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The Mathematics and Statistics of Quantitative Risk Management

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Abstract. It was the aim of this workshop to gather a multidisciplinary and international group of scientists at the forefront of research in areas related to the mathematics and statistics of quantitative risk management. The main objectives of this workshop were to break down disciplinary barriers that often limit collaborative research in quantitative risk management, and to communicate the state of the art research from the different disciplines, and to point towards new directions of research.

Mathematics Subject Classification (2000): 62 Statistics, 60 Probability.

Introduction by the Organisers

This meeting was well attended by more than 50 participants with broad geographic representation from five continents. The participants were statisticians, applied probabilists, financial mathematicians, econometricians, and economists. They represented rather different aspects of quantitative risk management

Through the last four years, the financial markets have been turbulent with frequent swings of unexpected magnitude. The recent financial crisis, paired with a wide economic downturn, is being analyzed by scientists, financial analysts, business leaders, and politicians. Among these persons with rather different opinions and interests there is agreement about the following fact: one reason for the crisis was bad risk management at the levels of the individual financial institutions and supervisory authorities.

In March 2008, when we held the first Oberwolfach Meeting on the *Mathematics and Statistics of Quantitative Risk Management,* the crisis had already evolved.

However, the dramatic collapse of the international financial system which happened shortly afterwards was hardly predictable. Since then, governments and financial institutions have been aiming at more stability and security in the national and international financial systems. A drastic improvement of the risk management of banks and insurance companies is definitely called for as well as a reform of those agencies which shall supervise individual institutions. Indeed regulators (FSA London, ECB Frankfurt, OCC Washington) are now organizing meetings with titles such as "An academic view on capital adequacy" and "Exploring statistical issues in financial risk modeling and banking regulation." Mathematicians are explicitly asked for their opinion.

In the last four years, various publications have blamed mathematicians and statisticians for unrealistic models and unreliable statistical procedures in risk management. In the Oberwolfach Report 15/2008 we mentioned the following: *A natural topic of the workshop was the recent worldwide crisis of credit portfolios. In the past, mathematical models have been designed to avoid the present situation and they are implemented in the framework of the Basel II accord. But they obviously have not been used successfully. Both formal and informal reasons for the present situation were discussed. Although it would be inappropriate to blame a mathematical model for its failure, there is evidence that various models are too simplistic and do not incorporate market information sufficiently fast. Further, it appears that the statistical analysis of the data was not conducted with sufficient care.*

At the 2008 Oberwolfach meeting on quantitative risk management we have reported about the high complexity of sophisticated mathematical and statistical methods needed for risk management. We mentioned areas of research such as the theory of high-dimensional data structures, rare event simulation, the theory of risk measures, multivariate and non-linear time series analysis, extreme event modeling and extreme value statistics, optimization, and linear, quadratic and convex programming. These are distinct areas with highly specialized experts who rarely communicate on a day-to-day basis. However, the present financial crisis indicates that specialists from different areas must collaborate to overcome the deficiencies of the models of quantitative risk management and their statistical implementation.

The theoretical results in quantitative risk management have immediate practical consequences for the financial and insurance industries as well as for supervisory authorities: they allow one to design mathematically tractable, practically relevant and statistically estimable risk measures.

Risks in insurance and finance are described by mathematical and stochastic models such as partial differential equations and stochastic differential equations describing the evolution of prices of risky assets (such as stock, composite stock indices, interest rates, foreign exchange rates, commodities) or difference equations describing the evolution of financial returns. The *Handbook of Financial Time Series* (Springer 2009), edited by R.A. Davis and T. Mikosch provides a comprehensive description of the state of the art in the area of financial time series analysis. Applications of these discrete and continuous time series models require advanced simulation and numerical methods and statistics plays a vital role in model building, from estimation to validation and model refinement. The models often depend on unknown parameters (possibly infinite dimensional) which have to be estimated from historical data.

Due to their complexity, problems of quantitative risk management require cross-disciplinary solutions. They involve functional analysts who design and analyze risk measures, probabilists who model with stochastic differential equations and time series, applied probabilists who solve the simulation problems, numerical analysts who deal with high-dimensional integration and optimization problems, and statisticians who fit stochastic models to the data and predict future values of risky assets.

Among the challenging problems that were discussed at the meeting are the following:

- **Statistical methods; dependence measures/high dimensional/copulae/extremal dependence** (Patton, Reiss, Segers, Rüschendorf, Neslehova, Nolan, Bücher, Stoev): Risk problems are often high-dimensional: a portfolio typically consists of several hundred assets. For example, the number of historical observations is often smaller than the number of parameters in the model. There is a growing statistics literature dealing with large-dimensional data and these modern techniques need to be adapted to problems in risk management. A number of modeling strategies were proposed for modeling dependence, from high-dimensional copula models to nonparametric and parametric inference procedures. While high-dimensional problems in this field remain a big challenge, there are now some serious attempts at dealing with these issues.
- **Financial time series modeling** (Teräsvirtä, Klüppelberg, Sørensen, Leucht, Davidson, Wintenberger, Nguyen, Kabluchko): Risks are dependent across the assets and through time. A key problem is the sensitivity of a particular modeling paradigm to model miss-specification of multivariate models. Robustness to parameter estimation does not quite fit the bill, since, for example, parameters coming from a particular copula (arising from a multivariate distribution) may be completely meaningless if the true model does not involve such quantities. Emphasizing this aspect of sensitivity to model miss-specification encompasses a number of the issues that were addressed at this workshop.
- **Pricing and hedging under nonstandard conditions** (Ruf, Becherer, Protter, Lindskog, Hernández-Hernández, Zhou): Many of the recent advances in financial mathematics study properties of models under nonstandard assumptions. These include, robust hedging under model uncertainty, expanding filtrations with applications to insider trading, hedging of options on exploding exchange rates, and optimal stopping when probability is distorted.

- **Foundational issues** (Rogers, Filipović, Teichmann): There was much discussion on the interaction between firms, banks, and households, and other topics such as term structure of interbank risks.
- **Insurance and reinsurance in a financial environment** (Barrieu, Blanchet, Albrecher, Steffensen): In this group of talks, classical insurance problems were embedded in a financial environment. Topics included generalized ruin concepts for a portfolio of insurance contracts, reinsurance and securitization of insurance risks in the presence of regulatory constraints, continuous time recursive utility, and modeling and simulation of stochastic risk networks.
- **High frequency/volatility modeling** (Tauchen, Todorov, Podolskij, Fasen): One of the topics that has attracted a great deal of attention is high-frequency modeling and estimation. New estimation procedures for the volatility density were proposed and new models were developed that pay closer attention to the interplay between jumps and extreme events. Refinements of techniques for high frequency data include Edgeworth expansions for functionals of continuous diffusion processes and limit theory for multivariate high-frequency data.
- **Algorithmic trading** (Cont, Hult, Yam): With advances of super high-speed trading, there has been increasing interest in the mathematical modeling of order-books. Even though at this point in time, the models are still in their infancy, a number of modeling scenarios were discussed. Super high-speed trading was a principal topic of discussion with the *Visiting Committee* consisting of political and business leaders. During this discussion, researchers expressed concern about the necessity of moving towards a super high-speed trading market and that there could be a number of unintended and uncontrollable consequences. Certainly, this is an issue that requires more collaboration between politicians, regulators, the banking industry, and of course, mathematicians. Other topics of discussion included an analysis of the flash crash and the use of game theory.

We summarize the main results of the workshop.

- Theory and statistical practice of quantitative risk management bear a multitude of challenging, partly contradictory problems which need to be discussed by mathematicians and statisticians in a rigorous way. The workshop aimed at emphasizing major problems in this area.
- The workshop brought together leading researchers from different disciplines to discuss successes, failures and limitations of present technology in quantitative risk management. Ultimately, the workshop facilitated communities and collaboration across disciplinary boundaries.
- The workshop aimed at setting the stage for future statistical and mathematical research in the area of quantitative risk management.

Workshop: The Mathematics and Statistics of Quantitative Risk Management

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Abstracts

A Relaxed Ruin Condition in Insurance

Hansjörg Albrecher
(joint work with H. Gerber and M. Wüthrich)

In this talk we introduce a generalized ruin concept for a portfolio of insurance contracts in collective risk theory, which can be interpreted as a smoothing of the classical criterion. Concretely, a level-dependent Poisson process is started at the beginning of every excursion of the surplus process below zero, and the portfolio is declared bankrupt at the first event of this Poisson process with negative surplus. It is shown that this formulation can lead to quite tractable calculations for bankruptcy probabilities, bankruptcy deficits, discounted dividend payments according to barrier strategies and related quantities. It also turns out that some classical identities in risk theory have similarly simple analogues in this more general setup. Finally it is discussed in some detail on how one can interpret the necessary safety loading contained in insurance premiums in the light of the current insurance regulatory framework, in particular in a cost-of-capital approach, considering the roles and objectives of policyholders, investors and regulators.

Reinsurance and Securitisation of Insurance Risk: the Impact of Regulatory Constraints

Pauline Barrieu
(joint work with Henri Loubergé)

The convergence of the insurance industry with capital markets has become ever more important over recent years (see, for instance, the papers by Cummins and coauthors [6], [7], [8] or [9] or the recent handbook by Barrieu and Albertini [1]). Such convergence has taken many forms. And of the many convergence attempts, some have been more successful than others. The first academic reference to the use of capital markets in order to transfer insurance risk was in a paper by Goshay and Sandor [12]. The authors considered the feasibility of an organized market, and how this could complement the reinsurance industry in catastrophic risk management. In practice, whilst some attempts have been made to develop an insurance future and option market, the results have, so far, been rather disappointing. In parallel to these attempts, however, the Insurance-Linked Securities (ILS) market has been growing rapidly over the last 15 years. There are many different motivations for ILS, including risk transfer, capital strain relief, boosting of profits, speed of settlement, and duration. Different motives mean different solutions and structures, as the variety of instruments on the ILS market illustrate.

Among the key challenges faced by the insurance industry, the management of longevity risk, i.e. the risk that the trend of longevity improvements significantly changes in the future, is certainly one of the most important. Ever more capital has to be accumulated to face this long-term risk, and new regulations in Europe, together with the recent financial crisis only amplify this phenomenon. Under the Solvency II rules, put forward by the European Commission, the more stringent capital requirements that have been introduced for banks should also be applied to insurance company operations (see Eling et al. [10]; Harrington [13]; and Geneva Association [11]). Moreover, in addition to this risk of observing a significant change in the longevity trend, the insurance sector is facing some basis risk, as the evolution of the policyholders mortality is usually different from that of the national population, due to selection effects. These selection effects have different impacts on different insurance companies' portfolios, as mortality levels and speeds of decrease and increase are very heterogeneous in the insurance industry. This makes it hard for insurance companies to rely on national, or even industry, indices, in order to manage their own longevity risk. Hence, it has become more and more important for insurance companies and pension funds to find a suitable and efficient way to deal with this risk. Recently, various risk mitigation techniques have been attempted. Reinsurance and capital market solutions, in particular, have received an accrued interest (see for instance Blake and Burrows [2] and Blake et al. [3]). Even if no Insurance-Linked Securitisation (ILS) related to longevity risk has yet been completed, the development of this market for other insurance risks has been experiencing a continuous growth for several years, mainly encouraged by changes in the regulatory environment and the need for additional capital from the insurance industry. Today, longevity risk securitisation lies at the heart of many discussions, and is widely seen as a potentiality for the future.

