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Algebraische Zahlentheorie

June 22nd – June 28th, 2003

The meeting was organised by Christopher Deninger (Münster), Peter Schneider (Münster) and Anthony Scholl (Durham).

The subject of the conference was Algebraic Number Theory and Arithmetic Algebraic Geometry. In particular there were lectures on recent developments in non-commutative Iwasawa theory and p-adic representations, on vector bundles over p-adic curves, arithmetic Chow groups, the Weil-étale topology for number fields, on regulators and on heights. Of particular interest were the proofs obtained independently by Ito and de Shalit of Grothendiecks monodromy weight conjecture in a special but arithmetically very important case.

Abstracts

Tetsushi Ito (MPI Bonn / University of Tokyo)

Weight-monodromy conjecture for p-adically uniformized varieties

The weight-monodromy conjecture (Deligne's conjecture on the purity of monodromy filtration) claims the coincidence of the weight filtration and the monodromy filtration, up to shift, on the l-adic étale cohomology of a proper smooth variety over a local field. The aim of my talk was to explain ideas how to prove this conjecture for varieties uniformed by Drinfel'd upper half spaces $\hat{\Omega}^d_K$.
Let $K/\mathbb{Q}_p$ be a p-adic field, $l \neq p$, $\Gamma \in \mathrm{PGL}_{d+1}(K)$ a cocompact discrete torsion free subgroup, $X_\Gamma :=$"$\hat{\Omega}^d_{\mathbb{K}}/\Gamma$", the algebraization of the quotient. On $H^w_{et}(X_\Gamma \otimes_K \bar{K}, \mathbb{Q}_l)$ we have the monodromy filtration $M_\bullet$ by Grothendieck's monodromy theorem.
Theorem: *$Gr^M_k(H^w_{et}(X_\Gamma \otimes_K \bar{K}, \mathbb{Q}_l))$ has weight $k+w$. (i.e. action of a lift of the geometric Frobenius has eigenvalues with complex absolute value $q^{\frac{k+w}{2}}$, $q =$cardinality of the residue field).*
The ingredients of the proof are to prove a special case of the Hodge standard conjecture, and apply an argument of Steenbrink, M. Saito to the weight spectral sequence of Rapoport-Zink. As an application, we have the following formula of the local zeta function of X_Γ: $\zeta(s, X_\Gamma) = (1-q^{-s})^{\mu(\Gamma)\cdot(-1)^{d+1}} \cdot \prod_{k=0}^d \frac{1}{1-q^{k-s}}$, where $\mu(\Gamma) = \dim_k H^d(\Gamma, \mathbb{K})$.
We can prove a p-adic version using log crystalline cohomology by the same method. Another proof of it was given by de Shalit and Alon.

Amnon Besser (Ben-Gurion University)

The Karoubi regulator and the syntomic regulator

Karoubi and Villamayor defined topological K-theory for a non-archimedean Banach algebra A by $K^{top}(A) = K(A(\Delta^\bullet))$ where $A(\Delta^n) = \{\sum a_I x_0^{i_0} - x_n^{i_n}, ||a_I|| \to 0\}/(\sum x_i = 1)$. Karoubi later defined relative K-theory as the mapping fibre of $K(A) \to K^{top}(A)$ and a regulator $K^{rel}(A) \to H^\bullet(MF(F^i\Omega^\bullet_A \to \Omega^\bullet_A))$.
We describe a partly conjectural picture that relates this regulator to syntomic regulators. The comparison is done via "syntomic K-theory" defined as the mapping fibre of $K(X) \to^{\psi_p - \phi^\times} K^{rig}(\mathcal{X}_{\mathbb{K}})$ for a smooth $\mathcal{X}/\mathcal{O}_{\mathbb{K}}$, where $\mathbb{K}$ is a p-adic field. K^{rig} is constructed from K^{top} by a standard argument and ψ_p is the p-th Adams operation.

