Mathematisches Forschungsinstitut Oberwolfach

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Topologie

September 14th – September 20th, 2003

The conference was organized by Cameron Gordon (Austin, Texas, USA), Wolfgang Lück (Münster, Germany), and Bob Oliver (Paris, France). It was attended by more than forty mathematicians from all over Europe and North America.

A highlight of the meeting was a series of three lectures by Ib Madsen (Århus, Denmark), and an expository talk by Michael Weiss (Aberdeen, UK), about their proof of the Mumford Conjecture on the stable cohomology of the mapping class group—surely one of the highest achievements in topology of the past years.

In line with the well-established tradition of the "Topologie Tagung", the remaining fifteen talks covered a wide variety of areas of current research in algebraic topology and related fields—such as algebraic K-theory, (stable) homotopy theory, p-compact groups, geometric group theory, L^2-cohomology, three and four dimensional manifolds, positive scalar curvature, and topological quantum field theory.

With an average of four one-hour lectures a day, the participants also had plenty of time for discussion and research. As usual the staff of the Mathematisches Forschungsinstitut Oberwolfach provided all the ideal conditions for a successful meeting.

Abstracts

Homotopy algebraic *K*-theory and groups acting on trees

Arthur Bartels

(joint work with Wolfgang Lück)

The isomorphism conjecture in algebraic K-theory predicts that the algebraic K-theory of a group ring $R\Gamma$ consists of two summands: the first comes from the algebraic K-theory of finite subgroups of Γ; the second comes from Nilgroups of virtually cyclic subgroups. Homotopy algebraic K-theory is a variant of algebraic K-theory that does not contain Nilgroups. The isomorphism conjecture for homotopy algebraic K-theory predicts then that the homotopy algebraic K-theory of a group ring comes from the homotopy algebraic K-theory of finite subgroups. We prove the following stability property of this conjecture: If a group Γ acts on a tree such that all isotropy subgroups satisfy the conjecture, then Γ satisfies the conjecture. A corollary is that the conjecture holds for one relator groups.

Nullification and cellularization of *H*-spaces with respect to *p*-torsion Eilenberg-MacLane spaces

Natàlia Castellana

(joint work with Juan A. Crespo and Jérôme Scherer)

A result obtained by C. Broto, J.A. Crespo and L. Saumell on the structure of Hopf spaces (H-spaces) shows that, under certain mod p cohomological finiteness conditions (p a prime number), p-completed H-spaces fit into fibrations

$$K(\mathbb{Z}_p^\wedge, 2)^m \times K(\pi, 1) \to X \to Y$$

where π is a finitely generated abelian p-group and Y is a p-completed $B\mathbb{Z}/p\mathbb{Z}$-null space (that is, $Map_*(B\mathbb{Z}/p\mathbb{Z}, X) \simeq *$).

The assumptions on X can be generalized in a way that we consider H-spaces such that $\Omega^m X$ is $K(B\mathbb{Z}/p\mathbb{Z}, n)$-null for some m, n.

Results of Bousfield allow us to generalize the previous statement for H-spaces X satisfying these conditions. It is shown that they fit into fibrations with $K(B\mathbb{Z}/p\mathbb{Z}, n)$-null base space and whose fibre is a p-torsion finite Postnikov piece.

In particular, these structure results allow us to compute the $K(B\mathbb{Z}/p\mathbb{Z}, m)$-cellularization of those H-spaces for $m \geq n$, which will be an $(m-1)$-connected finite Postnikov piece.

Decompositions of Hecke-von Neumann algebras and the L^2-cohomology of buildings

Mike Davis

(joint work with Jan Dymara, Tadeusz Januszkiewicz, and Boris Okun)

The reduced L^2-cohomology of a CW complex lies somewhere between its ordinary cohomology and its cohomology with compact support. One of the most interesting interpretations of the previous sentence involves the theory of "weighted L^2-cohomology" of the complex Σ associated to a Coxeter system (W, S). The weight function is of the form q^d, where d is the combinatorial distance to a base point and q is a positive real number. The corresponding weighted L^2-cohomology spaces are modules over the "Hecke-von Neumann algebra" associated to W with parameter q. Consequently, these spaces have a von Neumann dimension; the resulting real numbers are the *L^2_q-Betti numbers* of Σ. Dymara proved that the L^2_q-Betti numbers of Σ are equal to the L^2-Betti numbers of any building of type (W, S) and thickness $q+1$ (with respect to the von Neumann algebra of a chamber-transitive automorphism group G.) As q goes from 0 to ∞, these weighted L^2-cohomology spaces interpolate between the ordinary cohomology of Σ and its cohomology with compact support. There is a precise formulation of this involving the radius of convergence of the growth series of W.

