Mathematisches Forschungsinstitut Oberwolfach

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Singularitäten

September 21st – September 27th, 2003

The 2003 conference on Singularity theory in Oberwolfach was organised by J. Steenbrink (Nijmegen), D. van Straten (Mainz) and V. Vassiliev (Moskow). A total of 23 lectures was held on a variety of recent results and methods in the theory of singularities of mappings and spaces. Besides more classical questions on the topology of normal surface singularities and three-manifolds, applications of the newer techniques related to tropical geometry and motivic integration were discussed in various lectures. A couple of talks were held on topics in the neighbouring fields of classical algebraic geometry, homological algebra and mixed Hodge Theory. Some lectures were devoted to algorithmic aspects and applications of computer algebra.

Abstracts

A Tropical Approach to Enumeration of Singular Algebraic Curves

E. Shustin

The tropical algebraic geometry is an algebraic geometry over the tropical semiring $(\mathbb{R}, \odot, \oplus)$ with $a \odot b = a + b$, $a \oplus b = \max\{a, b\}$. For example, tropical curves in $\mathbb{R}^2$ are non-Archimedean amoebas, the images of algebraic curves in $(\mathbb{K}^*)^2$, $\mathbb{K}$ being an algebraically closed field of characteristic 0 with a non-Archimedean valuation $\nu : \mathbb{K}^* \to \mathbb{R}$ such that $\nu(ab) = \nu(a) + \nu(b)$, $\nu(a + b) \leq \max\{\nu(a), \nu(b)\}$, $\overline{\nu(\mathbb{K}^*)} = \mathbb{R}$ and the map is $(z_1, z_2) \in (\mathbb{K}^*)^2 \mapsto (\nu(z_1, z_2)) \in \mathbb{R}^2$. It is known that non-Archimedean amoebas are 1-dimensional graphs, the corner loci of convex piece-wise linear functions

$$N_f(x) = \max_{\omega}(x \cdot \omega + \nu(a_\omega))$$

where $f = \sum_\omega a_\omega z^\omega \in \mathbb{K}[z]$. Kontsevich proposed to count nodal algebraic curves passing through respective number of generic points via enumeration of "nodal" amoebas passing through generic points in $\mathbb{R}^2$. Mikhalkin realized this for nodal curves on toric surfaces associated with convex lattice polygons in $\mathbb{R}^2$, in the form: degree of the corresponding Severi variety is equal to the number of specific "nodal" amoebas passing through generic points in the plane and counted with weights. Furthermore, the counting of amoebas reduces to counting of certain lattice paths in the given Newton polygon. We suggest an algebraic-geometric proof of the theorem:

(1) We show that a singular curve over $\mathbb{K}$ is an equisingular family of complex curves.
(2) We define tropical limits of such families, which are curves on reducible surfaces, whose components corresponds to polygons in subdivisions of the given Newton polygon.
(3) The patchworking construction restores an equisingular family of curves out of the tropical data.

As an application we present

Theorem 1. *(Itenberg, Kharlamov, Shustin) Through any* $3d - 1$ *generic points in* $\mathbb{RP}^2$ *there exist* $\geq \frac{d!}{2}$ *real rational curves of degree* d.

Iterated vanishing cycles and a conjecture of J. Steenbrink

Michel Merle

(joint work with G. Guibert and F. Loeser)

Given an algebraic variety X over $\mathbb{C}$ and a function $f : X \to \mathbb{C}$ one can define, following Loeser and Denef, the motivic Milnor fibre $S_{f,x}$ of f at $x \in X$ for any point x where f vanishes. They also define $S^{\phi}_{f,x}$ as $(-1^{d-1})\,(S_{f,x} - [\mathbb{C}^*])$. $S_{f,x}$ is an element of a Grothendieck ring $\mathcal{M}^{\mathbb{C}^*}_{X\times\mathbb{C}^*}$ of varieties above $X \times \mathbb{C}^*$, with a $\mathbb{C}^*$-action and a map to $\mathbb{C}^*$, which is $\mathbb{C}^*$-equivariant.

If one is given another function $g : X \to \mathbb{C}$, $g(x) = 0$, one wants to define $S_g(S_f)$. This is an element of the Grothendieck ring $\mathcal{M}^{(\mathbb{C}^*)^2}_{X\times(\mathbb{C}^*)^2}$ of varieties with $(\mathbb{C}^*)^2$-action and a map to $X \times (\mathbb{C}^*)^2$, which is $(\mathbb{C}^*)^2$-equivariant.

