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Random Media

October 26th – November 1st, 2003

This workshop was organized by Jürgen Gärtner (Berlin), Stanislav Molchanov (Charlotte) and Charles Newman (New York). It was attended by about 50 scientists from Brazil, Chile, Germany, France, The Netherlands, Switzerland, the United Kingdom, and the United States.

The workshop brought together leading experts and younger researchers working in various fields of random media. In three introductory talks, aimed at non-experts, recent developments in some of the most rapidly changing areas were surveyed: spin glasses (Bovier), two-dimensional percolation (Camia), and directed polymers in random media (Comets). In a number of other talks, further active research themes were treated, like critical two-dimensional random systems and construction of conformally invariant fields, random walks on percolation clusters, ageing of random systems in statistical mechanics, the Sherrington-Kirkpatrick model and Parisi's hierarchical ansatz, random Schrödinger operators and Lifshitz tails, reconstruction of random sceneries, and others.

In a series of problem sessions at the end of the conference, new ideas and open problems were brought to the audience by experts and various ansatzes for solutions were discussed.

We are thankful to the staff of the Mathematische Forschungsinstitut Oberwolfach for the excellent and highly effective technical and logistic support prior to and during the workshop. This provided the base for a very pleasant and stimulating atmosphere.

Abstracts

Concerning the SK spin glass models

Michael Aizenman (Princeton)
(joint work with R. Sims and S. Starr)

The Sherrington-Kirkpatrick spin glass model has been of interest as a system with complex landscape of states, slow dynamical relaxation processes, and equilibrium state with rich structure. Our goal is to shed light on its structure, and in particular on the validity of Parisi's hierarchical ansatz through analysis which is both rigorous and expressed in terms of recognizable principles. Motivated by the cavity picture, we consider the process of amalgamation of a finite SK type system with a much larger reservoir. The reservoir is in the form of a random overlap structure (ROSt), examples of which are provided by large SK-type systems, as well as Ruelle-Derrida's GREM models. A functional $G(\mu)$ is introduced over the ROSt states, which expresses the change in the entropy resulting through the amalgamation process. To study it we employ an interpolation procedure which is akin to the one introduced in recent works by Guerra and Toninelli. We find that quite generally $G(\mu)$ is non-negative, and the condition for the entropy staying constant throughout the amalgamation procedure is that under coupling the system and the reservoir lock into a perfectly matched state, in terms of the joint distribution of the corresponding overlap functions. An example of such a match is found when the reservoir is taken to be an independent SK type model. When the reservoir is in any of the hierarchical GREM states, the entropy in the final state is calculable through a differential equation, which yields the Parisi functional. Thus, Parisi's solution-facilitating hierarchical ansatz is valid if and only if a perfect match can be met asymptotically (large N limit) when an SK system is coupled to a reservoir which is in a hierarchical (GREM) state. Such a match between a pair of coupled hierarchical ROSt's is possible if and only if they are identical. Thus the Parisi ansatz passes what may be viewed as a consistency test. A recent result announced by Talagrand appears to involve a stronger statement, which implies that under sufficient coupling the SK model can "lock in step" with more than one hierarchical ROSt state. From this perspective, the statement appears somewhat surprising.

Ageing for trap models

Gérard Ben Arous (New York)

I surveyed the recent work on ageing for Bouchaud trap models, on finite dimensional lattices $\mathbb{Z}^d$ (work of Fontes-Isopi-Newman in dimension 1, and joint work with Cerny-Mountford in dimension $d > 1$), as well on the complete graph and on the hypercube (dynamics of Derrida's Random Energy Model) which is a joint work with A. Bovier and V. Gayrard.

