Mathematisches Forschungsinstitut Oberwolfach

Report No. 12/2004

Algebraische Gruppen

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February 29th – March 6th, 2004

Introduction by the Organisers

The workshop was organized by Michel Brion (Grenoble) and Jens Carsten Jantzen (Aarhus). The schedule comprised twenty talks from a broad range of areas connected to algebraic groups, including but not limited to: arithmetic invariants of algebraic groups, compact Lie groups, embeddings of homogeneous spaces, Hilbert schemes, invariant theory, moduli spaces, nilpotent orbits, quantum groups, quotient singularities, rationality questions, reductivity and reducibility of algebraic groups, Schubert varieties, and symmetric varieties.

Workshop on Algebraische Gruppen

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Abstracts

LS-Galleries, the path model and MV-cycles
Peter Littelmann
(joint work with S. Gaussent)

The talk is a report on joint work [2] with Stéphane Gaussent (Nancy).

The aim of the work is to connect the combinatorics of the path model for representations of a complex semisimple algebraic group G [4] with the work of Mirković and Vilonen [6] on the intersection cohomology of Schubert varieties in the affine Grassmannian $\mathcal{G}$ of its Langlands dual group $\check{G}$.

Recall that $\mathcal{G}$ is the quotient $\mathcal{G} = \check{G}(\mathbb{C}((t)))/\check{G}(\mathbb{C}[[t]])$. As $\check{G}(\mathbb{C}[[t]])$–variety, $\mathcal{G}$ decomposes [3] into the disjoint union of orbits $\mathcal{G}_\lambda = \check{G}(\mathbb{C}[[t]]).\lambda$, where λ runs over all dominant characters of G (= co-characters of $\check{G}$).

The closure $X_\lambda = \overline{\mathcal{G}_\lambda}$ of such an orbit is a finite dimensional projective variety (in terms of Kac–Moody groups, it is a Schubert variety). The intersection cohomology of this variety is closely connected with the irreducible representation $V(\lambda)$ of G of highest weight λ. Lusztig [5] has shown that the Poincaré series of the stalks of the intersection cohomology sheaf in a point $x \in \mathcal{G}_\mu$, $\mu \preceq \lambda$, coincides with a q–version of the weight multiplicity of μ in $V(\lambda)$. Mirković and Vilonen construct in [6] a canonical basis of $I\!H^\bullet(X_\lambda)$, represented by certain cycles called MV–cycles in the following. This explicit basis has been used by Vasserot in [7] to construct an action of G on $I\!H^\bullet(X_\lambda)$ such that the latter is an irreducible representation of highest weight λ.

In our combinatorial setting, the language of paths is replaced by the language of galleries in an apartment, and LS-paths are replaced by LS–galleries. The translation between the two settings is rather straightforward.

Consider a Demazure–Hansen–Bott–Samelson desingularization $\hat{\Sigma}(\lambda)$ of X_λ. If λ is regular, fixing such a desingularization is equivalent to fixing a minimal gallery γ_λ joining the origin and λ. The homology of $\hat{\Sigma}(\lambda)$ has a basis given by Białynicki–Birula cells, which are indexed by the T–fixed points in $\hat{\Sigma}(\lambda)$. The connection with galleries is obtained as follows: by [1], the points of $\hat{\Sigma}(\lambda)$ can be identified with galleries of type γ_λ in the affine Tits building associated to $\check{G}$, and the T–fixed points correspond in this language to galleries of type γ_λ in the apartment fixed by the choice of T. We show that the retraction from $-\infty$ of the building onto the apartment induces on the level of galleries a map from $\hat{\Sigma}(\lambda)$ onto the set of galleries of type γ_λ, such that the fibres are precisely the Białynicki–Birula cells. We determine those galleries γ such that the associated cell has a non-empty intersection S_γ with $\mathcal{G}_\lambda$ (identified with an open subset of $\hat{\Sigma}(\lambda)$), and we show that the closure $\overline{S_\gamma} \subset X_\lambda$ is a MV-cycle if and only if γ is a LS-gallery. The galleries can also be used to derive more information about the cycles (dimension, affine open subsets of the form $\mathbb{C}^a \times (\mathbb{C}^*)^b, \ldots$).

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Algebras with trace and Clebsch-Gordan coefficients for quantum groups

Corrado De Concini
(joint work with Procesi, Reshetikhin, Rosso)

We have reported on joint work with Procesi, Reshetikhin and Rosso. All rings will be algebras over a field of characteristic zero. An associative algebra with trace, over a commutative ring A is an associative algebra R with a 1-ary operation

$$t : R \to R$$

which is assumed to satisfy the following axioms:

(1) t is $A-$linear.
(2) $t(a)b = b\,t(a), \quad \forall a, b \in R.$
(3) $t(ab) = t(ba), \quad \forall a, b \in R.$
(4) $t(t(a)b) = t(a)t(b), \quad \forall a, b \in R.$

This operation is called a formal trace. We denote $t(R) := \{t(a),\ a \in R\}$ the image of t. We have:

1) $t(R)$ is an $A-$subalgebra and that t is also $t(R)-$linear.
2) $t(R)$ is in the center of R.
3) t is 0 on the space of commutators $[R, R]$.

The basic example is the algebra of $n \times n$ matrices over a commutative ring B. For the algebra of matrices one has the Cayley Hamilton theorem:

Every matrix M satisfies its characteristic polynomial $\chi_M(t) := \det(t - M)$.

The main remark that allows to pass to the formal theory is that, in characteristic 0, there are universal polynomials $P_i(t_1, \ldots, t_i)$ with rational coefficients, such that:

$$\chi_M(t) = t^n + \sum_{i=1}^{n} P_i(tr(M), \dots, tr(M^i))t^{n-i}.$$

Thus we can consider the Cayley Hamilton polynomial of an element in an arbitrary algebra with trace and we are led to make the following.

Definition 1. An algebra with trace R is said to be an $n-$Cayley Hamilton algebra, or to satisfy the n^{th} Cayley Hamilton identity if:
1) $t(1) = n$.
2) $\chi_a^n(a) = 0, \ \forall a \in R$.

A structure of Cayley Hamilton algebra can be given in the following situation. Let A be a domain and assume that $A \subset R$ and R is an $A-$module of finite type. Furthermore assume that A is integrally closed in its quotient field F.

Set $S := R \otimes_A F$. S is a finite dimensional division ring.

Let Z denote the center of S Set $\dim_Z S = h^2$ and $p := [Z : F] = \dim_F Z$.

Consider the $F-$linear operator $a^L : S \to S, \ a^L(b) := ab$ and put $t_{S/F}(a) = \frac{1}{hk} tr(a^L)$.

Theorem 1. *The reduced trace $t_{S/F}$ maps R into A, so we will denote by $t_{R/A}$ the induced trace.*

The algebras R, S with their reduced trace are $n-$Cayley Hamilton algebras of degree $n = hp = [S : F] = [R : A]$ (we set $[R : A] := [S : F]$).

Assume now that we have two domains $R_1 \subset R_2$ over two commutative rings $A_1 \subset A_2 \subset R_2$, that each R_i is finitely generated as A_i module and that the two rings A_i are integrally closed. We thus have the two reduced traces t_{R_i/A_i}. We say that the two algebras are compatible if denoting by Z_1 the center of R_1, $Z_1 \otimes_{A_1} A_2$ is a domain.

Theorem 2. *Given two compatible algebras $R_1 \subset R_2$ we have that for a positive integer r:*

$$r \, t_{R_1/A_1} = t_{R_2/A_2} \quad \text{on } R_1$$

One can give various applications of these ideas. One is the following. Let $\mathfrak{g}$ denote a semisimple Lie algebra and let U_ε be the quantized enveloping algebra with deformation parameter specialized at a primitive ℓ-th root of 1 (ℓ odd and prime with 3 if there are G_2 factors). U_ε is a Hopf algebra with comultiplication

$$\Delta : U_\varepsilon \to U_\varepsilon \otimes U_\varepsilon.$$

One knows that the center Z of U_ε contains a Hopf subalgebra Z_0 and that U_ε is a finite free Z_0 module. In particular by taking central characters, one can associate to every irreducible representation V of U_ε an element $\pi(V)$ in the algebraic group $H = \mathrm{Spec} Z_0$. Applying Theorem 2 with $R_1 = \Delta(U_\varepsilon)$, $A_1 = \Delta(Z_0)$, $R_2 = U_\varepsilon \otimes U_\varepsilon$, $A_2 = Z \otimes Z$, we deduce:

Theorem 3. *Given two generic irreducible representations V and W of U_ε with $h = \pi(V),\ k = \pi(W)$,*

$$V \otimes W \simeq \bigoplus_{U \in \pi^{-1}(hk)} U^{\oplus \ell^{(\dim \mathfrak{g} - \mathrm{rk}\, \mathfrak{g})/2}}.$$

Projective normality of complete symmetric varieties

Andrea Maffei
(joint work with R. Chirivì)

The results of this talk have been obtained together with Rocco Chirivì of the University of Pisa.

Let G be an adjoint semisimple algebraic group over $\mathbb{C}$ and $\sigma : G \to G$ an involution of algebraic groups. Denote by H the subgroup of points fixed by σ. A wonderful G-equivariant compactification X of the symmetric variety G/H has been constructed by De Concini and Procesi [5] in characteristic zero and by De Concini and Springer [6] in general. Our main result is the following.

Theorem A ([2])**.** *If $\mathcal{L}$ and $\mathcal{M}$ are line bundles on X, generated by global sections, then the multiplication $\Gamma(X, \mathcal{L}) \otimes \Gamma(X, \mathcal{M}) \to \Gamma(X, \mathcal{L} \otimes \mathcal{M})$ is surjective.*

This generalizes a result of Kannan [8] on the wonderful compactification of groups, and in characteristic zero it answers a question of Faltings [7]. In positive characteristic De Concini has given a counterexample to the same theorem. It is maybe worth observing here that these varieties are Frobenius split (and probably canonical Frobenius split).

We say that a line bundle $\mathcal{L}$ is bigger or equal to a line bundle $\mathcal{M}$ if $\mathcal{L} \otimes \mathcal{M}^{-1}$ is generated by global sections. We call this the dominant ordering. The proof of the theorem is essentially by induction on the dimension of X and on the dominant order on line bundles. Using the description of the boundary of G/H in X given in [5] it is possible to reduce the claim to a few cases controlled by some special triples of weights of a root system that we call "low triples".