Ehud de Shalit (Hebrew University, Jerusalem)

Monodromy-weight conjecture for the Hyodo-Kato cohomology of p-adically uniformized varieties

If X_K is a proper smooth variety over a p-adic field K with semi-stable reduction, then Hyodo and Kato constructed $\forall 0 \leq m \leq 2\dim X_K$

$$\Phi = \text{Frob.} \curvearrowright D^m := H^m_{log-crys}(\underline{Y}/\underline{W}) \otimes_W K_0 \curvearrowleft N = \text{monodromy operator}$$

the m^{th} log-crystalline cohomology of the special fibre Y of X (with its induced log-structure). This is a (Φ, N)-module in the sense of Fontaine and it carries two increasing filtrations: the *monodromy* filtration, and the *weight* filtration. The latter was constructed by Mokrane, who also conjectured, in analogy with Deligne's MWC for l-adic cohomology, that the two coincide, up to a shift in the indexing.

We describe a proof of the WMC for X_K *which admit a rigid analytic uniformization* $X_K^{an} = \Gamma\backslash\mathcal{X}$ *by the Drinfel'd "upper half space"* modulo a discrete cocompact torsion-free $\Gamma \subset \mathrm{PGL}_{d+1}(K)$.
Since, by a result of E. Große-Klönne, the weight filtration for such an X coincides with the covering filtration on $H^m_{dR}(X_M)$ (after invoking the Hyodo-Kato comparison isomorphism) we are reduced to showing that $\forall r \leq \left[\frac{d}{2}\right]$

$$(\mathrm{gr}_\Gamma N)^{d-2r} : H^r(\Gamma, H^{d-r}_{dR}(\mathcal{X})) \tilde{\to} H^{d-r}(\Gamma, H^r_{dR}(\mathcal{X}))$$

is an isomorphism. In previous work we have identified $H^r_{dR}(\mathcal{X})$ with a certain space C^r_{har} of "harmonic" r-cochains on the Bruhat-Tits building $\mathcal{T}$ of $\mathrm{PGL}_{d+1}(K)$. With Gil Alon we have constructed a certain extension

$$0 \to C^{r-1}_{har} \to \tilde{C}^{r-1}_{har} \to C^r_{har} \to 0$$

so that the connecting homomorphism ν in Γ-cohomology induces an isomorphism

$$\nu^{d-2r} : H^r(\Gamma, C^{d-r}_{har}) \tilde{\to} H^{d-r}(\Gamma, C^r_{har}).$$

We use this to conclude the proof of the MWC by identifying $\mathrm{gr}_\Gamma N$ with ν.

Fargues Laurent (Orsay/CNRS)

Generalized Canonical subgroups and the boundary of Lubin-Tate spaces

The global geometry of the simple Shimura varieties used by M. Harris and R. Taylor suggest the existence of generalized canonical subgroups in some domains of the Lubin-Tate spaces. These spaces are in fact the p-adic tubes over supersingular points in these varieties. Outside these supersingular points the variety is stratified and on each local stratum the universal p-div. group is an extension of two non-trivial one. This suggests, going to the generic fibre, such extension should exist (at least at the level of truncated B.T. groups) by overconvergence on some domains of L. T. spaces.
I prove this by purely local methods (no shimura varieties). I give application to a conjecture of Prasad on autodual representations of division algebras which arise in the cohomology of these L. T. spaces. In fact, the canonical subgroups split the L. T. tower over strata.
I give another application to Strauch's project to achieve Carayol's program by purely local methods.

A. Agboola (University of California, Santa Barbara)

Anticyclotomic main conjectures for CM elliptic curves

(joint work with B. Howard (Harvard University))

We study the Iwasawa theory of a CM elliptic curve $E/\mathbb{Q}$ over the anticyclotomic $\mathbb{Z}_p$-extension of the CM field, where p is a prime of good, ordinary reduction. When the complex L-Function of $E_{|\mathbb{Q}}$ vanishes to odd order, work of Greenberg shows that the Pontryagin dual of the p-primary Selmer group of E over the anticyclotomic $\mathbb{Z}_p$-extension is not a torsion Iwasawa module. We show that the dual of the Selmer group has rank exactly one over the Iwasawa algebra, and we prove one divisibility of an Iwasawa main conjecture for the torsion submodule.