Limit Laws for Random Exponentials

Leonid Bogachev (Leeds)
(joint work with G. Ben Arous and S. Molchanov)

We study the limit distribution of exponential sums $S_N(t) = \sum_{i=1}^{N} e^{tX_i}$ as $t \to \infty$, $N \to \infty$, where (X_i) are i.i.d. random variables. Two cases are naturally distinguished, esssup $X_i = +\infty$ and esssup $X_i = 0$. The upper distribution tail of X_i is assumed to be of the form $\mathbb{P}\{X_i > x\} = \exp\{-h(\pm x^{\pm})\}$, where $h(\cdot)$ is regularly varying at infinity. An appropriate scale for the growth of $N = N(t)$ is given by $e^{\lambda H_0(t)}$, where the rate function $H_0(t)$ is a certain asymptotic version of the cumulant generating function $H(t) = \pm \log \mathbb{E}e^{tX_i}$, provided by the combined Kasahara–de Bruijn Tauberian theorem. We have found two critical points, $0 < \lambda_1 < \lambda_2 < \infty$, below which the Law of Large Numbers and the Central Limit Theorem, respectively, break down. For $\lambda \leq \lambda_2$ we impose a slightly stronger condition of normalized regular variation of h. The limit laws here appear to be stable, with characteristic exponent $\alpha = \alpha(\varrho, \lambda)$ ranging from 0 to 2 and with skewness parameter $\beta \equiv 1$.

From spin glasses to continuous state branching

Anton Bovier (Berlin)
(joint work with I. Kurkova)

In this talk I review recent work that has led to a rather complete description of the geometry of random Gibbs measures of a particular class of mean field spin glasses. These models, known as Derrida's Generalized Random Energy Models (GREM), can be characterized as Gaussian processes on the hypercube $\{-1, 1\}^N$ whose covariance is a function of the normalized lexicographic distance on this set. The thermodynamic limit of these measures is identified with a one-parameter family of probability measures related to a continuous state branching process introduced by Neveu. Using a construction introduced by Bertoin and Le Gall in terms of a coherent family of subordinators, we show how the Gibbs geometry of the limiting Gibbs measure is given in terms of the genealogy of this process via a deterministic time-change. This construction is fully universal in that all different models (characterized by the covariance of the underlying Gaussian process) differ only through that time change, which in turn is expressed in terms of Parisi's overlap distribution. The proof uses strongly the Ghirlanda-Guerra identities that impose the structure of Neveu's process as the only possible asymptotic random mechanism, and the duality between continuous state branching and coalescent processes on integer partitions, in our case the Bolthausen-Sznitman coalescent.

Two-dimensional critical percolation and its continuum scaling limit

Federico Camia (Eindhoven)

Percolation has received much attention from both physicists and mathematicians for being perhaps the simplest (non-mean-field) model displaying a phase transition with features such as scaling and universality. Scaling refers to the appearance of power laws and relations between their exponents. Moreover, such exponents are often seen to be the same experimentally for quite different materials and models, a phenomenon called universality.

In this talk, I will discuss two-dimensional percolation, and will introduce the continuum scaling limit, which can be seen as a mathematical tool for analyzing the behaviour of the critical point. Some recent progress on the fractal geometry and conformal invariance properties of the continuum scaling limit of two-dimensional critical percolation will also be discussed. This will include both the results of Schramm and Smirnov on the SLE_6 limit of the "exploration process" and the use of that by the speaker and C. Newman to analyze the "full scaling limit."

Directed polymers in random environment

Francis Comets (Paris)
(joint work with T. Shiga and N. Yoshida)

Directed polymers in random environment can be thought of as a model of statistical mechanics in which paths of stochastic processes interact with a quenched disorder (impurities), depending on both time and space. The ground state of the model is oriented percolation or more generally, first passage percolation. We review here main results which have been obtained during the last fifteen years. The material covers the diffusive behaviour of the polymers in weak disorder phase, and localization of the paths and exponents in strong disordered phase.

Convergence times and decay of the return probability for symmetric random walks with random rates in $\mathbb{Z}^d$.

Luiz Renato Fontes (Sao Paulo)
(joint work with P. Mathieu)

We consider symmetric random walks in the d-dimensional torus whose rates come from a family of i.i.d. random variables whose marginal distribution has a polynomial tail at the origin, and derive estimates for times of convergence to equilibrium, as functions of the volume, which are sharp in log scale. We also consider the decay of the return probability in infinite volume and obtain sharp estimates in the annealed case. Spectral techniques are key to all the derivations.

