Mathematisches Forschungsinstitut Oberwolfach

Report No. 10/2013

DOI: 10.4171/OWR/2013/10

Geophysical Fluid Dynamics

Organised by
Yoshikazu Giga, Tokyo
Matthias Hieber, Darmstadt
Edriss S. Titi, Irvine/Rehovot

17 February – 23 February 2013

Abstract. The workshop "Geophysical Fluid Dynamics" addressed recent advances in analytical, stochastic, modeling and computational studies of geophysical rotating fluids models. Of particular interest on the analytical and stochastic sides were the contributions concerning dispersive mechanism, regularity verses finite-time formation of singularities of certain viscous and inviscid geostrophic models, the primitive equations, Boussinesq approximation, boundary layers and fast rotating fluids. Model reductions, based on asymptotic, scaling analysis and variational methods, were presented. In addition, computational investigations were provided in support of the claim that three-dimensional geophysical turbulent flows exhibit two-dimensional features, at small Rosby numbers.

Mathematics Subject Classification (2010): 76-XX, 86A10, 35-XX, 60Hxx.

Introduction by the Organisers

This workshop was fostering the investigation of large classes of geophysical fluid models by means of techniques steaming from analysis, stochastics, modeling and computational sciences. The complexity of fluid models taking into account all relevant physical factors, such as fast rotation, thinness of the oceans and the atmosphere and moist, as well as spatial and temporal scales, arising in climate research, turbulent flows and meteorology show the strong need for accessible reliable reduced models. At the limit of small Rosby number, taking into account fast rotation and the smallness of the vertical to horizontal aspect ratio, these simplified models are reduced to, for instance, shallow water models capturing gravity waves, stratified three-dimensional fluids mimicking two-dimensional turbulent flows, geostrophic balanced models at planetary horizontal scales and primitive equations with vertical hydrostatic balance.

The mathematical tools for deriving these models rely often on sophisticated asymptotic analysis methods that take into consideration the spatial and time scales that are relevant to the physical phenomena. Let us stress that identifying these relevant scales is already a major mathematical challenge.

The mathematical investigation of these reduced models involves many modern tools ranging from harmonic analysis concerning oscillatory integrals, dispersive estimates, resonances, micro-local analysis, as well as nonlinear evolutionary partial differential equations and their stochastic counterpart. As a first step in this investigation one aims for proving the global well-posedness of these simplified models (or at least for the relevant time scales under which they are derived). Next, one attempts to provide rigorous justification and validation of the derived models. At the same time this is an important step toward the development of numerical and computational schemes for simulating these models. As it has been mentioned above, these models involve a wide spectrum of spatial and temporal scales that makes the computational aspects of these models still out of reach. Of particular challenge in this context is the present of fast rotating waves that require high temporal resolution. Consequently, a great effort is now being made to reduce the system further to mode out these fast waves. An additional approach for treating atmospheric motion is through the mathematical analysis and stability of boundary layers.

One of the main characteristics of this workshop was the bringing together leading experts from diverse scientific backgrounds such as analysis, modeling and numerics. This has ignited lively and productive interaction and exchange of interdisciplinary ideas. This was a very inspiring experience. The presence of younger participants was very visible all during this meeting. In particular, the workshop provided a platform for younger participants to play an active role in the meeting by encouraging them to present their own work in a special highly visible evening session which was fully attended by all participants. Topics covered in this special session are e.g., optimal fluid mixing, models for the aquaplanet, liquid crystals, and L^∞-estimates for the Navier-Stokes equations.

The lectures presented took 40 minutes which were followed by very lively and interactive discussions of 20 minutes. In addition, there were two special special evening tutorials of one hour on stochastic analysis with fast rotation and data assimilation.

The meeting brought together a very good mixture of various communities and several leaders from different disciplines met here for the first time in person. The gender diversity was good, especially among the younger participants.