Positive curvature and index theory

Anand Dessai

I wanted to discuss potential index theoretical obstructions to positive Ricci and positive sectional curvature. More precisely, I discussed what is known about the following

Problem. *Which characteristic numbers (i.e. linear combinations of Pontrjagin numbers) vanish on a closed manifold with positive sectional (resp. positive Ricci) curvature?*

At present this problem is wide open: Besides the vanishing of the $\hat{A}$-genus for spin manifolds of positive scalar curvature no other obstructions (in the above sense) are known—even if one restricts to manifolds of positive sectional curvature.

In the first part of the talk I recalled Stolz' conjecture for the Witten genus of string manifolds with positive Ricci curvature and gave an update on the evidence (in terms of examples) for this conjecture.

In the second part I discussed potential index theoretical obstructions to positive sectional curvature for a spin manifold M of dimension > 8. These obstructions come as a series given by an expansion of the elliptic genus. The first two terms are the $\hat{A}$-genus and $\hat{A}(M, TM)$, the index of the Dirac operator twisted with the tangent bundle. It is conceivable that $\hat{A}(M, TM)$ vanishes on any such manifold. The main evidence for this is the following result.

Theorem. *Let M be a spin manifold of dimension > 8 with positive sectional curvature. Then the twisted Dirac index $\hat{A}(M, TM)$ vanishes if*

(1) $b_2(M) = 0$ *and the dimension of the isometry group is* > 0 *or*
(2) *the dimension of the isometry group is* > 1.

On the topological side the proof relies on consequences of the rigidity theorem for cyclic group actions. On the geometrical side we use results of Frankel and Wilking about totally geodesic submanifolds in positive sectional curvature.

L^2-Betti numbers and measured equivalence relations

DAMIEN GABORIAU

When a group acts on a space X, it defines an equivalence relation: "to be in the same orbit", $\mathcal{R}_\alpha := \{(x, y) \in X \times X : \Gamma.x = \Gamma.y\}$.

We are interested in the situation where X is a standard Borel space with a probability measure μ, Γ is discrete countable and the action is measure preserving. Considering the equivalence relation amounts to forget about the action and the group, and the basic question is: "what does the equivalence relation remember from that?" Two actions of two groups Γ_1 and Γ_2 are said to be orbit equivalent (OE) if they define the same equivalence relation.

(1) I present a small account of the theory of OE, by first isolating the amenable world (Dye and Ornstein-Weiss theorems) and then describing some rigidity results connected with Kazhdan property (T) or higher rank in semi-simple Lie groups.

(2) The notion of ℓ^2 Betti numbers β_n, introduced by Atiyah (1976) for coverings of a compact manifold, has been gradually extended to other contexts: to measured foliations (Connes 1979), to any continuous action via singular ℓ^2 cohomology, with the consequence of being defined for every discrete group (Cheeger-Gromov 1986), and also to measured equivalence relations $\mathcal{R}$ (i.e. of the kind considered above) (G. 2000). I describe how to define the β_n for groups with finite $K(\Gamma, 1)$ and list their main properties.

(3) We focus on some applications of the theory of ℓ^2 Betti numbers for equivalence relations.
   - To foliations: when the leaves are contractible, the β_n only depend on the equivalence relation generated by the holonomy pseudogroup on a total transversal.
   - To group theory: proportionality of the β_n of any two lattices in a given l.c.s.c. group; vanishing of some β_n of a group, given finiteness conditions on a normal subgroup; a Schreier-like result: "a finitely generated normal subgroup of a group with $\beta_1 \neq 0$ has to be finite or of finite index".
   - To von Neumann algebras: due to a simple formula relating the β_n of $\mathcal{R}$ with the β_n of a restriction, it happens that the dynamical fundamental group $\mathcal{F}(\mathcal{R})$ of $\mathcal{R}$ (= a witness of the self-similarities of $\mathcal{R}$) is reduced to $\{1\}$ as soon as one of the β_n is different from 0 and ∞. This is the case, for instance, for the natural action α of $\mathrm{SL}(2, \mathbf{Z})$ on the 2-torus. S. Popa (2002) proved that in this example the associated von Neumann algebra M_α has its fundamental group $\mathcal{F}(M_\alpha)$ (= a witness of the self-similarities of M_α) that coincides with that of $\mathcal{R}_\alpha$. This leads to the first example of a factor von Neumann algebra with trivial fundamental group.

(4) I eventually introduce a joint work with N. Bergeron. Let K be a finite simplicial complex. We are interested in the asymptotic behaviour of the Betti numbers of a sequence of finite sheeted covers of K, when normalized by the index of the covers. W. Lück has proved that for regular covers, these sequences of numbers converge to the ℓ^2 Betti numbers of the associated (in general infinite) limit regular cover of K. For non normal finite coverings, the sequences of normalized Betti numbers still converge, but the "good" limit object is no longer the associated limit cover, but a lamination by simplicial complexes and its ℓ^2 Betti numbers.

