 0\} = \mathrm{SO}(3)U_1 \cup \mathrm{SO}(3)U_2 \cup \mathrm{SO}(3)U_3.$$

An "absolute" minimizer u is therefore a solution of the PDI

$$\begin{cases} Du \in K & \text{a.e. in } \Omega, \\ u(x) = Fx & \text{on } \partial\Omega. \end{cases}$$

The methods developed by Müller and Šverák ensure the existence of solutions if the compact set K admits an in-approximation with (relatively) open sets and if F lies in the (relative) interior of K^{rc}. Explicit formulae for K^{rc} are known for k-well problems in 2×2 matrices with equal determinant and for the general two-well problem in two dimensions.

The N-gradient problem

Agnieszka Kalamajska

We consider Partial Differential Inclusions of the form $Du \in K$, where $K = \{A_1, \ldots, A_N\} \subseteq M^{n\times m}$ consists of N pairwise rank–one disconnected matrices and ask when the solution to every such inclusion is necessarily an affine function. J.M. Ball and R.D. James showed in 1987 that this is true for $N = 2$. Their result was extended by V. Šverák in 1992 to $N = 3$.

We present recent work of M. Chlebík and B. Kirchheim from 2000 who proved that it is also true in the case $N = 4$. The proof uses dimensional and geometric arguments which allow reducing the problem to the case $n = m = 2$ and the following three situations:

(1) $A_1 = 0$ and $\det(A_i) > 0$ for $i = 2, 3, 4$
(2) $w||A_j v$ for $j = 1, \ldots, 4$ and some $w, v \in S^1$;
(3) $A_1, \ldots, A_4$ are symmetric and have the same determinant D

In Case 1) the result follows from Bojarski's theorem on quasiconformal mappings. In Case 2) one shows directly that Du is constant along certain lines and the problem reduces to the two gradient problem. The most nontrivial is Case 3): for $D > 0$ regularity follows from Šverák's result on regularity of the Monge–Ampère equation; the case $D \leq 0$ requires delicate and deep analysis based on maximum–like principles known in the theory of mappings of bounded distortion.

We also report on the recent result of B. Kirchheim and D. Preiss that there exist nonaffine solutions to the 5 gradient problem.

Convex integration of higher order PDE

Ursula Hamenstädt

We discuss how convex integration can be used to find solutions for the following second order PDE for maps $f = (f_1, f_2) \in C^2(U, \mathbb{R}^2)$, $U \subset \mathbb{R}^2$ open: write

$$\begin{aligned} X &= U \times \mathbb{R}^2 \\ X^{(1)} &= \text{space of 1-jets of sections } f \text{ of } X \\ X^c &= \text{space of "jets" of the form } j^c f = (j^1 f, \partial_x \partial_y f) \\ X^\perp &= \text{space of "jets" of the form } j^\perp f = (j^c f, \partial_x^2 f). \end{aligned}$$

Let $A_1 : X^c \mapsto \mathbb{R}$, $A_2 : X^\perp \mapsto (0, \infty)$ be continuous and

$$\partial_x^2 f_1 + (\partial_x^2 f_2)^3 - A_1(j^c f) = 0$$
$$(\partial_y^2 f_1)^2 + (\partial_y^2 f_2)^2 - A_2(j^\perp f) = 0.$$

The best regularity class of solutions produced by convex integration

David Spring

A general problem in Convex Integration Theory is to improve the smoothness class of C^r-solutions to closed differential relations in r-jet spaces and still preserve the C^{r-1}-density results with respect to the initial data consisting of suitable families of short solutions. In general we prove that the smoothness class cannot be improved to class C^{r+1}, while still maintaining C^{r-1}-density results with respect to the initial data. In case $r = 1$, we consider the following first order system associated to $f = (f_1, f_2) : K \to \mathbb{R}^2$, where K is a rectangle in $\mathbb{R}^2$ with coordinates $(x, y) \in \mathbb{R}^2$:

$$(*) \qquad \begin{aligned} (\partial_x f_1)^2 + (\partial_x f_2)^2 &= A(x, y, f) \\ (\partial_y f_1)^2 + (\partial_y f_2)^2 &= A(x, y, f). \end{aligned}$$

We assume $A > 0$. Let $\mathcal{S}ol_p$ be the space of C^p-solutions to the system $(*)$, $p \geq 1$. Let $\mathcal{S}h \subset C^1(K, \mathbb{R}^2)$ be the open subspace in the C^1-topology consisting of short solutions: the map $h = (h_1, h_2)$ is *short* if h satisfies the system $(*)$ with equality in both equations replaced by "$<$". Convex integration theory proves the C^0-density result: $\mathcal{S}h \subset \overline{\mathcal{S}ol_1}$.

If in addition $(A_x \pm A_y)^2 > 0$ on K then we prove that there is an open set $\mathcal{D} \subset \mathcal{S}h$ such that $\mathcal{D} \cap \overline{\mathcal{S}ol_2} = \emptyset$, i.e., the C^2-solutions of $(*)$ cannot satisfy the required C^0-density results on the space of short maps. Similar negative results can be proved for suitable rth order systems analogous to $(*)$, for all $r \geq 1$.

Edited by László Székelyhidi

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