Mathematisches Forschungsinstitut Oberwolfach

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Mathematical Aspects of Computational Fluid Dynamics

November 9th – November 15th, 2003

This conference followed the tradition of two preceding conferences with the same title in 1994 and 1997, this time organized by Endre Süli (Oxford), Miloslav Feistauer (Prague) and Rolf Rannacher (Heidelberg). The subjects have been current questions of modelling and numerical analysis in computational fluid dynamics (CFD). The conference had 47 participants from 11 different countries representing Applied and Numerical Analysis.

The frame of the program was set by seven invited survey lectures on the topics

- Compressible flow solvers;
- Modelling of non-Newtonian flow with applications;
- Higher-order methods in CFD;
- Adaptive DNS/LES: a new agenda in CFD;
- Residual-based stabilized FEM for non-isothermal incompressible flows;
- Numerical methods for low mach number flow;
- Numerics of fluid-structure interaction.

Supplementing these surveys, additionally 18 shorter talks have been given on more specialized topics. The participants have expressed there satisfaction with this format of the conference which left plenty of time for discussion.

A larger group of talks addressed computational issues raised in non-standard flow models such as non-Newtonian and free boundary flows as well as oceanic and atmospheric flows with multiple scales. Furthermore, flows with fluid-structure interaction and numerical flow control by adjoint techniques was considered.

A second group of talks discussed novel viscosity and multi-scale approaches to numerically modelling of turbulence. This subject has recently seen many promising developments based on new analytical approaches and improved techniques for adaptive error control in computations. Here, a major breakthrough seems possible by those new methods.

One central theme in CFD is that of numerical "stabilization". Accordingly, a third group of talks presented new ideas on stabilization of finite element schemes and possible causes and cures of unstable behaviour. Here, new edge-stabilization techniques seem to have very promising features.

The remaining talks dealt with special aspects of numerical schemes for viscous or inviscid flows such as high-resolution in multiple dimensions, discrete Korn's's inequality, splitting correction schemes and efficient multigrid methods. These are all areas of currently very active research.

Abstracts

Flow optimization with adaptive finite elements

Roland Becker

We give an overview over adaptive methods for optimization in the context of flow problems. The main ingredient is the development of a posteriori error estimators which take into account the specific needs originating from different types of optimization problems. The main application of the estimator is local mesh refinement; but it can obviously be used for the adaptation of other discretization parameters such as for example h-p refinement.

The first part of the talk describes three typical problems with different interpretation of the optimization framework. Once the action of the control for reduction of the drag coefficient is of primary interest, once the control itself is at the heart of the matter, for example when identifying the viscosity constant from measurements. The third example deals with the situation where the control parameters are used for model calibration, and the goal of the computation is not directly related to the control (but to another objective functional).

The presented examples lead us to consider the following general case: we are interest in computing an interest function $I(q, u)$ which depends on both control q and state variable u. The interest functional is independent of the optimization problem which determines q and u. The second part of the talk develops this idea. By specialization we obtain estimators for:

- error in the cost functional
- error in a functional of the controls
- error in an independent functional of the state
- norm of the controls

Beside the first estimator, the others require the solution of an additional problem involving the adjoint of the linearized state operator. The right-hand side of this problem depends on the special context.

In the last part of the talk, we show that the information given by the additional adjoint problem might be used for further purposes. We employ the well-known concept of condition numbers in order to produce answers to the following questions: which parameters have been most important, which measurements have been most important in computing the solution?

A numerical model for the wind-driven circulation of the North-Atlantic Ocean

Rodolfo Bermejo

I describe a numerical model to study the long-term wind driven circulation of the North-Atlantic Ocean. The model computes the vertically integrated velocity and pressure with a pressure correction projection scheme using the P_2/P_1 finite elements for space discretization. The first step of the projection scheme is further decomposed into two steps, namely, transport step and diffusive step. In the transport step the material derivative of the velocity is approximated by a second-order BDF-semi-Lagrangian scheme that uses P_2-quasi-monotone interpolation at the feet of the characteristics which go through the mesh points at the current time step.

Taking the climatological wind stress as the forcing of the model and the eddy viscosity coefficient A as a control parameter, I present some numerical experiments that illustrate the long-term behaviour of the circulation. Thus, it is shown that: (i) for a range of values of A there are multiple steady states; (ii) when A reaches a value which is sufficiently small the circulation undergoes a first Hopf-bifurcation; (iii) as A keeps decreasing the circulation will become rapidly quasi-periodic and for lower values it will reach a-periodic states. Another component of my talk is the POD post-processing of the numerical solution in order to calculate the equivalent reduced system of ordinary differential equations which allows to study the dynamical behaviour of the numerical solution.