Dissolving four manifolds and positive scalar curvature

BERNHARD HANKE
(joint work with Dieter Kotschick and Jan Wehrheim)

For every finite cyclic group G of odd order we construct closed smooth four manifolds M with fundamental group isomorphic to G and such that M does not admit a metric of positive scalar curvature, whereas the universal cover of M does admit such a metric. This contradicts a conjecture of J. Rosenberg (1986). Our construction uses results from Seiberg-Witten theory. Both spin and non spin examples can be obtained. For the spin case it is important to know the existence of a simply connected spin symplectic four manifold of zero signature which after taking the connected sum with $S^2 \times S^2$ is diffeomorphic to a connected sum of copies of $S^2 \times S^2$. This leads to the geography problem for almost dissolving simply connected spin symplectic four manifolds.

Derived functors of modular forms via group cohomology

HANS-WERNER HENN

Derived functors of modular forms form the E_2-term of a spectral sequence converging towards the homotopy groups of the spectrum TMF of topological modular forms introduced by Hopkins. After completing at a prime p (which is implicit in the notation below) this spectrum can be described as a pullback of the form

$$\begin{array}{ccc} TMF & \longrightarrow & L_{K(2)}TMF \\ \downarrow & & \downarrow \\ L_{K(1)}TMF & \longrightarrow & L_{K(2)}L_{K(1)}TMF \end{array}$$

where $L_{K(n)}$ denotes Bousfield localization with respect to the n-th Morava K-theory at the prime p. From a homotopy theoretic point of view TMF is particularly interesting at the primes $p = 2$ and $p = 3$. In this case $L_{K(2)}TMF$ can be identified with the "higher real K-theory" spectrum EO_2 of Hopkins and Miller, while $L_{K(1)}TMF$ can be identified with real K-theory with coefficients in p-adic modular forms. Conversely, it has been proposed that TMF can be constructed as pullback of the above diagram.

In this talk I presented an approach to a calculation of these derived functors (completed at $p = 2$ resp. $p = 3$) which reflects the construction of TMF in terms of this pullback diagram. In fact, there is a pullback diagram of p-complete graded comodule algebras over the Hopf algebroid of Weierstrass equations of the form

$$\begin{array}{ccc} A_* & \longrightarrow & B_* \\ \downarrow & & \downarrow \\ A_*[c_4^{-1}] & \longrightarrow & B_*[c_4^{-1}] \end{array}$$

where A_* represents the functor which associates to a commutative ring the set of Weierstrass equations of smooth elliptic curves together with an invariant differential on the curve. B_* is the completion of A_* at the ideal defining the locus of supersingular curves. If $p = 2$ resp. $p = 3$ this ideal is the maximal ideal of A_* and is generated by p and the modular form c_4.

The diagram should be thought of as a Mayer-Vietoris diagram corresponding to the covering of the "space of all Weierstrass equations" by a formal neighbourhood of the locus of supersingular curves and its open complement. The pullback diagram gives rise to a long exact Mayer-Vietoris sequence relating the cohomology of these comodules. The cohomology of A_* is, by definition, derived functors of modular forms. The cohomology of the other modules can be identified with the cohomology of suitable finite groups acting on suitable rings and then in turn with the E_2-terms of previously known homotopy fixed point spectra (which serve as candidates for the localizations of TMF in the homotopy theoretic pullback diagram above). For example, the cohomology of B_* can be identified with the cohomology of the automorphism group of "the" supersingular curve with coefficients in the ring classifying its deformations. It is the E_2-term of the homotopy fixed point spectral sequence converging towards the homotopy groups of the spectrum EO_2.

Isotopies of surfaces in triangulated 3-manifolds

Simon King

We presented proof techniques based on isotopies of surfaces in a closed orientable triangulated 3-manifold (M, T) that can be used in various contexts. The "thin position" technique is well-known: Roughly, a thin position isotopy $H : S \times [0, 1] \to M$ of a surfaces S yields some level surface $H(S \times \{\xi\})$ that interacts with T in a "nice" way. The "reduction" technique is in some sense complementary to thin position: We start with a "nicely" embedded surface $S \subset M$ and read off from $S \cap T^2$ an isotopy with useful properties.

The main application in the talk was a new and rather short proof of a result of Stocking [2000].

Theorem. *Any strongly irreducible Heegaard surface of a closed orientable triangulated 3-manifold is isotopic to either*

(1) *a 2-normal surface with exactly one octagon, or*
(2) *an almost 1-normal surface with exactly one unknotted tube.*

This theorem is part of Rubinstein's approach to prove that any non-Haken 3-manifold has only finitely many isotopy classes of Heegaard surfaces of minimal genus.