Workshop: Geophysical Fluid Dynamics

Table of Contents

Abstracts

Nonlinear ocean eddies

Roger M. Samelson

Two aspects of the nonlinear dynamics of ocean flows are discussed. First, the downscale transfer of energy from developing baroclinic waves to quasi-isotropic small-scale three-dimensional turbulence is examined in numerical calculations with a large-eddy simulation model based on the nonhydrostatic Boussinesq equations with a nonlinear eddy viscosity closure (joint work with E. Skyllingstad). These simulations resolve the full range of motions from rotationally dominated, growing baroclinic waves to quasi isotropic, three-dimensional shear instabilities. The results confirm a forty-year-old prediction that frontogenetic collapse of cross-frontal spatial scales, driven by baroclinic-wave deformation fields, will continue to the Kelvin-Helmholtz turbulent transition. This process of frontal collapse followed by K-H transition provides a mechanism for spontaneous loss of balance in an initially geostrophic flow, and a direct, spectrally non-local pathway for downscale energy transfer that is phenomenologically distinct from traditional concepts of turbulent cascades and can contribute substantially to total kinetic energy dissipation. Second, a recent global analysis of two decades of satellite altimeter observations of nonlinear ocean eddies are reviewed, and a simple stochastic model is presented of the mean altimeter-tracked eddy amplitude life cycles (joint work with D. Chelton, M. Schlax, J. Early). The stochastic model consists of thresholded subsequences of a Markov or first-order auto-regressive (AR1) model in which the random increments are drawn from a normal distribution. Normalized amplitude life cycles of altimeter-tracked with eddy lifetimes of 16 to 80 weeks are computed and found to be essentially independent of lifetime. Basic aspects of this approximately universal (lifetime-independent) mean structure of normalized altimeter-tracked eddy life cycles, including time-reversal symmetry and the simple structure of the mean amplitude time series, are reproduced with remarkable accuracy by the stochastic model. The dependence of dimensional amplitude statistics on eddy lifetime and the distribution of eddy numbers vs. eddy lifetime can also be partially reproduced by this model.

Dispersive estimates for the Euler and the Navier-Stokes equations with the Coriolis force

Ryo Takada
(joint work with Youngwoo Koh and Sanghyuk Lee)

We are interested in a dispersion phenomenon of the Coriolis force, arising in the incompressible Euler and Navier–Stokes equations in the rotational framework:

$$\text{(NSC)}\qquad \begin{cases} \dfrac{\partial u}{\partial t} - \nu\Delta u + \Omega e_3 \times u + (u\cdot\nabla)u + \nabla p = 0, & \text{in } \mathbb{R}_+ \times \mathbb{R}^3,\\ \operatorname{div} u = 0 & \text{in } \mathbb{R}_+ \times \mathbb{R}^3,\\ u(0,x) = u_0(x) & \text{in } \mathbb{R}^3, \end{cases}$$

where $\nu \geq 0$ denotes the kinetic viscosity coefficient and the constant $\Omega \in \mathbb{R}$ represents the speed of rotation around the vertical unit vector $e_3 = (0,0,1)$.

It is known that the semigroup $\{T_\Omega(t)\}_{t\geq 0}$ associated with the linearized problem of (NSC) has the explicit form

$$T_\Omega(t)f = \frac{1}{2}e^{i\Omega t\frac{D_3}{|D|}}\left[e^{\nu t\Delta}(I+\mathcal{R})f\right] + \frac{1}{2}e^{-i\Omega t\frac{D_3}{|D|}}\left[e^{\nu t\Delta}(I-\mathcal{R})f\right]$$

with some singular integral operator $\mathcal{R}$. Hence the dispersion effect of the Coriolis force is closely related to the dispersive estimates for the operator that is given by the Fourier integral

$$e^{\pm i\Omega t\frac{D_3}{|D|}}f(x) := \int_{\mathbb{R}^3} e^{i(x\cdot\xi \pm \Omega t\frac{\xi_3}{|\xi|})}\widehat{f}(\xi)d\xi, \quad (t,x)\in\mathbb{R}_+\times\mathbb{R}^3.$$

