Mathematisches Forschungsinstitut Oberwolfach

Report No. 45/2003

Mini-Workshop: Dimensional Reduction of Large-Scale Systems

October 19th – October 25th, 2003

The mini-workshop was organized by Peter Benner (Chemnitz), Gene Golub (Stanford), Volker Mehrmann (Berlin) and Danny C. Sorensen (Houston). The main goal of this mini-workshop was to discuss different approaches for model order reduction of large-scale systems with special emphasis on numerical aspects of model reduction algorithms. There were 16 talks, informal discussion sessions and an open problems session. The topics included spectral projection model reduction methods, proper orthogonal decomposition method for optimal control problems, balanced truncation model reduction for linear systems in descriptor form and systems of second order, Krylov subspace methods for structured systems, controller reduction problems, passivity preserving model reduction techniques as well as different application areas for model reduction. It was decided to organize a benchmark collection of real-wold problems for use in comparative studies and testing of model order reduction algorithms. A proceedings book will also be published that will contain survey papers on problems posed during the mini-workshop and their solutions.

Schedule of the mini-workshop

Monday, October 20

9.00 – 10.45	*Athanasios C. Antoulas.* An overview of approximation methods for large-scale systems.
10.45 – 12.15	*Michael Hinze.* Adaptive control strategies for time-dependent problems utilizing model reduction.
14.00 – 15.30	*Peter Benner.* Model reduction algorithms using spectral projection methods.
16.30 – 18.30	Discussion

Tuesday, October 21

9.00 – 10.45	*Paul Van Dooren.* Model reduction of second order systems.
10.45 – 12.15	*Karl Meerbergen.* The Q-Arnoldi algorithm for model reduction of second order systems.
14.30 – 15.30	*Zhaojun Bai.* A second-order Krylov subspace and its applications.
16.00 – 17.00	*Roland W. Freund.* Pade-type reduced-order modelling of higher-order systems.
17.00 – 17.30	*Volker Mehrmann.* System reduction via the transfer function.
17.30 – 18.30	Discussion

Wednesday, October 22

9.00 – 10.30	*Stefan Volkwein.* Model reduction of non-linear dynamical systems by POD.
10.30 – 12.15	*Tatjana Stykel.* Balanced truncation model reduction for descriptor systems: theory and numerical algorithms.
13.30 – 17.00	Hiking tour

Thursday, October 23

9.00 – 10.45	*Andras Varga.* Recent enhancements of controller reduction methods.
10.45 – 12.15	*Jan G. Korvink.* Engineering perspective on model order reduction.
13.30 – 14.30	*Athanasios C. Antoulas.* Gramians for structured systems.
14.30 – 15.30	Open problems and benchmarks.
16.00 – 17.00	*Jing-Rebecca Li.* Low order approximation of the spherical nonreflecting boundary kernel for the wave equation.
17.00 – 18.15	*Serkan Gugercin.* Projection methods for model reduction of large-scale dynamical systems.
19.30 – 20.30	Future work

Friday, October 24

9.15 – 11.00	*Danny C. Sorensen.* Passivity preserving model reduction.

Slides of the talks can be found at
http://www.math.tu-berlin.de/numerik/mt/NumMat/Meetings/0310_MFO/

Abstracts

An Overview of Approximation Methods for Large-Scale Systems

Athanasios C. Antoulas (Rice University)
(joint work with Danny C. Sorensen (Rice University))

In many applications one is faced with the task of simulating or controlling complex dynamical systems. Such applications include for instance, weather prediction, air quality management, VLSI chip design, molecular dynamics, active noise reduction, chemical reactors, etc. In all these cases complexity manifests itself as the number of first order differential equations which arise. For the above examples, depending on the level of modeling detail required, complexity may range anywhere from a few thousand to a few million first order equations, and above. Simulating (controlling) systems of such complexity becomes a challenging problem, irrespective of the computational resources available. In this talk we will first briefly describe some motivating examples, we will then define the problem in mathematical terms and sketch several methodologies for its solution. The presentation concludes with open problems and directions for future research.