When a variety A has two maps $u, v : A \to \mathbb{C}^*$ the convolution is defined as

$$\begin{array}{c} A \\ {}^{u}\swarrow \ast \searrow^{v} \end{array} = - \begin{bmatrix} A \backslash (u+v)^{-1}(0) \\ \downarrow{\scriptstyle u+v} \\ \mathbb{C}^* \end{bmatrix} + \begin{bmatrix} \mathbb{C}^* \times (u+v)^{-1}(0) \\ \downarrow{\scriptstyle pr_1} \\ \mathbb{C}^* \end{bmatrix}$$

If $A \in \mathcal{M}_{(\mathbb{C}^*)^2}^{(\mathbb{C}^*)^2}$, then the convolution belongs to $\mathcal{M}_{\mathbb{C}^*}^{\mathbb{C}^*}$.

Theorem 1. *Let X be an algebraic variety, $x \in X$, f, g algebraic maps to $\mathbb{C}$ such that $f(x) = g(x) = 0$. Then for N sufficiently large*

$$\ast \left[S_{g,x}(S_f^{\phi}) \right] = S_{f,x}^{\phi} - S_{f+g^N,x}^{\phi}$$

As an application, we give a new proof of Steenbrinks conjecture (first proved by M. Saito): If $\dim(Sing(f)) = 1$ and g, say, a linear form, compute the difference between the Spectrum(f, x) and Spectrum$(f + g^N, x)$ in terms of the sheaf of vanishing cycles of f on the smooth part of $Sing(f)$.

Global Euler obstruction and polar invariants

Mihai Tibar

(joint work with Pepe Seade and Alberto Verjovsky)

We define a global Euler obstruction $Eu(Y)$ for an affine singular variety $Y \subset \mathbb{C}^N$ of pure dimension d in a similar manner as the local Euler obstruction introduced by MacPherson, i.e., as the obstruction to extend a radial vector, defined on the link at infinity of Y, to a non-zero section of the Nash bundle.

We prove that $Eu(Y)$ can be expressed as an alternating sum

$$Eu(Y) = (-1)^d \alpha_Y^{(1)} + \ldots - \alpha_Y^{(d)} + \alpha_Y^{(d+1)} \qquad (1)$$

where $\alpha_Y^{(1)}$ is the number of Morse points of Y_{reg} of a Lefschetz pencil on Y and the following ones are similar numbers defined on successive generic hyperplane slices of Y. For instance, if Y is non-singular, then $Eu(Y) = \chi(Y)$.

The invariants $\alpha_Y^{(i)}$ can be viewed as global polar multiplicities. Local polar multiplicities where used by L and Teissier in the formula [LT, Annals of Math., '81] for the local Euler obstruction. Our proof of (1) has different flavour than L-Teissier's proof in the local case. It relies on the repeated use of the Lefschetz slicing method and on extending a radial vector field starting from a slice.

On a filtration defined by arcs on a variety

Wolfgang Ebeling

(joint work with Sabir M. Gusein-Zade)

Let $(V, 0)$ be a germ of a complex analytic variety and let $\mathcal{O}_{V,0}$ be the ring of germs of functions on it. We define a filtration on $\mathcal{O}_{V,0}$ which we call arc filtration. An arc φ on $(V, 0)$ is a germ of a complex analytic mapping $\varphi : (\mathbb{C}, 0) \to (V, 0)$. For a germ $g \in \mathcal{O}_{V,0}$

its order $v_\varphi(g)$ on the arc φ is defined as the order of the composition $g \circ \varphi$ at the origin. Let $v(g)$ be the minimum over all arcs φ on $(V,0)$ of the orders $v_\varphi(g)$. The arc filtration

$$\mathcal{O}_{V,0} = F_0 \supset F_1 \supset F_2 \supset \dots$$

on the ring $\mathcal{O}_{V,0}$ is the filtration by the ideals $F_i := \{g \in \mathcal{O}_{V,0} \mid v(g) \geq i\}$.

We compute the Poincarśeries of this filtration for the surface singularities from Arnold's lists including the simple and the uni- and bimodular ones. The classification of the unimodular singularities by these Poincaré series turns out to be in accordance with the hierarchy defined by E. Brieskorn using the adjacency relations. Besides that, we give a general formula for the Poincaré series of the arc filtration for isolated surface singularities which are stabilizations of plane curve singularities.

Poincaré series and zeta functions

Jan Stevens

Several cases are known when the ζ-function of the monodromy of a singularity is related to the Poincaré series of its coordinate ring. The first instance of this phenomenon was observed by Campillo, Delgado and Gusein-Zade: For an irreducible plane curve singularity, the ζ-function equals its Poincaré series. There are several ζ-functions around in singularity theory. The monodromy is a direct analogue to ζ-functions from number theory.