Tadashi Ochiai (University of Tokyo)

Iwasawa theory for nearly ordinary Galois deformation spaces

Our subject is a generalization of Iwasawa theory to Galois deformation spaces which do not necessarily come from $\mathbb{Z}_p^d$-extensions of a number field. (This kind of generalization was first proposed by R. Greenberg in early 90's).
In this talk, I gave a construction of two-variable p-adic L-function $L_p(J) \in \mathbb{Z}_p[[X_1, X_2]]$ for a Hida family $J \cong \mathbb{Z}_p[[X_1, X_2]]^{\otimes 2} \curvearrowleft G_{\mathbb{Q}}$ by means of Coleman map and Beilinson-Kato element. By using this two variable p-adic L-function, we propose our main conjecture.
Iwasawa Main Conjecture:

$$\mathrm{char}_{\mathbb{Z}_p[[X_1,X_2]]}(\mathrm{Sel}_J)^{\vee} = (L_p(J)).$$

Under certain assumptions, we prove one of the divisiblity

$$\mathrm{char}_{\mathbb{Z}_p[[X_1,X_2]]}(\mathrm{Sel}_J)^{\vee} \supset (L_p(J)).$$

For a proof, it is important to establish the theory of Euler systems which works for Galois deformation spaces. We establish such theory via a method which we call "specialization principle". This is roughly a principle which recovers the characteristic ideal of a given torsion Iwasawa module from the behaviour of its specializations (at prime ideals of $\mathbb{Z}_p[[X_1, X_2]]$). Using this "specialization principle" (+ some other technique), we give an example of the equality of the main conjecture and alg. structures of the two variable Selmer group (for the case of Ramanujan's cusp form $\Delta = q\prod_{n\geq 1}(1-q^n)^{24}$ with $p = 11$).

Otmar Venjakob (Universität Heidelberg)

Characteristic elements in non-commutative Iwasawa theory

In the classical Iwasawa theory over the cyclotomic $\mathbb{Z}_p$-extension k_{cyc} of a number field k there is a precise relation between the characteristic polynomial of a torsion Iwasawa-module M and its Γ-Euler characteristic $\chi(\Gamma, M)$ where Γ denotes the Galois group $G(k_{cyc}/k)$. In fact, by results of Perrin-Riou and Schneider independently, for the Pontryagin dual $X_f(k_{cyc})$ of the p-primary Selmer group $\mathrm{Sel}_p(E/k_{cyc})$ of an elliptic curve E defined over k the Euler characteristic $\chi(\Gamma, M)$ equals – under certain conditions – the p-BSD-constant and is related to the value $L(E, 1)$ of 1 of the Hasse-Weil L-function of E.
In the talk we discuss the problem of extending this picture to the case of a p-adic Lie extension k_∞ over k, i.e. $G(k_\infty/k)$ is a (compact) p-adic Lie group. Assuming that k_{cyc} is contained in k_∞ and under further assumptions on G, we suggest a definition of a characteristic element F_M for such $\Lambda(G)$-modules M which are finitely generated over $\Lambda(H) \subseteq \Lambda(G)$ where H denotes the Galois group $G(k_\infty/k_{cyc})$. The existence of $F_M \in \Lambda_T^\times$ relies on the existence of the localisation Λ_T of Λ with respect to a certain set T, i.e. T has to satisfy the Ore-condition, which – in contrast to commutative ring theory – is a highly delicate question. Furthermore we use the identification $K_0(\Lambda, \Lambda_T) \simeq \frac{(\Lambda_T^\times)^{ab}}{\mathrm{im}(\Lambda^\times)}$ and we observe that any such M gives rise to a class in this relative K-gr! oup á la Swan. We go on showing that by evaluation of F_M at "0" we reobtain the G-Euler characteristic $\chi(\Gamma, M)$ (if defined) of M. In certain arithmetic situation we are able to relate the image $\pi_H(F_{X_f(k_\infty)})$ of the characteristic element $F_{X_f(k_\infty)}$ of the dual of Selmer $X_f(k_\infty)$ over k_∞ under the natural projection $\pi_H : \Lambda(G) \to \Lambda(\Gamma)$ to the characteristic polynomial $\mathrm{char}_\Gamma(X_f(k_{cyc}))$ of the $\Lambda(\Gamma)$-module $X_f(k_{cyc})$ considered before.
Finally we discuss the possible shape for the formulation of a Iwasawa Main Conjecture (over p-adic Lie extensions) using our definition of characteristic elements.