A Second-Order Krylov Subspace and its Applications

Zhaojun Bai University of California
(joint work with Yangfeng Su (Fudan University))

A second-order Krylov subspace $G_n(A, B; u)$ based on a pair of square matrices A and B and a vector u is presented in this paper. The subspace is spanned by a sequence of vectors defined via a second-order linear homogeneous recurrence relation with coefficient matrices A and B and an initial vector u. It extends the well-known Krylov subspace $K_n(A; v)$, which is spanned by a sequence of vectors defined via a first-order linear homogeneous recurrence relation with a single coefficient matrix A and an initial vector v. A second-order Arnoldi (SOAR, in short) procedure for generating an orthonormal basis of $G_n(A, B; u)$ is given. As applications, a Rayleigh-Ritz orthogonal projection method is proposed for solving a large-scale quadratic eigenvalue problem (QEP) and for reduced-order modelling of a second-order dynamical system. This method is applied directly to the QEP and the second-order system. Hence it preserves essential structures and properties of these problems. Numerical examples demonstrate that the new method attains and sometimes outperforms superior convergence behaviours of the Krylov subspace-based Arnoldi method applied to the linearized QEP and the dynamical system.

Model Reduction Algorithms Using Spectral Projection Methods

Peter Benner (Technische Universität Chemnitz)

We discuss model reduction methods based on balancing-related techniques and spectral projection methods. We show how full-rank factors of the Gramians of linear time-invariant systems can be efficiently computed using a particular spectral projection method: the sign function method. This leads to a significant reduction of the cost for computing the reduced-order model in methods related to balanced truncation. We also show how this technique can be used for stochastic truncation and other more involved balancing-related model reduction methods. Spectral projection techniques will also be employed for model reduction of unstable systems and descriptor systems. Moreover, we will discuss how a scalable implementation of balanced truncation based on the sign function/full-rank factor approach can be obtained using the concept of hierarchical matrices and formatted arithmetic. We will conclude with some remarks on a parallel implementation of the sign function-based model reduction algorithms and demonstrate the efficiency and reliability of the algorithms by showing the results of various numerical experiments.

Pade-Type Reduced-Order Modelling of Higher-Order Systems

Roland W. Freund (Bell Laboratories)

A standard approach to reduced-order modelling of higher-order linear dynamical systems is to rewrite the system as an equivalent first-order system and then employ Krylov-subspace techniques for reduced-order modelling of first-order systems. While this approach results in reduced-order models that are optimal in a Pade sense, in general, these models do not preserve the form of the original higher-order system.

In this talk, we present a new approach to reduced-order modelling of higher-order systems based on projections onto suitably partitioned Krylov basis matrices that are obtained by applying Krylov-subspace techniques to an equivalent first-order system. We show that the resulting reduced-order models preserve the form of the original higher-order system. Moreover, possible additional properties such as passivity or reciprocity are also preserved. While the resulting reduced-order models are no longer optimal in the Pade sense, we show that they still satisfy a Pade-type approximation property. We also discuss some implementation details and present some numerical examples.

Projection Methods for Model Reduction of Large-Scale Dynamical Systems

Serkan Gugercin (Virginia Tech)
(joint work with Athanasios C. Antoulas and Danny C. Sorensen (Rice University))

In this talk, we focus on projection methods to efficiently construct reduced order models for large linear dynamical systems. Three different classes of methods have been examined: SVD based methods, Krylov based methods and SVD-Krylov based methods.

We first study the modified cyclic low-rank Smith method to compute low-rank approximations to solutions of large-scale Lyapunov equations. Unlike the original cyclic low-rank Smith method of Penzl, the number of columns in the modified approximate does not necessarily increase at each step and is much lower. The resulting low-rank gramians are then used to apply approximate balanced reduction, which is an SVD based method.

In the second part of the talk, based on an exact expression for the H_2 norm, we show that the H_2 error is due to the mismatch at the mirror images of the poles of the original and reduced systems. Hence for the rational Krylov method, we propose choosing the mirror images as the interpolation points.

Finally, the least-squares model reduction algorithm is studied. The method is a projection and combines Krylov and SVD based methods. The reduced model is asymptotically stable, matches a certain number of moments; and minimizes a weighted H_2 error in the discrete-time case.

Various numerical experiments are presented.

Adaptive Control Strategies for Time-Dependent Problems Utilizing Model Reduction

Michael Hinze (Technische Universität Dresden)

We present an effective control method for mathematical models governed by systems of nonlinear time-dependent partial differential equations. It takes account of the fact that control inputs may alter the regime of the underlying physical process. The method in an adaptive manner constructs a hierarchy of appropriate low dimensional approximations to the mathematical model which then are used as subsidiary condition in the related optimization problem. We discuss different possibilities to construct low dimensional systems and the related modes (eigenfunctions of stationary problem, eigenfunctions of the linearized model and snapshot form of proper orthogonal decomposition).