We can give the definition of root system for an abstract root system. Let Φ be a root system, Δ a basis of simple roots and Λ^+ the corresponding monoid of dominant weights and indicate with $\leq$ the dominant order. Given $\lambda, \mu, \nu \in \Lambda^+$, we say that (λ, μ, ν) is a *low triple* if the following conditions hold: (i) if $\lambda', \mu' \in \Lambda^+$ satisfy $\lambda' \leq \lambda$, $\mu' \leq \mu$ and $\nu \leq \lambda' + \mu'$, then $\lambda' = \lambda$, $\mu' = \mu$; (ii) $\nu + \sum_{\alpha \in \Delta} \alpha \leq \lambda + \mu$. We have the following classification which suffices to finish the proof of Theorem A.

Theorem B ([2])**.** *A triple (λ, μ, ν) of dominant weights is a low triple if and only if λ and μ are minuscule weights, $\mu = -w_0\lambda$ and $\nu = 0$, for the longest element w_0 of the Weyl group of Φ.*

Since X is smooth, Theorem A implies that for all line bundles $\mathcal{L}$ generated by global sections the cone over the image of X in $\mathbb{P}(\Gamma(X,\mathcal{L})^*)$ is normal.

Together with Corrado De Concini, we have applied Theorem A to the investigation of normality of cones of other immersions of X. Consider an irreducible representation V of $Lie\, G$ such that H has a fixed point h in $\mathbb{P}(V)$. Let C_V be the cone over the closure of the G-orbit X_V through h. The natural map $G/H \longrightarrow X_V$ induced by $g \longmapsto gh$ extends to X and defines a line bundle $\mathcal{L}_V$ generated by global sections. We have obtained the following description of the normalization of C_V.

Theorem C ([3]). *The integral closure of the coordinate ring of C_V is the ring $\bigoplus_{n\geq 0}\Gamma(X,\mathcal{L}_V^{\otimes n})$.*

In particular (by Theorem A above and the description of the sections of a line bundle given in [5]), we can classify the representations for which C_V is normal. generalizing the results obtained in [4] in the case of the compactification of a group.

A simple generalization of this result allows us to give a uniform proof of the normality of some classical varieties that appear in Lie theory.

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Complete reducibility and strong reductivity

Gerhard Röhrle

(joint work with M. Bate and B. Martin)

Abstract. Let G be a connected reductive linear algebraic group and let H be a closed subgroup of G. Our main result shows that H is G-completely reducible if and only if H is strongly reductive in G. As a consequence we provide an affirmative answer to a question posed by J.-P. Serre, whether a normal subgroup of a G-completely reducible

subgroup of G is again G-completely reducible. Apart from this we discuss other applications. In particular, we prove a converse to Serre's question, namely that H is G-completely reducible if and only if its normalizer $N_G(H)$ is.

This is a report on joint work with M. Bate and B. Martin.

Let G be a connected reductive linear algebraic group defined over an algebraically closed field k. Let H be a closed subgroup of G. Following R.W. Richardson, we say that H is *strongly reductive in* G provided H is not contained in any proper parabolic subgroup of $C_G(S)$, where S is a maximal torus of $C_G(H)$, [2, Def. 16.1]. Observe that this notion does not depend on the choice of the maximal torus S of $C_G(H)$. Richardson introduced this notion in order to characterize closed orbits for the diagonal action of G on the direct product of a finite number of copies of G or its Lie algebra $\operatorname{Lie} G$, [2, Thm. 16.4]. In [2, Lem. 16.2] Richardson showed that a closed subgroup H of $\mathrm{GL}(V)$ (where V is a finite dimensional k-space) is strongly reductive if and only if V is a semisimple H-module. Our aim is to extend this result to arbitrary reductive groups. For that purpose we require the notion of G-complete reducibility due to J.-P. Serre, [3]. Following Serre, a subgroup H of G is called *G-completely reducible* (G-cr) provided that whenever H is contained in a parabolic subgroup P of G, it is contained in a Levi subgroup of P. In case $G = \mathrm{GL}(V)$ a subgroup H is G-cr exactly when V is a semisimple H-module.

The principal result of this talk is

Theorem 1. *Let G be reductive and suppose H is a closed subgroup of G. Then H is G-completely reducible if and only if H is strongly reductive in G.*

The notion of G-complete reducibility is part of the philosophy developed by J.-P. Serre, J. Tits and others to extend standard results from representation theory to algebraic groups by replacing representations $H \to \mathrm{GL}(V)$ with morphisms $H \to G$, where the target group is an arbitrary reductive algebraic group. Theorem 1 is an example of such an extension.

Using Theorem 1 and existing results on strong reductivity, we immediately get new results on G-complete reducibility.

The following result which follows readily from Theorem 1 and [1, Thm. 2] gives an affirmative answer to a question posed by J.-P. Serre, [3, p. 24]. The special case when $G = \mathrm{GL}(V)$ is just a particular instance of Clifford theory.

Theorem 2. *Let G be reductive and let H be a closed subgroup of G with closed normal subgroup N. If H is G-completely reducible, then so is N.*

Serre proves a converse to Theorem 2 in [3, Property 5] under the assumption that the index of N in H is prime to $\operatorname{char} k$. Examples show that this restriction cannot be removed. For instance, let U be a finite unipotent subgroup of G contained in a Borel subgroup of G. Then, by a construction due to Borel and Tits there exists a parabolic subgroup P of G so that $U \subseteq R_u(P)$. In particular, U is not G-cr, but clearly $U^0 = \{1\}$ is. In Theorem 4 below we give a converse of Theorem 2 without characteristic restrictions but with the additional assumption that H contains the centralizer in G of N. In particular, we derive that a closed subgroup H of G is G-completely reducible if and only if its normalizer $N_G(H)$ is, cf. Corollary 5.

Much of the work in this paper is based on the following result:

Proposition 3. *Let $x_1, \dots, x_n \in G$ (for $n \in \mathbb{N}$) and let H be the subgroup of G (topologically) generated by $x_1, \dots, x_n$. Then H is G-completely reducible if and only if the orbit of $(x_1, \dots, x_n)$ under the diagonal action of G on G^n by simultaneous conjugation is closed.*

Proposition 3 allows us to use methods from geometric invariant theory to study G-completely reducible subgroups. E.g. it is crucial for our next

Theorem 4. *Let H be a closed G-completely reducible subgroup of G and suppose K is a closed subgroup of G satisfying $HC_G(H) \subseteq K \subseteq N_G(H)$. Then K is G-completely reducible.*

The following are immediate consequences of Theorems 2 and 4.

Corollary 5. *Let H be a closed subgroup of G. Then H is G-completely reducible if and only if $N_G(H)$ is.*

Corollary 6. *Let H be a closed subgroup of G. If H is G-completely reducible, then so is $C_G(H)$.*

Time permitting we shall discuss other applications of Theorem 1 and new results for G-completely reducible subgroups of G

Finally, we will indicate Serre's approach to G-complete reducibility by means of the homotopy type of the fixed point subcomplex of the building of G.

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On the canonical embeddings of certain homogeneous spaces

Dmitri A. Timashev

(joint work with I. V. Arzhantsev)

This is a joint work with I. V. Arzhantsev, see [3]. Let G be a connected reductive algebraic group over an algebraically closed field $\Bbbk$ of characteristic 0, and H its closed subgroup. The subgroup H is said to have the *Grosshans property* [1] if the homogeneous space G/H is quasiaffine and the coordinate algebra $\Bbbk[G/H]$ is finitely generated. In this situation among all equivariant open affine embeddings $X \hookleftarrow G/H$ one can distinguish a minimal one $X = \operatorname{Spec} \Bbbk[G/H]$, called the *canonical embedding*. The study of the canonical embedding is a geometric way to examine the properties of the coordinate algebra of G/H.

It is well known [2] that the unipotent radical P_{u} of a parabolic subgroup P of G is a Grosshans subgroup. We study the canonical embeddings of the spaces G/P_{u}. This interesting class of affine varieties includes the universal affine embedding of G/U, where U is a maximal unipotent subgroup of G, the space of linear maps to a symplectic vector space with isotropic image, etc. Our main results include: the description of the orbital decomposition for the canonical embedding $X \hookleftarrow G/P_{\mathrm{u}}$; computing the modality of the G-action; classification of the smooth canonical embeddings; construction of the minimal ambient G-module $V \supset X$ (in the algebraic language this is equivalent to the description of a minimal generating set for $\Bbbk[G/P_{\mathrm{u}}]$).

Our approach works for a wider class of affine embeddings of G/P_{u}. The idea is to consider G/P_{u} as a homogeneous space under $G \times L$, where the Levi subgroup $L \subseteq P$ acts by right translations. It is clear that this $(G \times L)$-action extends to the canonical embedding. More generally, we consider arbitrary $(G \times L)$-equivariant affine embeddings $X \hookleftarrow G/P_{\mathrm{u}}$. Several interesting varieties such as varieties of complexes belong to this class.

Such affine embeddings are classified by finitely generated semigroups S of G-dominant weights having the property that all highest weights of tensor products of simple L-modules with highest weights in S belong to S, too. Furthermore, every choice of the generators $\lambda_1, \ldots, \lambda_m \in S$ gives rise to a natural G-equivariant embedding $X \hookrightarrow \mathrm{Hom}(V^{P_{\mathrm{u}}}, V)$, where V is the sum of simple G-modules of highest weights $\lambda_1, \ldots, \lambda_m$. The convex cone Σ^+ spanned by S is nothing else but the dominant part of the cone Σ spanned by the weight polytope of $V^{P_{\mathrm{u}}}$. The variety X is normal iff S is the semigroup of all lattice points of Σ^+.

We prove that the $(G \times L)$-orbits in X are in bijection with the faces of Σ whose interiors contain dominant weights, orbit representatives being given by the projectors onto the subspaces of $V^{P_{\mathrm{u}}}$ spanned by eigenvectors of eigenweights in a given face. Also we compute the stabilizers of these points in $G \times L$ and in G, and the modality of the action $G : X$.

These results are applied to canonical embeddings as follows. The semigroup S here consists of all dominant weights, and Σ is the span of the dominant Weyl chamber by the Weyl group of L. The $(G \times L)$-orbits in X are in bijection with the subdiagrams in the Dynkin diagram of G such that no connected component of such a subdiagram is contained in the Dynkin diagram of L. In terms of these diagrams, we compute the stabilizers and the modality of $G : X$.

We prove that the only essential cases of smooth embeddings in the considered class are: $X_1 = G$, $X_2 = \mathrm{Mat}(n, n-1)$, $X_3 = \mathrm{Mat}(n, n)$, all other smooth cases being given by a product construction. The first two examples are canonical embeddings with $P = G$ for X_1; $G = SL(n)$, P the stabilizer of a hyperplane in $\Bbbk^n$ for X_2; $G = P = GL(n)$ for X_3.

The techniques used in the description of affine $(G \times L)$-embeddings of G/P_{u} are parallel to those in the study of equivariant embeddings of reductive groups

[4]. This analogy becomes more transparent in view of the bijection between these affine embeddings $G/P_u \hookrightarrow X$ and algebraic monoids M with the group of invertibles L, given by $X = \operatorname{Spec} \Bbbk[G \times^P M]$.