Edge stabilization: an interior penalty method for conforming and non-conforming Galerkin approximations

Erik Burman

We discuss stabilized Galerkin approximations in a new framework, widening the scope from the usual dichotomy of the discontinuous Galerkin method on the one hand and Petrow-Galerkin methods such as the SUPG method on the other. The idea is to use interior penalty terms as a means of stabilizing the finite element method using conforming or non-conforming approximation, thus circumventing the need of a Petrow-Galerkin type choice of spaces. This is made possible by adding a higher order penalty term giving L^2-control of the jumps in the gradients between adjacent elements. In this talk we give a general outline of how to obtain stability and convergence proofs for this kind of method in the case of convection-diffusion equations. We discuss what approximation spaces that may be used and how the method relates to classical stabilization methods. The extension of the theoretical results to the Oseen's equations is then sketched. Numerical examples are given showing that the a priori error estimate derived for the convection–diffusion equation is sharp. Finally some preliminary computational results on the Navier-Stokes equations are presented.

Numerical instabilities in the finite element approximation of rotating flows

Ramon Codina

In this talk we discuss the instability encountered in flows dominated by the Coriolis force. In the first part, we obtain the expression for the acceleration of a fluid particle when it is referred to a rotating frame of reference. This development is absolutely standard and basic, but will serve to motivate the numerical method introduced in the third part of the presentation.

Once the Navier-Stokes equations are written in a rotating reference system and discretized in time, a simple energy estimate is obtained. Again, this is a standard development, but serves to present the instability encountered when rotation dominates at the same point and with the same arguments as the instabilities due to convection dominated flows and the pressure instability, which leads to the need to satisfy an inf-sup condition for the velocity and pressure spaces.

As an extension of the well known Characteristic Galerkin Method, the Inertial Galerkin Method is then introduced as a remedy to deal with the instabilities due to convection and rotation dominated flows at the same time. However, rather than integrating the equations along the characteristics and referring the time levels to a same inertial basis, a Taylor expansion is performed to obtain a problem posed in the current reference and the current spatial points. The outcome is the appearance of some stabilizing terms that will enhance stability.

In the fourth and final part of the talk, an error estimate is presented for the case of the linearized stationary problem. Even though it is based on a subgrid scale formulation, it introduces the same stabilizing terms as the Inertial Galerkin Method. The inspection of what happens in some limiting cases indicates that the error estimate behaves adequately.

Compressible flow solvers

Vit Dolejsi

(joint work with Miroslav Feistauer)

We present a survey of numerical methods for the solution of inviscid as well as viscous compressible flows with the emphasis on modern methods represented by discontinuous Galerkin methods (DGM).

After the problem formulation, we start with the use of the finite volume method for the solution of inviscid flow problems. We mention various numerical fluxes and two main approaches for higher order scheme. Then we develop the application of DGM, we mention main types of time discretization and describe the application of semi-implicit scheme.

We proceed with the solution of viscous compressible flow, present the use of finite volume (FV) methods, finite element (FE) methods, combined FV – FE methods and DGM. We discuss several approaches of the application of DGM and derive the discontinuous Galerkin finite element method with nonsymmetric treatment of diffusive stabilization terms with interior and boundary penalty for the solution of the compressible Navier-Stokes equations. Several numerical examples are presented. At the end some conclusion and possible future research are given.

Modelling of non-Newtonian flow with applications

Giovanni P. Galdi

We begin to give an overview of some fundamental experiments that do not find explanation in the (Newtonian) Navier-Stokes theory. They include the "rod-climbing" effect and several still unexplained experiments on fluid-particle interaction. In the second part of the talk, we introduce some popular non-Newtonian fluid models and point out the limit of applicability of each of them. Finally, we show how some of these models furnish a mathematical explanation of certain experiments on sedimentation of particles in visco-elastic liquids.

Convergence of discrete adjoint approximations in the presence of shocks

Michael Giles and Stefan Ulbrich

In the first part of this talk, we look at the convergence of discrete approximations to linearized equations when there are shocks in the underlying nonlinear solution. Numerical examples show that the convergence of a nonlinear functional due to the use of a conservative discretization does NOT guarantee the convergence of linear sensitivities. However, convergence of the linearized functional can be proved for a simple discretization of Burgers equation in which a) the linear discretization is a linearization of the nonlinear discretization, and b) the nonlinear discretization increases the number of points across the shock as $h \to 0$.