The classical and standard framework of risk sharing in the insurance industry, as studied, for instance, by Karl Borch ([4] and [5]), involves two types of agents: primary insurers and a pool of reinsurers. The risk is shared among different agents of the same type, but with both differing sizes and utility functions. The possible financial consequences of some risks, such as large-scale catastrophes or dramatic changes in longevity trends, however, make this sharing process difficult to conduct within a reinsurance pool. In this case, capital markets may improve the risk-sharing process. Indeed, non-diversifiable risks for the insurance industry may be seen as a source of diversification for financial investors, such as a new asset class, enhancing the overall diversification of traditional investment portfolios, particularly in case of low correlations with overall market risk. Even if the correlation is not necessarily low, which may be the case for changes in longevity, the non-diversifiable insurance risks may be shared by a larger population of financial investors, instead of being assumed by reinsurers only. In the first section of the paper, we focus on some insurance risks (for instance, longevity and mortality risks), and, from a general point of view, study the optimal strategy of risk-sharing and risk-transfer between three representative agents (an insurer, a reinsurer and an investor), taking into account pricing principles in insurance and finance within a unified framework. Comments on an optimal securitisation process and, in particular, on the design of an appropriate alternative risk transfer are made. In the second section, we focus on the impact of the regulation upon risk transfer, by differentiating reinsurance and securitisation in terms of their impact upon reserves. More precisely, we will study the bias introduced by the regulatory framework, and the subsequent impact upon the aforementioned risk transfer techniques.

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On Robust Hedging under Model Uncertainty

Dirk Becherer

In the (over-)idealized model world of a complete Black-Scholes financial market, any financial risk can always be eliminated completely by dynamic hedging. If we drop this unrealistic assumption, then a bank holding some risky position X has to realize that it may face some inevitable residual risk that cannot be "hedged away", whatever hedging strategy were to be applied. Nonetheless, the firm may still dynamically hedge its risk the best that is can, minimzing the risk even if the minimum may well be greater than zero.

Assume the bank uses some good dynamic coherent risk measure which may be interpreted as a monetary capital requirement imposed by regulation. Such can be represented by the maximum of conditional expectations $E_t^Q[X]$ over an m-stable family of equivalent valuation-measures Q (generalized scenarios). The simplest parametrization of this family may be by radial or elliptical constraints on the Girsanov kernels of the measures Q, which gives rise to no-good-deal bounds as being bounds on (conditional) expected growth rates. It is natural to define the hedging strategy as minimizer of the risk measure of the outstanding position after (partial) hedging. We show that the good-deal valuation bounds then arise as the market-consistent transformation of the risk measure and how they and the hedging strategy depend on the market price of risk, and thereby on precise knowledge about the objective probability measure P.

Under model uncertainty, we do not have such precise knowledge about the unique real-world probability measure $\{P\}$. Instead we do only know that it should lie in some confidence region $\{P^\nu : \nu \in V\}$ of possible probabilities. Parameterizing that region as before by constraints on Grisanov kernels, we show that in general there is a duality gap and the robust hedging strategy under model uncertainty is not arising from a saddle point. Despite there being a worst case probability scenario $\bar{\nu} \in V$ which maximizes the good deal bounds, the respective model-specific hedging strategy would not ensure a supermartingale property for hedging errors uniformly over all scenario valuation measures to be considered. A robust hedging strategy which ensures such property uniformly under uncertainty is different in general since it must not rely on knowledge that is not available to us under model uncertainty. Instead, it robust hedging corresponds to a robust risk measure that in general may be even higher than the one corresponding to the worst-case scenario $\bar{\nu}$. If uncertainty however is big enough in relation to the growth bounds for no-good-deals, we show that there is a saddle point and that the robust hedging strategy is the risk minimzing strategy under the valuation measure from the saddlepoint and does not involve a speculative component anymore.

All risk measures appearing and the respective hedging strategies can be described and analyzed conveniently on a Wiener space by using the classical theory of backward stochastic differential equations with Lipschitz generators.

Modeling and Efficient Rare Event Simulation of Systemic Risk in Insurance-Reinsurance Networks

Jose Blanchet
(joint work with Yixi Shi)

In this paper we develop efficient simulation methodology for risk assessment in the context of multiple insurance and / or financial entities with correlated exposures to each others risks and to systematic market factors. We also introduce a modeling framework for insurance / reinsurance networks that evolves according to equilibrium settlements at the time of default of companies. These settlements are computed as the solution of an associated linear program at each time period. Our types of models are closely related to and, in fact, inspired by network models that have been analyzed in the literature in recent years, for example [5], [6], [1], [8] and [9], to name a few.

Our interest lies in efficiently computing the conditional expected amount of the losses in the entire system, given the failure of a selected set of market participants. We say a *market or system dislocation* occurs when a specific group of participants fails. Using our results and simulation procedures we aim at characterizing the features that dictate a significant change in the nature of the system's exposures given market dislocation. For instance, if a specific set of market participants is not sufficiently capitalized to fulfill their obligations, what is the most likely reason for such a situation, a systemic shock in the market or a sequence of specific idiosyncratic events pertaining to the specific set of participants?

Because of the various levels of dependence present in our model, and the structure of rare-events of interest (involving several companies defaulting) it turns out that the design of efficient simulation procedures for rare events in our setting typically involves more than one jump, whereas most of the rare-event simulation literature dealing with heavy tailed models involves single-jump events (see [7], [4], and [3]). The challenge in this situation lies in the fact that we are conditioning on rare events (involving several market participants) whose occurrence could most likely be caused by several large jumps. Also, given the integer programming formulation that we provide in one of our main results in the paper, obtaining the large deviations behavior involves dealing with a combinatorial problem.

Our goal is to provide a simulation framework that can be rigorously shown to achieve *strong optimality* properties (in terms of designing estimators with bounded coefficient of variation uniformly as the event of interest becomes more and more rare, see [2]), and yet it is simple to implement in practice. Our contributions can therefore be summarized as follows:

a) We propose a dynamic network model that allows to deal with counterparty default risks with a particular aim of capturing cascading losses at the time of company defaults by means of the solution of a *linear programming problem* that can be interpreted in terms of an equilibrium. The formulation allows to define the evolution of reserve processes in the network throughout time.
b) The linear programming formulation and therefore the associated equilibrium of settlements at the time of default recognizes: 1) the correlations among the risk factors, which are assumed to follow a linear factor model, 2) the contractual obligations among the companies, which are assumed to follow popular contracts in the insurance industry (such as stop-loss and retrocession), and 3) the interconnectedness of the network.
c) Our model allows to obtain asymptotic results and a description of the asymptotic most likely way in which the default of a specific group of participants can occur. This description indicated is fleshed out explicitly, by means of an *integer programming problem* (a Knapsack problem with multiple knapsacks). Such a description emphasizes the impact of the interactions between the severity of the exogenous claims, their dependence structure, and the interconnectedness of the companies on the systemic risk landscape of the entire network under consideration.

d) We propose a class of strongly efficient estimators for computing the expected loss of the network at the time of dislocation conditioning on the event that a specific set of market participants fails to meet their obligations. In addition, these estimators allow to compute associated conditional distributions of the network exposures given the dislocation caused by a set of specific players. The estimation of these conditional distributions is performed with a computational cost (as measured by the number of simulation replications) that remains bounded even if the dislocation event of interest becomes more and more rare.

This is the first paper to the best of our knowledge that constructs provably efficient estimators in the setting of heavy-tailed risk networks. We have formulated our results in terms of regularly varying distributions for simplicity. Deriving logarithmic asymptotics with basically the same qualitative conclusions under other types of tail distributions is straightforward. Our asymptotic results are obtained with the intention of gaining qualitative insight in the form of approximations that are correct up to a constant in the regularly varying setting. The role of the simulation algorithms, then, is to endow these asymptotic approximations with a computational device that allows one to efficiently obtain quantitatively accurate results.

Now, as the connections in the network increase, one must account for all possibilities in which failure can occur. We have aimed at laying out a program to obtain estimators that have uniform relative error, for a fixed network architecture, as the probability of a failure event becomes more and more rare. At the same time, we have settled for estimators that are relatively easy to implement with the indicated performance guarantee. When the networks have more connections, the relative variance (even though uniformly bounded as rare events of interest become more and more rare) could grow. The question of designing rare-event simulation algorithms in which both uniformity in the size of the network and the underlying large deviations parameter are ensured is certainly important but too open-ended at this point. We plan to investigate this avenue in future research.

We envision that our model and our computational approach, based on efficient simulation, can serve as a prototype for the analysis of other types of risk networks. The philosophy behind this paper is that in the presence of network risk models, the settlements and the evolution of the associated risk reserve processes should obey equilibrium constraints that dictate the cascading effect when default occurs. These constraints can effectively be modeled in terms of linear programs, which, coupled with a linear factor heavy-tailed model, allow to describe qualitatively the most likely way in which simultaneous defaults occur. Efficient simulation, in the form of provably efficient Monte Carlo estimators, should then be used to make more precise quantitative statements.

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Multiplier Bootstrap of Tail Copulas

Axel Bücher

In the problem of estimating the lower and upper tail copula we propose two bootstrap procedures for approximating the distribution of the corresponding empirical tail copulas. The first method uses a multiplier bootstrap of the empirical tail copula process and requires estimation of the partial derivatives of the tail copula. The second method avoids this estimation problem and uses multipliers in the two-dimensional empirical distribution function and in the estimates of the marginal distributions. For both multiplier bootstrap procedures we prove consistency.

For these investigations we demonstrate that the common assumption of the existence of continuous partial derivatives in the the literature on tail copula estimation is so restrictive, such that the tail copula corresponding to tail independence is the only tail copula with this property. We solve this problem and prove weak convergence of the empirical tail copula process under nonrestrictive smoothness assumptions which are satisfied for many commonly used models.

Modeling Endogenous Risk

Rama Cont
(joint work with Lakshithe Wagalath)

The traditional approach to statistical modeling in finance models markets risks as "exogenous' stochastic processes, whose dynamics is unaffected -or marginally affected- by trading strategies of market participants. This picture fits well with fragmented markets where many small participants act more or less independently, leading to a random outcome in price behavior, but does not describe the type of extreme market fluctuations and systemic risk which results from large capital flows generated by a few institutional investors, as witnessed in the recent crisis. We argue that such endogenous risks may be modeled mathematically and that taking them into account challenges the traditional perspectives on quantitative risk management and estimation of risk parameters such as volatility and correlation.

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Modelling Extreme Rainfall in Space and Time

Anthony Davison
(joint work with Raphaël Huser)

This talk will describe the space-time modelling of extreme hourly rainfall based on max-stable processes fitted to data at a number of spatial locations. The ingredients are large amounts of data, random set modelling and max-stable processes fitted using composite threshold likelihoods. Though essentially a conceptual model, the approach seems to work surprisingly well.