Finally, returning to the case of the canonical embedding $X \hookleftarrow G/P_u$, we describe the G-module structure on the tangent space of X at the G-fixed point, assuming that G is simply connected simple.
This space is obtained from $\bigoplus_i \operatorname{Hom}(V_i^{P_u}, V_i)$, where V_i are the fundamental simple G-modules, by removing certain summands according to an explicit algorithm. The tangent space at the fixed point is at the same time the minimal ambient G-module for X.

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Categorification of Weyl groups and Lie algebras

Raphaël Rouquier
(joint work with J. Chuang)

It is classical that various actions of Weyl groups or Lie algebras on vector spaces come from functors acting on abelian or triangulated categories of algebraic or geometric origin, whose Grothendieck group is that space. We want to explain that the natural transformations between these functors should satisfy certain algebraic relations, leading to a better control of the triangulated categories acted on. Namely, we believe there is a "canonical" categorification of a number of classical algebras or groups, in particular Kac-Moody algebras, Weyl groups, braid groups (a monoidal category given by generators and relations).

In a joint work with Joseph Chuang, we explain the setting for $\mathfrak{sl}_2$, which leads to a construction of equivalences of derived categories between blocks of Hecke algebras of type A.

An $\mathfrak{sl}_2$-*categorification* of an abelian category is the data of adjoint exact endofunctors E and F inducing an $\mathfrak{sl}_2$-action on the Grothendieck group and the data of endomorphisms X of E and T of E^2 satisfying the defining relations of (degenerate) affine Hecke algebras.

We prove a categorified version of the relation $[e, f] = h$. We construct divided powers of E and F and a categorification Θ of the simple reflection (following a construction of Rickard). Our main result is a proof that Θ is a self-equivalence at the level of derived categories.

We construct a minimal categorification of the simple $\mathfrak{sl}_2$-representations and show that the proof of the results above can be reduced to this case of a minimal categorification.

We apply these results to the sum of the module categories of all Hecke algebras of type A at an e-th root of unity in characteristic 0 (there are similar results in characteristic $p > 0$).

Recall that there is an action of $\hat{\mathfrak{sl}}_e$ on the sum of Grothendieck groups of categories of modules over Hecke algebras of type A at an e-th root of unity. The action of the generators e_i and f_i come from exact functors between modules ("i-restriction" and "i-induction"). The adjoint action of the simple reflections of the affine Weyl group can then be categorified as inversible endofunctors of the derived category, since every i leads to an $\mathfrak{sl}_2$-categorification. As a consequence, two blocks in the same affine Weyl group orbit have equivalent derived categories.

McKay equivalence for symplectic quotient singularities

Roman Bezrukavnikov
(joint work with D. Kaledin)

Let K be an algebraically closed field of characteristic 0, let V be a finite-dimensional K-vector space equipped with a non-degenerate skew-symmetric form $\omega \in \Lambda^2(V^*)$, and let $\Gamma \subset Sp(V)$ be a finite subgroup. Suppose that we are given a resolution of singularities of the quotient variety $\pi : X \to V/\Gamma$ such that the symplectic form on the smooth part of V/Γ extends to a non-degenerate closed 2-form $\Omega \in H^0(\Omega^2_X)$. In a joint work with D. Kaledin, see [BK], we prove the following

Theorem. *There exists an equivalence of $\mathcal{O}_V^\Gamma$-linear triangulated categories*

$$D^b(Coh(X)) \cong D^b(Coh^\Gamma(V)).$$

A conjecture of this type was first made by M. Reid [R]; a more general statement was conjectured by A. Bondal and D. Orlov, [BOr, §5].

When $\dim(V) = 2$ such an equivalence is well-known, [KV]; in fact, our argument relies on these results. Recently a similar statement was established by T. Bridgeland, A. King and M. Reid [BKR] for crepant resolutions of Gorenstein quotients of vector spaces of dimension 3. The result of [BKR] does not follow from our theorem, because a symplectic vector space can not be 3-dimensional. Notice though that our additional assumption on the resolution is not restrictive — every crepant resolution X of a symplectic quotient singularity in fact carries a non-degenerate symplectic form (see e.g. [Ka]).

Our proof uses reduction to positive characteristic, and quantization of the symplectic variety X_{k} over a field k of characteristic $p > 0$. Our method is suggested by the results of [BMR], where D-module technique is applied to study representations of simple Lie algebras in positive characteristic.

The key ingredient of the proof is a quantization of X_{k} whose global sections coincide with the standard quantization of $H^0(\mathcal{O}_X) = H^0(V, \mathcal{O}_V)^\Gamma$ (the role of this quantization in our picture is similar to the role played by the (crystalline) differential operators in [BMR]). By quantization we mean a deformation of the structure sheaf $\mathcal{O}_X$ to a sheaf of non-commutative $\mathsf{k}[h]$-algebras $\mathcal{O}_h(X)$ such that the algebra of global sections $H^0(X, \mathcal{O}_h)$ is identified to the subalgebra $\mathcal{W}^\Gamma \subset \mathcal{W}$ of Γ-invariant vectors in the (completed) Weyl algebra $\mathcal{W}$ of the vector space V.

It turns out that the reduction of $\mathcal{O}_h(X)$ at a non-zero value of the deformation parameter h (e.g. at $h = 1$) is an Azumaya algebra on $X_{\mathsf{k}}^{(1)}$ (a parallel statement for the ring of differential operators was discovered by Mirkovic and Rumynin, see [BMR]). The category of modules over the latter is the category of coherent sheaves on some gerb over $X^{(1)}$.

One then argues that the above Azumaya algebra on $X^{(1)}$ is *derived affine*, i.e. the derived functor of global sections provides an equivalence between the derived category of sheaves of modules, and the derived category of modules over its global sections; this algebra of global sections is identified with the algebra W^Γ, where W is the reduction of the Weyl algebra at $h = 1$.

Furthermore, for large p we have a Morita equivalence between W^Γ and $\mathsf{W}\#\Gamma$, the smash-product of W and Γ.
Thus we get an equivalence between $D^b(\mathcal{W}\#\Gamma\text{-mod}^{\mathsf{fg}})$ and the derived category of modules over the Azumaya algebra on $X^{(1)}$. The algebra W is an Azumaya algebra over $V^{(1)}$; thus, roughly speaking, the latter equivalence differs from the desired one by a twist with a certain gerb. We then use the norm map on Brauer groups, and Gabber's Theorem [G] to pass from sheaves over a gerb to coherent sheaves on the underlying variety.

Then the equivalence over k of large positive characteristic is constructed; by a standard procedure we derive the desired statement over a field of characteristic zero.

The above Theorem implies, more or less directly, that any crepant resolution X of the quotient V/Γ is the moduli space of Γ-equivariant Artinian sheaves on V satisfying some stability conditions (what is known nowadays as *G-constellations*).

In the case when $X = Hilb^n(\mathbb{A}^2)$ is the Hilbert scheme of n points on the affine plane our argument reproves some of the results by M. Haiman, which constitute a part of his proof of the $n!$ Conjecture.

Also, our methods were used by Finkelberg and Ginzburg to relate representation theory of (a graded version of) Cherednik double affine Hecke algebra (a.k.a. the symplectic reflection algebra) in characteristic p to geometry of the Hilbert scheme; this is explained in the talk by Victor Ginzburg at this conference.

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Cherednik algebras and Hilbert schemes in characteristic p

Victor Ginzburg
(joint work with R. Bezrukavnikov, M. Finkelberg)

We prove a localisation theorem for the type A rational Cherednik algebra $H_c = H_{1,c}$ over an algebraic closure of the finite field F_p. In the most interesting special case where the parameter c takes values in F_p, we construct an Azumaya algebra A_c on $Hilb^n$, the Hilbert scheme of n points in the plane, such that the algebra of global sections of A_c is isomorphic to H_c. Our localisation theorem provides an equivalence between the bounded derived categories of H_c-modules and sheaves of coherent A_c-modules on the Hilbert scheme, respectively. Furthermore, we show that the Azumaya algebra splits on the formal completion of each fiber of the Hilbert-Chow morphism. This provides a link between our results and those of Bridgeland-King-Reid and Haiman.

Arithmetic birational invariants of linear algebraic groups over some geometric fields

Boris Kunyavskii
(joint work with M. Borovoi)

We discuss two birational invariants: the set of classes of R-equivalence $G(k)/R$, and the unramified Brauer group $\mathrm{Br}_{nr}\, G$. Our goal is to extend some results from the arithmetic case (where k is a number field) to the case where k is a field of cohomological dimension two. More precisely, for k of one of the following types:

(i) $k = k_0(X)$, $\dim X = 2$, $k_0 = \overline{k_0}$, $\mathrm{char}\, k_0 = 0$;
(ii) k = fraction field of a 2-dimensional, excellent, henselian local domain with residue field k_0;
(iii) $k = l((t))$, $\mathrm{c.d.}(l) = 1$, $\mathrm{char}\, l = 0$,

which were earlier studied by Colliot-Thélène, Gille, and Parimala, we prove that $G(k)/R$ and $\mathrm{Br}_{nr}\, G/\operatorname{Br} k$ can be expressed through the algebraic fundamental group $\pi_1(G)$. More precisely, in the above set-up, let

$$0 \to Q_G \to P \to \pi_1(G) \to 0$$

be a coflasque resolution of $\pi_1(G)$, that is, P is a permutation $\mathrm{Gal}(\overline{k}/k)$-module, and $\mathrm{H}^1(\Gamma', Q_G) = 0$ for all open $\Gamma' \subset \mathrm{Gal}(\overline{k}/k)$. Then $G(k)/R \cong \mathrm{H}^1(k, F_G)$, where F_G is the k-torus with cocharacter module Q_G, and $\mathrm{Br}_{nr}\, G/\operatorname{Br} k \cong \mathrm{H}^1(k, Q_G^\vee)$, where $Q_G^\vee = \mathrm{Hom}(Q_G, \mathbb{Z})$ is the dual module.

Furthermore, if $G \hookrightarrow V$ is a smooth compactification of G, $N_G = \mathrm{Pic}(V \times_k \overline{k})$ is the Picard module, S_G is the Néron-Severi torus (= the torus with character module N_G), then $G(k)/R \cong \mathrm{H}^1(k, S_G)$. This shows that the group $G(k)/R$ is a birational invariant of G.

To appear in J. of Algebra, 2004 (with an appendix by P. Gille).

A non rational group variety of type E_6

Philippe Gille

Let G/k be a semisimple algebraic group defined over a field k. The question whether the group variety G/k is k-rational (i.e. birational to an affine space) has been investigated for classical groups by several authors: Platonov, Yanchevskii, Merkurjev, Chernousov...