As numerical example we present tracking-type control of the incompressible Navier-Stokes system as mathematical model for periodic flow around a cylinder. The numerical results of the adaptive approaches for different modes are compared. Furthermore, they are compared to the result of the optimal control approach applied to the full Navier-Stokes system. It turns out that the quality of the controls obtained from the suboptimal approaches compares to that obtained by optimal control, and the computational costs for the optimal approach are at least one order of magnitude larger.

Engineering Perspective on Model Order Reduction

Jan G. Korvink (Albert-Ludwigs-Universität Freiburg)
(joint work with Evgenii B. Rudnyi (Albert-Ludwigs-Universität Freiburg))

In this presentation we focus on the engineering needs from model order reduction (MOR), and in particular those of microsystems. The basic idea is that engineers require simulation tools as support for design processes. As a result, very accurate yet very fast simulations of ODEs resulting from CAD packages are what is most needed. However, in performing MOR, engineers have additional requirements that relate to reuse of these often very complex models throughout the design lifetime of a subsystem, and hence have traditionally tended to hand-build the so-called compact models. MOR can play a large role here, but for success it is important to understand how compact modelling is used in the design process. These aspects, as well as experience with a number of MOR techniques to microsystems, are discussed in more detail.

Low Order Approximation of the Spherical Nonreflecting Boundary Kernel for the Wave Equation

Jing-Rebecca Li (INRIA)

We find low order approximations to the spherical nonreflecting boundary kernel for the wave equation in three dimensions. First we express the Laplace transform of the kernel as a rational function by solving for the zeros of a modified Bessel function. Then we formulate a linear time-invariant dynamical system whose transfer function is this rational function. Finally we use the Balanced Truncation method to generate low order approximations. We compare our approach with a direct L^2 minimization approach, where a rational approximation is expressed as the ratio of two polynomials.

The Q-Arnoldi Algorithm for Model Reduction of Second Order Systems

Karl Meerbergen (Free Field Technologies)

We present the Q-Arnoldi algorithm, which is an Arnoldi algorithm for the solution of the quadratic eigenvalue problem and linear systems with a quadratic parameter that exploits the structure of the linearized problem. This allows us to reduce the memory requirements by about a half. We compare the numerical stability of the Arnoldi and Q-Arnoldi algorithms by a theoretical analysis and numerical examples. The theory is illustrated by examples from applications.

System Reduction via the Transfer Function

Volker Mehrmann (Technische Universität Berlin)

We discuss a new approach for the construction of low dimensional model approximation. Instead of first discretizing in space and then performing model reduction, we propose to directly discretize the transfer operator from the inputs to the outputs.

Passivity Preserving Model Reduction

Danny C. Sorensen (Rice University)

An algorithm is developed for passivity preserving model reduction of linear time-invariant systems. Implementation schemes are described for both medium scale (dense) and large scale (sparse) applications. The algorithm is based upon interpolation at selected spectral zeros of the original transfer function to produce a reduced transfer function that has the specified roots as its spectral zeros. These interpolation conditions are satisfied through the computation of a basis for a selected invariant subspace of a certain blocked matrix which has the spectral zeros as its spectrum. Explicit interpolation is avoided and passivity of the reduced model is established, instead, through satisfaction of the necessary conditions of the Positive Real Lemma. It is also shown that this procedure indirectly solves the associated controllability and observability Riccati equations and how to select the interpolation points to give maximal or minimal solutions of these equations. From these, a balancing transformation may be constructed to give a reduced model that is balanced as well as passive and stable.

Balanced Truncation Model Reduction for Descriptor Systems: Theory and Numerical Algorithms

Tatjana Stykel (Technische Universität Berlin)

We present a generalization of balanced truncation model reduction methods for linear time-invariant descriptor systems. These methods are closely related to the proper and improper controllability and observability Gramians and Hankel singular values of descriptor systems. Important properties of the balanced truncation approach are that the asymptotic stability is preserved in the reduced order system and there is an a priori bound on the approximation error. The Gramians of a descriptor system can be computed by solving projected generalized Lyapunov equations. We discuss numerical solution of large-scale projected generalized Lyapunov equations and demonstrate the application of balanced truncation model reduction to a semidiscretized Stokes equation.