Limit Theory for High Frequency Sampled MCARMA Models

Vicky Fasen

This talk is based on the papers Fasen [4, 5]. Multivariate continuous-time ARMA (MCARMA) processes $\mathbf{V} = (\mathbf{V}(t))_{t\geq 0}$ are the continuous-time versions of the well known multivariate ARMA processes in discrete time having short memory. They are important for stochastic modelling in many areas of application as, e.g., signal processing and control, econometrics, high-frequency financial econometrics, and financial mathematics. Starting at least with Doob in 1944, Gaussian CARMA processes under the name Gaussian processes with rational spectral density appeared, where the driving force is a Brownian motion. To obtain more realistic marginal distributions and dynamics Brockwell (cf. [1, 2]) analyzed Lévy driven CARMA models, which were extended by Marquardt and Stelzer [6] to the multivariate setting; see [3] for an overview and a comprehensive list of references.

Lévy processes are defined to have independent and stationary increments, and are characterized by their Lévy-Khintchine representation. An $\mathbb{R}^m$-valued Lévy process $(\mathbf{L}(t))_{t\geq 0}$ has the Lévy-Khintchine representation $\mathbb{E}(\mathrm{e}^{i\Theta'\mathbf{L}(t)}) = \exp(-t\Psi(\Theta))$ for $\Theta \in \mathbb{R}^m$, where Θ' is the transpose of Θ and

$$\Psi(\Theta) = -i\gamma_{\mathbf{L}}'\Theta + \frac{1}{2}\Theta'\mathbf{\Sigma}_{\mathbf{L}}\,\Theta + \int_{\mathbb{R}^m}\left(1 - \mathrm{e}^{i\mathbf{x}'\Theta} + i\mathbf{x}'\Theta 1_{\{\|\mathbf{x}\|^2\leq 1\}}\right)\nu_{\mathbf{L}}(d\mathbf{x})$$

with $\gamma_{\mathbf{L}} \in \mathbb{R}^m$, $\mathbf{\Sigma}_{\mathbf{L}}$ a positive semi-definite matrix in $\mathbb{R}^{m\times m}$ and $\nu_{\mathbf{L}}$ a measure on $(\mathbb{R}^m, \mathcal{B}(\mathbb{R}^m))$, called the *Lévy measure*, which satisfies $\int_{\mathbb{R}^m}\min\{\|\mathbf{x}\|^2, 1\}\,\nu_{\mathbf{L}}(d\mathbf{x}) < \infty$ and $\nu_{\mathbf{L}}(\{\mathbf{0}_m\}) = 0$. The triplet $(\gamma_{\mathbf{L}}, \mathbf{\Sigma}_{\mathbf{L}}, \nu_{\mathbf{L}})$ is called *characteristic triplet*, because it characterizes completely the distribution of the Lévy process. A two-sided

Lévy process $(\mathbf{L}(t))_{t\in\mathbb{R}}$ is then a composition of two independent and identically distributed Lévy processes $(\mathbf{L}^{(1)}(t))_{t\geq 0}$ and $(\mathbf{L}^{(2)}(t))_{t\geq 0}$ in

$$\mathbf{L}(t) = \begin{cases} \mathbf{L}^{(1)}(t) & \text{for } t \geq 0, \\ \mathbf{L}^{(2)}(-t-) & \text{for } t < 0. \end{cases}$$

We refer to the excellent monograph of Sato [8] for more details on Lévy processes.

Then the MCARMA(p,q) model can be interpreted as the solution to the p-th-order d-dimensional stochastic differential equation

$$\mathbf{P}(D)\mathbf{V}(t) = \mathbf{Q}(D)D\mathbf{L}(t) \quad \text{for } t \in \mathbb{R},$$

where D is the differential operator,

$$\mathbf{P}(z) := \mathbf{I}_{d\times d}z^p + \mathbf{P}_1 z^{p-1} + \ldots + \mathbf{P}_{p-1}z + \mathbf{P}_p \tag{1}$$

with $\mathbf{P}_1,\ldots,\mathbf{P}_p \in M_{d\times d}(\mathbb{R})$ is the auto-regressive polynomial and

$$\mathbf{Q}(z) := \mathbf{Q}_0 z^q + \mathbf{Q}_1 z^{q-1} + \ldots + \mathbf{Q}_{q-1}z + \mathbf{Q}_q \tag{2}$$

with $\mathbf{Q}_0,\ldots,\mathbf{Q}_q \in M_{d\times m}(\mathbb{R})$ is the moving-average polynomial. Since a Lévy process is not differentiable, this definition can not be used, however, it can be interpreted to be equivalent to the following.

Definition 1. *Let $(\mathbf{L}(t))_{t\in\mathbb{R}}$ be an $\mathbb{R}^m$-valued Lévy process and let the polynomials $\mathbf{P}(z),\mathbf{Q}(z)$ be defined as in* (1) *and* (2) *with $p,q\in\mathbb{N}_0$, $q<p$, and $\mathbf{Q}_0 \neq \mathbf{0}_{d\times m}$. Moreover, define*

$$\mathbf{\Lambda} = -\begin{pmatrix} \mathbf{0}_{d\times d} & \mathbf{I}_{d\times d} & \mathbf{0}_{d\times d} & \cdots & \mathbf{0}_{d\times d} \\ \mathbf{0}_{d\times d} & \mathbf{0}_{d\times d} & \mathbf{I}_{d\times d} & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \mathbf{0}_{d\times d} \\ \mathbf{0}_{d\times d} & \cdots & \cdots & \mathbf{0}_{d\times d} & \mathbf{I}_{d\times d} \\ -\mathbf{P}_p & -\mathbf{P}_{p-1} & \cdots & \cdots & -\mathbf{P}_1 \end{pmatrix} \in M_{pd\times pd}(\mathbb{R}),$$

$\mathbf{E} = (\mathbf{I}_{d\times d}, \mathbf{0}_{d\times d},\ldots,\mathbf{0}_{d\times d}) \in M_{d\times pd}(\mathbb{R})$ *and* $\mathbf{B} = (\mathbf{B}_1' \cdots \mathbf{B}_p')' \in M_{pd\times m}(\mathbb{R})$ *with*

$$\begin{aligned} \mathbf{B}_1 &:= \ldots := \mathbf{B}_{p-q-1} := \mathbf{0}_{d\times m} \quad \textit{and} \\ \mathbf{B}_{p-j} &:= -\sum_{i=1}^{p-j-1} \mathbf{P}_i \mathbf{B}_{p-j-i} + \mathbf{Q}_{q-j} \quad \textit{for } j = 0,\ldots,q. \end{aligned}$$

Assume $\mathcal{N}(\mathbf{P}) = \{z\in\mathbb{C} : \det(\mathbf{P}(z)) = 0\} \subseteq (-\infty,0)+i\mathbb{R}$. Furthermore, the Lévy measure $\nu_{\mathbf{L}}$ of $\mathbf{L}$ satisfies

$$\int_{\|\mathbf{x}\|>1} \log\|\mathbf{x}\|\,\nu_{\mathbf{L}}(d\mathbf{x}) < \infty.$$

Then the $\mathbb{R}^d$-valued causal MCARMA(p,q) process $(\mathbf{V}(t))_{t\in\mathbb{R}}$ *is defined by the state-space equation*

$$\mathbf{V}(t) = \mathbf{E}\mathbf{Z}(t) \quad \textit{for } t \in \mathbb{R}, \tag{3}$$

where

$$\mathbf{Z}(t) = \int_{-\infty}^{t} e^{-\mathbf{\Lambda}(t-s)}\mathbf{B}\, d\mathbf{L}_1(s) \quad \text{for } t \in \mathbb{R} \tag{4}$$

is the unique solution to the pd-dimensional stochastic differential equation $d\mathbf{Z}(t) = -\mathbf{\Lambda}\mathbf{Z}(t)\, dt + \mathbf{B}\, d\mathbf{L}(t)$. *The function* $\mathbf{f}(t) = \mathbf{E}e^{-\mathbf{\Lambda}t}\mathbf{B}1_{(0,\infty)}(t)$ *for* $t \in \mathbb{R}$ *is called kernel function.*

This definition of a MCARMA(p, q) process is well-defined; see Marquardt and Stelzer [6]. In particular, the MCARMA$(1, 0)$ process and $\mathbf{Z}$ in (4) are multivariate Ornstein-Uhlenbeck processes. Moreover, Lemma 3.8 of Marquardt and Stelzer [6] says that the set $\mathcal{N}(\mathbf{P})$ is equal to the set of eigenvalues of $-\mathbf{\Lambda}$, which means that for a MCARMA(p, q) process the eigenvalues of $\mathbf{\Lambda}$ have strictly positive real parts. The class of MCARMA processes is huge.

Before we can present the main results, we recall the definition of multivariate regular variation.

Definition 2. *A random vector* $\mathbf{U} \in \mathbb{R}^m$ *is multivariate regularly varying with index* $-\alpha < 0$ *if and only if there exists a non-zero Radon measure* μ *on* $(\overline{\mathbb{R}}^m \setminus \{\mathbf{0}_m\}, \mathcal{B}(\overline{\mathbb{R}}^m \setminus \{\mathbf{0}_m\}))$ *with* $\mu(\overline{\mathbb{R}}^m \setminus \mathbb{R}^m) = 0$ *and a sequence* $(a_n)_{n\in\mathbb{N}}$ *of positive numbers increasing to* ∞ *such that*

$$n\mathbb{P}(a_n^{-1}\mathbf{U} \in \cdot) \stackrel{v}{\Longrightarrow} \mu(\cdot) \quad \text{as } n \to \infty \quad \text{on } \mathcal{B}(\overline{\mathbb{R}}^m \setminus \{\mathbf{0}_m\}).$$

The limit measure μ *is homogenous of order* $-\alpha$, *i.e.,* $\mu(uB) = u^{-\alpha}\mu(B)$ *for* $u > 0$, $B \in \mathcal{B}(\overline{\mathbb{R}^m}\setminus\{\mathbf{0}_m\})$. *We write* $\mathbf{U} \in \mathcal{R}_{-\alpha}(a_n, \mu)$.

If the representation of the limit measure μ or the norming sequence $(a_n)_{n\in\mathbb{N}}$ does not matter we also write $\mathcal{R}_{-\alpha}(a_n)$ and $\mathcal{R}_{-\alpha}$, respectively. For further information regarding multivariate regular variation of random vectors we refer to Resnick [7].

Definition 3. *Let* $\mathbf{U}$ *be an* $\mathbb{R}^m$*-valued random vector,* $\alpha \in (0, 2]$, $(a_n)_{n\in\mathbb{N}}$ *be an increasing sequence of positive constants tending to* ∞, μ *be a Radon measure on* $(\overline{\mathbb{R}}^m \setminus \{\mathbf{0}_m\}, \mathcal{B}(\overline{\mathbb{R}}^m \setminus \{\mathbf{0}_m\}))$ *with* $\mu(\overline{\mathbb{R}}^m \setminus \mathbb{R}^m) = 0$ *and* $\mathbf{\Sigma} \in M_{m\times m}(\mathbb{R})$ *be a positive semi-definite matrix. We write* $U \in DA(\alpha, a_n, \mathbf{\Sigma}, \mu)$ *if either*

(a) $\alpha < 2$, $\mathbf{\Sigma} = \mathbf{0}_{m\times m}$, μ *is non-zero and* $\mathbf{U} \in \mathcal{R}_{-\alpha}(a_n, \mu)$, *or*
(b) $\alpha = 2$, $a_n = n^{1/2}$, $\mu = 0$ *and* $\mathbb{E}\|\mathbf{U}\|^2 < \infty$ *with* $\mathbb{E}(\mathbf{U}\mathbf{U}') = \mathbf{\Sigma}$.