The talk deals with the rationality question for exceptional groups. For trialitarian groups of type D_4 and groups of type F_4, the rationality question is open. Using the Bruhat-Tits theory [T], we have found a simply connected group of type E_6 which is not a k-rational variety. The field k is then a 2-iterated power series field over some Merkurjev's suitable field. The proof of the non-rationality goes by a specialization argument involving Chow groups of 0-cycles on G/k.

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Rational points and curves on flag varieties

Emmanuel Peyre
(joint work with A. Chamber-Loir)

1. Heights

It is well known that there are many analogies between the rational points on a variety V defined over a number field K and the rational curves on a variety V over $\mathbf{C}$ and that one of the simplest way to make these links more precise is to consider rational points on a global field of finite characteristic.

In this talk we shall consider the three settings simultaneously:

(1) Over $\mathbf{Q}$ we may define several natural heights on the projective space, for example the height $H_N : \mathbf{P}^N(\mathbf{Q}) \to \mathbf{R}$ defined by

$$H_N((x_0 : \ldots : x_N)) = \sqrt{x_0^2 + \cdots + x_N^2},$$

if $x_0, \ldots, x_N$ are coprime integers. The corresponding logarithmic height is $h_N = \log H_N$.

More generally, if K is a number field, let M_K be the set of places of K. For any place v of K, we denote by K_v the completion of K for the topology defined by v and the absolute value $|\cdot|_v$ is normalized by $\mathrm{d}(ax)_v = |a|_v \mathrm{d}x_v$ for any Haar measure $\mathrm{d}x_v$. We then choose v-adic norms $\|\cdot\|_v : K_v^{N+1} \to \mathbf{R}$, for example we may define the norm $\|(x_0, \ldots, x_N)\|_v$ as $\sup_{0 \leq i \leq N} |x_i|_v$ if v is a finite place, as $\sqrt{\sum_{i=0}^N x_i^2}$ if K_v is isomorphic to $\mathbf{R}$, and as $\sum_{i=0}^N x_i \overline{x}_i$ if K_v is isomorphic to $\mathbf{C}$. Then $H_N : \mathbf{P}^N(K) \to \mathbf{R}$ is defined by

$$H_N(x_0 : \ldots : x_N) = \prod_v \|(x_0, \ldots, x_N)\|_v$$

and $h_N = \log H_N$.

(2) If $K = \mathbf{F}_q(\mathcal{C})$ where $\mathcal{C}$ is a smooth projective curve of genus g over $\mathbf{F}_q$, then there is a bijection from the set of points in the projective space $\mathbf{P}^N(K)$ to the set $\mathrm{Mor}(\mathcal{C}, \mathbf{P}^N_{\mathbf{F}_q})$. Let us denote by $\tilde{x}$ the image of a point x. Then

$$h_N(x) = \deg(\tilde{x}^*(\mathcal{O}(1)))$$

where $\tilde{x}^*(\mathcal{O}(1))$ belongs to the Picard group of the curve $\mathcal{C}$. We also put $H_N = q^{h_N}$.

(3) Similarly, if $K = k(\mathcal{C})$ where $\mathcal{C}$ is a smooth projective curve over a field k, we define

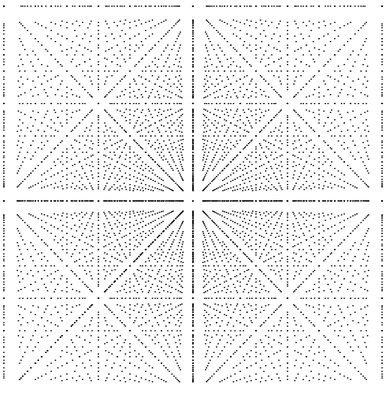
$$h_N(x) = \deg(\tilde{x}^*(\mathcal{O}(1)))$$

where $\tilde{x}^*(\mathcal{O}(1))$ belongs to the Picard group of the curve $\mathcal{C}$.

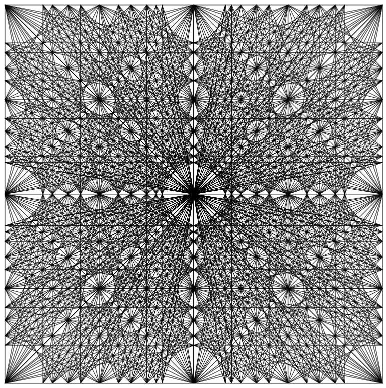
In all settings, if V is a variety over K, any morphism $\phi : V \to \mathbf{P}^N_K$ induces a map $h : V(K) \to \mathbf{R}$ defined by $h = h_N \circ \phi$. We want to study asymptotically the set

$$\{\, x \in V(K) \mid h(x) < \log(B) \,\}$$

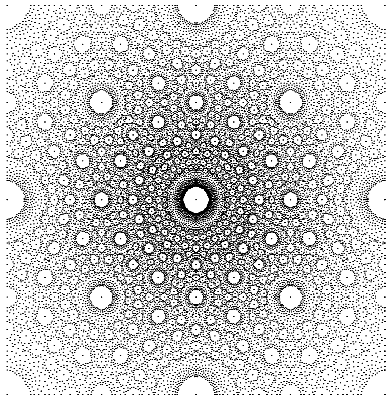
as B goes to $+\infty$. To illustrate this, I represented such sets as points on the projective plane, as lines on the plane and as points in $\mathbf{P}^1_{\mathbf{Q}(i)}$.



$\mathbf{P}^2_{\mathbf{Q}}$



Lines in $\mathbf{P}^2_{\mathbf{Q}}$



$\mathbf{P}^1_{\mathbf{Q}(i)}$

2. Height zeta functions

One of the main tool to study the asymptotic behavior of the number of points of bounded height is the height zeta function.

(1) Over a number field, it is defined for any open subset U of V by

$$\zeta_{U,H}(s) = \sum_{x \in U(K)} \frac{1}{H(x)^s}$$

where this series converges.

(2) Similarly, over $\mathbf{F}_q(T)$, for any open subset U of V

$$Z_{U,h}(T) = \sum_{x \in U(K)} T^{h(x)} \quad \text{and} \quad \zeta_{U,H}(s) = Z_{U,h}(q^{-s}).$$

(3) In the functional setting, we are in fact interested in moduli spaces of morphisms from the curve $\mathcal{C}$ to the variety V. Let $\mathcal{M}_k$ be the group generated by symbols $[V]$ for V variety over k with the relations $[V] = [V']$ if V and V' are isomorphic and

$$[V] = [F] + [V - F]$$

for any closed subset F of V.

If U is an open subset of V, for any integer n, there exists a variety U_n over k such that for any extension k' of k, there is a functorial bijection from $U_n(k')$ to the set of points of $U(k'(\mathcal{C}))$ of height n. The motivic height zeta function is the formal series in $\mathcal{M}_k[[T]]$ defined by

$$Z^{\mathrm{mot}}_{U,h}(T) = \sum_{n \in \mathbf{N}} [U_n] T^n.$$

If k is a finite field, one may go from the functional setting to the classical one by using the map

$$\begin{array}{rcl} \mathcal{M}_k & \to & \mathbf{Z} \\ [V] & \mapsto & \sharp V(\mathbf{F}_q). \end{array}$$

This map sends $Z^{\mathrm{mot}}_{U,h}$ to the classical zeta function $Z_{U,h}$.

3. The case of flag varieties

For flag varieties, one may use the fact, first discovered by Franke, Manin and Tschinkel [FMT] that, in that case, the height zeta function coincides with an Eisenstein series. One may then apply the difficult and deep results obtained for Eisenstein series by Langlands over number fields [Lan], by Harder [Harder] and Morris ([Mo1] and [Mo2]) over global fields of finite characteristic and by Kapranov [Ka] in the functional setting.

Notations 3.1. *Let G be a split semi-simple simply-connected algebraic group over K, let P be a smooth parabolic subgroup of G, let B be a Borel subgroup of G contained in P and let T be a split maximal torus of G contained in B. We denote by Φ the root system of T in G, by Φ^+ the positive roots corresponding to B and by Δ the corresponding basis of the root system. Let Φ_P be the roots of T in the Lie algebra $\mathrm{Lie}(R_u(P))$ of the unipotent radical of P. The set Φ_P is contained in the set of positive roots. We also put $\Delta_P = \Phi_P \cap \Delta$.*

Let $V = G/P$. There exists a canonical isomorphism from the character group $X^*(P)$ of P to $\mathrm{Pic}(V)$ sending the character χ to the line bundle $\mathcal{L}_\chi = G \times^P \mathbf{A}^1_K$ where P acts on the affine line via χ. There is also an injective restriction map $\mathrm{res} : X^*(P) \to X^*(T)$. Let ρ_P (resp. ρ_B) be the half-sum of the roots in Φ_P (resp. Φ^+) then $2\rho_P$ belongs to the image of $X^*(P)$ and we denote also by $2\rho_P$ its inverse image in $X^*(P)$. The line bundle $\mathcal{L}_{2\rho_P}$ is isomorphic to the anticanonical line bundle ω_V^{-1} and is very ample. From now on, all the heights used will be relative to ω_V^{-1}.

(1) In the number field case, it is possible to choose the height on V so that the height zeta function coincides with the value of an Eisenstein series:

$$\zeta_{V,H}(s) = \sum_{x \in G/P(K)} H(x)^{-s} = E^G_P((2s-1)\rho_P, e).$$

Franke, Manin and Tschinkel then applied the work of Langlands and have proven the following results:

- $\zeta_{V,H}(s)$ converges for $\mathrm{Re}(s) > 1$,
- It extends to a meromorphic function on the projective plane,
- It has a pole of order $t = \mathrm{rk}\,\mathrm{Pic}(V)$ at $s = 1$,
- There is a explicit formula for the leading term of the development of $\zeta_{V,H}(s)$ in Laurent series at $s = 1$:

$$\lim_{s\to 1}(s-1)^t \zeta_{V,H}(s) = \prod_{\alpha \in \Phi_P - \Delta_P} \frac{\xi_K(\langle \check{\alpha}, \rho_B\rangle)}{\xi_K(\langle \check{\alpha}, \rho_B\rangle + 1)} \prod_{\alpha\in\Delta_P} \frac{\mathrm{res}_{s=1}\,\xi}{\xi_K(2)\langle\check{\alpha}, 2\rho_P\rangle},$$

where

$$\xi_K(s) = d_K^{s/2}(\pi^{-s/2}\Gamma(s/2))^{r_1}((2\pi)^{-s}\Gamma(s))^{r_2}\zeta_K(s),$$

r_1 being the number of real places of K and r_2 the number of complex places. This limit may be reinterpreted as

$$\lim_{s\to 1}(s-1)^t\zeta_{V,H}(s) = \prod_{\alpha\in\Delta_P}\frac{1}{\langle\check{\alpha},2\rho_P\rangle}\boldsymbol{\omega}_H(V(\boldsymbol{A}_K))$$

where $\boldsymbol{\omega}_H$ is a Tamagawa measure on $V(\boldsymbol{A}_K)$.