For quasi-homogeneous complete intersections, a formula of Ebeling and Gusein-Zade computes the Poincaré series (multiplied with an orbit invariant) in terms of the ζ-functions of the function f_j on the zero set of $f_1, \dots, f_{j-1}$.

¿From this we derive the original formula for irreducible curves. Key ingredients are that the monomial curve with the same semi-group is a complete intersection and that the plane curve is a deformation of it. This enables us to give a model for the monodromy knowing the singular quasi-homogeneous Milnor fibre and the local Milnor fibres of its singularities.

In this way, the quasi-homogeneous formula becomes the primary object, which has to be explained conceptually.

Betti numbers of semi- and subalgebraic sets

A. Gabrielov

(joint work with N. Vorobjov (Bath, UK) and T. Zell (Purdue))

Spectral sequences associated with surjective maps and Hausdorff limits allow one to compute or at least to obtain an upper bound for the Betti numbers of sets defined by expressions with semialgebraic conditions and quantifiers, preserving additional structure such as sparsity of the semialgebraic conditions.

Degeneration of the Leray spectral sequence for certain quotient mappings

J. Steenbrink

(joint work with Chris Peters)

We consider an affine complex algebraic group G acting on a smooth algebraic variety X with geometric quotient $\phi : X \to Y$. We give geometric conditions ensuring that the Leray spectral sequence of ϕ in rational cohomology degenerates at E_2. We show that these are fulfilled for $X = V \setminus \Sigma$, $V = \mathbb{C}[z_0, \dots, z_n]_d$, Σ = discriminant, $G = GL(n+1, \mathbb{C})$, $d \geq 3$. We communicate the result of Orsola Tommasi, proved using similar methods: The Betti numbers b_i of the moduli space $\mathcal{M}_4$ of smooth Riemann surfaces of genus four are equal to one if $i \in \{0, 2, 4, 5\}$ and zero else.

Hurwitz numbers of generalized polynomials

S. Shadrin

A Hurwitz number is the number of coverings with fixed ramification types over fixed points in the target. We give some relations for Hurwitz numbers coming from the intersection theory of the moduli spaces of curves. In fact, these relations are just geometrical interpretation of some formulas. The initial formulas for intersection numbers on the moduli spaces of curves are a powerful tool for computation of concrete integrals. We generalize this approach to give an algorithm for calculation of the simplest Hodge integrals.

Monodromy and "Dessin d'Enfants"

Norbert A'Campo

A generic relatively immersed curve P in the unit disc $D^2 \subset \mathbb{R}^2$ defines by $L(P) = TP \cap S^3$ a knot or link in S^3. Indeed, think of the tangent space TP as a subset in $\mathbb{R}^4$ via the chain of inclusions $TP \subset TD \subset T\mathbb{R}^2 = \mathbb{R}^4$ and S^3 as unit sphere in $\mathbb{R}^4$. Links of type $L(P)$ are very special: (We assume P to be connected)

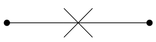
(1) The complement $S^3 \backslash L(P)$ is fibred over S^1. The monodromy is a product of $\mu = h_1(fibre)$ positive Dehn twists. The position of the core curves of the twists on the fibre is read off from the combinatorics of $P \subset D$.
(2) The contact structure of the fibred link as constructed by Emanuel Gicoux is tight.
(3) If P is the image of $[0, 1]$ in D^2, then the unknotting number $u(L(P))$ of the knot $L(P)$ equals $\delta(P) :=$ number of double points of P. The 4-genus $g_4(L(P)) = \delta(P)$.

The construction $\{P \subset D^2\} \to \{L(P) \subset S^3\}$ of classical links fits with singularity theory: The saddle points level $P_{\widetilde{f}}$ on $\mathbb{R}^2 \cap D_\epsilon$ of a real morsification $\widetilde{f}$ of a plane curve singularity $\{f = 0\}$ with real equation and with Milnor ball $B_\epsilon \subset \mathbb{C}^2$ is an immersed curve to which the construction applies. We get

Theorem 1. *The links $L(P_{\widetilde{f}}) \subset S^3$ and $\{f = 0\} \cap \partial B_\epsilon \subset \partial B_\epsilon$ are equivalent as oriented links.*

As topological application we have

Theorem 2. *The higher Milnor linking invariants of the link $L(P)$ of $P \subset D^2$ with three or more components vanish.*

A graph in D^2 as Γ defines an immersed curve $P(\Gamma)$ by putting on each edge an "X" to obtain . It is a lot of fun to draw these curves.