We also presented applications to knot theory. Let T be a triangulation of S^3 with n tetrahedra, and let $L \subset T^1$ be a link formed by edges of T. With a generalization of the bridge number, $b(\cdot)$, to the 1-skeleton T^1 (which is a spatial graph), we obtain

Theorem. $b(L) \leq b(T^1) < 2^{200n^2}$, *and there is no general sub-exponential upper bound for $b(L)$ in n.*

Note that Lickorish [1991] and Armentrout [1994] obtained $b(L) < const \cdot n$, under the strong additional hypothesis that T or its dual is *shellable*.

Surprisingly, $b(T^1)$ is closely related to notions from Discrete Geometry. This led to the following estimate for the crossing number, $cr(\cdot)$.

Theorem. $cr(L) < const \cdot (b(T^1))^4 < 2^{const \cdot n^2}$.

Such a bound holds for only finitely many links, and was out of reach even in the case of shellable triangulations.

The stable moduli space of Riemannian surfaces

Ib Madsen

The three lectures outlined the proof of the Madsen-Weiss theorem about the stable cohomological structures of Riemann surfaces or, what is roughly the same thing, the stable mapping class group.

Let $F = F_{g,b}$ be a surface of genus g and with b (parametrised) boundary circles, and let $\Gamma_{g,b}$ be the associated mapping class group:

$$\Gamma_{g,b} = \pi_0 \operatorname{Diff}(F_{g,b}).$$

Here $\operatorname{Diff}(F_{g,b})$ is the topological group of orientation preserving diffeomorphisms of F that keeps the boundary pointwise fixed. There is a map

$$\rho\colon B\Gamma_{g,b} \to \mathcal{M}(F_{g,b})$$

into the moduli space of hyperbolic surfaces of type $F_{g,b}$. $H_*(\rho; \mathbf{Z})$ is an isomorphism when $b > 0$ and $H_*(\rho; \mathbf{Q})$ is an isomorphism for $b = 0$.

The Harer-Ivanov stability theorem tells us that the natural maps

$$B\Gamma_{g,b} \to B\Gamma_{g+1,b}\ , \qquad B\Gamma_{g,b} \to B\Gamma_{g,b-1} \qquad (b > 0)$$

induce isomorphisms on $H_*(-; \mathbf{Z})$ for $2* < g - 1$. Take $b = 2$ for convenience. Gluing along a boundary component gives a pairing $\Gamma_{g,2} \times \Gamma_{h,2} \to \Gamma_{g+h,2}$ and $\coprod B\Gamma_{g,2}$ becomes a topological monoid with group completion

$$\mathbf{Z} \times B\Gamma^+_{\infty,2} = \Omega B\Big(\coprod_{g\geq 0} B\Gamma_{g,2}\Big).$$

Main Theorem (Madsen-Weiss). *There is a homotopy equivalence*

$$j^1\colon \mathbf{Z} \times B\Gamma^+_{\infty,2} \xrightarrow{\simeq} \Omega^\infty \mathbf{C}P^\infty_{-1}.$$

The source is an infinite loop space (U. Tillmann) and j^1 is an infinite loop map (Madsen-Tillmann). The target $\Omega^\infty \mathbf{C}P^\infty_{-1}$ is the infinite loop space associated with the spectrum $\mathbf{C}P^\infty_{-1}$. There is a fibration sequence

$$\Omega^\infty \mathbf{C}P^\infty_{-1} \to \Omega^\infty S^\infty(\mathbf{C}P^\infty_+) \to \Omega^{\infty+1} S^\infty$$

and since $\Omega^{\infty+1} S^\infty$ is rationally a point (Serre), it is easy to see that

$$H^*(\Omega^\infty \mathbf{C}P^\infty_{-1}; \mathbf{Q}) = \mathbf{Q}[\kappa_1, \kappa_2, \ldots], \qquad \deg \kappa_i = 2i.$$

The main theorem thus implies Mumford's conjecture about the stable cohomology of the mapping class group.

There is a geometric interpretation of the spaces in the theorem. Consider the following two sets associated with a smooth m-manifold X^m. The first set $\mathcal{V}(X)$ consists of pairs (π, f) with:

1) $\pi\colon E^{m+3} \to X^m$ a submersion,
2) $f\colon E^{m+3} \to \mathbf{R}$ fibrewise regular (i.e. $d(f|E_x)$ is surjective for all $x \in E$ and $E_x = \pi^{-1}(x)$),
3) $f \times \pi\colon E \to X \times \mathbf{R}$ proper,
4) $\partial E = X \times \{0,1\} \times S^1 \times \mathbf{R}$; $f|\partial E = \mathrm{pr}_{\mathbf{R}}$, $\pi|\partial E = \mathrm{pr}_X$, and
5) $f^{-1}(0) \cap E_x$ connected.