(2) The connection with Eisenstein series is also valid for global fields of finite characteristic and we may apply the work of Harder and Morris to get the following results:

- $Z_{V,h}(z)$ converges for $|z| < q^{-1}$,
- $Z_{V,h}(T)$ is a rational function,
- $Z_{V,h}(z)$ has a pole of order t at $z = q^{-1}$
- the leading term at $z = q^{-1}$, that is $\lim_{z\to q^{-1}}(z-q^{-1})^t Z_{V,h}(z)$ is given by

$$q^{\dim(V)(1-g)}\prod_{\alpha\in\Phi_P-\Delta_P}\frac{Z_K(q^{-\langle\check{\alpha},\rho_B\rangle})}{Z_K(q^{-\langle\check{\alpha},\rho_B\rangle-1})}\prod_{\alpha\in\Delta_P}\frac{\operatorname{res}_{z=q^{-1}} Z_K}{Z_K(q^{-2})\langle\check{\alpha},2\rho_P\rangle}$$

which may be reinterpreted as in the number field case.

(3) In the functional setting, we need to use an extension of $\mathcal{M}_k$ constructed by Denef and Loeser to give an analog to the last assertion. Let $\mathcal{M}_k^{\mathrm{loc}}$ be the ring $\mathcal{M}_k[\mathbf{L}^{-1}]$ and, for any integer n, let $F^n\mathcal{M}_k^{\mathrm{loc}}$ be the subgroup of $\mathcal{M}_k^{\mathrm{loc}}$ generated by the elements of the form $\mathbf{L}^{-i}[V]$ where $i - dim(V) \geq m$. Then $\widehat{\mathcal{M}}_k$ is the completion of $\mathcal{M}_k^{\mathrm{loc}}$ for this filtration.

- The varieties V_n verify:

$$\overline{\lim_{n\to+\infty}}\frac{\dim(V_n)}{n}\leq 1,$$

- $Z_{V,h}^{\mathrm{mot}}(T)$ is a rational function,
- the formal series

$$\Big(\prod_{\alpha\in\Delta_P}\big(1-(\mathbf{L}T)^{\langle\check{\alpha},2\rho_P\rangle}\big)\Big)Z_{V,h}^{\mathrm{mot}}(T)$$

converges in $\widehat{\mathcal{M}}_k$ at $T = \mathbf{L}^{-1}$,
- at this point it takes the value

$$\mathbf{L}^{\dim(V)(1-g)}\prod_{\alpha\in\Phi_P-\Delta_P}\frac{Z_K^{\mathrm{mot}}(\mathbf{L}^{-\langle\check{\alpha},\rho_B\rangle})}{Z_K^{\mathrm{mot}}(\mathbf{L}^{-\langle\check{\alpha},\rho_B\rangle-1})}\prod_{\alpha\in\Delta_P}\frac{Z_K^{\mathrm{mot}}(T)(1-\mathbf{L}T)(\mathbf{L}^{-1})}{Z_K^{\mathrm{mot}}(\mathbf{L}^{-2})}$$

where Z_K^{mot} is the zeta function of the field defined by

$$Z_K^{\mathrm{mot}}(T) = \sum_{n\in\mathbf{N}}[\mathcal{C}^{(n)}]T^n,$$

$\mathcal{C}^{(n)}$ being the n-th symmetric power of $\mathcal{C}$. Kapranov proved that Z_K^{mot} verifies

$$Z_K^{\mathrm{mot}}(T) = \frac{P(T)}{(1-T)(1-\mathbf{L}T)}$$

for a polynomial P in $\mathcal{M}_k[T]$ of degree $2g$ which satisfies a functional equation. Once again this may be interpreted in terms of a Tamagawa number in a motivic setting.

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Tangent cones to Schubert varieties

Jochen Kuttler
(joint work with J. B. Carrell)

Suppose G is a connected semisimple algebraic group over $\mathbb{C}$. Fix $T \subset B \subset G$, a maximal torus and a Borel subgroup respectively. For every $w \in W = N_G(T)/T$ (which is in one to one correspondence with $(G/B)^T$) denote by $X(w) \subset G/B$ the associated Schubert variety $\overline{Bw}$. Following ideas of Dale Peterson we obtain a generalization to Peterson's ADE-Theorem, which states that in types ADE every rationally smooth point of $X(w)$ is in fact smooth (see [1] for more details). If G has no component of type G_2, we generalize this to: $x \in X(w)^T$ is smooth if and only if for all $y \geq x$ (with respect to the Bruhat-Chevalley ordering on $(G/B)^T$) the reduced tangent cone $\mathcal{T}_y(X(w))$ is linear, ie. its linear span $\Theta_y(X(w))$ satisfies $\dim \Theta_y(X(w)) = \dim X$. Furthermore, we give a method to compute $\Theta_x(X(w))$ at a maximal singularity, provided G has no G_2 factor, which is assumed for the remainder of this text.

For this, let $TE(X(w), x)$ denote the span of tangent lines to the T-stable curves containing x, and let $\mathbb{T}_x(X(w))$ denote the B_x-submodule of $T_x(X(w))$ generated by $TE(X(w), x)$ with $B_x = \{b \in B \mid bx = x\}$. Then

$$TE(X(w), x) \subset \mathbb{T}_x(X(w)) \subset \Theta_x(X(w)).$$

Let μ, ϕ be two negative long orthogonal roots occurring as weights of $TE(X(w), x)$. Then $\{\mu, \phi\}$ is called an *orthogonal B_2-pair* (for $X(w)$ at x) if $\{\mu, \phi\}$ is contained in a copy of $B_2 \subset \Phi$, the roots of G, and if the following holds: suppose α, β are the (unique) positive generators of this B_2 with α short, then we require $r_\alpha x < x$ and $r_\alpha r_\beta x \leq w$. Here $r_\alpha \in W$ is the reflection associated to a root α.

We show that every T-weight of $\Theta_x(X(w))/\mathbb{T}_x(X(w))$ arises as $\frac{1}{2}(\mu + \phi)$ where $\{\mu, \phi\}$ is an orthogonal B_2-pair, whenever x is a maximal singularity.

A second method to compute $\Theta_x(X(w))$ is given by using so called *Peterson translates*: following an idea of Dale Peterson, to a T-stable curve C containing x we associate

$$\tau_C(X(w),x) = \lim_{\substack{z\to x\\ z\neq x}} T_z(X(w)),$$

a T-stable linear subspace of $T_x(X(w))$, which can be computed explicitly at maximal singularities x, whenever $C^T = \{x,y\}$ with $y > x$. We then have

$$\Theta_x(X(w)) = \sum_{\substack{C\\ C^T=\{x,y\}\\ \text{for some } y>x}} \tau_C(X(w),x),$$

where, again, x is a maximal singularity. A more thorough investigation of the schematic tangent cone itself yields a description of the T-fixed points in the fiber of the Nash blowing up over a maximal singular point: each such fixed point is a Peterson translate, provided G is simply laced or the number of T-stable curves containing x equals $\dim X(w)$. This is proved by showing that the blowing up $B_x(S)$ with center x of the (T-stable affine) slice S of $X(w)$ at x is nonsingular, admitting an equivariant surjective map to $N(S)$, the Nash blowing up of S ([3]): Since every T-fixed point in $B_x(S)$ lies on $B_x(C)$ for some T-stable curve $C \subset S$ ([2]), it follows that each T-fixed point of $N(S)$ lies on a T-stable curve which lifts a curve in S, and therefore is a Peterson translate.

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Torsion in intersection cohomology of Schubert varieties

Tom Braden

To a reduced word $a = (s_{i_1}, s_{i_2}, \dots, s_{i_k})$ for an element w in the Weyl group W of a semisimple complex algebraic group G, we can associate a Bott-Samelson variety

$$BS(a) = P_{i_1} \times_B P_{i_2} \times_B \cdots \times_B P_{i_k}/B.$$

The multiplication map π to the flag variety G/B makes $BS(a)$ a resolution of singularities of the Schubert variety $X_w = \overline{BwB/B}$.

We study the question: does the statement of the Decomposition Theorem, proved by Beilinson, Bernstein, Deligne, and Gabber for sheaves with coefficients in $k = \mathbb{Q}$, hold for other coefficient rings k? In other words, is there an isomorphism

$$R\pi_* k_{BS(a)} \cong \bigoplus_\alpha \mathbf{IC}^\bullet(X_{y_\alpha}; k)[n_\alpha], \qquad (1)$$

where $y_\alpha \in W$ and $n_\alpha \in \mathbb{Z}$? Soergel has shown that an affirmative answer for k an algebraically closed field of characteristic p would prove Lusztig's conjectured character formula for modular representations of a simply connected split algebraic group of the same type as G, for weights around the Steinberg weight.

We have the following criterion to decide if a splitting (1) is possible. For an element $x \in W$ with $x \leq w$, we let $i_x : C_x \to X_w$ be the inclusion.

Theorem 1. *Let a, k be as above. A decomposition of the form* (1) *is possible if and only if every prime p for which p-torsion appears in the cokernel of the natural map*

$$\mathbb{H}^\bullet(i_x^! R\pi_* \mathbb{Z}_{BS(a)}) \to \mathbb{H}^\bullet(i_x^* R\pi_* \mathbb{Z}_{BS(a)}) \tag{2}$$

for some $x \leq w$ is a unit in k.

Furthermore, if $y \in W$, then there is a decomposition (1) *for every $w \leq y$ and every reduced word a for w if and only if for all $w \leq y$, the stalks and costalks of $\mathbf{IC}^\bullet(X_w; k)$ are free k-modules which vanish in odd degrees and whose ranks are given in the usual way by Kazhdan-Lusztig polynomials.*

The theorem that rationally smooth Schubert varieties are smooth in types A, D, E proved by Deodhar [D] (type A), Peterson, and Carrell-Kuttler [CK] might suggest that for these groups the decomposition should hold for all rings and all characteristics. However, we have the following examples where the decomposition theorem fails for $\mathbb{Z}$ or $\mathbb{Z}/2$ coefficients.

Let G be the group $SL_8(\mathbb{C})$; then $W = S_8$ is the symmetric group on 8 letters. Let s_i denote the transposition of i and $i+1$. Then we consider the *hexagon permutations*:

$$w = s_4^c s_3 s_2 s_1 s_5 s_4 s_3 s_2 s_6 s_5 s_4 s_3 s_8 s_7 s_6 s_5 s_4^d, \ c, d \in \{0, 1\}.$$

Billey and Warrington [BW] have shown that avoiding these four permutations along with the longest element $s_1 s_2 s_1$ in S_3 characterizes the permutations for which X_w has a small Bott-Samelson resolution. For this choice of w, we let $x = s_4^c s_2 s_3 s_4 s_5 s_6 s_5 s_4^d$.