The integrated density of states of non-symmetric Jacobi matrices

Ilya Ya. Goldsheid (London)
(joint work with B. A. Khoruzhenko)

Let (a_j, b_j, c_j) be a sequence of i.i.d. random vectors satisfying certain moment conditions. By

$$H_n = \begin{pmatrix} b_1 & c_1 & & & \\ a_2 & b_2 & c_2 & & \\ & \ddots & \ddots & \ddots & \\ & & a_{n-1} & b_{n-1} & c_{n-1} \\ & & & a_n & b_n \end{pmatrix}$$

we denote a tri-diagonal Jacobi matrix, and by $\gamma(z)$ the top Lyapunov exponent of the product $S_n = g_n \dots g_1$ of matrices $g_j = \begin{pmatrix} \frac{z-b_j}{c_j} & \frac{-a_j}{c_j} \\ 1 & 0 \end{pmatrix}$.

Theorem. *Let μ_n be the normalized eigenvalue counting measure of H_n, that is, $\mu_n = \frac{1}{n}\sum_{l=1}^{n} \delta_{z_l}$, where $z_1, \dots z_n$ are the eigenvalues of H_n. Then:*

(a) *With probability one, μ_n converges weakly, as $n \to \infty$, to*

$$\mu = \frac{1}{2\pi}\Delta\gamma. \tag{1}$$

(b) *(Thouless formula) $\gamma(z) = \int_{\mathbb{C}} \log|w-z| \, d\mu(w) + \mathbb{E}(c_1)$.*

(c) *The limiting eigenvalue counting measure μ is log-Hölder continuous. More precisely, for any $B_{z_0,\delta} = \{z : |z - z_0| \le \delta\}$, $0 < \delta < 1/2$,*

$$\mu(B_{z_0,\delta}) \le \frac{C(z_0,\delta)}{\log\frac{1}{\delta}}, \tag{2}$$

where $C(z_0,\delta) \to 0$ as $\delta \to 0$.

A few questions about ageing

Alice Guionnet (Lyon)
(joint work with C. Mazza)

During the last decade, the study of physical systems out of equilibrium has received a great interest in physics, but also very recently in mathematics. A system is said to age if the older it gets, the longer it will take to forget its past. Such a phenomenon was experimentally observed in diluted media or granular matter for instance. Its mathematical study is still restricted to very few models, since it deals in general with non Markovian, non linear processes whose understanding is still very much model dependent.

In my talk, I detailed the challenge of understanding ageing for the Sherrington-Kirkpatrick (SK) model of spin glass. Then, I described the analysis of ageing for a much simpler model of spin glass, the so-called spherical model of SK. The study of more general systems as described by the p-spins spherical model of SK raises interesting questions. One of them is to understand the long time behaviour of the solutions of the following equations given for $s \ge t \ge 0$, by

$$\begin{aligned}
\partial_s R(s,t) &= -U'(C(s,s))R(s,t) + \int_t^s R(u,t)R(s,u)\nu''(C(s,u))du \\
\partial_s C(s,t) &= -U'(C(s,s))C(s,t) + \int_0^t R(t,u)\nu'(C(s,u))du \\
&\quad + \int_0^s R(s,u)C(t,u)\nu''(C(s,u))du,
\end{aligned}$$

where R is an auxiliary unknown called the response function such that $R(t,t) = 1$ and ν is a polynomial function depending on the model. The analysis of such a system is a completely open problem, even on a non rigorous ground where scenarios could only be checked by L. Cugliandolo and J. Kurchan. With C. Mazza, I undertook the analysis of the first equation being given C. We can then predict the asymptotic behaviour of R for a variety of asymptotic behaviour for C.

Localization transition for a copolymer in an emulsion

Frank den Hollander (Eindhoven)
(joint work with S. Whittington)

In this talk we look at a copolymer near an interface separating two immiscible fluids. The copolymer consists of a random concatenation of hydrophobic (A) and hydrophilic (B) monomers. The interface separates oil (A) and water (B). The energy of the copolymer is $-\alpha$ times the number of AA-matches plus $-\beta$ times the number of BB-matches, where $\alpha, \beta \geq 0$ are parameters. Depending on the values of α, β, the copolymer either localizes near the interface or wanders away from it.