In the second part of this talk, as a model problem, we consider the initial control of a scalar hyperbolic conservation law with convex flux and a tracking-type output functional. We present differentiability results for the output functional with respect to the control and derive an adjoint-based gradient formula. The properties of the adjoint equation are discussed. For the discretization we consider a particular difference scheme with associated adjoint scheme. A key point is that the discretization increases the number of points across the shock as the grid is refined. It is proved that this gives L^p-convergence of the discrete adjoint and uniform convergence outside of the extreme backward characteristics confining the shock funnel. The theoretical findings are illustrated by numerical results.

A tentative definition of large eddy simulation (LES) of turbulent flows and illustration on a hyperviscosity spectral model

Jean-Luc Guermond

I propose to call LES solution of NS any sequence $(u_\epsilon, p_\epsilon)_{\epsilon>0}$ such that $u_\epsilon \to u$, $p_\epsilon \to p$ in the usual class of Laray's weak solutions and (u,p) is a "suitable" solution in the sense of V. Scheffer, Caffarelle-Khon-Nierenberg, i.e. $\int_0^T \int_\Omega (\partial_t u + u \cdot \nabla u - \Delta u + \nabla p - f) \cdot \phi u \leq 0$ for all $\phi \in D((0,T) \times \Omega)$, $\phi \geq 0$. This definition is constructive, in the sense that requiring a "numerical" approximation to converge to a suitable solution requires the numerical scheme to be non-trivial, i.e. "Galerkin" is not enough. In general adding a so-called "LES model" is necessary to have a suitable weak solution at the limit.

The spatially periodic Navier-Stokes equations

John Heywood

Engineering applications of the Navier-Stokes equations are invariably set in domains with boundaries. The study of spatially periodic solutions provides a venue for certain aspects of turbulence theory, dynamical systems theory, and a posteriori existence theory, and for attempting to prove the global regularity of solutions. The nonlinear interactions within such solutions can be studied by Fourier Transforms.

Noting (as have others) that the Fourier Transform of the nonlinear term implies a parallelogram law, I have written a code based on just that, utilizing observations about the physics of triadic interactions and an adaptive listing of active modes. These modes are found explicitly as sinusoidal shear waves, with directions orthogonal to their wave numbers. The parallelogram law for their nonlinear interactions is based on identities for trigonometric integrals, and on dot products between their wave numbers and wave directions. We make some observations about their triadic interactions that may be relevant to the problem of global regularity. In particular, we find a family of globally existing solutions whose Dirichlet norms grow at most exponentially.

Higher-order methods in CFD

Vincent Heuveline and Paul Houston

We present an overview of some recent developments concerning the design and mathematical analysis of *hp*-version finite element methods for CFD problems. In particular, we review both stabilized conforming as well as non-conforming discontinuous Galerkin finite element methods for a range of problems, including first-order hyperbolic problems, convection-diffusion problems, and incompressible and compressible flow problems. Here, the emphasis will be on the choice of various stabilization parameters arising within these schemes, as well as the choice of suitable (stable) approximation spaces, which ensures that simultaneously h- and p-optimal error bounds may be established. The final part of this talk will be devoted to the design of reliable and efficient *hp*-adaptive algorithms. Numerical experiments highlighting the practical performance of automatic *hp*-adaptive algorithms will be presented.

On a new type of a variational multiscale method

Volker John

The talk starts with the consideration of the classical LES approach where the large scales are defined by space filtering. It is shown that already the first step in deriving equations for the large scales leads in the case of a bounded domain to an additional term whose practical treatment is completely unclear. This motivates a different definition of the large scales, namely be projection into subspaces as in the variational multiscale method (VMS). The second part of the talk presents a new type of VMS where the fluctuations (small scales) can cross mesh cell boundaries. A finite element error analysis of this method is given, its implementation is discussed and numerical studies will be presented.

Adaptive DNS/LES: a new agenda in CFD

Claes Johnson

(joint work with Johan Hoffmann)

We present a new approach to computational turbulence modeling based on Adaptive DNS/LES. We show an example of the computations of drag of a surface mounted cube at Reynolds number $40,000$ performed on a PC using $100,000 - 250,000$ mesh points.