The main results are the following.

Theorem 1. *Let* $(\mathbf{V}(t))_{t\in\mathbb{R}}$ *be an* $\mathbb{R}^d$*-valued causal MCARMA*(p, q) *process as given in Definition 1 driven by the* $\mathbb{R}^m$*-valued Lévy process* $(\mathbf{L}(t))_{t\in\mathbb{R}}$ *with* $\mathbf{L}(1) \in DA(\alpha, a_n, \mu, \mathbf{\Sigma})$ *and* $\mathbb{E}(\mathbf{L}(1)) = \mathbf{0}_m$ *if* $\alpha > 1$. *If* $\alpha = 1$ *we assume additionally that* $\mathbf{L}(1)$ *is symmetric.*
(a) Let $(\mathbf{S}(t))_{t\geq 0}$ *be an* $\mathbb{R}^m$*-valued* α*-stable Lévy process with characteristic triplet* $(\int_{\|\mathbf{x}\|\leq 1} \mathbf{x}\mu(d\mathbf{x}), \mathbf{\Sigma}, \mu)$ *if* $\alpha \in (0, 1]$ *and* $(-\int_{\|\mathbf{x}\|>1} \mathbf{x}\mu(d\mathbf{x}), \mathbf{\Sigma}, \mu)$ *if* $\alpha > 1$. *Suppose the sequence of positive constants* $(h_n)_{n\in\mathbb{N}}$ *satisfies* $h_n \downarrow 0$ *as* $n \to \infty$ *and*

$\lim_{n\to\infty} nh_n = \infty$. *Then as* $n \to \infty$,

$$h_n a_{nh_n}^{-1} \sum_{k=1}^{n} \mathbf{V}(kh_n) \Longrightarrow \left(\int_0^\infty \mathbf{f}(s)\, ds\right) \mathbf{S}(1).$$

(b) Let $h > 0$ *and let* $(\mathbf{S}_{\mathbf{f},h}(t))_{t\geq 0}$ *be an* $\mathbb{R}^d$*-valued* α*-stable Lévy process with characteristic triplet* $(\int_{\|\mathbf{x}\|\leq 1} \mathbf{x}\mu_{\mathbf{f},h}(d\mathbf{x}), \mathbf{\Sigma}_{\mathbf{f},h}, \mu_{\mathbf{f},h})$ *if* $\alpha \in (0,1]$ *and* $(-\int_{\|\mathbf{x}\|>1} \mathbf{x}\mu_{\mathbf{f},h}(d\mathbf{x}), \mathbf{\Sigma}_{\mathbf{f},h}, \mu_{\mathbf{f},h})$ *if* $\alpha > 1$, *where*

$$(5)\quad \mu_{\mathbf{f},h}(B) = \int_0^h \int_{\mathbb{R}^m} 1_B\left(\sum_{k=0}^{\infty} \mathbf{f}(kh+s)\mathbf{x}\right)\mu(d\mathbf{x})ds \qquad \text{for } B \in \mathcal{B}(\mathbb{R}^d\backslash\{\mathbf{0}_d\}),$$

$$(6)\quad \mathbf{\Sigma}_{\mathbf{f},h} = \int_0^h \left(\sum_{k=0}^{\infty} \mathbf{f}(kh+s)\right)\mathbf{\Sigma}\left(\sum_{k=0}^{\infty} \mathbf{f}(kh+s)\right)' ds.$$

Suppose $\mathbb{E}\|\mathbf{L}(1)\|^r < \infty$ *for some* $r > 2$ *if* $\alpha = 2$. *Then as* $n \to \infty$,

$$a_n^{-1} \sum_{k=1}^{n} \mathbf{V}(kh) \Longrightarrow \mathbf{S}_{\mathbf{f},h}(1).$$

Theorem 2. *Let* $(\mathbf{V}(t))_{t\geq 0}$ *be an* $\mathbb{R}^d$*-valued MCARMA*(p,q) *process as given in Definition 1 driven by the* $\mathbb{R}^m$*-valued Lévy process* $(\mathbf{L}(t))_{t\in\mathbb{R}}$ *with* $\mathbf{L}(1) \in DA(\alpha, a_n, \mu, \mathbf{\Sigma})$.
(a) Let $(\mathbf{S}(t))_{t\geq 0}$ *be an* $\mathbb{R}^m$*-valued* α*-stable Lévy process with characteristic triplet* $(\mathbf{0}_m, \mathbf{\Sigma}, \mu)$. *Suppose the sequence of positive constants* $(h_n)_{n\in\mathbb{N}}$ *satisfies* $h_n \downarrow 0$ *as* $n \to \infty$ *and* $\lim_{n\to\infty} nh_n = \infty$. *Then as* $n \to \infty$,

$$h_n a_{nh_n}^{-2} \sum_{k=1}^{n} \mathbf{V}(kh_n)\mathbf{V}(kh_n)' \Longrightarrow \int_0^\infty \mathbf{f}(s)[\mathbf{S},\mathbf{S}]_1\mathbf{f}(s)'\, ds,$$

which is equal to $\mathbb{E}(\mathbf{V}(0)\mathbf{V}(0)')$ *if* $\alpha = 2$. *In particular, this means for a one-dimensional CARMA process* $(V(t))_{t\geq 0}$ *with* $f = \mathbf{f}$, $L = \mathbf{L}$ *and* $S = \mathbf{S}$ *that as* $n \to \infty$,

$$h_n a_{nh_n}^{-2} \sum_{k=1}^{n} V(kh_n)^2 \Longrightarrow \left(\int_0^\infty f(s)^2\, ds\right)[S,S]_1.$$

(b) Let $h > 0$ *and let* $(\mathbf{S}_{\mathbf{f},h}(t))_{t\geq 0}$ *be an* $\mathbb{R}^d$*-valued* α*-stable Lévy process with characteristic triplet* $(\mathbf{0}_d, \mathbf{\Sigma}_{\mathbf{f},h}, \mu_{\mathbf{f},h})$ *where* $\mu_{\mathbf{f},h}$ *and* $\mathbf{\Sigma}_{\mathbf{f},h}$ *are given as in* (5) *and* (6), *respectively. Then as* $n \to \infty$,

$$a_n^{-2} \sum_{k=1}^{n} \mathbf{V}(kk)\mathbf{V}(kh)' \Longrightarrow [\mathbf{S}_{\mathbf{f},h}, \mathbf{S}_{\mathbf{f},h}]_1,$$

which is equal to $\mathbf{\Sigma}_{\mathbf{f},h}$ *if* $\alpha = 2$.

Thus, if $\mathbb{E}\|\mathbf{L}(1)\|^2 < \infty$, the sample autocovariance is a consistent estimator.

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The Term Structure of Interbank Risk

Damir Filipović

(joint work with Anders Trolle)

We use the term structure of spreads between rates on interest rate swaps indexed to LIBOR and overnight indexed swaps to infer a term structure of interbank risk. Using a dynamic affine term structure model, we decompose the term structure of interbank risk into default and non-default components. The default component is modeled by assuming the credit quality of a individual borrowing bank in the LIBOR panel may deteriorate from the one of the continually refreshed LIBOR panel over the duration of the loan. This is similar to the approach used in [1]. We then posit a multiplicative residual factor to capture the component of the interest rate swap spread that is not due to default risk. An multivariate affine factor specification yields closed form expressions for LIBOR, interest rate swap, overnight indexed swap, and credit default rates. We estimate the model by quasi maximum likelihood in conjunction with unscented Kalman filtering.

We find that, on average, from August 2007 to January 2011, the fraction of total interbank risk due to default risk increases with maturity. At the short end of the term structure, the non-default component is important in the first half of the sample period and is correlated with various measures of funding liquidity and market liquidity. Further out the term structure, the default component is the dominant driver of interbank risk throughout the sample period. These results hold true in both the USD and EUR markets and are robust to different model parameterizations and measures of interbank default risk. The analysis has implications for monetary and regulatory policy as well as for pricing, hedging, and risk-management in the interest rate swap market.

The talk is based on the joint paper [2] with Anders Trolle.

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Dual Formulation of the Robust Efficient Hedging Problem

Daniel Hernández-Hernández
(joint work with Erick Treviño-Aguilar)

1. The hedging problem

Let $W := \{W_t\}_{0\leq t\leq T}$ be a two dimensional Brownian motion defined in the canonical Wiener space $\nu = (\Omega, \mathcal{F}, \mathbb{F} = \{\mathcal{F}_t\}_{t\in[0,T]}, R)$. W has components W^1 and W^2. The filtration $\mathbb{F}$ is the augmented filtration generated by W. The discounted[1] price process of a risky asset X^1 in a financial market has dynamics

$$dX^1 = X^1 b(X^2)dt + X^1\sigma(X^2)dW^1 \tag{1}$$

$$dX^2 = g(X^2)dt + \alpha dW^1 + \beta dW^2. \tag{2}$$

The positive constants α and β satisfy $\alpha+\beta = 1$. The coefficients of this system satisfy the

Assumption A

The functions b, σ and g together with the corresponding derivatives are bounded. Moreover $\sigma_0 := \inf_{x\in\mathbb{R}} \sigma(x) > 0$.

The process X^2 represents an exogenous source of risk driving the price process X^1. The market represented by X^1 is free of arbitrage opportunities in the sense that the family of equivalent martingale measures:

$$\mathcal{M} := \{P \sim R \mid X^1 \text{ is a local martingale under } P\}$$

is non empty. In the next definition a class of random variables which are going to represent discounted payoffs of European options is introduced.

Definition 1. *A non negative random variable $\widehat{H}$ represents the discounted payoff of a* European option *if it is $\mathcal{F}_T$-measurable and* the cost of superhedging $\pi_{\sup}(\widehat{H})$ *is finite:*

$$\pi_{\sup}(\widehat{H}) := \sup_{P\in\mathcal{M}} E_P[\widehat{H}] < \infty.$$

We fix a European option $\widehat{H} = H(X^1)$ satisfying Definition 1. The payoff function $H : [0,\infty) \to \mathbb{R}^+$ is continuous. In the next definition we introduce admissible hedging strategies.

[1] For simplicity we are taking an interest rate $r = 0$.

Definition 2. An admissible hedging strategy *is a pair* (c, ξ) *where* $c \in \mathbb{R}_+$ *is a non negative constant and* $\xi := \{\xi_t\}_{0 \le t \le T}$ *is a* $\mathbb{F}$*-predictable process such that the stochastic integral*

$$V_t^\xi := \int_0^t \xi_s dX_s^1$$

is well defined for all $t \in [0, T]$ *and the corresponding value process*

$$V^{c,\xi} := c + V_t^\xi$$

is R*-a.s. non negative for all* $t \in [0, T]$.

In this case, we say that the stochastic process ξ *is* c-admisible *and the family of such processes is denoted by* Ad_c.