John Coates (University of Cambridge)

Fine Selmer groups of elliptic curves over p-adic Lie extensions

(joint work with R. Sujatha)

Let E be an elliptic curve over a finite extension F of $\mathbb{Q}$, and let F_∞ be a p-adic Lie extension of F with Galois group G. We assume that G is pro-p and has no element of order p. The lecture discussed evidence for the following two conjectures. Let $R(E/F_\infty)$ be the submodule of the Selmer group of E over F_∞ consisting of all elements which map to 0 in $H^1(F_{\infty,\nu}, E_{p^\infty})$ for all ν dividing p. Let $Y(E/F_\infty)$ be the Pontrjagin dual of $R(E/F_\infty)$, which is a finitely generated module over the Iwasawa algebra $\Lambda(G)$ of G. Let F^{cyc} denote the cyclotomic $\mathbb{Z}_p$-extension of F. Then we conjecture that $Y(E/F^{cyc})$ is always a finitely generated $\mathbb{Z}_p$-module. We also conjecture that $Y(E/F_\infty)$ is a pseudo- null $\Lambda(G)$-module whenever G has dimension > 1.

Stephen Lichtenbaum (Brown University)

The Weil-étale topology for number fields

Let X be a scheme of finite type over Spec $\mathbb{Z}$. Let n be a non-negative integer and let $\zeta^*(X,-n)$ be the coefficient of the leading term of the Laurent series for $\zeta(X,s)$ at $s=-n$. We attempt to construct a cohomology theory for which the following formula is valid:

$$\zeta^*(X,-n) = \pm\chi_c(X \times \mathbb{A}^n, \mathbb{Z})$$

where χ_c denotes the Euler characteristic of cohomology with compact support.
This cohomology theory should have the following properties:

(1) $H^i_c(X,\mathbb{Z})$ is finitely generated and zero for i large.
(2) $H^i_c(X,\mathbb{Z}) \otimes \mathbb{R} \cong H^i_c(X,\mathbb{R}_{top})$, where $\mathbb{R}_{top}$ denotes the real numbers with its usual topology.
(3) There exists a universal cohomology operation $\cup\psi : H^i_c(X,\mathbb{R}_{top}) \to H^{i+1}_c(X,\mathbb{R}_{top})$ such that $(\cup\psi)^2 = 0$.
(4) The complex $(H^*_c(X,\mathbb{R}_{top}), \cup\psi)$ is acyclic.

We then define the Euler characteristic χ_c as the quotient:

$$\frac{\prod(\# H^i_c(X,\mathbb{Z})_{tor}^{(-1)^i})}{\det(H^i_c(X,\mathbb{Z}), H^i_c(X,\mathbb{R}), \cup\psi)}$$

where "det" refers to the determinant of an exact sequence of finite-dimensional vector spaces with given bases.
We construct a candidate for a Grothendieck topology giving the above cohomology theory by taking a variant of étale topology where we replace Galois groups by Weil groups.