After reviewing earlier results with E. Bolthausen, M. Biskup and M. Wüthrich, we focus on a model where the oil/water medium has a percolation-like structure: $\mathbb{Z}^2$ is divided into blocks of size L_n, and each block has a probability p to be oil and $1-p$ to be water. The copolymer is a directed random walk of length that is allowed to enter and exit blocks only at opposite corners. We derive a variational expression for the quenched free energy per step, $f(\alpha, \beta; p)$, in the limit as $n \to \infty$ when $L_n \to \infty$ and $L_n/n \to 0$. We show that, for each p, $f(\alpha, \beta; p)$ is non-analytic along a curve $\mathcal{C}(p)$ in the (α, β)-plane. If $p \geq p_c$, with p_c the critical probability for oriented bond percolation on $\mathbb{Z}^2$, then $\mathcal{C}(p)$ does not depend on p, and we obtain its qualitative properties. On the other hand, if $p < p_c$, then $\mathcal{C}(p)$ does depend on p, and is different in nature. Its qualitative properties are still ill understood in this regime.

Old and new tales about Lifshitz tails

Werner Kirsch (Bochum)

We give an overview on mathematical results about the low energy behaviour of the density of states for random Schrödinger operators. This characteristic behaviour is known as "Lifshitz tails", after the physicist I. Lifshitz who found this behaviour on the basis of convincing but non rigorous arguments. There are various mathematical techniques to solve this problem. At the core of any of these methods is a large deviation result making Lifshitz argument rigorous, that low energies should come from the rare event that the random potential is extraordinary small. Besides a historical the talk concentrates on recent results about non homogeneous potential and/or homogeneous magnetic fields.

Universality in the parabolic Anderson model

Wolfgang König (Berlin)
(joint work with R. van der Hofstad and P. Mörters)

The parabolic Anderson model is the Cauchy problem for the heat equation on $(0, \infty) \times \mathbb{Z}^d$ with random potential. It describes a random mass transport through a random potential of sinks and sources, where the mass is exponentially decreased and increased, respectively. This model also appears in the description of branching processes in a random field of branching rates; furthermore, it serves as a simplified model for chemical reactions.

Our main concern is the analysis of the large-time behaviour, for i.i.d. potential, of the total mass accumulated in the space. Here the effect of *intermittency* is of particular importance. It roughly states that the main bulk of the mass is produced in few small islands which are far away from each other.

We explain the heuristics about the large-time behaviour of the model and survey existing rigorous results. The particular case of the so-called double-exponential distribution (investigated by Gärtner and Molchanov in 1998) is presented in some detail. Finally, two mild regularity assumptions on the upper tails of the field distribution are presented under which there are only three universality classes of asymptotic behaviours. Two of these classes are already known and have been worked out, and the third one is currently under investigation in joint work with Remco van der Hofstad (Eindhoven) and Peter Mörters (Bath).

Transport by random flows

Leonid Koralov (Princeton)
(joint work with D. Dolgopyat and V. Kaloshin)

Consider the equation $\frac{dx}{dt} = v(x,t)$, where the right hand side is a random field (or it may contain stochastic differentials). We overview the results concerning the asymptotic behaviour of a single particle (CLT for one point motion). We then discuss the results (joint with D. Dolgopyat and V. Kaloshin), which describe the evolution of sets carried by the finite dimensional stochastic flow, the CLT for measures, which holds for almost every realization of the randomness, and the theorem on the limiting shape of the linearly scaled image of a set carried by the flow.

A random necklace model

Vadim Kostrykin (Aachen)
(joint work with R. Schrader)

We consider a Laplace operator on a random graph consisting of infinitely many loops joined symmetrically by intervals of unit length. The arc lengths of the loops are considered to be independent, identically distributed random variables. The integrated density of states of this Laplace operator is shown to have discontinuities provided that the distribution of arc lengths of the loops has a nontrivial pure point part. The Lyapunov exponent and localization are also discussed.