The concordance classes $\mathcal{V}[X]$ of these structures form a representable functor in the homotopy category. Its representing space is $|\mathcal{V}| \simeq \coprod B\Gamma_{g,2}$.

The second set $h\mathcal{V}(X)$ consists of pairs $(\pi, \hat{f})$ with π as above and

$$\hat{f}\colon T_\pi E \to \mathbf{R} \qquad \text{fibrewise affine, non-singular,}$$

i.e., $\hat{f}_z = f(z) + \ell_z$ for $z \in E$ with $f \in C^\infty(E, \mathbf{R})$ and ℓ_z a non-zero linear map. (π, f) is required to satisfy 3) and 4) above. The associated classifying space $|h\mathcal{V}|$ is homotopy equivalent to $\Omega^\infty \mathbf{C}P^\infty_{-1}$. The map

$$j^1\colon \mathcal{V}(X) \to h\mathcal{V}(X), \qquad j^1(\pi, f) = (\pi, f + df)$$

induces the map of the theorem, which can therefore be interpreted as an "integrability result up to group completion".

Inspired by Vassiliev's "first main theorem" one enlarges $\mathcal{V}(X)$ resp. $h\mathcal{V}(X)$ from fibrewise regular functions to fibrewise Morse functions reps. Morse type tangential maps. Call these sets $\mathcal{W}(X)$ and $h\mathcal{W}(X)$. The map

$$j^2\colon \mathcal{W}(X) \to h\mathcal{W}(X), \qquad j^2 f = \text{“}f + d_\pi f + d_\pi^2 f\text{”}$$

induces a map of classifying spaces, and

Theorem 2. $j^2\colon |\mathcal{W}| \to |h\mathcal{W}|$ *is a homotopy equivalence.*

(The proof uses Vassiliev's theorem and an interpretation of $\mathcal{W}[X]$ and $h\mathcal{W}[X]$ as concordance classes of "Steenrod type coordinate bundles".)

Given $(\pi, f) \in \mathcal{W}(X)$ we have the singularity set

$$\Sigma(\pi, f) = \bigcup_{x \in X} \Sigma(f_x).$$

It is a codimension 3 submanifold of E, and

$$p = \pi|\Sigma\colon \Sigma(\pi, f) \to X$$

is a local diffeomorphism (étale).

A parametrised form of the Morse lemma tells us that the normal bundle $V = T_\pi E|\Sigma(\pi, f)$ of $\Sigma(\pi, f) \subset E$ has a Riemannian metric $\langle -, - \rangle$, an orthogonal decomposition $V = V^+ \oplus V^-$, and a tubular embedding $\lambda\colon V \to E$ so that

$$f \circ \lambda(v) = f(z) + |v_+|^2 - |v_-|^2 \qquad \text{(for small } |v|\text{)}.$$

This local information is collected in the set $\mathcal{W}_{\text{loc}}(X)$. There is also an analogous set $h\mathcal{W}_{\text{loc}}(X)$. These constructions lead to the diagram

$$\begin{array}{ccccc}
|\mathcal{V}| & \longrightarrow & |\mathcal{W}| & \longrightarrow & |\mathcal{W}_{\text{loc}}| \\
\downarrow{\scriptstyle j^1} & & \downarrow{\scriptstyle j^2} & & \downarrow{\scriptstyle j} \\
|h\mathcal{V}| & \longrightarrow & |h\mathcal{W}| & \longrightarrow & |h\mathcal{W}_{\text{loc}}|
\end{array}$$

The two right-hand j-maps are homotopy equivalences. The lower sequence is (by calculation) a homotopy fibration, and finally we have the key result, based in part on the Harer-Ivanov stability theorem:

Theorem 3. *The homotopy fibre of $|\mathcal{W}| \to |\mathcal{W}_{\mathrm{loc}}|$ is homology equivalent to $\mathbf{Z} \times B\Gamma_{\infty,2}$.*

But $|h\mathcal{W}|$ turns out to be an infinite loop space, and the same will be the case for $|\mathcal{W}|$ according to Theorem 2. It follows that the homotopy fibber in Theorem 3 is an infinite loop space. It is homology equivalent to $\mathbf{Z}\times B\Gamma_{\infty,2}$, thus homotopy equivalent to $\mathbf{Z}\times B\Gamma^{+}_{\infty,2}$. A five-lemma argument completes the proof of the Main Theorem.

All the above represents joint work with Michael Weiss, cf. `arXiv math. AT/0212321`.