Chandrashekhar Khare (University of Utah / TIFR)

New approaches to modularity of p-adic Galois representations

I presented new approaches to proving modularity of 2-dimensional p-adic Galois representations $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{Z}_p)$ assuming ρ mod p is irreducible and modular, ρ semistable and ρ Barsotti-Tate at p ($p > 3$), and the primes ramified in ρ are not 1 mod p. This reproves by quite different method cases of results of Wiles and Taylor. The proof is by p-adic approximation: one proves that ρ is the limit of modular representations ρ_i. I ended by studying "pro-modular" Galois representations $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{Z}_p)$, i.e., those that arise as limits from classical Galois representations associated to newforms. I gave examples (joint work with Ramakrishna) of such that are infinitely ramified and crystalline at p (proved earlier by Ramakrishna under GRM) and a general result that for a continuous, semisimple representation $\rho : G_{\mathbb{Q}!} \to \mathrm{GL}_n(\mathbb{Q}_p)$, the set of primes that ramify in ρ has density 0 (joint with Rajan).

Niko Naumann (Universität Münster)

Algebraic independence in $K_0(\mathrm{Var}_{\mathbb{F}_q})$

Using weights in étale cohomology we construct a "motivic measure" for varieties over finite fields, i.e. a homomorphism on $K_0(\mathrm{Var}_{\mathbb{F}_q})$, the Grothendieck ring of varieties over the finite field $\mathbb{F}_q$. We use it to derive sufficient cohomological conditions on given varieties to be algbraically independent in $K_0(\mathrm{Var}_{\mathbb{F}_q})$ and as an example exhibit a polynomial sub-ring in infinitely many variables inside $K_0(\mathrm{Var}_{\mathbb{F}_q})$.

G. Böckle (Universität Duisburg-Essen)

A conjecture of de Jong

(joint work with C. Khare (Utah))

Suppose X is a smooth geometrically connected curve over a finite field of characteristic p, and let l be a prime number different from p. Denote by $\pi_1^{geom}(X) \subset \pi_1(X)$ the geometric and arithmetic fundamental groups of X, respectively. In this situation, de Jong conjectures the following: For any representation $\rho : \pi_1(X) \to \mathrm{GL}_n(\overline{\mathbb{F}_l[[T]]})$, the group $\rho(\pi_1^{geom}(X))$ is finite.
For $n \leq 2$ the conjecture was proved by de Jong by generalizing a reciprocity law of Drinfel'd to $\overline{\mathbb{F}_l[[T]]}$-coefficients. The conjecture as well as its proof for $n = 2$ were motivated by a conjecture of Deligne on irred. lisse l-adic sheaves with trivial determinant and its subsequent proof for $n = 2$ by Drinfel'd. To explain some consequences of the conjecture, fix a continuous representation $\bar{\rho} : \pi_1(X) \to \mathrm{SL}_n(\mathbb{F})$, $\mathbb{F}$ of characteristic l, finite, and such that the restriction $\bar{\rho}|\pi_1^{geom}(X)$ is absolutely irreducible. In this situation on obtains a universal ring $R_{X,\bar{\rho}}$ for deformations of $\bar{\rho}$, by results of Mazur. The above conjecture implies that a) $R_{X,\bar{\rho}}$ is finite flat over $\mathbb{Z}_l$ and b), using Lafforgue's recent results on global Langlands, that $\bar{\rho}$ is "automorphic". Using recent lifting techniques of Ramakrishna and methods used in the modularity proofs by Wiles, Taylor, Skinner et al., we can prove a) and b) in many new instances.

Alexander Goncharov (Brown University)

Galois groups and modular varieties

We study the action of the absolute Galois group on the pro-l completion of the fundamental group of the scheme X_N, defined as P^1 minus zero, infinity and all N-th roots of unity. Namely, the Galois action provides a collection of representations, indexed by a pair of positive integers $w \geq m \geq 1$:

$$\phi_{w,m}^{(l)}(\mu_N) : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{Aut}\ \pi_1^{(l)}(X_N, \nu_\infty)/(\mathcal{F}_{-w-1}^W \cdot \mathcal{F}_{-m-1}^D),$$