For our other example, let G be a group of type D_4. Order the simple reflections $\{s_1, s_2, s_3, s_4\}$ so that s_1, s_2, and s_3 all commute. Then we consider the pair of elements in W: $w = s_1 s_2 s_3 s_4 s_3 s_2 s_1$, and $x = s_1 s_2 s_3$.

Theorem 2. *Let w, x be any of the five pairs described above. Then the cokernel of the map* (2) *has 2-torsion.*

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Normality of nilpotent varieties

Eric Sommers

For the classical groups, Kraft and Procesi have resolved the question of when the closure of a nilpotent orbit is normal, except in the case of some of the orbits for a special orthogonal group which are not invariant under the full orthogonal group. For example, the normality of the closure of an orbit with Jordan block sizes $(4, 4, 2, 2)$ can not be decided using previously known methods.

In this talk we show that these remaining orbits do have normal closure by showing that the regular functions on these orbits are naturally a quotient of the regular functions on an orbit whose closure is known to be normal. Along the way we prove and use a new result concerning the vanishing of the higher cohomology of vector bundles on flag varieties.

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Standard Monomial basis for nilpotent orbit closures

V. Lakshmibai
(joint work with V. Kreiman, P. Magyar, and J. Weyman)

We shall work over the base field $k := \mathbb{C}$. Let $G = GL_n(k), \mathcal{N} =$ the variety of nilpotent $n \times n$ matrices. For the action of G on $\mathcal{N}$ given by conjugation, the G-orbits are indexed by partitions of n. For a partition λ of n, let $\mathcal{N}_\lambda$ denote the corresponding orbit closure. Note that $\mathcal{N} = \mathcal{N}_\lambda$, where $\lambda = (n, 0, \cdots, 0)$. We construct a basis for $k[\mathcal{N}_\lambda]$, the ring of regular functions on $\mathcal{N}_\lambda$. For this construction, we use the following result of Lusztig:

Theorem ([5]). *A nilpotent orbit closure $\mathcal{N}_\lambda$ gets identified with an open subset of a certain affine Schubert variety in the affine Grassmannian.*

We first recall the following classical result of Hodge on the Grassmannian.

1. The Grassmannian & its Schubert varieties

Let us fix the integers $1 \leq d < n$ and let $V = k^n$. The *Grassmannian* $G_{d,n}$ is the set of d-dimensional subspaces $U \subset V$; with respect to a basis $a_1, \ldots, a_d$ of U, where

$$a_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \cdots \\ a_{nj} \end{pmatrix}, \text{ with } a_{ij} \in k, \text{for } 1 \leq i \leq n,\ 1 \leq j \leq d$$

(here, each vector a_j is written as a column vector with respect to the standard basis of k^n), U may be represented by the $n \times d$ matrix $A = (a_{ij})$ (of rank d), whose columns are the vectors $a_1, \ldots, a_d$.

1.1. **Plücker embedding and Plücker co-ordinates.** Let

$$p : G_{d,n} \to \mathbb{P}(\wedge^d V)$$

be the *Plücker embedding.* Define the set

$$I_{d,n} := \{\underline{i} = (i_1, \ldots, i_d) \mid 1 \leq i_1 < \cdots < i_d \leq n\}$$

For $\underline{i} \in I_{d,n}$, the $\underline{i}$-th component of p is denoted by $p_{\underline{i}}$; the $p_{\underline{i}}$'s, with $\underline{i} \in I_{d,n}$, are called the *Plücker coordinates*. If a point U in $G_{d,n}$ is represented by the $n \times d$ matrix A (as above), then $p_{i_1,\ldots,i_d}(U) = det(A_{i_1,\ldots,i_d})$, where $A_{i_1,\ldots,i_d}$ denotes the $d \times d$ sub matrix of A consisting of the rows with indices $i_1, \ldots, i_d$.

1.2. **Schubert Varieties of $G_{d,n}$.** For $1 \leq t \leq n$, let V_t be the subspace of V spanned by $\{e_1, \ldots, e_t\}$. For each $\underline{i} \in I_{d,n}$, the *Schubert variety associated to $\underline{i}$* is defined to be

$$X_{\underline{i}} = \{U \in G_{d,n} \mid \dim(U \cup V_{i_t}) \geq t,\ 1 \leq t \leq d\}.$$

Remark 1.2.1. We have that under the set-theoretic bijection between the set of Schubert varieties and the set $I_{d,n}$, the partial order on the set of Schubert varieties given by inclusion induces the partial order $\geq$ on $I_{d,n} : \underline{i} \geq \underline{j} \Leftrightarrow i_t \geq j_t, \forall t$.

1.3. **Standard Monomial Basis for Schubert varieties in $G_{d,n}$.** Let R be the homogeneous co-ordinate ring of $G_{d,n}$ for the Plücker embedding, and for $\tau \in I_{d,n}$, let $R(\tau)$ be the homogeneous co-ordinate ring of the Schubert variety $X(\tau)$.

Definition 1.3.1. A monomial $f = p_{\tau_1} \cdots p_{\tau_m}$ is said to be *standard* if

$$(*) \qquad \tau_1 \geq \cdots \geq \tau_m.$$

Such a monomial is said to be *standard on* $X(\tau)$, if in addition to condition (*), we have $\tau \geq \tau_1$.

Theorem 1.3.2 ([2, 3]). *Standard monomials on $X(\tau)$ of degree m give a basis for $R(\tau)_m$.*

As a corollary, we obtain that if L denotes the tautological line bundle on $\mathbb{P}(\wedge^d V)$ (as well as its restriction to $X(\tau)$), then the standard monomials on $X(\tau)$ of degree m form a basis for $H^0(X(\tau), L^m)$.

2. The Affine Grassmannian & its Schubert varieties

Let $F = k((t))$, the field of Laurent series, $A = k[[t]]$, the ring of formal power series. Let $G = SL_n(k), B$ the Borel subgroup consisting of upper triangular matrices, and T the maximal torus consisting of diagonal matrices. Let $\mathcal{G} = SL_n(F), \mathcal{P} = SL_n(A), \mathcal{B} = ev^{-1}(B)$, where ev is the evaluation map $ev : SL_n(A) \to SL_n(k), t \mapsto 0$. Let $\hat{W}$ be the affine Weyl group. Then $\mathcal{G}/\mathcal{B}$ is an ind-variety; further, $\mathcal{G}/\mathcal{B} = \cup_{w \in \hat{W}} \mathcal{B}w\mathcal{B}\,(mod\,\mathcal{B})$. Set $X(w) = \cup_{\tau \leq w} \mathcal{B}\tau\mathcal{B}\,(mod\,\mathcal{B})$. Then, $X(w)$ is the *affine Schubert variety* associated to w. Even though, $\mathcal{G}/\mathcal{B}$ is infinite dimensional, $X(w)$ is a finite dimensional projective variety. Similarly, one defines Schubert varieties inside the *affine Grassmannian* $\mathcal{G}/\mathcal{P}$.

2.1. **Sketch of the construction of the basis:** The first step in our construction is to give a matrix presentation for the elements of $\mathcal{G}/\mathcal{P}$, the affine Grassmannian (as in the case of the classical Grassmannian). Once we have a matrix presentation, then we could talk about Plücker co-ordinates on the affine Grassmannian; we could then define standard monomials in the Plücker co-ordinates on the affine Grassmannian (as well as on its Schubert varieties) similar to the classical situation.

Let $k^\infty = \overline{span}\{e_i, i \in \mathbb{Z}\}$, where by $\overline{span}$, we mean that we allow for rightward-infinite linear combinations. For $i \in \mathbb{Z}$, let $E_i = \overline{span}\{e_j, j \geq i\}$. For a subset S such that $\mathbb{N}_p \supset S \supset \mathbb{N}_q$, for some $p, q \in \mathbb{Z}$, let $E_S = \overline{span}\{e_j, j \in S\}$ (here, $\mathbb{N}_p = \{p, p+1, \cdots\}$); note that $E_p \supset E_S \supset E_q$. Define Gr_∞, the *infinite Grassmannian* $= \{$subspaces $E \subset k^\infty | \, E_p \supset E \supset E_q$, for some $p, q \in \mathbb{Z}\}$. Let $GL_\infty = \{A = (a_{ij})_{\mathbb{Z}\times\mathbb{Z}} \,|$ all but a finite number of $a_{ij} - \delta_{ij}$ are 0, and $detA \neq 0\}$. Then GL_∞ acts transitively on Gr_∞, the isotropy at $E_1 (= \overline{span}\{e_j, j \geq 1\})$ being a certain parabolic subgroup P_∞. Let $\sigma : k^\infty \to k^\infty$ be the map, $e_j \mapsto e_{j+n}$. Define $\hat{Gr}_n$, the *affine Grassmannian* $= \{\sigma-$stable subspaces of $k^\infty\}$.

Identify $k^\infty \cong F^n, e_j \mapsto t^c e_i$, where c and i are given by $j = i + nc, 0 \leq i \leq n-1$ (here, we denote the standard basis for F^n by $\{e_0, \cdots, e_{n-1}\}$). Let $\tau : F^n \to F^n$ be the map $v \mapsto tv$. Via this identification, $\hat{Gr}_n$ may be identified with $\{A-$lattices in $F^n\}$.

Connected components of $\hat{Gr}_n$:

For $i \in \mathbb{Z}$, let $\hat{Gr}^i_n := \{A-$lattices $L \,|\, dim_k(L/L \cap L_0) = dim_k(L_0/L \cap L_0)\}$, where L_0 is the A-lattice $Ae_0 \oplus \cdots \oplus Ae_{n-1}$. The $\hat{Gr}^i_n$'s, $i \in \mathbb{Z}$ give the connected components of $\hat{Gr}_n$. We have an identification $\mathcal{G}/\mathcal{P} \cong \hat{Gr}^0_n, g\mathcal{P} \mapsto gL_0 (= Age_0 \oplus \cdots \oplus Age_{n-1})$. Thus via the embedding $\mathcal{G}/\mathcal{P} \hookrightarrow Gr_\infty$, we obtain a $\mathbb{Z} \times \mathbb{Z}$ matrix presentation for elements of $\mathcal{G}/\mathcal{P}$. Let

$$Y_0 = \{S \,|\, \mathbb{N}_p \supset S \supset \mathbb{N}_q, |S \setminus \mathbb{N}| = |\mathbb{N} \setminus S|\}.$$

For $S \in Y_0$, let p_S denote the Plücker co-ordinate on $\hat{Gr}^0_n$ defined in the obvious way.