We now specify the robust loss functional $L(\cdot)$. The loss function l satisfies the **Assumption B**
The function $l : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable, strictly convex and strictly increasing. Moreover, $l(0) = l'(0) = 0$ and

$$\lim_{x \to \infty} l'(x) = \infty.$$

We now specify the penalty function γ. We fix a continuous coercive[2] function $h : \mathbb{R}^2 \to \mathbb{R}_+$ with $h(0) = 0$. Given $Q \ll R$, there exists a process η such that

$$Z^Q := \frac{dQ}{dR} = \mathcal{E}_T(\eta \cdot W),\ Q - a.s.$$

due to Lemma 3. We penalize Q by

$$\gamma(Q) := E_Q\left[\int_0^T h(\eta_s) ds\right]. \tag{3}$$

Remark 1. *We define a convex risk measure* ρ *in* $L^\infty(R)$ *by*

$$\rho(X) = \sup_{Q \ll R} \left\{E_Q[-X] - \gamma(Q)\right\}.$$

The function γ *is the minimal penalty function of* ρ. *Moreover,* ρ *is continuous from below*

Hereon $L(\cdot)$ will denote the robust loss functional with penalization (3). Given a nonegative bounded random variable X, it is defined as

$$L(X) = \sup_{Q \ll R} \left\{E_Q[l(X) - \gamma(Q)\right\}$$

Definition 3. *Let* c *be a constant satisfying* $0 \le c \le \pi_{\sup}(H)$. The value of the partial hedging problem at cost c for the European option H with respect to the loss functional $L(\cdot)$ *is defined by*

$$EH(c) := \inf_{\xi \in Ad_c} L((H - V_T^{c,\xi})^+).$$

[2] $h(\eta) \ge k \|\eta\|^2 + c$.

A c-admissible strategy $\xi^* \in Ad_c$ *is optimal if it attains the value* $EH(c)$ *in the following sense*

$$EH(c) = L((H - V_T^{c,\xi^*})^+).$$

2. Density of probability measures

Definition 4. *A probability measure* P *is a localized martingale measure if* X^1 *is a* P*-local martingale and* P *is locally equivalent to* R*. We denote this class by* $\mathcal{M}^{loc}$*. Note that a martingale measure* $P \in \mathcal{M}$ *is a localized martingale measure.*

The next lemmas hold true in the canonical Wiener space.

Lemma 2. *Let* $P \in \mathcal{M}^{loc}$ *be a localized martingale measure. Then, there exists a* $\mathbb{F}$*-progressively measurable process* η^3 *such that*

$$\int_0^{\cdot} (\eta_t^3)^2 dt < \infty,\ R - a.s. \, locally$$

and the local martingale

$$Z_t^P := \mathcal{E}_t(-\theta(X^2) \cdot W^1 - \eta^3 \cdot W^2),\ \textit{for all}\ t \in [0, T], \tag{4}$$

is locally the density process of P *with respect to* R*. Here* $\theta(x) := \frac{b(x)}{\sigma(x)}$ *is the Sharp ratio.*

The next lemma explicitly describes the dynamics of absolutely continuous probability measures.

Lemma 3. *Let* $Q \ll R$ *be an absolutely continuous probability measure. Then, there exist* $\mathbb{F}$*-progressively measurable processes* η^i *for* $i = 1, 2$ *such that*

$$\int_0^T (\eta_t^i)^2 dt < \infty, Q - a.s.$$

and the local martingale

$$Z_t^Q := \mathcal{E}_t(\eta^1 \cdot W^1 + \eta^2 \cdot W^2), \tag{5}$$

is a continuous version of the density process of Q *with respect to* R*.*

3. The stochastic control problem associated to the dual function DH

Instead of constructing the optimal strategy ξ^* from the mixed problem of Definition 3, we are going to start from the dual representation of the next proposition.

Proposition 4. *The value function* EH *can be represented as*

$$EH(c) = \sup_{\lambda > 0} \{DH(\lambda) - \lambda c\}, \tag{6}$$

where $DH : (0, \infty) \to \mathbb{R}$ *is the concave function defined by*

$$DH(\lambda) := \sup_{h \in \mathcal{D}} \sup_{Q \ll R} \left\{ E_Q \left[u\left(\widehat{H}, \lambda \frac{h}{Z^Q}\right)\right] - \gamma(Q) \right\}.$$

The convex family $\mathcal{D}$ is a closed subset of $L^0(R)$ and "extends" the family of martingale measures. The conjugate function u of l is defined by

$$u(h,\lambda) := \inf_{0\le v\le h} \left\{ l((h-v)^+) + \lambda v \right\}.$$

Let us introduce some notation. Let $\overline{x} = (x^1, x^2, x^3)$ be a vector in $\mathbb{R}^3$, which typically will represent the starting point of a controlled process $\overline{X}$. The vector $\overline{\eta} = (\eta^1, \eta^2, \eta^3)$ will denote the three components of a control process valuated in $\mathbb{R}^3$. We are going to set

$$\eta := (\eta^1, \eta^2),$$

so that, by a slight abuse of notation, $\overline{\eta} = (\eta, \eta^3)$. Given a probability measure $Q \ll R$ we denote by Z^Q the density in $\mathcal{F}_T$.

The goal in this section is to study the value function

$$V(t,\overline{x}) := \operatorname{ess\,sup}_{\overline{\eta}\in\mathcal{A}_t} J(t,\overline{x};\overline{\eta}), \tag{7}$$

with cost functional

$$J(t,\overline{x};\overline{\eta}) := E_Q\left[u\left(H(X_T^1), X_T^3\right) \mid \mathcal{F}_t \right] - \gamma_t(Q). \tag{8}$$

Here the t-penalty function $\gamma_t(Q)$ is given by

$$\gamma_t(Q) = \begin{cases} E_Q[\int_t^T h(\eta_s)ds \mid \mathcal{F}_t], & \text{if } Q \ll R \\ +\infty, & \text{in other case} \end{cases}$$

Algorithmic Trading with Markov Chains

Henrik Hult

An order book consists of a list of all buy and sell offers, represented by price and quantity, available to a market agent. The order book changes rapidly, within fractions of a second, due to new orders being entered into the book. The volume at a certain price level may increase due to limit orders, i.e. orders to buy or sell placed at the end of the queue, or decrease because of market orders or cancellations.

In this talk a high-dimensional Markov chain is used to represent the state and evolution of the entire order book. The design and evaluation of optimal algorithmic strategies for buying and selling is studied within the theory of Markov decision processes. General conditions are provided that guarantee the existence of optimal strategies. Moreover, a value-iteration algorithm is presented that enables finding optimal strategies numerically.

As an illustration a simple version of the Markov chain model is calibrated to high-frequency observations of the order book in a foreign exchange market. In this model, using an optimally designed strategy for buying one unit provides a significant improvement, in terms of the expected buy price, over a naive buy-one-unit strategy.

Extremes of Independent Gaussian Processes

Zakhar Kabluchko

Let $X_1, X_2, \ldots$ be independent stationary Gaussian processes on $\mathbb{R}^d$. Consider the pointwise maximum $M_n(t) = \max\{X_1(t), \ldots, X_n(t)\}$. We discuss necessary and sufficient conditions under which the maximum M_n converges, as $n \to \infty$ and after suitable normalization procedures, to a non-trivial limiting max-stable process. The class of limiting processes is described in terms of their Poisson process representations.

Modelling Energy Markets: Spot Prices, Futures and Risk Premiums

Claudia Klüppelberg

(joint work with Fred E. Benth and Gernot Müller and Linda Vos)

We present a new model for the electricity spot price dynamics, which is able to capture seasonality, low-frequency dynamics and the extreme spikes in the market; for details see [1]. The work continues [2, 5, 6]. Background on the electricity market can be found in [3]. Instead of the usual purely deterministic trend, we introduce a Lévy process for the low-frequency dynamics, and model the large fluctuations by a non-Gaussian α-stable CARMA process, which is able to capture the extreme behavior of electricity spot prices. For background on CARMA processes we refer to [5], and for α-stable processes in general to [8].

This means we assume the spot price dynamics

$$S(t) = \Lambda(t) + Z(t) + Y(t), \quad t \geq 0,$$

where Λ is a deterministic trend/seasonality function and Z is a Lévy process with zero mean. The process Z models the low-frequency non-stationary dynamics of the spot, and can together with Λ be interpreted as the long-term factor for the spot price evolution. The process Y accounts for the stationary short-term variations. We will assume that Y and Z are independent processes.

We propose a statistical method to calibrate the suggested spot and futures model to real data. The calibration is done using spot and futures data together, where we applied futures prices in the far end of the market to filter out the non-stationary factor in the spot. Besides standard parameter estimation, an estimation procedure is suggested, where we fit the non-stationary trend using futures data with long time until delivery. Our data suggest that futures curves and spot prices are driven by a common stochastic trend, and it turns out that this is very well described by a normal inverse Gaussian Lévy process. After having removed deterministic trend and seasonality as well as the slow-frequency dynamics Lévy process, we fit an α-stable CARMA(2,1) model, which perfectly describes the high peaks in the spot price (cf. [4, 6]). With a robust L^1-filter we find the states of the CARMA process. Our model leads to realistic predictions of the futures prices. The estimation also involves the empirical and theoretical risk premiums which – as a by-product – are also estimated.

We apply the Esscher transform to produce a parametric class of market prices of risk for the non-stationary term. The α-stable Lévy process driving the CARMA-factor is transformed into a tempered stable process in the risk neutral setting; cf. [7]. The spot price dynamics and the chosen class of risk neutral probabilities allow for analytic pricing of the futures. A crucial insight in the futures price dynamics is that the stationary CARMA effect from the spot price is vanishing for contracts far from delivery, where prices essentially behave like the non-stationary long-term factor.

We apply this procedure to data from the German electricity exchange EEX, where we split the empirical analysis into base load prices (24 hours 7 days a week) and peak load prices (only weekdays from 8:00 to 20:00). Moreover, in order to gain full insight into the risk premium structure in this market, we study both peak load and base load futures contracts with delivery over one month. The base load futures are settled against the hourly spot price over the whole delivery period, while the peak load contracts only deliver against the spot price in the peak hours on working days. Our model and estimation technique seem to work well in both situations.

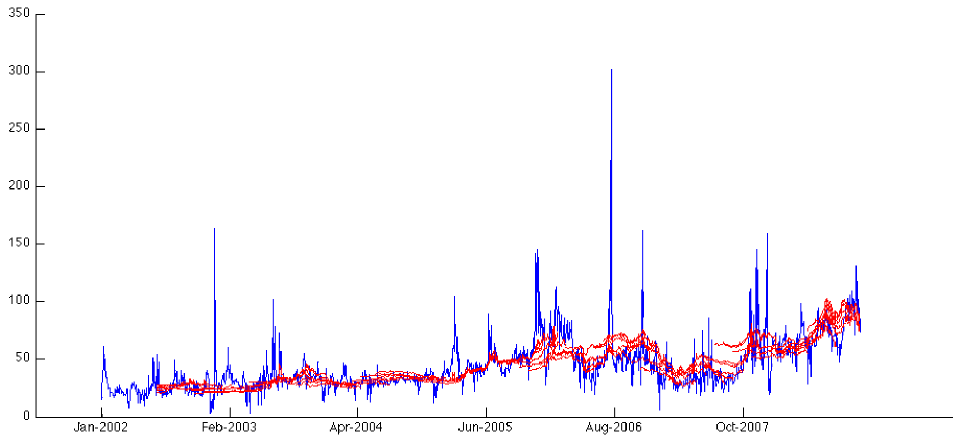

Figure 1. German EEX daily spot (base load) data together with futures prices.