Here, if $H(n)$ denotes the lower central series of a group H, we set

$$\mathcal{F}_{-w}^W := \pi_1^{(l)}(X_N, \nu_\infty)(w),$$

and $\mathcal{F}^D_{-w} = I_N(m)$, where I_N is defined via the exact sequence

$$0 \to I_N \to \pi_1^{(l)}(X_N, \nu_\infty) \to \pi_1^{(l)}(\mathbb{G}_m) \to 0.$$

Let $\mathbb{Q}_w$ be the field stabilized by $\mathrm{Ker}\phi^l_{w,w}(\mu_N)$ and $\mathbb{Q}_{w,m}$ the field stabilized by $\mathrm{Ker}\phi^l_{w,m}(\mu_N)$. $\mathcal{G}^l_w = \mathrm{Gal}(\mathbb{Q}_w/\mathbb{Q}_{w-1}) \otimes \mathbb{Q}_l$, then $\mathcal{G}^j_\bullet(\mu_N) = \bigotimes_{w\geq 1} \mathcal{G}_w$ has a graded Lie algebra structure.

Theorem:

$$H^m_{(m)}(\mathcal{G}^l_\bullet(\mu_p)) \simeq H_{m-1}(Y_1(m;p);\mathbb{Q}_l)$$

where $Y_1(m;N) = \Gamma_1(m;N)\backslash GL_2(\mathbb{R})/\mathcal{O}_m \cdot \mathbb{R}^\times_+$.

Similarly one defines a bigraded Lie algebra $\mathcal{G}^l_{\bullet,bullet}(\mu_N)$. We proved that the degree (m,m) piece of the i-th cohomology of the bigraded Lie algebra $\mathcal{G}^l_{\bullet,bullet}(\mu_N)$ are related to the $2m-2-i$th Borel Moore homology homology of the modular variety $Y_1(m;p)$ for $m = 1,2,3,4$

J. Bellaïche (Université de Nice)

A LEVEL-RAISING THEOREM FOR $U(3)$ AND APPLICATIONS

I presented and proved the following theorem: Let $G = U(3)$ be the unitary group over $\mathbb{Q}$ attached to a quadratic imaginary extension $E/\mathbb{Q}$, which is compact at infinity. Let $B = I_p \times K^p \subset G(\mathbb{A}_{\mathbb{Q},p})$ where I_p is a parahoric subgroup of $G(\mathbb{Q}_p)$ and K^p is a compact open subgroup. Denote by $O_{B,R}$ and $N_{B,R}$ the R-module of automorphic forms of level B, defined over R, old and new respectively. Let η be a character of the Hecke-algebra in $O_{B,R}$ with values in a d.v.r. $R \subset \mathbb{C}$, of maximal ideal μ. Then if $\eta(I_p) \equiv p(p^3+1)[\mu^n](n \in \mathbb{N})$, then their exists $f \in O_{B,R}$, eigenform of character η, and $g \in N_{B,R}$ s.t. $f \not\equiv g[\mu^{\lceil \frac{n}{2} \rceil}], f \not\equiv 0[\mu]$. The theorem works also for E a CM extension, and when replacing $O_{B,R}, N_{B,R}$ by spaces of automorphic forms with fixed height and Bushnell-Kutsko types.
As an application, we prove the compatibility of the Blasius-Rogavski construction on a Galois representation on $\bar{\mathbb{Q}}_l$ attached to an automorphic representation on G with the local Langlands correspondence (we have to exclude a set of density 0 of l) and the Ramanujan conjecture for G.

F. Jarvis (University of Sheffield)

MODULARITY OF CERTAIN ELLIPTIC CURVES OVER TOTALLY REAL FIELDS.