Model Reduction of Second Order Systems

Paul Van Dooren (Université catholique de Louvain)
(joint work with Y. Chahlaoui, D. Lemonnier, K. Meerbergen, A. Vandendorpe)

The objectives of this paper is to present a new method for model reduction of a second order linear time-invariant system of the type

$$M\ddot{x}(t) + C\dot{x} + Kx(t) = f(t), \tag{1}$$

where the matrix $M \in \Re^{N\times N}$ is assumed to be invertible. Models of mechanical systems are often of this type since (1) then represents the equation of motion of the system. For such a system $M = M^T$, $C = C^T$ and $K = K^T$ are respectively the mass, damping and stiffness matrices $f(t) \in \Re^{N\times 1}$ is the vector of external forces, and $x(t) \in \Re^{N\times 1}$ is the vector of internal generalized coordinates.

In civil engineering or aeronautics, the size N of the model (obtained using, for instance, finite elements techniques) is often so high that many analysis and design problems can not be solved anymore within a reasonable computing time. It is then advisable to construct a reduced order model that nevertheless keeps the "mechanical" structure of the system. Since (1) is a particular case of a linear time-invariant system, one may consider its corresponding (linearized) state-space model and apply the techniques of model reduction known for state-space models. In doing so, the reduced-order system is generally not of the same type anymore and the symmetry of the data is lost. Since from a physical point of view it makes sense to impose the reduced-order system to be of the same type, we propose in this paper new methods of model reduction that preserve the second order form and (if needed) its symmetry. The scheme that seems to behave the best is based on the notation of contrained Gramians, which were already defined in a certain sense in the work of Meyer et al.

Recent Enhancements of Controller Reduction Methods

Andras Varga (DLR Oberpfaffenhofen)

Applying modern control synthesis methodologies leads often to controllers whose orders are too large for their practical use. In such cases it is necessary to perform controller reduction by determining a lower order approximation of the original controller. Controller reduction problems are often formulated as particular frequency-weighted model reduction problems, where the frequency-weights are chosen to enforce closed-loop stability and an acceptable performance degradation when the low order controller replaces the original high order one. Many stability/performance preserving controller reduction problems have therefore very special structures which can be exploited when developing numerical methods for controller reduction. An overview of the latest developments in controller reduction methods is presented. First, the frequency-weighted model reduction is addressed by means of the frequency-weighted balanced truncation method. Improved grammian selection schemes and application of enhanced accuracy techniques are key aspects for the efficient use of this approach. Them we discuss the main stability/performance preserving controller reduction techniques, including coprime factor reduction based approaches. The recent enhancements of these methods focused on the efficient computation of frequency-weighted grammians and the extension of square-root techniques for the computation of Cholesky factors of these grammians. Efficient computational schemes have been developed for several stability/performance preserving coprime factor reduction based coprime factor controller reduction approaches.

Model Reduction of Non-Linear Dynamical Systems by POD

Stefan Volkwein (University of Graz)

Proper orthogonal decomposition (POD) provides a method for deriving reduced-order models of non-linear dynamical systems, where a so-called POD basis is computed by means of a singular value decomposition and these functions are used in a Galerkin ansatz for the non-linear dynamics. Due to the good approximation properties of POD the question for error analysis arises. In the talk error estimates for Galerkin-POD based methods for non-linear equations arising in fluid dynamics are presented. The resulting error bounds depends on the number of POD basis functions and on the time discretization.

An optimal non-linear boundary control problem is considered to illustrate the application of the reduced order modelling with POD. It turns out that including dual (or adjoint or observability) information into the POD basis, the POD based optimal control works numerically very efficiently.

In last part of the presentation three different suboptimal feedback strategies for non-linear evolution problems are introduced. The numerical realization of closed loop control for distributed parameter systems is still a significant challenge and in fact infeasible unless specific structural techniques are employed. In the first approach POD is utilized to design an reduced-order (or static) output feedback controller. Secondly, a feedback strategy is given that is based on Bellman's optimality principle. Finally, a combination of POD model reduction techniques with the numerical treatment of the Hamilton-Jacobi-Bellman equation for infinite horizon optimal control problems is shown. The feasibility of the proposed methodologies are demonstrated numerically by means of optimal control problems for non-linear parabolic problems.

Edited by Tatjana Stykel

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