Asymptotic analysis and numerical methods for atmospheric flows

Rupert Klein

Observations of the atmosphere reveal a multitude of different length and time scales. Theoretical meteorology has developed a hierarchy of simplified models that explain the phenomena associated with specific combinations of these scales. The derivations provided in textbooks and research papers are often acceptable only to the meteorologically trained reader. In this presentation I demonstrate how one may derive a large collection of these simplified model equations through a particular distinguished limit of scale-independent parameters and a general multiple-scales-asymptotic ansatz. The derivations show how most simplified models are associated with singular asymptotic limits giving rise to numerical stiffness in any attempt to solve the full 3D compressible equations in theses flow regimes. By one example I demonstrate how the insight gained on the asymptotics may be used to construct robust numerical methods.

Is a uniform Korn's inequality necessary for proving convergence of nonconforming finite elements?

Petr Knobloch
(joint work with Lutz Tobiska)

We investigate the Korn first inequality for low order nonconforming finite elements and clarify the dependence of the constant in this inequality on the discretization parameter h. Then we use the nonconforming elements for approximating the velocity in a discretization of the Stokes equations with boundary conditions involving surface forces. We show that, in some cases, convergence results can be proved even if the Korn inequality does not hold uniformly with respect to the discretization parameter h.

High-resolution finite element schemes for convection-dominated flows

Dmitri Kuzmin

A new family of local extremum diminishing finite element schemes is introduced. The monotonicity of the Galerkin discretization is enforced at the discrete level by adding artificial diffusion edge-by-edge so as to eliminate negative off-diagonal matrix entries. The resulting low-order operator enjoys the M-matrix property and acts as a 'preconditioner' within an outer defect correction loop. A generalization of classical TVD concepts is used to design the antidiffusive fluxes which are incorporated into the defect. The proposed methodology is truly multidimensional and applicable in conjunction with both explicit and implicit time-stepping schemes. Its potential is demonstrated by application to incompressible flow problems, the Euler equations of gas dynamics and turbulent bubbly flows in gas-liquid reactors.

Residual-based stabilized FEM for non-isothermal incompressible flows

Gert Lube

The goal of the lecture is a survey of selected theoretical results for stabilized finite element methods for incompressible flow problems and a discussion of open problems from the view-point of applications. The numerical simulation of incompressible and non-isothermal flows with a two-equation turbulence model serves as typical applied problem.

We combine an implicit semidiscretization in time of the coupled nonlinear problem with a first or second order scheme as the outer loop with a block Gauß-Seidel decoupling together with fixed point or Newton-type iteration as an inner loop in each time step. The resulting auxiliary problems are the linear(iz)ed advection-diffusion-reaction problem and an Oseen-type problem as a simplified version of the linearized Navier-Stokes model.

First we review theoretical results for the h- and p-variants of the Galerkin-FEM with residual stabilization of SUPG-type applied to the scalar auxiliary problem. Moreover, we discuss a residual-based shock-capturing variant of the SUPG scheme. Then we consider stabilization techniques for the saddle point problem of Oseen type. Besides the classical SUPG/PSPG-stabilization of equal-order pairs of velocity and pressure, we discuss corresponding stabilization techniques for divergence-stable pairs. Then we discuss how the numerical analysis can be extended to anisotropic meshes in boundary layers.

Then it is shown that the proposed approach an be applied to the numerical simulation of indoor air flows in practical applications. Moreover, we discuss some problems occurring if an higher-order implicit time discretization scheme is combined with stabilization techniques in space-time. Finally, we discuss how standard stabilization methods fit into the recently proposed concept of variational multiscale methods. Such methods provide a very promising framework for the treatment of turbulent flow problems.

Numerical methods for low mach number flow

Claus-Dieter Munz

In the approximation of fluid flow for small Mach numbers several difficulties appear. This is due to the fact that in the limit of vanishing Mach numbers a change of the type of the equations occurs. This incompressible limit of a compressible fluid is addressed and discussed in detail. Because the propagation rate of the pressure waves tends to infinity, the equations become stiff and an accuracy problem may occur. The different classes of numerical schemes and their construction is discussed. The extension of explicit compressible methods is usually based on preconditioning. Another approach that may be used to extent incompressible methods to the compressible regime is the MPV-method or a rescaling approach. Here, the pressure is decomposed into a thermodynamic and a hydrodynamic part. If this pressure decomposition is inserted into the equations, they converge formally to the incompressible equations when the Mach number tends to zero. The pressure correction idea as developed for the incompressible regime can be extended to the weakly compressible regime. To capture shock waves as well the whole procedure should be based on the conservation form of the compressible equations.