(joint work with Jayanta Manoharmayum)

C. Skinner and A. Wiles have demonstrated the modularity of many elliptic curves with "nearly ordinary" Galois representations over totally real fields (and also some with reducible mod p Galois representations). Following methods of R. Taylor, and refining arguments originally presented by Wiles and Taylor-Wiles, we state and prove a theorem concerning modularity of deformations of mod p Galois representations whose form is that of the mod p Galois representation associated to elliptic curves with good supersingular reduction, at least if p is unramified in the totally real field. We present one application to the modularity of elliptic curves with supersingular reduction. We remark that the modularity of all semistable elliptic curves over totally real fields implies the modularity of all elliptic curves over totally real fields, and give an example of a field $\mathbb{Q}(\sqrt{2})$ over which our results imply modularity of all semistable curves. An application to the Fermat equation over this field is also given.

P. Autissier (Université d'Orsay)

Correspondences and heights

The Néron-Tate height on an abelian variety is known to be constructed from any Weil height by a dynamical process. Similarly, we wish to obtain (dynamically) the Faltings height on a moduli space of abelian varieties by iterating a Hecke correspondence. In this direction, we study the convergence of metrices under this iteration. As an application, we give formulas on the average height of isogenous elliptic curves (resp. QM abelian surfaces).

U. Kühn (Humboldt-Universität Berlin)

On cohomological arithmetic Chow groups and applications.

(joint work with J.-I. Burgos (UB Barcelona) and J. Kramer (Humboldt-Universität Berlin))

We constructed abstract versions of arithmetic Chow groups. The main idea in this new approach to higher dimensional Arakelov theory is that one replaces the analytical constructions in the work of Gillet-Soulé by homological methods. Using this newly defined cohomological arithmetic Chow groups we are able to obtain generalizations of the usual $\widehat{\mathrm{CH}}(X)$ that can be used to calculate arithmetic Chern numbers for compactifications of Shimura varieties. At the end we discussed some examples.

Annette Werner (Universität Münster)

Vector bundles and p-adic representations

(joint work with C. Deninger)

Let X be a smooth, projective, geometrically connected curve over a finite extension K of $\mathbb{Q}_p$. We define a full subcategory $\mathcal{B}_{X_{\mathbb{C}_p}}$ of the category of vector bundles on $X_{\mathbb{C}_p}$ containing all line bundles of degree zero and all extensions of the trivial line bundle with itself, and a functor $\rho : \mathcal{B}_{X_{\mathbb{C}_p}} \to \mathrm{Rep}_{\pi_1(X_{\overline{K}},\overline{x})}(\mathbb{C}_p)$ in the category of continuous representations of $\pi_1(X_{\overline{K}},\overline{x})$ on finite-dimensional $\mathbb{C}_p$-vector spaces. This functor ρ is compatible with direct sums, tensor products, internal Homs, pullbacks, exact sequences and Galois-action. ρ induces a homomorphism on Yoneda-Ext groups which can be identified with the Hodge-Tate map $H^1(X,\mathcal{O})\otimes\mathbb{C}_p \to H^1_{et}(X_{\overline{K}},\mathbb{Q}_p)\otimes\mathbb{C}_p$. We also define a category of vector bundles on abelian varieties with good reduction giving rise to p-adic representations of the Tate module and discuss its relation to the Hodge-Tate decomposition.

J. Teitelbaum (University of Illinois at Chicago)

Distribution algebras of p-adic analytic groups

(joint work with P. Schneider (Münster))

Motivated by questions in representation theory we study the algebra $D(G,K)$ of locally analytic distributions on a compact L-analytic group where $L/\mathbb{Q}_p$ is finite and K/L is discretely valued. We show that $D(G,K)$ is a "Fréchet-Stein Algebra". This means that $D(G,K)$ is a projective limit of noetherian Banach algebras with flat transition maps. A Fréchet-Stein algebra has an associated abelian category of "coadmissible" modules that includes all finitely presented modules and is closed under passage to finitely generated

submodules. Thus, for example, finitely generated left ideals in $D(G, K)$ are coadmissible; and as a consequence closed.
The methods of proof involve Lazard's theory of the completed group ring of p-valued groups and the theory of filtered rings.

Edited by Gabriel Herz

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