Definition 2.1.1. Let $S = (s_1 < s_2 < \cdots) \in Y_0$. We say, S is *admissible* if $s_{i+1} - s_i \leq n$.

Theorem 2.1.2 (cf.[1])**.** $\{p_S, S$ admissible$\}$ *is a basis for* $H^0(\mathcal{G}/\mathcal{P}, L), L$ *being the basic line bundle on* $\mathcal{G}/\mathcal{P}$.

We define operators $\{e_\alpha, f_\alpha, \alpha$ simple$\}$ similar to Kashiwara's crystal operators (cf. [4]), using which we associate to each $S \in Y_0$ a canonical pair $(\lceil S \rceil > \lfloor S \rfloor)$ of elements of $\hat{W}$, and call these respectively the *ceiling* and the *floor* of S.

Definition 2.1.3. A monomial $p_{S_1} \cdots p_{S_m}, S_i \in Y_0$ is standard on $X(\tau)$ if $\tau \geq \lceil S_1 \rceil \geq \lfloor S_1 \rfloor \geq \lceil S_2 \rceil \geq \cdots \geq \lfloor S_m \rfloor$.

Theorem 2.1.4. *Monomials in p_S's standard on $X(\tau)$ of degree m form a basis of $H^0(X(\tau), L^m)$.*

As a corollary, we obtain a basis for $k[\mathcal{N}_\lambda]$.

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Singularities of moduli spaces of vector bundles in char. 0 and char. p

Vikram Mehta

(joint work with V. Balaji)

We study the singularities of the moduli spaces of vector bundles on curves. These moduli spaces over arbitrary base have been constructed by Seshadri. If $U_W \to W$ is the relative moduli space over W, then $U_W \otimes_W \overline{K}$ gives the "correct" moduli space over $\overline{K}$, as this is a flat base change. We prove that $U_W/(p)$ also gives the correct moduli space in characteristic p.

This is achieved by studying the action of $\mathrm{Aut}(V)$ on the local moduli space. The key point is that these local moduli spaces have a good filtration relative to $\mathrm{Aut}(V)$. This enables us to conclude that these invariants in char. 0 specialize to the invariants in char. p. Also we prove that the moduli spaces in char. p are strongly F-regular and consequently, the moduli spaces in char. 0 have canonical singularities.

We conclude with some remarks on the moduli spaces of principal G-bundles in arbitrary characteristic.

Good quotients for reductive group actions

J. Hausen

Good quotients. In the sequel, G is a reductive complex algebraic group, and X is a normal complex algebraic G-variety. A *good quotient* for X is a morphism $p\colon X \to X/\!\!/G$ to an algebraic space $X/\!\!/G$ such that for every affine étale neighbourhood $V \to X/\!\!/G$ the inverse image $p^{-1}(V)$ is affine, G-invariant, and the structure sheaf $\mathcal{O}_V$ equals the sheaf of invariants $p_*(\mathcal{O}_{p^{-1}(V)})^G$.

We are interested in the family $\mathfrak{F}^G$ of all G-invariant open subsets admitting a good quotient $p\colon U \to U/\!\!/G$. Inside $\mathfrak{F}^G$, we will consider subfamilies with prescribed properties on the quotient space, e.g., the family $\mathfrak{F}^G_{\mathrm{sep}} \subset \mathfrak{F}^G$ of subsets with a separated quotient space, or the family $\mathfrak{F}^G_{\mathrm{qp}} \subset \mathfrak{F}^G$ of subsets with a quasiprojective quotient space.

Let "$*$" stand for a property imposed on the quotient space, e.g., separatedness or quasiprojectivity. Then a set $U_1 \in \mathfrak{F}_*^G$ is called *maximal* if there exists no $U_2 \in \mathfrak{F}_*^G$ containing U_1 as a proper subset such that U_1 is saturated with respect to the quotient map $p_2 \colon U_2 \to U_2/\!\!/G$, i.e., satisfies $U_1 = p_2^{-1}(p_2(U_1))$. By a basic result of A. Białynicki-Birula, there are only finitely many maximal $U \in \mathfrak{F}_{\mathrm{sep}}^G$.

The central task of the theory of good quotients is to describe or even construct all maximal sets $U \in \mathfrak{F}_*^G$. In the sequel, we report on some results obtained since the appearance of Białynicki-Birula's survey article [2].

Around Mumford's GIT. Here we consider the family $\mathfrak{F}_{\mathrm{qp}}^G \subset \mathfrak{F}^G$ of open subsets with quasiprojective quotient spaces. In his fundamental book, D. Mumford introduces the notion of a G-linearized line bundle L on X, and to any such L he associates a set of semistable points $X^{ss}(L)$. The main features of this construction are well known:

- $X^{ss}(L) \in \mathfrak{F}_{\mathrm{qp}}^G$ holds, that means that there is a good quotient $X^{ss}(L) \to X^{ss}(L)/\!\!/G$, and the quotient space is quasiprojective;
- if X is smooth and $U \in \mathfrak{F}_{\mathrm{qp}}^G$ is maximal, then $U = X^{ss}(L)$ for some G-linearized line bundle L on X;
- for projective X and ample L, the Hilbert–Mumford Criterion characterizes semistability in terms of one parameter subgroups $\mathbb{C}^* \to G$;
- for projective X, the sets $X^{ss}(L)$ with L ample correspond order reversingly to the cones of a fan subdivision of the G-ample cone, see [4] and [9].

In the case of a projective X and an ample L, the quotient space $X^{ss}/\!\!/L$ is projective, but even for smooth projective X, there may exist $U \in \mathfrak{F}_{\mathrm{qp}}^G$ with $U/\!\!/G$ projective that do not arise from ample bundles. Moreover, if X is not smooth, then there may exist maximal $U \in \mathfrak{F}_{\mathrm{qp}}^G$ that do not arise from any G-linearized line bundle.

To overcome the latter problem, we propose in [8] the following approach: consider a Weil divisor D on X, the subsemigroup $\Lambda \subset \mathrm{WDiv}(X)$ generated by D, and the data

$$\mathcal{A} := \bigoplus_{E \in \Lambda} \mathcal{O}_X(E), \qquad \widehat{X} := \mathrm{Spec}(\mathcal{A}).$$

Then a G-linearization of D is a certain lifting of the G-action to $\widehat{X}$, and $x \in X$ is semistable, written $x \in X^{ss}(D)$, if there is a G-invariant $f \in \Gamma(X, \mathcal{O}_X(nD))$, where $n > 0$, such that X_f is affine, $x \in X_f$ holds, and D is Cartier on X_f. The basic features are the following:

- $X^{ss}(D) \in \mathfrak{F}_{\mathrm{qp}}^G$ holds, and, conversely, if $U \in \mathfrak{F}_{\mathrm{qp}}^G$ is maximal, then $U = X^{ss}(D)$ for some G-linearized Weil divisor D on X;
- given a maximal torus $T \subset G$, we have a generalized Hilbert-Mumford Criterion

$$X^{ss}(D, G) = \bigcap_{g \in G} g \cdot X^{ss}(D, T).$$

Divisorial quotient spaces. Borelli calls a prevariety Y *divisorial* if any $y \in Y$ admits an affine neighbourhood $Y \setminus D$ with an effective Cartier divisor D on Y. This concept comprises quasiprojective as well as smooth varieties. It turns out that a prevariety is divisorial if and only if it is the quotient of a quasiaffine variety by a free torus action.

Let $\mathfrak{F}^G_{\mathrm{div}} \subset \mathfrak{F}^G$ denote the family of subsets with a divisorial quotient space. We discuss the construction of such sets presented in [5] and [8]. Consider a finitely generated subgroup $\Lambda \subset \mathrm{WDiv}(X)$ of the group of Weil divisors of the G-variety X. Then we have the data

$$\mathcal{A} := \bigoplus_{D \in \Lambda} \mathcal{O}_X(D), \qquad \widehat{X} := \mathrm{Spec}(\mathcal{A}).$$

Similarly as before, a G-linearization of Λ is a certain lifting of the G-action to $\widehat{X}$. A point $x \in X$ is semistable, written $x \in X^{ss}(\Lambda)$, if there is a G-invariant homogeneous $f \in \Gamma(X, \mathcal{O}_X(\Lambda))$ such that X_f is affine with $x \in X_f$, all $D \in \Lambda$ are Cartier on X_f, and almost all $D \in \Lambda$ admit an invertible $h \in \Gamma(X_f, \mathcal{A}_D)^G$. The basic features are

- $X^{ss}(\Lambda) \in \mathfrak{F}^G_{\mathrm{div}}$ holds, and, conversely, if $U \in \mathfrak{F}^G_{\mathrm{div}}$ is maximal, then $U = X^{ss}(\Lambda)$ for some G-linearized group Λ of Weil divisors on X;
- given a maximal torus $T \subset G$, we have a generalized Hilbert-Mumford Criterion:

$$X^{ss}(\Lambda, G) = \bigcap_{g \in G} g \cdot X^{ss}(\Lambda, T).$$

An application of this construction is the following algebraicity criterion for orbit spaces, see [5]: Suppose that X is $\mathbf{Q}$-factorial and that G acts properly. Then the algebraic space X/G is an algebraic variety if and only if the induced action of the Weyl group $W(T)$ on the algebraic variety X/T has an algebraic variety as orbit space.

CombinaTorics. Here, X is a toric variety, and T is a subtorus of the big torus $T_X \subset X$. In this setting, J. Święcicka observed that any maximal $U \in \mathfrak{F}^T_{\mathrm{sep}}$ is already T_X-invariant. Thus, the description of the maximal sets $U \in \mathfrak{F}^T_{\mathrm{sep}}$ becomes a purely toric problem.

A first approach is the language of fans. Let X arise from a fan Σ in the lattice of one parameter subgroups of T_X. Then the maximal $U \in \mathfrak{F}^T_{\mathrm{sep}}$ correspond to the subfans $\Sigma' \subset \Sigma$ that are maximal with the property that any two maximal cones admit a separating linear form, which is invariant under the lattice of one parameter subgroups of the small torus $T \subset T_X$, see e.g. [6].

Another approach is the language of bunches of cones in the character lattice of the small torus, compare [3] and [1]. Suppose that $X = \mathbb{C}^n$ holds. Then T acts via

$$t \cdot z = (\chi_1(t) z_1, \ldots, \chi_n(t) z_n).$$

A weight cone is a cone generated by some of the weights $\chi_i \in \mathrm{Char}(T)$. A *bunch* is a collection Φ of weight cones such that for any arbitrary weight cone σ we have:

$$\sigma \in \Phi \iff \emptyset \neq \mathrm{relint}(\sigma) \cap \mathrm{relint}(\tau) \neq \mathrm{relint}(\tau) \text{ for all } \sigma \neq \tau \in \Phi.$$

The possible bunches are in one to one correspondence with the maximal $U \in \mathfrak{F}^T_{\mathrm{sep}}$. The translation from the language of bunches to the language of fans is based on a linear Gale transformation. Moreover, the language of bunches has an analogue for $X = \mathbb{P}_n$, obtained by replacing the weight cones with weight polytopes, see [2].