Since the function $\xi_3/|\xi|$ which is the source of dispersion is homogeneous of degree 0, by the Littlewood–Paley decomposition and the scaling argument, the matter reduces to the case in which $\widehat{f}$ is supported in the annulus $\{\xi\in\mathbb{R}^3 \mid 1/2 \leq |\xi| \leq 2\}$. We show the two–dimensional dispersive estimates for the linear group $e^{\pm i\Omega t\frac{D_3}{|D|}}$. More precisely, we prove that

$$\left\| e^{\pm i\Omega t\frac{D_3}{|D|}}f\right\|_{L^\infty} \leq C(1+|\Omega|t)^{-1}\|f\|_{L^1}$$

for all $t\geq 0, \Omega\in\mathbb{R}$ and $f\in L^1(\mathbb{R}^3)$ with $\operatorname{supp}\widehat{f}\subset\{\xi\in\mathbb{R}^3 \mid 1/2\leq|\xi|\leq 2\}$. As an application to the Euler equations, we prove that the lifespan of the solution can be taken arbitrarily large provided the speed of rotation is sufficiently high.

Multiscale regimes in atmospheric flows

Rupert Klein

My presentation first summarized the unified approach to meteorological modelling based on multiple scales asymptotics that I have developed over the past decade and which is summarized in [1]. In the context of the present workshop, I have emphasized the roles of formal asymptotic analysis as both a “language” for framing phenomenological descriptions of physical processes of interest in systematic mathematical terms and as a means for the generation of interesting hypotheses and conjectures that would await rigorous proof.

Next I have pointed out three typical examples of multiscale problems for atmospheric flows that are of high interest to the meteorological community and for which it would be very much worthwhile to develop mathematical theory beyond formal asymptotics.

The first example concerns a recent theory, [2], for nearly axisymmetric tilted vortices in the "gradient wind regime". This flow regime covers vortices from strong tropical storms to weak hurricanes or taiphoons. The development involves matched asymptotic expansions for vortices that are axisymmetric to leading order in each horizontal plane. At the same time the vortices are strongly tilted in that the connecting line of the leading order vortex centers features horizontal displacements comparable to the typical vortex core size of about 200 km. The theory yields coupled evolution equations for the vortex centerline and the leading order axisymmetric primary circulation in the same spirit as earlier analyses of slender three-dimensional vortices in engineering fluid flows, see [3] and references therein.

The second example presents a somewhat unusual problem from a theoretical point of view. In [4] we investigate the regime of validity of the anelastic and pseudo-incompressible (sound-proof) approximations for atmospheric flows. We point out that, asymptotically speaking, for realistic stratifications of potential temperature (entropy), the full compressible flow equations represent an asymptotic three timescale problem, in which sound waves are fastest, internal waves have an intermediate time scale, and advection is the slowest process (we have neglected rotation in this analysis). The sound-proof models are designed to maintain internal waves and advection but to suppress sound waves. Thus the challenge is to analyze an asymptotic three-scale problem and to obtain insight into the remaining two-scale problem – the sound-proof models – when only the fastest time scale is removed by asymptotic arguments.

The last example involves strong storm fronts or "squall lines". Through dimensional analysis of observational data we, that is Verena Molina, [5], Mitch Moncrieff (National Center for Atmospheric Research, Boulder, CO, USA), and the author, were led to set up an unusual multiscale asymptotic regime for such processes. The data reveal that a relatively narrow storm front that extends laterally over several hundred kilometers produces strong precipitation in a band of 10-30 km thickness. Within this band, the actual precipitation is induced by strong updrafts which themselves are concentrated in narrow convective towers with a typical diameter of 1 km. These towers, in turn, are *sparsely* distributed within the precipitation band with characteristic distances of just a few kilometers. In nondimensional asymptotic terms this can be phrased by assuming convective tower diameters of order $O(\epsilon)$, characteristic tower-to-tower distances of order $O(\sqrt{\epsilon})$, and a bulk front-normal thickness of the precipitation band of order $O(1)$. Thus, this problem clearly demands an asymptotic multiple scales formulation, albeit one that does not assume the small-scale processes to be "space filling". Rather, the narrow towers are sparsely distributed, and this requires modifications in the formulation of sublinear growth conditions. At the same time, due to the fact that the towers are separated by distances large compared with their diameter, the method of matched asymptotic expansions must be invoked to describe their structure. Finally, since the development of an individual convective tower is triggered by local flow instabilities, the spacio-temporal distribution of the towers should be modelled by a stochastic process akin to suggestions by Plant & Craig [6]. In conclusion, a reasonably comprehensive model for the main precipitation band will – besides boundary layer and related process descriptions – combine matched and nonstandard multiscale asymptotic expansions with a stochastic process to model the appearance of convective towers.