We study especially immersed curves of planar trees in D. It seems that links of planar trees Γ_1, Γ_2 that are related by the action of $Gal(\overline{\mathbb{Q}}/\mathbb{Q})$ on isotopy types of planar trees have much in common. The action of $Gal(\overline{\mathbb{Q}}/\mathbb{Q})$ on isotopy types of trees is defined in the theory of "dessin d'enfants", especially by using Belyi's theorem. We speculate that the knot groups of $L(P_{\Gamma_1})$ and $L(P_{\Gamma_2})$ are isomorphic after profinite completion, if Γ_1 and Γ_2 are conjugated by $Gal(\overline{\mathbb{Q}}/\mathbb{Q})$. We give a construction to support this speculation without proving it. Recent work of A. Shumakovitch shows that the second Vassiliev invariant $v_2(L(P))$ is related to the $J^{\pm}$ and "strangeness" invariants of Arnold of the underlying immersed curve P.

The Casson Invariant Conjecture

Walter Neumann
(joint work with J. Wahl)

The **Casson Invariant Conjecture** asserts that if $(V,0)$ is an isolated complete intersection surface singularity, F its Milnor fibre and Σ its link, and if Σ is an integral homology sphere, then

$$\lambda(\Sigma) = \frac{1}{8} sign(F)$$

where λ is the Casson invariant.

We formulated this conjecture 15 years ago and proved some specail cases. At the time, finding explicit examples was difficult as well as then confirming the conjecture. The conjecture can be strengthened by weakening "complete intersection" to "Gorenstein" and replacing the equation by

$$\lambda(\Sigma) = -p_g(V,0) - \frac{1}{8}(c_1^2 + c_2 - 1)(\widetilde{V})$$

where $\widetilde{V}$ is a resolution.

However, we conjecture:

Classification conjecture: If $(V,0)$ is a Gorenstein surface singularity with homology sphere link Σ, then $(V,0)$ is in fact a complete intersection and even of "splice type".

"Splice type" is a natural generalization of Brieskorn-Pham complete intersections. Very many, but not all homology sphere singularity links occur as links of splice type complete intersections. The name "splice type" comes from the fact that the relevant homology spheres are classified by certain weighted trees called "splice diagrams" (Eisenbud-Neumann, Ann. Math. Studies 110, 1985).

Among the results discussed:

Theorem 1. *The Casson Invariant Conjecture holds for a splice type singularity whose splice diagram has all its nodes in a line.*

Theorem 2. *If $(V,0)$ has a homology sphere link and each knot $K \subset \Sigma$ corresponding to a leaf of the splice diagram is cut out by a function $\{f(x) = 0\}$ on V then $(V,0)$ is a complete intersection of splice type.*

Theorem 3. *We have a conjectured topological description of the Milnor fibre of a complete intersection with homology sphere link Σ just in terms of Σ which would imply the Casson Invariant Conjecture and which is valid at least for suspension hypersurfaces: $z^n = f(x,y)$.*

Some other cases of the Casson Invariant Conjecture have been proved by Collin and Saveliev. Nemethi and Nicolaescu have a generalization to $\mathbb{Q}$-homology sphere links.

Invariants of normal surface singularities

Andrs Nmethi
(joint work with L. Nicolaescu)

Assume that $(X, 0)$ is a normal surface singularity whose link M is a rational homology sphere. If $\widetilde{X}$ is a resolution with s irreducible exceptional divisors and characteristic class K, then $K^2 + s$ is independent of the resolution, it is a topological invariant of $(X, 0)$.

Let $sw^*_{M,can}$ be the (or any candidate for) Seiberg-Witten invariant of M associated with the canonical $spin^c$-structure. (Here, one can consider the modified topological Seiberg-Witten invariant; or the Turaev-Reidemeister torsion normalized by the Casson-Walker invariant, or the Ozsváth-Szab invariant. Conjecturally, these are all equal.) Finally, let p_g be the geometric genus of $(X, 0)$. Then the following facts holds conjecturally:

Conjecture 1. (1) $p_g \leq -sw^*_{M,can} - \frac{K^2+s}{8}$

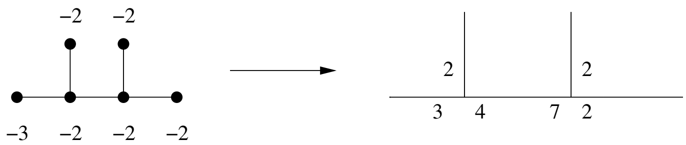
(2) *the right hand side is an* ***optimal*** *topological upper bound: if* $(X, 0)$ *is* $\mathbb{Q}$*-Gorenstein, then* $p_g = -sw^*_{M,can} - \frac{K^2+s}{8}$.