The calibration is done using spot and futures data together, where we applied futures prices in the far end of the market to filter out the non-stationary factor in the spot. We choose a threshold for what is sufficiently “far out” on the futures curve by minimizing the error in matching the theoretical risk premium to the empirical. In this minimization over thresholds, we need to re-estimate the whole model until the minimum is attained.

We find that the base load futures contracts have a risk premium which is close to linearly decaying with time to delivery. The risk premium is essentially governed by the long term factor. There is evidence of a positive premium in the short end of the futures curve. For peak load contracts, which are much more sensitive to spikes, the positive premium in the short end is far most distinct, but also here the premium decays close to linearly in the long end of the market. These observations are in line other theoretical and empirical studies of risk premia in electricity markets, which argue that the risk premia in power markets are driven by hedging needs.

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Bootstrap-Aided Hypothesis Tests for Time Series

Anne Leucht

We establish two consistent L_2-type tests for time series data. Besides a test for symmetry, a test for the parametric class of the marginal distribution based on the empirical characteristic function is considered. In particular, the second one is of special interest in financial mathematics since distributions of financial time series often have a complicated density or cumulative distribution function while their characteristic function is simple. Examples include NIG and VG distributions. Our test statistics can be approximated by degenerate V-statistics. Their asymptotics are then derived invoking recent results of [1].

The limit distributions of the test statistics have a complicated form and depend on unknown parameters and the underlying dependence structure in a complicated way. Therefore, (asymptotic) critical values of the tests cannot be derived directly. We propose certain bootstrap methods to overcome these difficulties.

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A Model for the Conditional Density Process

Filip Lindskog
(joint work with Henrik Hult and Johan Nykvist)

Given a set of option contracts with the same maturity, we consider the modeling of the simultaneous evolution of the prices over time such that the corresponding forward price processes are martingales. The model considered is a model for the evolution over time of the density of the future value on which the options are written, conditional on the flow of information on the market. At any given point, the conditional density process evaluated at that point is constructed to be a martingale and the price processes are defined as expected payoffs with respect to the conditional density process.

The conditional density process is set up to have the following properties:

- Recalibration of the model is unlikely - the future realized prices are possible for the model to produce.
- The model allows for exact and straightforward calibration to the initial option prices.
- The model gives the price processes characteristic properties of option price data.
- The filtration representing the flow of information on the market equals the filtration generated by the price processes.

Beyond Simplified Pair-Copula Constructions

Johanna Nešlehová
(joint work with Elif F. Acar and Christian Genest)

Pair-copula constructions (PCCs) offer great flexibility in modeling multivariate dependence. For inference purposes, however, conditional pair copulas are often assumed to depend on the conditioning variables only indirectly. The authors show here that this assumption can be misleading. To assess its validity in trivariate PCCs, they propose a visual tool based on a local likelihood estimator of the conditional copula parameter which does not rely on the simplifying assumption. They establish the consistency of the estimator and assess its performance in finite samples via Monte Carlo simulations. They also provide a real data application.

Inference about Tail Measures: Where Does the Tail Begin?

Tilo Nguyen
(joint work with Gennady Samorodnitsky)

The quality of estimation of tail parameters, such as tail index in the univariate case, or the spectral measure in the multivariate case, depends crucially on the part of the sample included in the estimation. A simple approach involving sequential statistical testing is proposed in order to choose this part of the sample. We establish consistency of the Hill estimator when used in conjunction with the proposed method, as well describe its asymptotic fluctuations.

Functions Associated with Multivariate Stable Laws

John Nolan

The lack of explicit formulas for distribution functions and densities for stable distributions make them difficult to use in practice. We describe a family of functions that are used to express these and other quantities for univariate and multivariate stable laws. The special case of elliptically contoured stable laws is computationally accessible for high dimensions. For the general multivariate stable case, we describe Zolotarev integrals and programs to compute these functions for moderate dimensions.

Modelling Dependence in High Dimensions with Factor Copulas

Andrew J. Patton

(joint work with Dong Hwan Oh)

This paper presents new models for the dependence structure, or copula, of economic variables, and asymptotic results for new simulation-based estimators of these models. The proposed models are based on a factor structure for the copula and are particularly attractive for high dimensional applications, involving fifty or more variables. Estimation of this class of models is complicated by the lack of a closed-form likelihood, but estimation via a simulation-based method using rank statistics is simple, and we provide asymptotic results that show the consistency and asymptotic normality of such estimators. We analyze the finite-sample behavior of these estimators in an extensive simulation study. We apply the model to a group of 100 daily stock returns and find evidence of statistically significant tail dependence, and that the dependence between these assets is stronger in crashes than booms.

Edgeworth Expansion for Functionals of Continuous Diffusion Processes

Mark Podolskij

(joint work with Nakahiro Yoshida)

In this talk we present new results on the second order Edgeworth expansion for high frequency functionals of continuous diffusion processes. We derive asymptotic expansions for weighted functionals of the Brownian motion and for power variation of diffusion processes.

Let us consider an Itô semimartingale of the form

$$X_t = X_0 + \int_0^t b_s dW_s + \int_0^t a_s ds,$$

where b is a *volatility* process, a is a *drift* process and W is a Brownian motion, defined on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$. A typical (and most useful) example of a high frequency statistics is the *realised volatility* RV_n defined as

$$RV_n = \sum_{i=1}^{[1/\Delta_n]} |\Delta_i^n X|^2, \qquad \Delta_i^n X = X_{i\Delta_n} - X_{(i-1)\Delta_n}.$$

It is well known that the convergence $RV_n \xrightarrow{\mathbb{P}} \langle X \rangle = \int_0^1 b_s^2 ds$ holds as $\Delta_n \to 0$, and we obtain a stable central limit theorem

$$\Delta_n^{-1/2}\Big(RV_n - \langle X \rangle\Big) \xrightarrow{st} MN\Big(0, 2\int_0^1 b_s^4 ds\Big),$$

where $MN(0, V^2)$ denotes a mixed normal distribution with conditional variance V^2. The latter result can be transformed into a feasible central limit theorem via

$$\frac{\Delta_n^{-1/2}\Big(RV_n - \langle X \rangle\Big)}{\sqrt{\frac{2}{3}\Delta_n^{-1}\sum_{i=1}^{[1/\Delta_n]} |\Delta_i^n X|^4}} \xrightarrow{d} N(0,1).$$

The main aim of our talk is derive a second order Edgeworth expansion associated with mixed normal limits and with the associated studentized statistics. For this purpose we consider the quantity

$$Z_n = M_n + r_n N_n,$$

where $r_n \to 0$ (typically $r_n = \Delta_n^{1/2}$) and $N_n = O_{\mathbb{P}}(1)$. The basic idea is to embed the main part M_n into a martingale framework. We assume that there exist continuous martingales $(M_t^n)_{t\in[0,1]}$, $(M_t)_{t\in[0,1]}$ with $M_0^n = M_0 = 0$ such that

$$M_n = M_1^n, \qquad M = M_1,$$

$$C_t^n = \langle M^n \rangle_t, \qquad C_t = \langle M \rangle_t,$$

$$C_n = \langle M^n \rangle_1 \xrightarrow{\mathbb{P}} C = \langle M \rangle_1,$$

$$M_t = MN(0, C_t), \qquad M = MN(0, C).$$

We consider another sequence of random variables F_n with $F_n \xrightarrow{\mathbb{P}} F$. Typically F_n serves as a consistent estimator of the random variance C (i.e. $F = C$). We set

$$\widehat{C}_n = r_n^{-1}(C_n - C),$$

$$\widehat{F}_n = r_n^{-1}(F_n - F).$$

The main assumption for the validity of the Edgeworth expansion is

$$(\star) \qquad (M_t^n, N_n, \widehat{C}_n, \widehat{F}_n) \xrightarrow{st} (M_t, N, \widehat{C}, \widehat{F}).$$

In order to compute the asymptotic expansion of the density of Z_n we need to introduce two random symbols $\underline{\sigma}$ and $\overline{\sigma}$. First, we define the functions $\widetilde{C}(z)$, $\widetilde{F}(z)$ and $\widetilde{N}(z)$ via

$$\begin{aligned}\widetilde{C}(z) &= \mathbb{E}[\widehat{C}|M=z],\\ \widetilde{F}(z) &= \mathbb{E}[\widehat{F}|M=z],\\ \widetilde{N}(z) &= \mathbb{E}[N|M=z].\end{aligned}$$

The random symbol $\underline{\sigma}$ is given by

$$\underline{\sigma}(z,iu,iv)=\frac{(iu)^2}{2}\widetilde{C}(z)+iu\widetilde{N}(z)+iv\widetilde{F}(z).$$

The second random symbol $\overline{\sigma}$ is more involved. For any continuous martingale H, we denote by $\mathcal{E}(H)_t$ the exponential martingale associated with H, i.e.

$$\mathcal{E}(H)_t=\exp\Big(H_t-\frac{1}{2}\langle H\rangle_t\Big)=1+\int_0^t\mathcal{E}(H)_s dH_s.$$

Next, we define the random variable

$$\Phi_n(u,v)=\mathbb{E}\left[\exp\Big(-\frac{u^2}{2}C+ivF\Big)\Big(\mathcal{E}(iuM^n)_1-1\Big)\right],\qquad (u,v)\in\mathbb{R}^2.$$

Let $\alpha=(\alpha_1,\alpha_2)\in\mathbb{N}^2$ denote a multi-index and $|\alpha|=\alpha_1+\alpha_2$. We write

$$\partial^\alpha=i^{-|\alpha|}d^\alpha,\qquad d^\alpha=d_{x_1}^{\alpha_1}d_{x_2}^{\alpha_2},$$

where d^kf denotes the kth derivative of a function f. We assume that $\Phi^\alpha(u,v):=\lim_{n\to\infty}r_n^{-1}\partial^\alpha\Phi_n(u,v)$ (if it exists) admits the representation

$$\Phi^\alpha(u,v)=\partial^\alpha\mathbb{E}\left[\exp\Big(-\frac{u^2}{2}C+ivF\Big)\cdot\overline{\sigma}(iu,iv)\right],$$

where the random symbol $\overline{\sigma}(iu,iv)$ has the form

$$\overline{\sigma}(iu,iv)=\sum_j c_j\ (iu)^{m_j}(iv)^{n_j}\qquad\text{(finite sum)}$$

We remark that the random symbol $\sigma=\underline{\sigma}+\overline{\sigma}$ is a polynomial in (iu,iv) and it admits the representation

$$\overline{\sigma}(iu,iv)=\sum_j \widetilde{c}_j(z)\ (iu)^{m_j}(iv)^{n_j}\qquad\text{(finite sum)}$$

Now, the approximative density of (Z_n,F_n) is defined as

$$p_n(z,x)=\mathbb{E}[\phi(z;0,C)]p^F(x)+r_n\sum_j(-d_z)^{m_j}(-d_x)^{n_j}\Big(\mathbb{E}\left[\widetilde{c}_j(z)\phi(z;0,C)|F=x\right]p^F(x)\Big),$$

where p^F denotes the density of F and $\phi(\cdot;0,C)$ denotes the density of $N(0,C)$. The main result is given as follows.