The connection of incompressible flow calculation to noise generation and propagation is discussed with respect to a perturbation approach. In this expansion about the incompressible flow field the linear Euler equations are obtained as perturbation equations describing the noise propagation with source terms corresponding to the noise production calculated from the incompressible flow data. Numerical results for several applications and test problems are shown.

Numerics of fluid-structure interaction

Michael Schäfer

An overview of numerical aspects relevant for the simulation of fluid-structure interaction problems is given. We consider aspects of modelling, discretization, mesh dynamics and coupled solution techniques. By several numerical applications aspects of efficiency and accuracy of the computations are discussed. Here, a major focus is on the efficient involvement of multigrid approaches.

Adaptive wavelet methods for modelling and computing turbulent flows

Kai Schneider

(joint work with Marie Farge)

Recently we have introduced a new wavelet-based method, called Coherent Vortex Simulation (CVS), to compute turbulent flows. The main idea is to split the flow into two orthogonal parts, a coherent flow and an incoherent background flow, using nonlinear filtering of vorticity. We show that the coherent flow is responsible for the nonlinear dynamics, and thus its evolution is deterministically computed in an adaptive wavelet basis, while the incoherent flow is noise-like, structureless and decorrelated. Therefore its influence is statistically modelled. Since the coherent part is described by only few wavelet modes, it is possible to reduce the computational cost, both in terms of memory requirement and CPU time. The developed adaptive wavelet solver for the 2D Navier-Stokes equations uses a semi-implicit time discretization and a Petrov-Galerkin scheme in space with wavelets as trial-functions and vaguelettes (operator adapted wavelets) as test-functions. To take into account no-slip boundary conditions and complex geometries we coupled the adaptive scheme with a volume penalization technique. In the talk we present different applications of coherent vortex extractions out of 2D and 3D turbulent flows and different CVS computations of 2D flows past bluff bodies and a 3D turbulent mixing layer.

One the accuracy of the splitting schemes for incompressible flows

Jie Shen
(joint work with J.L. Guermond)

I review various splitting schemes for solving time dependent Navier-Stokes equations. These schemes share the same advantage that one only needs to solve a sequence of decoupled Poisson-type equations at each time step and have been widely used in practice due to their efficiency and simplicity. However, since the splitting involves non-commutative operators, how to design accurate and stable splitting schemes is a very subtle issue. I present error estimates for two class of splitting schemes, namely pressure-correction and velocity-correction schemes and show that they all suffer from an irreducible splitting error. I then present a new class of truly consistent splitting schemes. Finally, I discuss the influence of open boundary conditions on the accuracy of the splitting schemes.

Black-box multigrid preconditioning for steady incompressible flows

David Silvester

Discretization of PDEs using mixed approximation leads to symmetric indefinite or unsymmetric indefinite linear system of equations. We outline a generic block preconditioning technique for such systems with the property that the eigenvalues of the preconditioned matrices are contained in intervals that are bounded independently of the mesh size. The attractive feature of our technique is that the basis of the preconditioning is a readily available building block; namely, a scalar diffusion or convection-diffusion solve based on a geometric or algebraic multigrid V-cycle. Some numerical results are presented showing the effectiveness of this approach in the context of anisotropic diffusion equations arising in modelling ground-water flow, and the Navier-Stokes equations that arise in incompressible flow modelling.

On application of finite element method to problems of aeroelasticity

Petr Sváček

We are concerned with mathematical modelling of an interaction of viscous incompressible fluid and a wing. The real situation is three dimensional, but for our purposes we restrict us only to a situation in two dimensional cuts of the original domain. For such a situation the airfoil can be considered as a solid body with two degrees of freedom - one is vertical displacement and the other one angle of rotation. The motion of the airfoil can be either prescribed or obtained by a solution of a coupled fluid-structure model. The structure model is described by a system with two ordinary differential equations. The fluid flow is simulated by the Navier-Stokes system of equations, which is rewritten in the Arbitrary Lagrangian-Eulerian formulation. The space discretization is done with the aid of finite element method. Because of a high Reynolds number an appropriate stabilization method has to be used. We discuss the SUPG stabilization of the FEM, coupled fluid-structure model as well as the solution of the discrete problem.