The conjecture was verified for quotient singularities, suspension singularities ($f(a, b) + z^n = 0$, f irreducible) and singularities with good $\mathbb{C}^*$-action. The case of singularities with $\mathbb{C}^*$-action was discussed in some details.

Splice diagram complete intersection singularities

Jonathan Wahl
(joint work with Walter Neumann)

Suppose $(V, 0)$ is a normal surface singularity whose link Σ is a $\mathbb{Q}$HS ("rational homology sphere"), i.e. $H = H_1(\Sigma, \mathbb{Z})$ is finite. The UAC ("universal abelian covering") $\widetilde{\Sigma} \to \Sigma$ is realized by a map $(\widetilde{V}, 0) \to (V, 0)$, an H-covering off the 0, and is also called the UAC. From the resolution diagram Γ of V (a tree of rational curves), one associates another graph Δ called a "splice diagram", e.g.



Most of the time, one can associate equations to Δ, defining "splice diagram complete intersection singularities" $X(\Delta)$.

Further, "much of the time" there is an action of the group H on $X(\Delta)$. Specifically, there are conditions on Γ that equations and an H-action exists.

Theorem 1. *$X(\Delta)$ has an isolated singularity, H acts freely on $X(\Delta) \setminus \{0\}$, $X(\Delta) \to X(\Delta)/H$ is the universal abelian covering and Γ is a resolution dual graph for $X(\Delta)/H$.*

In other words, we construct equations of a singularity with given topological type (determined by Γ).

Conjecture 2. *Let $(V, 0)$ be $\mathbb{Q}$-Gorenstein, with $\mathbb{Q}HS$ link. Then the UAC of $(V, 0)$ is a complete intersection, an equisingular deformation of $X(\Delta)$.*

This generalizes a 1982 theorem of W. Neumann: if $(V, 0)$ is in addition weighted homogenous, the UAC is a Brieskorn complete intersection (a special case of $X(\Delta)$)).

Algorithmic resolution of singularities from a practical point of view

Anne Frhbis-Krger
(joint work with Gerhard Pfister)

In the 1990s, algorithmic proofs of resolution of singularities were found independently by Bierstone and Milman and by Villamayor. But it is still one more step from an algorithmic proof to an implementable algorithm. In this talk, we have a look at the elements of Villamayor's proof and consider the modifications that are necessary to obtain an implementation which can handle interesting examples. On a few examples, the SINGULAR library containing the (current state of) the implementation is presented. (work in progress)

Polynomial Lie algebras and versal deformations

V. Buchstaber
(joint work with D. V. Leikin)

We introduce and study a special class of infinite-dimensional Lie algebras that are modules of finite type over a polynomial ring. They have canonical representations as moving frames with polynomial structure functions. For simple singularities, the fields form moving frames with polynomial connection on the subspace of parameters of positive weight.

A particular result is the direct computation of convolution of matrices. Convolution matrices define moving frames related to the potential fields coming from the Vita mapping. Under this map, we may loose the polynomiality of the connection coefficients, however, the structure functions stay polynomial. The nature of this kind of results is due to the fact that the theory of polynomial Lie algebras combines the properties of Lie algebroids (studied in differential geometry) and free divisors (studied in singularity theory).

Pencils of K3-surfaces with maximal Picard number

A. Sarti
(joint work with W. Barth)

I describe three particular pencils of K3-surfaces with maximal Picard number. More precisely the general member in each pencil has Picard number 19 and each pencil contains four surfaces with Picard number 20. These surfaces are obtained as the minimal resolution of quotients X/G, where $G \subset SO(4,\mathbb{R})$ is some finite subgroup and $X \subset \mathbb{P}_3(\mathbb{C})$ denotes a G-invariant surface. The singularities of X/G come from fix points of G on X or from singularities of X. In any case the singularities on X/G are $A-D-E$ surface singularities. The rational curves which resolve them give almost all the generators of the Neron-Severi group of the minimal resolution.