Continuous integral kernels for Schrödinger semigroups with Gaussian random potentials

Hajo Leschke (Erlangen)
(joint work with K. Broderix (1962–2000) and P. Müller)

This is a joint work with Kurt Broderix (1962–2000) and Peter Müller. It builds on a short paper by Barry Simon published in 2000. Therein he singled out a maximal class of negative (non-random) potentials $V\colon \mathbb{R}^d \to \mathbb{R}$ such that the corresponding Schrödinger operator $H := -\nabla^2 + V$ on the Hilbert space $L^2(\mathbb{R}^d)$ is self-adjoint and unbounded from below, but the exponential $\exp\{-tH\}$ nevertheless acts as an integral operator with a kernel $k_t(x,y)$ given by a Feynman-Kac formula for arbitrarily large (time) parameter $t > 0$. By suitably extending Simon's result we show, among other things, that $k_t(x,y)$ is continuous in $(x,y) \in \mathbb{R}^d \times \mathbb{R}^d$ and that any spectral-projection operator $\chi_I(H)$ of H associated to a Borel set $I \subset \mathbb{R}$ with $\sup I < \infty$ also possesses an integral kernel which is continuous (and given in terms of $k_t(x,y)$). It is pointed out that such results hold true almost surely for a wide class of random potentials, including Gaussian ones. This leads to a rigorous justification of an integral-kernel formula for the corresponding integrated density of states which is frequently used in the physics literature on disordered systems.

Two-colour scenery reconstruction see along the path of a random walk with jumps

HEINRICH MATZINGER (BIELEFELD/ATLANTA

A scenery is a colouring of the integers $\xi\colon \mathbb{Z} \to \{0, 1, \ldots, C-1\}$. Let $\{S(k)\}_{k\geq 0}$ be a recurrent random walk on $\mathbb{Z}$ starting at the origin. Let χ designate the colour record obtained by observing the scenery ξ along the path of the random walk $\{S(k)\}_{k\geq 0}$:

$$\chi := (\xi(S(0)), \xi(S(1)), \xi(S(2)), \ldots).$$

Here is the scenery reconstruction problem: given an unknown scenery ξ, can we "reconstruct" ξ if we can only observe χ ? Does one path realization of the process $\{\chi(k)\}_{k\geq 0}$ a.s. uniquely determine ξ? Not every scenery can be reconstructed but a lot of typical sceneries can. For this we take the scenery to be random itself and show that with the right measure almost every scenery can be reconstructed up to shift and reflection. How to solve such a reconstruction problem depends a lot on the specific properties of the random walk and the distribution of the scenery. We present some of the general methods and ideas.

Edge-reinforced random walk on a ladder

FRANZ MERKL (LEIDEN)
(joint work with S. Rolles)

In 1987, Persi Diaconis introduced edge-reinforced random walks as a model for a stochastic process in a self-generated stochastic environment. In spite of its elementary definition, it is hard to derive properties of these random walks. In particular, the question asked by Diaconis in the late eighties, whether the edge reinforced random walk on $\mathbb{Z}^d$, $d \geq 2$ is recurrent, remains still open. Recently, Silke Rolles and F.M. have proven that the edge-reinforced random walk on the ladder $\mathbb{Z} \times \{1, 2\}$ with initial weights $a > 3/4$ is recurrent. The proof uses a representation of the edge-reinforced random walk on a finite piece of the ladder as a random walk in a random environment. This environment is given by a marginal of a multi-component Gibbsian process. A transfer operator technique and entropy estimates from statistical mechanics are used to analyse this Gibbsian process. The talk will explain these techniques and show how they are applied to derive properties of the edge reinforced random walk.

Transition from quenched to annealed asymptotics for random walks among random traps

Alejandro F. Ramirez (Santiago)
(joint work with G. Ben Arous and S. Molchanov)

We present a natural transition mechanism describing the passage from a quenched (almost sure) regime to an annealed (in average) one, for some stochastic processes in random media. We investigate a particular example of a random walk on i.i.d. random obstacles. Let $p_x(t)$ be the survival probability at time t of the random walk, starting from site x, and $L(t)$ be some increasing function of time. We show that the empirical average of $p_x(t)$ over a box of side $L(t)$ has different asymptotic behaviours depending on $L(t)$. There are constants $0 < \gamma_1 < \gamma_2$ such that if $L(t) \geq e^{\gamma t^{d/(d+2)}}$, with $\gamma > \gamma_1$, a law of large numbers is satisfied and the empirical survival probability decreases like the annealed one; if $L(t) \geq e^{\gamma t^{d/(d+2)}}$, with $\gamma > \gamma_2$, also a central limit theorem is satisfied. If $L(t) \ll t$, the averaged survival probability decreases like the quenched survival probability. If $t \ll L(t)$ and $\log L(t) \ll t^{d/(d+2)}$ we obtain an intermediate regime. Furthermore, when the dimension $d = 1$ it is possible to describe the fluctuations of the averaged survival probability when $L(t) = e^{\gamma t^{d/(d+2)}}$ with $\gamma < \gamma_2$: it is shown that they are infinitely divisible laws with a Lévy spectral function which explodes when $x \to 0$ as stable laws of characteristic exponent $\alpha < 2$. These results show that the quenched and annealed survival probabilities correspond to a low and high temperature behaviour of a mean field type phase transition mechanism.