New finite loop spaces

Erik Pedersen

(joint work with Kasper Anderson, Tilman Bauer, and Jesper Grodal)

In their paper, *Finite H-spaces and algebras over the Steenrod algebra*, Adams and Wilkerson mention the following problem as an old problem in Homotopy Theory: Does there exist an H-space with a classifying space so that the rational cohomology is different from the rational cohomology of every Lie-group? It was mostly believed that such H-spaces do not exist. We do however construct such an H-space. The rank, the number of exterior generators in the rational cohomology, in our example is 66, which by a computer calculation is minimal. The dimension is 1254. We do know an example of higher rank but dimension 1250. We do not know the minimal dimension.

Eta invariants and applications to positive scalar curvature and homotopy invariants

Thomas Schick

Given the Dirac operator D on a closed spin manifold M and a finite dimensional representation σ of the fundamental group, we can define the eta-invariant of D_σ, D twisted by the flat bundle associated to the representation. Using the regular representation (which is infinite dimensional if π is infinite) and an appropriate normalization procedure, in a similar way the L^2-eta invariant is defined.

The difference of the L^2-eta invariant and the ordinary eta-invariant is the L^2-rho invariant $\rho_{(2)}(D)$.

We discussed the following two theorems.

Theorem. *If π contains odd torsion and the dimension of M is congruent to 3 mod 4, then there are infinitely many different bordism classes of metrics of positive scalar curvature on M, provided at least one such metric exists.*

Theorem. *If π is torsion free and the assembly map $KO(B\pi) \to KO(C^*_{max}\pi)$ is an isomorphism, then the L^2-rho invariant vanishes.*

Key ingredients are appropriate versions of the Atiyah-Patodi-Singer index theorem, together with Gromov-Lawson surgery constructions for metrics of positive scalar curvature.

Units of ringspectra and their traces in algebraic K-theory

Christian Schlichtkrull

When R is a discrete ring the natural map from the units of R to the algebraic K-theory is realized by a map of spaces

$$BGL_1(R) \to K(R)$$

and if R is commutative this is split by the determinant. In the case of a ring spectrum the above map still exists, but is in general not split even if R is commutative. In this talk we identify the composition

$$BGL_1(R) \to K(R) \to THH(R) \to \Omega^\infty(R)$$

when R is a commutative ring spectrum. As a corollary we show that classes in $\pi_i R$ not annihilated by the stable Hopf map give rise to non-trivial classes in $\pi_i K(R)$.

From sporadic simple groups to exotic p-local finite groups

Antonio Viruel

The theory of p-local finite groups is a generalization of the classical theory of finite groups in the sense that every finite group leads to a p-local finite group, but there exist exotic p-local finite groups which are not associated to any finite group. Therefore, the classification of p-local finite groups has interest not only by itself but as an opportunity to enlighten one of the highest mathematical achievements of the last decades: The Classification of Finite Simple Groups. This Classification of Finite Simple Groups provides 26 mathematical gems, 26 sporadic finite simple groups that enjoy an intriguing property: if G is a sporadic finite simple group with p-Sylow $S \leq G$, $p > 2$, of order p^3, then S is isomorphic to the extraspecial group of order p^3 and exponent p, denoted by p_+^{1+2}, and $p \leq 13$. This fact, partially explained in this lecture, is the starting point for the classification of p-local finite group over the p-groups of type p_+^{1+2}.

Outer space, graph homology, and the rational cohomology of $Out(F_n)$

Karen Vogtmann

Outer space is a contractible space on which the group $Out(F_n)$ acts with finite stabilizers. It was introduced in the mid-1980's and has proved very useful in particular for studying the cohomology of $Out(F_n)$. In this talk I will recall the definition of Outer space and several of the major cohomological applications. I will then describe two other aspects of Outer space. First, I will define Kontsevich's graph homology, and show how to identify the graph homology chain complex with the relative chains of Outer space modulo its simplicial boundary, twisted by the determinant action of $Out(F_n)$ on the reals. In joint work with Jim Conant, we show how this geometric realization can be used to produce a chain complex quasi-isomorphic to the graph homology chain complex. This new chain complex carries a Lie bialgebra structure, and is considerably smaller than the graph homology chain complex, so can be used to simplify the computation of graph homology. I will then define surface subcomplexes of Outer space, each of which can be identified with the ribbon graph complex of a surface with punctures. These subcomplexes cover Outer space, and it is a theorem of M. Horak that intersections are either empty or contractible. Thus the action of $Out(F_n)$ on the nerve of this cover produces a spectral sequence converging to the cohomology of $Out(F_n)$. The stabilizer of a surface subcomplex is the mapping class group of the associated surface, so that this spectral sequence relates the cohomology of the mapping class subgroups of $Out(F_n)$ with the cohomology of $Out(F_n)$.