Exterior Algebra Methods and Applications

Frank-Olaf Schreyer

(joint work with D. Eisenbud and G. Fløstad)

It is well known that the derived category of coherent sheaves on $\mathbb{P}^n$ is equivalent to the stable module category of graded modules over the exterior algebra (Bernstein, Gel'fand, Gel'fand, 1979):

$$\underline{mod}(E) \;\cong\; \underline{D}^b(\mathbb{P}^n)$$

$$\text{by} \qquad P \longmapsto \widetilde{L(P)} \quad \text{with}$$

$$\widetilde{L(P)} \;:\; \quad \ldots \to P_i \otimes \mathcal{O}(-i) \to P_{i-1} \otimes \mathcal{O}(-i+1) \to \ldots$$

In this talk we answer the question how to find an element $P \in \underline{mod}(E)$ corresponding to a sheaf and which P correspond to sheaves.

The second part is an application to Chow forms and resultants. For example, the Sylvester matrix for the resultant of two binary forms of the same degree can be thought of as the syzygy matrix of the Bzout determinant of the resultant.

Def $\neq$ Diff for 1-connected Surfaces

Fabizio Catanese

(joint work with B. Wajnryb)

Def = Diff was a speculation of Friedman-Morgan in the 80's, namely that two smooth algebraic surfaces are deformation equivalent iff they are diffeomorphic. Indeed, $X \sim_{def} Y$ implies the existence of a diffeomorphism $\phi : X \to Y$ s.t.

$$\phi^*(K_Y) = K_X \qquad (*)$$

where K_X is the class of the canonical divisor in $H^2(X, \mathbb{Z})$.

Counterexamples were given by Manetti in '97, later by Kharlamov-Kulikov and myself. The later are obtained by exhibiting surfaces with $S \not\sim_{def} \bar{S}$, and indeed have the drawback that there does not exist a diffeomorphism ϕ satisfying $(*)$. Moreover, in these examples, there is a finite etale cover $\hat{S}$ of S for which there is only one deformation type. I reported on the following:

Theorem (C, Wajnryb). *$\forall r\, \exists r$ different deformation types with the same differentiable type, and moreover they are 1-connected, i.e. $\pi_1(S) = 0$.*

The examples are very simple, they are given by taking equations

$$z^2 = f_{2a,2b}(x, y)$$
$$w^2 = g_{2c,2b}(x, y)$$

yielding a $(\mathbb{Z}/2)^2$-cover of $\mathbb{P}^1 \times \mathbb{P}^1$ denoted by $S(a, b, c)$. Here $f_{n,m}(x, y)$ denotes a bihomogeneous polynomial of bidegree (n, m).

Theorem 2 (C, Wajnryb). *$S' = S(a + 1, b, c - 1)$ is diffeomorphic to $S = S(a, b, c)$.*

Theorem 1 (C, Manetti; through a series of papers). *Assume $a \geq 2c + 1$, $a \geq b + 2$, $c \geq b + 2$ and a, b, c even. Then the family of natural deformations*

$$z^2 = f + w\psi$$
$$w^2 = g$$

yields a complete deformation class.

Note: $\mathbb{Z}/2$ acts by $z \mapsto -z$, and by $w \mapsto -w$ on the quotient. I sketched the ideas behind the proof: For theorem 1 one uses that if $S_t \to S_0$, the limit $M_t \cong \mathbb{P}^1 \times \mathbb{P}^1$, $M_t = \big((S_t)/(\mathbb{Z}/2)\big)/(\mathbb{Z}/2) \to M_0$.

A classification of the singularities of M_0, plus the fact that we have a $\mathbb{Q}$-Gorenstein smoothing and the Milnor fibre is $\subset \mathbb{P}^1 \times \mathbb{P}^1$ shows that M_0 is smooth.

For theorem 2 we observe that S, S' are fibre sums $N_1 \# N_2$ and $N_1 \#_\psi N_2$ of 2 SLF (Symplectic Lefschetz Fibrations) over $\bar{\Delta}$.

A lemma of Auroux shows that the fibre sum is independent of ψ if the monodromy on $\partial\Delta$ is trivial and ψ is a product of the Dehn twists associated to $N_1 \to \bar{\Delta}$. I showed some pictures of ψ and of the vanishing cycles.

Limits of Hodge structures in several variables

Taro Fujisawa

I talked about a generalization of the famous results by J. Steenbrink on limits of Hodge structures. Here I treat a morphism over a higher dimensional polydisc which is proper, surjective and satisfies some conditions (something like semi-stable degeneration). I explained how to construct a cohomological mixed Hodge complex which gives the limit mixed Hodge structure. L.-H. Tu's previous work suggested a candidate which I proved to be the correct one for the $\mathbb{C}$-structure level. But I had to construct a new $\mathbb{Q}$-structure. I talked about the construction of the $\mathbb{Q}$-structure by using the log-structure associated to the morphism which I considered.