Theorem 1. *For any function $h : \mathbb{R}^2 \to \mathbb{R}$ we set*

$$\Delta_n(h) = \left| \mathbb{E}[h(Z_n, F_n)] - \int h(z,x) p_n(z,x) dz dx \right|.$$

Let

$$\Lambda(K,\gamma) = \{h : \mathbb{R}^2 \to \mathbb{R} |\ h \text{ is measurable and } |h(z,x)| \leq K(|z| + |x|)^\gamma\}$$

for some $K, \gamma > 0$. Then, under assumption $(\star)$ and certain integrability conditions, we have that

$$\sup_{h \in \Lambda(K,\gamma)} \Delta_n(h) = o(r_n).$$

As an illustration we present an application of the theorem to realised volatility RV_n.

A Systematic Approach to the Expansion of Filtrations, Inspired by Applications to Insider Trading

Philip Protter
(joint work with Younes Kchia)

We study progressive filtration expansions with càdlàg processes. Using results from the weak convergence of σ-fields theory, we first establish a semimartingale convergence theorem. Then we apply it to the framework of a filtration expansion with a process setting. In so doing, we give sufficient conditions for a semimartingale of the base filtration to remain a semimartingale in the expanded filtration. Detailed studies of examples are given. We also show how this might hep to understand mathematically the effects of insider trading.

Covariation Estimation: Unexpected Effects due to Noise

Markus Reiss
(joint work with Markus Bibinger)

We consider the problem of estimating the multivariate covariation (or covolatility) matrix based on high-frequency observations under microstructure noise. Based on a fundamental Gaussian model, we discuss first efficiency of estimators for the integrated and spot volatility in the scalar case, using asymptotic Le Cam equivalence with a Gaussian shift model. We then discuss specific features in the multivariate setting, in particular allowing for asynchronous observations. Employing again asymptotic equivalence concepts, we show that asynchronicity is (asymptotically) not an issue under non-vanishing microstructure noise. Moreover, we discuss the surprising result that multivariate observations can even improve on estimators for the univariate volatility. This is in contrast to to the classical i.i.d. situation and is fundamentally due to non-commutative Fisher information matrices along different frequency scales. The estimation procedure is explicitly constructed using spectral covolatility estimators with local weights, applying a local likelihood ansatz.

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Firms, Banks and Households

Leonard Chris G. Rogers

This paper sets up and analyses a continuous-time equilibrium model with firms, households and a bank. The model allows us to study the inter-relation of production, consumption, levels of working, interest rates, debt, inflation and wage levels.

Hedging Options On Exploding Exchange Rates

Johannes Ruf

(joint work with Peter Carr and Travis Fisher)

Recently strict local martingales have been used to model exchange rates. In such models, put-call parity does not hold if one assumes minimal superreplicating costs as contingent claim prices. I will illustrate how put-call parity can be restored by changing the definition of a contingent claim price. More precisely, I will discuss a change of numeraire technique when the underlying is only a local martingale. Then, the new measure is not necessarily equivalent to the old measure. If one now defines the price of a contingent claim as the minimal superreplicating costs under both measures, then put-call parity holds. I will discuss properties of this new pricing operator.

Risk bounds, worst case dependence, and optimal claims and contracts

Ludger Rüschendorf

Main subject of the talk is the description of the influence of stochastic dependence on risk functionals. The first part is concerned with some new developments on bounds for the distribution function of the joint portfolio or equivalently for the value at risk (joint work with Giovanni Puccetti). The second part is dealing with the question of generalizing comonotonicity (i.e. worst case dependence structure) to the higher dimensional case. In the third part dependence bounds are applied to the construction of optimal claims and (re-)insurance contracts.

Nonparametric Estimation of Pair-Copula Constructions with the Empirical Pair-Copula

Johan Segers
(joint work with I. Hobæk Haff)

A pair-copula construction is a decomposition of a multivariate copula into a structured system, called regular vine, of bivariate copulae or pair-copulae. The standard practice is to model these pair-copulae parametrically, which comes at the cost of a large model risk, with errors propagating throughout the vine structure. The empirical pair-copula proposed in the paper provides a nonparametric alternative still achieving the parametric convergence rate. It can be used as a basis for inference on dependence measures, for selecting and pruning the vine structure, and for hypothesis tests concerning the form of the pair-copulae.

Martingale Estimating Functions for Diffusions with Jumps

Michael Sørensen

A diffusion with jumps is a stochastic process given by a stochastic differential equation driven not only by a Wiener process, but also by another stochastic mechanism that causes the process to make jumps. This other mechanism can be a Lévy process, or more generally, a random measure on a suitable space. When the data are continuous time observations, likelihood inference for diffusions with jumps has long been well understood; see e.g. Sørensen (1991). However, for discrete time observations the likelihood function is not explicitly known and usually extremely difficult to calculate numerically. Therefore alternatives like estimating functions are even more useful for jump diffusions than for classical diffusions. We present a highly flexible class of diffusions with jumps for which explicit optimal martingale estimating functions of the type introduced by Kessler and Sørensen (1999) are available. These are based on eigenfunctions of the generator of the diffusion. The class of Pearson diffusions, investigated in Forman and Sørensen (2008), has the property that the generator maps polynomials into polynomials. Therfore it is easy to find polynomial eigenfunctions. Here we generalize these ideas and consider a class of diffusions with jumps for which the generator has the same property using ideas from Zhou (2003). The generator of a diffusion with jumps is a differential-integral operator. However, it turns out that a simple condition on the compensator of the jump measure is enough to ensure that the generator maps polynomials into polynomials, and hence that explicit optimal martingale estimating functions can be found. We illustrate the general theory by several concrete examples.

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On the Theory of Continuous-Time Recursive Utility

Mogens Steffensen

We establish a connection between continuous-time recursive utility and the notion of consistency studied, in particular, in connection with the non-linear objective mean-variance. We propose a time-global optimization problem and show that the optimal time-consistent solution to this time-global problem is closely related to the standard recursive utility solution which is defined via differential equations. The two approaches are, to some extent, equivalent. Standard continuous-time recursive utility was developed for Brownain markets, and a generalization is complicated. Our approach contributes with insight in the notion of recursive utility and lightens up a relatively simple path that potentially gives access to results for general Markovian markets.

Decomposability for Stable Processes

Stilian A. Stoev

We characterize all possible independent symmetric α-stable (SαS) components of a non–Gaussian SαS process, $0 < \alpha < 2$. In particular, we characterize the independent stationary SαS components of a stationary SαS process. One simple consequence of our characterization is that all stationary components of the SαS moving average processes are trivial. We obtain parallel characterization results for the components of max-stable processes by using the notion of association.

Inverting Realized Laplace Transforms

George Tauchen
(joint work with Viktor Todorov)

We develop a nonparametric estimator of the stochastic volatility density of a discretely-observed Ito semimartingale in the setting of an increasing time span and finer mesh of the observation grid. There are two steps. The first is aggregating the high-frequency increments into the realized Laplace transform, which is a robust nonparametric estimate of the underlying volatility Laplace transform. The second step is using a regularized kernel to invert the realized Laplace transform. The two steps are relatively quick and easy to compute, so the nonparametric estimator is practicable. We derive bounds for the mean squared error of the estimator. The regularity conditions are sufficiently general to cover empirically important cases such as level jumps and possible dependencies between volatility moves and either diffusive or jump moves in the semimartingale. Monte Carlo work indicates that the nonparametric estimator is reliable and reasonably accurate in realistic estimation contexts. An empirical application to 5-minute data for three large-cap stocks, 1997-2010, reveals the importance of big short-term volatility spikes in generating high levels of stock price variability over and above that induced by price jumps. The application also shows how to trace out the dynamic response of the volatility density to both positive and negative jumps in the stock price.

Finite-Dimensional Realizations for the CNKK-Volatility Surface Model

Josef Teichmann

We show that parametrizations of volatility surfaces (and even more involved multivariate objects) by time-dependent Lévy processes (as proposed by Carmona-Nadtochiy-Kallsen-Kröhner) or even semimartingales lead to quite tractable arbitrage-free term structure problems, i.e. the drift condition can be easily understood and implemented and is comparable to the HJM-drift condition. For the purposes of risk management arbitrage free term structure models, which due to their simple structure allow for statistical estimation, are of particular importance, since usually term structure problems involve highly dependent assets, where misspecifications of drifts quickly lead to misspecifications of risk. In this talk we follow another direction of research: we introduce the corresponding term structure SPDE and an appropriate state Hilbert space. In this context we can then ask whether the corresponding term structure SPDE allows for (regular) finite dimensional realization, which necessarily leads to models driven by an affine factor process. This is another confirmation that affine processes play a particular role in mathematical finance. The analysis is based on a careful geometric analysis of the term structure equations by methods from foliation theory.

Conditional Correlation Models of Autoregressive Conditional Heteroskedasticity with Nonstationary GARCH Equations

Timo Teräsvirta
(joint work with Cristina Amado)

In this paper we investigate the effects of careful modelling the long-run dynamics of the volatilities of stock market returns on the conditional correlation structure. To this end we allow the individual unconditional variances in Conditional Correlation GARCH models to change smoothly over time by incorporating a nonstationary component in the variance equations. In our case this means that the covariance matrix of the errors, or returns if we assume the conditional mean to be identically equal to a null vector, contains a deterministic time-varying component. To the best of our knowledge, the only comparable study in which the covariance matrix of the errors contains a smoothly evolving deterministic component is Hafner and Linton (2010).

The modelling technique to determine the parametric structure of this time-varying component is based on a sequence of specification Lagrange multiplier-type tests derived in Amado and Teräsvirta (2011). The variance equations combine the long-run and the short-run dynamic behaviour of the volatilities. The structure of the conditional correlation matrix is assumed to be either time independent or to vary over time.

We discuss the specification and estimation of the individual GARCH equations before turning to the (empirical) question of whether or not careful specification of the GARCH equation changes correlation estimates compared to the case in which standard GARCH or asymmetric (GJR-) GARCH equations are used to characterize heteroskedasticity in the return series.

We apply our model to pairs of seven daily stock returns belonging to the S&P 500 composite index and traded at the New York Stock Exchange. The results suggest that accounting for deterministic changes in the unconditional variances considerably improves the fit of the multivariate Conditional Correlation GARCH models to the data. The effect of careful specification of the variance equations on the estimated correlations is variable: in some cases rather small, in others more discernible. As a by-product, we generalize news impact surfaces to the situation in which both the GARCH equations and the conditional correlations contain a deterministic component that is a function of time.