Navier-Stokes-α turbulence models

Edris Titi

It is well known that one can derive the Euler equations of inviscid incompressible fluid flow from the Hamilton Principle, where the Lagrangian equals to $\int_0^T \int_{R^n} |u(x,t)|^2 \, dx \, dt$ subject to the incompressibility constraint $\nabla \cdot u = 0$. Holm, Marsden and Ratin derived the so-called Euler-α equation using the same principle with the Hamiltonian involved $\int_0^T \int_{R^n} \left(|u(x,t)|^2 + \alpha^2 |\nabla u(x,t)|^2\right) dx \, dt$, subject to the constraint $\nabla \cdot u = 0$. Adding a viscosity to the Euler-α model one reaches to the Navier-Stokes-α model (also known as the viscous Camassa-Holm equations or averaged Navier-Stokes-α model):

$$\partial_t v - \nu \Delta v - u \times (\nabla \times v) + \nabla p = f, \quad v = u - \alpha \Delta u, \quad \nabla \cdot u = 0.$$

In this talk we show global well-posedness of this model and show that the dimension of its global attractor is of the order of $(L/l_d)^3$, where l_d is the Kolmogorov small dissipation scale. Then, we use this model as a closure model for the Reynolds averaged equations and compare our analytic solution to empirical data in turbulent channels and pipes, and demonstrate a very good agreement. We also show that its energy spectrum obeys the Kolmogorov $k^{-5/3}$ power law for wave number $\alpha k \ll 1$, and the k^{-3} power law for $\alpha k \gg 1$. We also introduce the Leray-α model, which is in spired by the above work:

$$\partial_t v - \nu \Delta v - (u \cdot \nabla) v + \nabla p = f, \quad u - \alpha \Delta u = v, \quad \nabla \cdot u = 0,$$

and show that it gives similar results. Most interestingly, we show that the dimensions of the global attractors for the Leray-α model is of the order $(L/l_d)^{12/7}$.

Free boundary value problems in ferrohydrodynamics

Lutz Tobiska

Ferrohydrodynamics deals with the mechanics of fluid motion influenced by magnetic fields. Ferrofluids are colloidal suspensions of ferromagnetic particles in a nonmagnetic carrier liquid (oil, water). Due to the small size of about 10nm the particles are magnetic monodomains and the fluid behaves like super-parametic material. The interaction between the magnetizable fluid and an external magnetic field gives rise to several interesting phenomena like instabilities, field dependent viscosities and viscoelastic effects. The possibility to modulate the hydrodynamic parameters of the fluid with an external magnetic field also opens the way for a variety of technical and medical applications, like zero-leakage rotary shaft seals, liquid-cooled loudspeakers and magnetic drug targeting.

We first present modelling aspects to describe the interaction between magnetic fields and fluids. The basic model consists of the incompressible Navier-Stokes equations coupled with the Maxwell equations for a nonconducting fluid. Then we focus on numerical solution strategies for solving flow problems with free boundaries under the influence of external magnetic fields. In particular, we consider three examples: the behavior of a ferrofluid drop in a rotary shaft seal under different operating conditions, the instability of a ferrofluid in a narrow gap between two parallel plates, and the Rosensweig instability of spontaneous generation of an ordered pattern of surface protuberances when the field exceeds a critical value. In the static case of the Rosensweig instability no flow calculation is needed and the problem consists in solving a 3D problem for the magnetostatic potential and a 2D problem to determine the free surface iteratively. In the dynamic case, full 3D calculation of the flow and magnetic fields are needed. The ALE approach and an advanced 3D Navier-Stokes solver has been used to handle the time-dependence of the fluid domain. We discuss splittings and simplifications of the three examples of coupled nonlinear partial differential equations into smaller subproblems and their robust and accurate numerical solution. Details on the numerical solution of selected subproblems as well as results on test cases will be provided.

Numerical simulation of powder flow

Stefan Turek

In this talk, we present some of our recent results concerning flows with pressure and shear dependent viscosity. Based on a continuum approach, we derive a general powder flow model for slow granular material which can be embedded into the framework of generalized incompressible Navier-Stokes equations. Mathematical approaches regarding FEM discretization, nonlinear Newton solvers and multigrid techniques for the resulting saddle-point-like problems, which are nonstandard, are discussed and specified for the so-called Schaeffer model (by D. Schaeffer 1987). Moreover, we present a hierarchy of compressible extensions of this model (due to G. Tardos) and discuss corresponding mathematical and numerical challenges.

Edited by Rolf Rannacher

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