Patchworking Singular Algebraic Curves

Ilya Tyomkin
(joint work with E. Shustin)

In the talk we will discuss a method (called Geometric Patchworking) for constructing algebraic varieties with prescribed "local" geometry. This method can be traced back to Viro's method (and its modifications suggested by Shustin). From our point of view Patchworking (both Viro's and Shustin's versions) is equivalent to the study of deformations of pairs consisting of a surface X_t and a curve $C_t \subset X_t$ (either real or complex; either smooth or singular, depending on the context). More precisely, we study deformations of surfaces (equipped with a line bundle $\mathcal{L}$) having reducible central fiber and given generic fiber, i.e. the surface X_0 is a union of some surfaces Σ^i and the generic fiber is the given surface Σ. We construct a curve in the central fiber, given by a section of $\mathcal{L}_0$ and having required geometry, and we ask whether one can deform this curve into a curve on the generic fiber *preserving the geometry*. As a result we will generalize some results of Greuel, Lossen, Shustin, Keilen, Ciliberto and Chiantini.

Symplectic singularities from the Poisson point of view

Dmitry Kaledin

Symplectic singularities are a new kind of singularities recently introduced by A. Beauville; very important contributions were also made by Y. Namikawa. By definition, X has symplectic singularities iff

(1) There is a non-degenerate closed two-form Ω on the smooth locus $U \subset X$ (we assume X to be a normal algebraic variety over $\mathbb{C}$).

(2) The form Ω extends without poles to a smooth resolution $\widetilde{X} \to X$.

This definition does not depend on the resolution; it is easy to see that symplectic singularities are Gorenstein, canonical and rational. Examples include:

(1) Quotient singularities V/G, $G \subset Sp(V)$
(2) so-called quiver varieties of Nakajima
(3) the nilpotent cone $\mathcal{N} \subset \mathfrak{g}$ of a semi-simple Lie algebra $\mathfrak{g}$

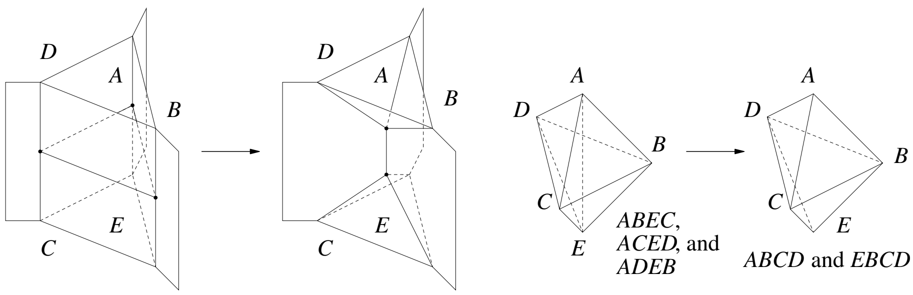
There is vague hope that all symplectic singularities are of "this type", but it is too early to formulate a precise conjecture.

There is one thing common to all the examples: In all the cases X admit a stratification X_i such that all the open strata X_i° are smooth and symplectic. The goal of the talk is to produce such a stratification in the general setting. To be able to tie together all the symplectic structures on all strata we make use of a Poisson scheme. Unlike the symplectic form, the Poisson bracket requires no assumption of smoothness. It is easy to see that a symplectic singularity is Poisson. We call a Poisson scheme holonomic if the induced Poisson structure on every Poisson subscheme is non-degenerate in the generic point. Our main result states that every symplectic singularity is holonomic as a Poisson scheme. The stratification we are looking for then follows easily: The singular locus $Sing(X) \subset X$ is a Poisson subscheme, which is also holonomic. We take $X \backslash Sing(X)$ as the first open stratum and stratify $Sing(X)$ by induction.

Matveev-Piergallini theorem and singularities of cut loci

Sergei Anisov

A spine P of a 3-manifold M is a subpolyhedron of dimension ≤ 2 such that the manifold M (either with boundary or punctured in one point) can be collapsed onto P. Special spines of M^3 (spines that satisfy some genericity conditions) inherit all the information about M^3. A manifold always has a special spine (infinitely many ones, in fact). Transformations T and T^{-1} are local surgeries converting special spines to other special spines of the same 3-manifold:



In 1988, Matveev and Piergallini independently have proved that all special spines of the same 3-manifold can be converted into each other by several $T^{\pm 1}$-moves. This fact is crucial, e.g. for the construction of Turaev-Viro invariants: a state sum defined for a spine is an invariant of a manifold if $T^{\pm 1}$-moves do not change it. Both Matveev and Piergallini proofs are rather sophisticated, technically as well as conceptually.