Minimal generating sets of Fuchsian groups

Richard Weidmann

The rank of a group G is the minimal number of elements needed to generate G. In this talk we discuss the rank of Fuchsian group. For Fuchsian groups not containing reflections the rank was computed by Zischang, Rosenberger and Peczynski, furthermore all 2-generated Fuchsian groups were classified by Klimenko and Sakuma. In this talk we show how to compute the rank of Fuchsian groups generated by reflections, i.e the rank of planar Coxeter groups. This also gives a complete solution for non-cocompact Fuchsian groups.

We reduce the computation of the rank of a planar Coxeter group to the computation of a simple combinatorial complexity of an associated labeled graph. This reduction is achieved by approximating the given Coxeter group with fundamental groups of graphs of groups that are related by folding moves as discussed by Stallings, Bestvina-Feighn and Dunwoody.

Vassiliev's theorem on spaces of functions with moderate singularities

Michael Weiss

This expository talk was about a theorem due to V.Vassiliev. It served as background to the three talks given by Ib Madsen at this meeting. Vassiliev's theorem is one of the most important ingredients in the proof of the Mumford conjecture given by Ib and myself.

To state Vassiliev's theorem we need positive integers m, n, k and a closed semi-algebraic subset $\mathfrak{A}$ of the *jet space* $J^k(\mathbb{R}^m, \mathbb{R}^n)$. The jet space $J^k(\mathbb{R}^m, \mathbb{R}^n)$ can be described as the space of pairs (z, f) where $z \in \mathbb{R}^m$ and f is a polynomial map of degree at most k from $\mathbb{R}^m$ to $\mathbb{R}^n$. But it is better to view f as an equivalence class of germs of maps from $\mathbb{R}^m$ to $\mathbb{R}^n$, defined near z; two such are equivalent if their k-th Taylor expansions at z agree. Then it is clear that the group of smooth automorphisms of $\mathbb{R}^m$ acts on $J^k(\mathbb{R}^m, \mathbb{R}^n)$. See the book by Golubitsky and Guillemin, *Stable mappings and their singularities*, for more details on jets.

We now require that $\mathfrak{A}$ be invariant under the action of the group of smooth automorphisms of $\mathbb{R}^m$. For a smooth closed m-manifold M without boundary, let $\mathfrak{A}(M) \subset J^k(M, \mathbb{R}^n)$ consist of those jets which, in local coordinates near $z \in M$, belong to $\mathfrak{A}$. We say that a smooth $f : M \to \mathbb{R}^n$ has *no $\mathfrak{A}$-singularities* if its k-jet prolongation $j^k f$, which is a section of the jet bundle projection $J^k(M, \mathbb{R}^n) \to M$, avoids $\mathfrak{A}(M)$. Now we have a comparison map Φ, given by $f \mapsto j^k f$, from the space of smooth maps $f : M \to \mathbb{R}^n$ which have no $\mathfrak{A}$-singularities to the space of sections of $J^k(M, \mathbb{R}^n) \to M$ which avoid $\mathfrak{A}(M)$.

Theorem. *The map Φ induces an isomorphism in integer cohomology if the codimension of $\mathfrak{A}$ in $J^k(\mathbb{R}^m, \mathbb{R}^n)$ is at least $m + 2$.*

It follows easily that Φ is a weak homotopy equivalence if that codimension is at least $m+3$. There is also a version for compact smooth M with boundary; for that case we fix a smooth $\varphi : M \to \mathbb{R}^n$ which has no $\mathfrak{A}$-singularities in the interior of M near ∂M, and allow only smooth maps $f : M \to \mathbb{R}^n$ which agree with φ near ∂M. The theorem was announced in 1989 with an outline of proof; more details can be found in Vassiliev's book on *Complements of discriminants of smooth maps*. It is a typical h-principle in the sense of Gromov. But Vassiliev's proof is untypical. It relies on the method which has made Vassiliev's name so popular in knot theory: learn something about a space of "good" smooth maps from a manifold M to $\mathbb{R}^n$ by studying and stratifying its complement in the space of all smooth maps from M to $\mathbb{R}^n$. (Here the good maps are the maps $M \to \mathbb{R}^n$ without $\mathfrak{A}$-singularities; in knot theory, they are the embeddings from $\mathbb{S}^1$ to $\mathbb{R}^3$.)