Finally, consider an arbitrary $\mathbf{Q}$-factorial T-variety X that has the A_2-property, i.e., any two $x, x' \in X$ have a common affine neighborhood in X. Then there are only finitely many maximal $U_1, \dots, U_r \in \mathfrak{F}^T_{A_2}$, and X admits a T-equivariant closed embedding into a toric variety Z such that $U_i = X \cap W_i$ for some maximal open $W_i \subset Z$ having a good quotient $W_i \to W_i /\!\!/ T$ with $W_i /\!\!/ T$ separated, see [6].

Reduction Theorems. Now G is a connected reductive group, and $T \subset G$ is a maximal torus. Consider a given family $\mathfrak{F}^T_*$, let $U \in \mathfrak{F}^T_*$ be maximal, and set

$$W(U) := \bigcap_{g \in G} g \cdot U.$$

Then Białynicki-Birula asks when we have (a) $W(U) \in \mathfrak{F}^G$, or (b) $W(U) \in \mathfrak{F}^G_*$, or even when (c) $W(U)$ is maximal in $\mathfrak{F}^G_*$.

A first couple of results can be derived using Mumford's GIT and the generalizations presented before: Suppose that G is semisimple and that U is invariant under the normalizer $N(T) \subset G$. Then

- for $U \in \mathfrak{F}^T_{\mathrm{proj}}$, one has (c), see [2] for smooth X, and [8] for normal X;
- for $U \in \mathfrak{F}^T_{\mathrm{qp}}$, one has (b), see [8];
- for a maximal $U \in \mathfrak{F}^T_{\mathrm{div}}$, one has (b), see [8].

An interesting question is that of complete quotients — here only for $G = \mathrm{SL}_2$ is something known, see [2]. Further reduction theorems are the following:

- for $U = X \in \mathfrak{F}^T_{\mathrm{sep}}$, one has (a), see [2];
- for $X = \mathbb{P}_n, \mathbb{C}^n$ and $U \in \mathfrak{F}^T_{\mathrm{sep}}$, one has (a), see [2];
- for $\mathbf{Q}$-factorial X and $U \in \mathfrak{F}^T_{A_2}$, one has (a), see [7].

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Universal denominators of invariant rings

Harm Derksen

Suppose that R is a finitely generated graded ring and M is a finitely graded module. It is not always true that the denominator of the Hilbert series of M divides the denominator of the Hilbert series of R. (By Hilbert's Syzygy Theorem this is true if R is a polynomial ring.) This observation leads to my notion of "the universal denominator of a module". Usually the universal denominator of a module is the denominator of the Hilbert series but there are exceptions. The universal denominator behaves much nicer. I could present various formulas for universal denominators of Hilbert series. There are various applications:

(a) I can prove a statement that is very close to one of the conjectures of Dixmier about the denominator of the Hilbert series for invariants of binary forms.
(b) In case the coefficients of the Hilbert series have combinatorial interpretation, one can prove properties about these combinatorial numbers. For example, Jerzy Weyman and I proved the polynomiallity of Littlewood-Richardson numbers. The notion of the universal denominator sheds new light on this result.
(c) Universal denominators can be used to bound the denominator of invariant rings. For example, Nolan Wallach's computation of the Hilbert series for the invariants of 4 qubits can be simplified by using better bounds for the denominator.

Alternating signs of quiver coefficients

Anders S. Buch

Let $E_0 \to E_1 \to \cdots \to E_n$ be a sequence of vector bundles and bundle maps over a non-singular variety X. A set of *rank conditions* for this sequence is a collection $r = \{r_{ij}\}$ of non-negative integers, for $0 \leq i \leq j \leq n$, such that r_{ii} is the rank of E_i for every i. This data defines the *quiver variety*

$$\Omega_r = \{x \in X \mid \operatorname{rank}(E_i(x) \to E_j(x)) \leq r_{ij} \ \forall i < j\} \,.$$

I demand that the rank conditions can *occur*, i.e. they describe an orbit in a quiver representation, and that the bundle maps are sufficiently general, so that the quiver variety obtains its expected codimension $d(r) = \sum_{i<j}(r_{i,j-1} - r_{ij})(r_{i+1,j} - r_{ij})$.

In earlier work with Fulton [3], we proved a formula for the cohomology class of the quiver variety Ω_r in the cohomology ring of X. I later generalized this to the following formula for the *Grothendieck class* of Ω_r in the Grothendieck ring of algebraic vector bundles on X [1]:

$$[\mathcal{O}_{\Omega_r}] = \sum_{\mu} c_\mu(r)\, G_{\mu_1}(E_1; E_0) \cdot G_{\mu_2}(E_2; E_1) \cdots G_{\mu_n}(E_n; E_{n-1}) \in K(X)\,.$$

The sum is over sequences μ of partitions, and the elements G_{μ_i} are K-theoretic generalizations of Schur determinants called *stable Grothendieck polynomials*. The *quiver coefficients* $c_\mu(r)$ appearing in this formula are uniquely determined by the fact that the formula is true for all varieties X, as well as the condition that these coefficients do not change when the same number is added to all the rank conditions.

The quiver coefficients are indexed by sequences of partitions μ for which the sum of the weights is greater than or equal to the codimension $d(r)$. The coefficients $c_\mu(r)$ for which $\sum |\mu_i| = d(r)$ also appear in the cohomology formula and are called *cohomological quiver coefficients*. It was conjectured that cohomological quiver coefficients are non-negative and that the general quiver coefficients have signs that alternate with codimension, that is

$$(-1)^{\sum |\mu_i| - d(r)} c_\mu(r) \geq 0\,.$$

The conjecture for cohomological quiver coefficients has been proved by Knutson, Miller, and Shimozono [4]. In my talk (based on [2]) I present a proof of the general conjecture, which furthermore results in a combinatorial formula for K-theoretic quiver coefficients. My main result is a K-theoretic generalization of the *component formula* of [4]. It writes the Grothendieck class of a quiver variety as an alternating sum of products of stable Grothendieck polynomials given by permutations:

$$[\mathcal{O}_{\Omega_r}] = \sum (-1)^{\sum \ell(u_i) - d(r)}\, G_{u_1}(E_1; E_0) \cdot G_{u_2}(E_2; E_1) \cdots G_{u_n}(E_n; E_{n-1})\,.$$

This sum is over certain sequences of permutations $(u_1, \ldots, u_n)$ which I call *KMS-factorizations* for the rank conditions r. Since Lascoux has proved that any stable Grothendieck polynomial given by a permutation is an alternating linear combination of stable Grothendieck polynomials for partitions [5], this formula implies that K-theoretic quiver coefficients have alternating signs.

I also introduce and prove some new variants of the *factor sequences conjecture* from [3], and I prove Knutson, Miller, and Shimozono's conjecture that their *double ratio formula* agrees with the original quiver formulas.

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On the values of the characters of compact Lie groups

Jean-Pierre Serre

The lecture discussed three loosely related theorems on the characters of a compact Lie group G.

1. A generalization of a theorem of Burnside

Theorem 1. *Let χ be the character of an irreducible complex representation of G. Assume $\chi(1) > 1$. Then there exists an element x of G, of finite order, with $\chi(x) = 0$.*

When G is finite, this is a well-known result of Burnside.

2. Positive characters with mean value equal to 1

Let f be a virtual character of G having the following two properties:

(a) $f(x)$ is real ≥ 0 for every $x \in G$.
(b) The mean value $\langle f, 1\rangle$ of f is equal to 1.

There are many examples of such characters when G is finite (e.g. permutation characters relative to a transitive action). Not so when G is connected. More precisely:

Question. If G is connected and simply connected, is it true that every character f having properties (a) and (b) is equal to $\chi.\bar{\chi}$, where χ is an irreducible (complex) character of G?

Theorem 2. *The answer to the question above is* "yes" *when G is of rank 1, i.e. when $G = \mathbf{SU}_2(\mathbf{C})$.*

3. The character of the adjoint representation

Consider the adjoint representation $\mathrm{Ad} : G \to \mathrm{Aut}\ (\mathrm{Lie}\ G)$.

Theorem 3. *One has* $\mathrm{Tr}\ \mathrm{Ad}(x) \geq -\ \mathrm{rank}(G)$ *for every* $x \in G$.

The minimal value of $\mathrm{Tr}\ \mathrm{Ad}(x)$ can be determined explicitly:

Choose a maximal torus T of G ; let N be its normalizer and let W be the quotient N/T (so that W is the Weyl group if G is connected). For any $w \in W$, let $\mathrm{Tr}_T(w)$ be the trace of w acting on Lie T. Theorem 3 can be refined as:

Theorem 3$'$. *One has* $\inf_{x\in G} \operatorname{Tr} \operatorname{Ad}(x) = \inf_{w\in W} \operatorname{Tr}_T(w)$.

This shows in particular that inf Tr Ad(x) is *an integer*, a fact which was not *a priori* obvious. It also shows that inf Tr Ad(x) is equal to $-$ rank(G) if and only if W contains an element which acts on T by $t \mapsto t^{-1}$.

When G is connected and simple, Theorem 3$'$ gives:

$$\inf \operatorname{Tr} \operatorname{Ad}(x) = -\operatorname{rank}(G) \text{ if } G \text{ is of type } A_1, B_n, C_n, D_n \ (n \text{ even}), G_2, F_4, E_7, E_8 \ ,$$

$$\inf \operatorname{Tr} \operatorname{Ad}(x) = \begin{cases} -1 & \text{if } G \text{ is of type } A_n \ (n \geq 1) \\ 2-n & \text{if } G \text{ is of type } D_n \ (n \text{ odd} \geq 3) \\ -3 & \text{if } G \text{ is of type } E_6 \ . \end{cases}$$

4. Proofs

They are not published yet. Here are some hints for the interested reader :

Theorem 1: Use the properties of the "principal A_1 subgroup" of G.

Theorem 2: An exercise on positive-valued trigonometric polynomials.

Theorem 3$'$: If $w \in W$ is such that $\operatorname{Tr}_T(w)$ is minimum, any representative x of w in N is such that Tr Ad$(x) = \operatorname{Tr}_T(w)$; this proves the inequality inf Tr Ad$(x) \leq$ inf $\operatorname{Tr}_T(w)$. The opposite inequality can be checked by a case-by-case explicit computation. The classical groups are easy enough, but F_4, E_6, E_7 and E_8 are not (especially E_6, which I owe to Alain Connes). I hope there is a better proof.

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