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Parametric Inference, Testing and Dynamic State Recovery from Option Panels with Fixed Time Span

Viktor Todorov

(joint work with Torben G. Andersen and Nicola Fusari)

We develop a new parametric estimation procedure for option panels observed with error. Our inference techniques exploit asymptotic approximations under the assumption of an ever increasing set of observed option prices in the moneyness-maturity (cross-sectional) dimension, but with a fixed time span. The framework allows for considerable heterogeneity over time in the quality of the information inherent in the option data. We develop consistent estimators of the parameter vector as well as the dynamic realization of the state vector that governs the option price dynamics. We show that the estimators converge stably to a mixed-Gaussian law and provide feasible estimators for the limiting covariance matrix. We also provide feasible semiparametric tests for the option price dynamics based on the distance between the diffusive (stochastic) volatility state extracted from the options and the one obtained nonparametrically from high-frequency return data for the underlying asset. In addition, we construct formal tests for the fit of the option pricing model for a specific region of the volatility surface over a given time period as well as for the stability of the risk-neutral dynamics, or parameter vector, over time. In an empirical application to S&P 500 index options we extend the double-jump stochastic volatility model of [1], popular in option pricing applications, to allow for time-varying risk premia of extreme events, i.e., jumps, as well as a more flexible relation between the risk premia and the level of risk. We show that both extensions provide a significantly improved characterization, both statistically and economically, of observed option prices and their covariation with the underlying asset price.

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Precise Large Deviations for Dependent Regularly Varying Sequences

Olivier Wintenberger
(joint work with Thomas Mikosch)

The aim of this talk is to present *precise large deviation probabilities* for sequences of dependent and heavy-tailed random variables. To make the notion of heavy tails precise, we assume that the stationary sequence (X_t) has regularly varying finite-dimensional distributions. A particular consequence is that the distribution of a generic variable X of this sequence has regularly varying tails. This means that there exist $\alpha > 0$, $p, q \geq 0$ with $p + q = 1$ and a slowly varying function L such that

$$\frac{\mathbb{P}(X > x)}{\mathbb{P}(|X| > x)} \sim p\,\frac{L(x)}{x^\alpha} \quad \text{and} \quad \frac{\mathbb{P}(X \leq -x)}{\mathbb{P}(|X| > x)} \sim q\,\frac{L(x)}{x^\alpha}\,, \quad x \to \infty. \tag{1}$$

The latter condition is often referred to as a *tail balance condition.*

In the case of an iid sequence satisfying (1) one can derive precise asymptotic bounds for the tails of the random walk (S_n) with step sequence (X_t) given by

$$S_0 = 0 \quad \text{and} \quad S_n = X_1 + \cdots + X_n\,, \quad n \geq 1\,.$$

We recall a classical result which can be found in the papers of A.V. and S.V. Nagaev [7, 8] and Cline and Hsing [3]. x

Theorem 1. *Assume that (X_i) is an iid sequence with a regularly varying distribution in the sense of* (1). *Then the following relations hold for $\alpha > 1$ and suitable sequences $b_n \uparrow \infty$:*

$$\lim_{n\to\infty} \sup_{x \geq b_n} \left| \frac{\mathbb{P}(S_n - \mathbb{E}S_n > x)}{n\,\mathbb{P}(|X| > x)} - p \right| = 0 \tag{2}$$

and

$$\lim_{n\to\infty} \sup_{x\ge b_n} \Big| \frac{\mathbb{P}(S_n - \mathbb{E}S_n \le -x)}{n\,\mathbb{P}(|X|>x)} - q \Big| = 0\,. \tag{3}$$

If $\alpha > 2$ *one can choose* $b_n = \sqrt{an\log n}$, *where* $a > \alpha - 2$, *and for* $\alpha \in (1,2]$, $b_n = n^{\delta+1/\alpha}$ *for any* $\delta > 0$. *For* $\alpha \le 1$, (2) *and* (3) *remain valid with* $\mathbb{E}S_n$ *replaced by* 0 *and one can choose* $b_n = n^{\delta+1/\alpha}$ *for any* $\delta > 0$.

As a matter of fact, precise large deviation principles can be derived for iid heavy-tailed sequences more general than regularly varying ones, e.g. for the general class of random walks (S_n) with subexponential steps; see e.g. Cline and Hsing [3]. Theorem 1 serves as a benchmark result for the purposes of this paper.

In this talk we extend Theorem 1 to suitable regularly varying stationary sequences (X_t). For linear processes $X_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j}$, $t \in \mathbb{Z}$, under suitable assumptions on the sequence of real numbers (ψ_j) (ensuring the existence of the infinite series) and assuming regular variation of the iid innovations (Z_t) was proved in Mikosch and Samorodnitsky [6]. The limiting constants p and q in (2) and (3), respectively, had to be replaced by quantities depending on p, q and the sequence (ψ_j). The region (b_n, ∞), where the large deviation principle holds, remains the same as for an iid regularly varying sequence. Similar results were obtained in Konstantinides and Mikosch [5] for solutions to the stochastic recurrence equation $X_t = A_t X_{t-1} + B_t$, $t \in \mathbb{Z}$, with iid $((A_t, B_t))_{t\in\mathbb{Z}}$ with a generic element (A, B), $A, B \ge 0$ a.s., B regularly varying with index $\alpha > 0$ and $EA^\alpha < 1$. For the same type of stochastic recurrence equation with B not necessarily positive, Buraczewski et al. [2] proved precise large deviation principles. The main difference to [5] is the assumption that (X_t) is regularly varying with some positive index α while (A_t, B_t) has moments of order $\alpha + \delta$ for some positive δ. It is shown in [2] that the relation

$$\limsup_{n\to\infty} \sup_{x\ge b_n} \frac{\mathbb{P}(S_n > x)}{n\,\mathbb{P}(|X|>x)} < \infty$$

holds for suitable sequences $b_n \to \infty$ such that $b_n^{-1} S_n \xrightarrow{P} 0$. Again, the sequences (b_n) are close to those in Theorem 1. However, uniform relations of type (2) and (3) are not true in the unbounded regions (b_n, ∞) but in bounded regions (b_n, c_n) such that $b_n \to \infty$ and $c_n = \mathrm{e}^{s_n}$ for $s_n \to \infty$ and $s_n = o(n)$.

In this talk we approach the problem of precise large deviations from a more general point of view. The following inequality is crucial for proving the results of this paper, see also Jakubowski [4]: for every $k \ge 2$, some constant b_+,

$$\begin{aligned} &\Big|\frac{\mathbb{P}(S_n > x)}{n\,\mathbb{P}(|X|>x)} - b_+\Big| \\ &\le \Big|\frac{\mathbb{P}(S_n > x) - n\,(\mathbb{P}(S_{k+1} > x) - \mathbb{P}(S_k > x))}{n\,\mathbb{P}(|X|>x)}\Big| \\ &\quad + \Big|\frac{\mathbb{P}(S_{k+1} > x) - \mathbb{P}(S_k > x)}{\mathbb{P}(|X|>x)} - b_+\Big|\,. \end{aligned} \tag{4}$$

Regular variation of (X_t) ensures that the second quantity in (4) is negligible, by first letting $x \to \infty$ and then $k \to \infty$. The first expression in (4) provides a link between the asymptotics of the tail $\mathbb{P}(S_n > x)$ for increasing values of n, $x \geq b_n$ and the regularly varying tails $\mathbb{P}(S_k > x)$ and $\mathbb{P}(S_{k+1} > x)$ for every fixed k. Thus the tail asymptotics of $\mathbb{P}(S_n > x)$ are derived from the known tail asymptotics for finite sums, again by first letting $n \to \infty$ and then $k \to \infty$.

Under regular variation and anti-clustering conditions we show precise large deviation principles of the following type:

$$\lim_{n\to\infty} \sup_{x\in\Lambda_n} \Big| \frac{\mathbb{P}(S_n > x)}{n\,\mathbb{P}(|X| > x)} - b_+ \Big| = 0\,, \tag{5}$$

for some non-negative constant b_+ and a sequence of sets $\Lambda_n \subset (0,\infty)$ such that $b_n = \inf \Lambda_n \to \infty$. We apply the large deviation principle (5) to a variety of important regularly varying time series models, including the stochastic volatility model, solutions to stochastic recurrence equations and Markov chains. These are examples of rather different dependence structures, showing that the large deviation principle does not depend on a particular mixing condition or on the Markov property.

However, we give special emphasis to Markov chains satisfying a polynomial drift condition, exploiting a sophisticated exponential bound for partial sums of Markov chains due to Bertail and Clémencon [1]. It yields an intuitive interpretation of relation (5) in terms of the regeneration property of $(X_t)_{t=1,\ldots,n}$. Given an atom A of the chain, one can split the chain into a random number $N_A(n)$ of iid random cycles. Denoting the block sum of the X_t's over the ith cycle by $S_{A,i}$, it will be shown that the iid $S_{A,i}$'s inherit regular variation from X, and then we can apply the classical result of Theorem 1 to $\mathbb{P}\Big(\sum_{i=1}^{N_A(n)-1} S_{A,i} > x\Big)$. If $b_+ > 0$ the tails $\mathbb{P}_A(S_{A,1} > x)$ and $\mathbb{P}(|X| > x)$ are equivalent. There is a major difference between an iid sequence and a dependent Markov chain (X_t): if the first generation time τ_A is larger than n, it has significant influence on the region Λ_n, where (5) holds. It turns out that one has for any $x \geq b_n$,

$$\frac{\mathbb{P}(S_n > x)}{n\mathbb{P}(|X| > x)} \sim b_+ + \frac{\mathbb{P}(S_n > x, \tau_A > n)}{n\mathbb{P}(|X| > x)}\,,$$

and the second term is in general not negligible, leading to the fact that (5) may only be valid in a bounded region (b_n, c_n). Thus we found an explanation for the same observation we experienced in the case of a Markov chain given by a stochastic recurrence equation; see the discussion above.

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Linear Quadratic Mean Field Games

Phillip Yam
(joint work with A. Bensoussan, K. C. J. Sung and S. P. Yung)

As an organic combination of mean field theory in statistical physics and (non-zero sum) stochastic differential games, Mean Field Games (MFGs) has become a very popular research topic in the fields ranging from physical and social sciences to engineering applications, see for example the earlier studies by Huang, Caines and Malhamé (2003), and that by Lasry and Lions (2006a, b and 2007). In this paper, we introduce a study of a general class of mean field games in the linear quadratic framework. We adopt the adjoint equation approach to investigate the existence and uniqueness of equilibrium strategies of these Linear-Quadratic Mean Field Games (LQMFGs). Due to the linearity of the adjoint equations, the optimal mean field term satisfies a forward-backward ordinary differential equation. For the one dimensional case, we show that the equilibrium strategy always exists uniquely. For dimension greater than one, by choosing a suitable norm and then applying the Banach Fixed Point Theorem, a sufficient condition for the unique existence of the equilibrium strategy is provided, which is independent of the coefficients of controls and is always satisfied whenever those of the mean-field term are vanished (and therefore including the classical Linear Quadratic Stochastic Control (LQSC) problems as special cases).

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Optimal Stopping when Probability is Distorted

Xunyu Zhou

We formulate an optimal stopping problem for a geometric Brownian motion where the probability scale is distorted by a general nonlinear function. The problem is inherently time inconsistent due to the Choquet integration involved. We develop a new approach, based on a reformulation of the problem where one optimally chooses the probability distribution or quantile function of the stopped state. An optimal stopping time can then be recovered from the obtained distribution/quantile function, either in a straightforward way for several important cases or in general via the Skorokhod embedding. This approach enables us to solve the problem in a fairly general manner with different shapes of the payoff and probability distortion functions. We also discuss economical interpretations of the results. In particular, we justify several liquidation strategies widely adopted in stock trading, including those of “buy and hold”, “cut loss or take profit”, “cut loss and let profit run”, and “sell on a percentage of historical high”.

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