¿From the singularity point of view, spines are cut loci for appropriate metrics on M^3. This observation leads to a knew proof of the Matveev-Piergallini theorem, still not very short but more transparent.

Multipoint Seshadri Constants

Joaquim Roé
(joint work with B. Harbourne)

Working over $\mathbb{C}$ and formalizing and sharpening approaches introduced by Xu, Szemberg and Tutaj-Gasińska, we give a method for verifying when a divisor on a blow up of $\mathbb{P}^2$ at general points is nef. The method is useful both theoretically and when doing computer computations. The main application is to obtaining lower bounds on multipoint Seshadri constants on $\mathbb{P}^2$. In combination with methods previously developed to estimate the degree of singular curves, significantly improved explicit lower bounds are obtained.

Engel-like identities characterizing finite solvable groups

Gert-Martin Greuel
(joint work with T. Bandman, F. Grunewald, B. Kunyavskii, G. Pfister, E. Plotkin)

We report on a result by the above six (!) authors, characterizing finite solvable groups by an inductively defined Engel-like sequence of two-variable identities.

Let G be a group and $x, y \in G$. Define

$$u_1(x,y) := x^{-2}y^{-1}x$$

$$\text{and} \quad u_{n+1}(x,y) := [xu_n(x,y)x^{-1}, yu_n(x,y)y^{-1}] \qquad \text{for } n \geq 2,$$

where $[a,b] = aba^{-1}b^{-1}$ is the commutator for all $x, y \in G$.

Theorem 1. *A finite group G is solvable if and only if for some n the identity $u_n(x,y) = 1$ holds for all $x, y \in G$.*

Note that this theorem is analogous to Zorn's result which characterizes the finite nilpotent groups by the condition that for some n, $e_n(x,y) = 0$ for all $x, y \in G$, where $e_n(x,y)$ is the Engel sequence defined by $e_1(x,y) = [x,y]$, $e_{n+1}(x,y) = [e_n(x,y), y]$. The above theorem was conjectured by Plotkin in a slightly modified form.

Clearly, in every solvable group the identities $u_n(x,y) = 1$ are satisfied for all $n \geq$ some n_0. The non-trivial "if" part will be deduced from the following.

Theorem 2. *Let G be one of the following groups:*

(1) *$G = PSL(2, \mathbb{F}_q)$, where $q \geq 4$ ($q = p^n$, p a prime),*
(2) *$G = Sz(2^n)$, $n \geq 3$ and odd,*
(3) *$G = PSL(3, \mathbb{F}_3)$.*

Then there are $x, y \in G$ such that $u_1(x,y) \neq 1$ and $u_1(x,y) = u_2(x,y)$.

Since the groups in theorem 2 contain Thompsons list of finite simple groups all of whose subgroups are solvable, theorem 1 follows easily from theorem 2.

For small groups from the list in theorem 2 it is an easy computer exercise to verify the statement. The general proof of theorem 2 is however surprisingly complex and involves not only group-theoretic methods but also methods from algebraic geometry, arithmetic geometry and computer algebra, in particular the computer algebra systems SINGULAR and MAGMA. Not only proofs but even the precise statements of our results would hardly have been found without extensive computer experiments.

The general idea is roughly as follows: For G in the above list, use a matrix representation over $\mathbb{F}_q$ and interpret solutions of the equation $u_1(x,y) = u_2(x,y)$ as $\mathbb{F}_q$-rational points of an algebraic variety. To ensure that $u_1(x, y) \neq 1$ holds, we take x, y from appropriate Zariski-closed subsets only. In the $PSL(2, \mathbb{F}_q)$-case, we obtain a curve defined over $\mathbb{Z}$ for which the Hasse-Weil bound guaranties $\mathbb{F}_q$-rational points if q is big enough and if the reduction mod q is absolutely irreducible. The explicit bound for q and the absolute irreducibility (for all q!) are proved by Grbner basis methods using SINGULAR. The Suzuki groups $Sz(q)$, $q = 2^n$, n odd, provide the most difficult case. Indeed, we construct a 2-dimensional variety $V \subset \mathbb{A}^8$ defined over $\mathbb{F}_2$ which is affine, smooth, absolutely irreducible and which is α-invariant where $\alpha : \mathbb{A}^8 \to \mathbb{A}^8$ is the square root of the Frobenius sucht that non-zero fixed points of α^n give rise to solutions of $1 \neq u_1(x, y) = u_2(x, y)$ in $Sz(2^n)$. The existence of such fixed points follows from the Lefschetz trace formula as conjectured by Deligne and proved by Fujiwara. To apply all this, SINGULAR was an indispensible tool.

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