I concentrated on the special case where M is closed and tried to emphasize the importance of interpolation theory and transversality theory in Vassiliev's proof. According to a point of view which is explained in an article by Glaeser (Proceedings of Liverpool Singularities Symposium, SLN 192), an interpolation problem consists in specifying a $C^\infty(M,\mathbb{R})$-submodule Y of $C^\infty(M,\mathbb{R}^n)$ which has finite $\mathbb{R}$-codimension c, and sometimes an element of the finite dimensional vector space $C^\infty(M,\mathbb{R}^n)/Y$. We look for "canonical" representatives in $C^\infty(M,\mathbb{R}^n)$ of the prescribed element of $C^\infty(M,\mathbb{R}^n)/Y$. More to the point, we might want to solve many interpolation problems using relatively few canonical elements. So we fix the codimension c and search for a single finite dimensional $\mathbb{R}$-linear subspace D of $C^\infty(M,\mathbb{R}^n)$ which has the transversality property $D + Y = C^\infty(M,\mathbb{R}^n)$ for *every* codimension c submodule Y as above. It turns out that many such D exist for given c. This can be used to define a topology on the set of the codimension c submodules Y of $C^\infty(M,\mathbb{R}^n)$. Conversely, the fact that this topology is compact Hausdorff implies that the set of finite dimensional $\mathbb{R}$-linear subspaces D of $C^\infty(M,\mathbb{R}^n)$ which are transverse to all codimension c submodules Y as above is "open". This combines well with the generic nature of other good properties which such a D might have. For example, we might want the evaluation-prolongation map $(f,z) \mapsto j^k f(z)$ from $D \times M$ to $J^k(M,\mathbb{R}^n)$ to be transverse to $\mathfrak{A}(M)$, assuming or hoping that $\mathfrak{A}(M)$ has a good stratification inside $J^k(M,\mathbb{R}^n)$.

Rozansky-Witten theory: Lie algebras and complex manifolds

Simon Willerton

(joint work with Justin Roberts)

A 3-*dimensional topological quantum field theory* (3-d TQFT) is a mathematical object comprising a 3-manifold invariant, knot invariants, and representations of mapping class groups of surfaces. *Chern-Simons theory* is a 3-d TQFT associated to a fixed compact Lie group (such as SU(2)), with origins in physics. This gives rise to quantum invariants of knots such as the Jones polynomial.

Rozansky-Witten theory is a non-rigorously defined TQFT associated to a fixed hyper-kähler (or holomorphic symplectic) manifold. Here a *hyper-kähler* manifold is a $4n$-dimensional Riemannian manifold with three complex structures, I, J and K, each compatible with the metric and satisfying the quaternion relations, such as $IJ = K$; and a *holomorphic symplectic manifold* is a complex manifold with a non-degenerate, holomorphic two-form. Any hyper-kähler manifold gives a holomorphic symplectic manifold by fixing one of the complex structures and using the other two to form a symplectic form. Conversely any *compact* holomorphic symplectic manifold can be given a hyper-kähler metric.

A *weight system* is in some sense an infinitesimal knot invariant. It is a combinatorial invariant of certain graphs and in Chern-Simons theory is obtained from the structure constants and Killing form of the Lie algebra. The *Rozansky-Witten weight systems* were rigorously written down by Rozansky and Witten in terms of the curvature tensor of the hyper-kähler manifold. Kapranov showed that in the holomorphic symplectic world they could be defined using the holomorphic symplectic form and the *Atiyah class* of the holomorphic tangent bundle. Where the *Atiyah class* $\alpha_E \in H^1(X, T^* \otimes \mathrm{End}(E))$ of a holomorphic vector bundle $E \to X$ is a characteristic class which can be viewed as a holomorphic version of curvature.

The bounded *derived category* of coherent sheaves on a complex manifold X is the category whose objects are complexes of coherent sheaves and whose morphism sets are homotopy classes of maps, with the quasi-isomorphisms formally inverted. A sheaf E can be considered as an object in the derived category which is concentrated in degree zero, similarly $E[-i]$ denotes the sheaf concentrated in degree i. If E and F are sheaves on X then the morphism set has the interpretation: $\mathrm{Mor}(E[-i], F) \cong \mathrm{Ext}^i(E, F) \cong H^i(X, E^* \otimes F)$.

Theorem. *Suppose X is a complex manifold, T is the holomorphic tangent bundle, α_T is the Atiyah class.*

(1) *The Atiyah class $\alpha_T \in \mathrm{Mor}(T[-1] \otimes T[-1], T[-1])$ makes $T[-1]$ into a Lie algebra object in the derived category of X.*
(2) *If E is an object of the derived category then its Atiyah class in $\mathrm{Mor}(E \otimes T[-1], E)$ makes E into a module over $T[-1]$.*
(3) *If ω is a holomorphic symplectic form on X, then, considered as an element of $\mathrm{Mor}(T[-1] \otimes T[-1], \mathcal{O}_X[-2])$, gives an "invariant inner product" on $T[-1]$.*

¿From this the derived category can be considered to be the *representation category* of $T[-1]$, and the Rozansky-Witten weight systems can be recovered exactly as they are in Chern-Simons theory. One can then try to do with this object $T[-1]$ whatever one would do with usual Lie algebras.

¿From here, with Justin Sawon, we are trying to rigorously construct the entire Rozansky-Witten TQFT.

Edited by Marco Varisco

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