Large deviations for random walks in a mixing random environment, and other (non-Markov) random walks

Firas Rassoul-Agha (Columbus)

In [3], we extend a recent work by S.R.S. Varadhan [1] on large deviations for random walks in a product random environment, to include many non-Markovian random walks. In particular, several classes of reinforced random walks, and of random walks in Gibbs fields are included. We use the method of the point of view of the particle that we have used before in [2]. We bound the Radon-Nykodim derivative of two environments, as viewed from two independent walkers, by how close the future path passes by the past of the two walkers.

References

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Localization for random word models

Bob Sims (Princeton)

We consider Schrödinger operators in $\ell^2(\mathbb{Z})$ whose potentials are obtained by randomly concatenating words from an underlying set $\mathcal{W}$ according to some probability measure ν on $\mathcal{W}$. Our assumptions allow us to consider models with local correlations, such as the random dimer model or, more generally, random polymer models. We prove spectral localization and, away from a finite set of exceptional energies, dynamical localization for such models. These results are obtained by employing scattering theoretic methods together with Furstenberg's theorem to verify the necessary input to perform a multiscale analysis.

On the largest eigenvalue of a random subgraph of the hypercube

Alexander Soshnikov (Davis)
(joint work with B. Sudakov)

The problem discussed below came from mathematical biology ([1], [2]). In mathematical terms we consider a random subgraph G of the n-cube where each edge appears randomly and independently with probability $p = p(n)$. In the limit $n \to \infty$ we prove ([3]) that the largest eigenvalue of the adjacency matrix of G is almost surely

$$\lambda_{max} = \max(\Delta^{1/2}(G), \bar{d})(1 + o(1)),$$

where $\Delta(G)$ is the maximal degree of G and $\bar{d}$ is the average degree, assuming that $p(n)$ is at least not exponentially small in n (the case of $p(n)$ exponentially small is rather trivial and can be treated separately). This is the joint result with Benny Sudakov (Princeton).

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Quenched results for some random walks in random media

Alain-Sol Sznitman (Zurich)
(joint work with V. Sidoravicius)

In this talk we discuss random walk on the supercritical infinite cluster of bond percolation on $\mathbb{Z}^d$, and random walk among random conductances between nearest neighbour edges of $\mathbb{Z}^d$. We describe functional central limit theorems for these walks valid for almost every environment, when d is bigger or equal to 4 in the case of percolation, and when d is bigger or equal to 1 in the percolation case. The strategies employed when proving the two results are quite different.

Brownian percolation

Wendelin Werner (Orsay)
(joint work with G. Lawler and O. Schramm)

After recalling some relevant known or conjectured facts from the theory of critical two-dimensional systems from statistical physics, we construct the Brownian loop soup (joint work with Greg Lawler), a conformally invariant Poissonian cloud of Brownian loops. Restricting the loop-soup to those loops that stay in a given domain D gives a random countable family of (overlapping) loops that all stay in D. There is a natural parameter c in this model: The intensity of the Poissonian realization. Using also the family of sets satisfying the conformal restriction property (constructed with Greg Lawler and Oded Schramm), we then show that when $c \leq c_0$ (a heuristic, that can probably be made rigorous shows that $c_0 = 1$), there exist disjoint clusters of Brownian loops, while for $c > c_0$, one single cluster is dense in D. Furthermore, the outer boundaries of these clusters are SLE type loops, (i.e. Schramm-Loewner Evolutions with paramater κ related to c). This allows to give a direct construction of conformal fields as observables of the loop-soup, and the central charge of the model is simply the intensity c.

Edited by Wolfgang König

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