To conclude, atmospheric flow processes are a rich source of intriguing applied mathematics problems that often deviate from established problem types and classes.

References

[1] Klein R., *Scale-Dependent Asymptotic Models for Atmospheric Flows*, Ann. Rev. Fluid Mech., **42**, 249–274 (2010)

[2] Päschke E., Marschalik P., Owinoh A.Z., Klein R., *Motion and structure of atmospheric mesoscale baroclinic vortices: dry air and weak environmental shear*, J. Fluid Mech., **701**, 137–170. (2012)

[3] Ting L., Klein R., Knio O.M., *Vortex Dominated Flows: Analysis and Computation for Multiple Scales*, Series in Applied Mathematical Sciences, **161**, Springer Verlag, (2007)

[4] Klein R., Achatz U., Bresch D., Knio O.M., Smolarkiewicz P.K., *Regime of Validity of Sound-Proof Atmospheric Flow Models*, J. Atmos. Sci., **67**, 3226–3237 (2010)

[5] Molina V., *Multiple-scales analysis of deep convection in a squall line*, Diploma Thesis, Freie Universität Berlin (2010)

[6] Plant R.S., Craig G.C., *A stochastic parameterization for deep convection based on equilibrium statistics*, J. Atmos. Sci., **65**, 87–105 (2008)

Vortex formation in stably stratified turbulence

Yoshifumi Kimura
(joint work with Jackson R. Herring)

Recent numerical results on vortex formation in stably stratified turbulence is presented. In particular, the relation between the power-law transition in energy spectra and production and destabilization of Kelvin-Helmholtz billows in stably stratified turbulence is studied. Kelvin-Helmholtz billows, often observed in the oceans and the atmosphere, are thought to be an important mechanism of mixing and turbulence in stably stratified flows. We integrated the Navier-Stokes equations under the Boussinesq approximation pseudo-spectrally using up to 2048^3 grid points. Our method is to integrate the equations from the zero total energy initial condition with horizontal forcing imposed in a narrow wave number band. Recent computations demonstrate that a power-law transition in the horizontal energy spectrum, which has been observed in the atmosphere[1], exists for stably stratified turbulence even without rotation[2]. In the course of development for a stationary shape, the horizontal spectrum undergoes some different stages. At the first stage, it shows a single steep power-law ($k_\perp^{4-5}$). By this time, we observe that many wedge vortices are produced and they move horizontally (like dipoles) in random directions. This stage lasts a long period of time, and then the tail part of the spectrum begins to rise to show the Kolmogorov-type slope ($k_\perp^{-5/3}$). During the time of this stage, the wings of the wedges become thinner and thinner while translating, and finally detach to be almost independent vortex layers. This thinning mechanism makes the vertical shear stronger and eventually local Richardson number small to develop Kelvin-Helmholtz billows. We will show that the horizontal breaking of the Kelvin-Helmholtz billows results in the Kolmogorov-type slope in the spectrum.

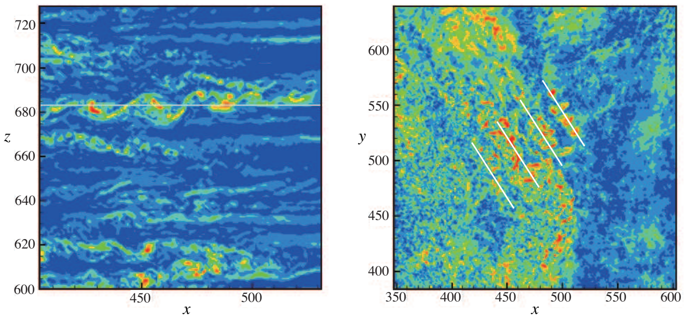


FIGURE 1. Left: Kelvin-Helmholtz billows in stably stratified turbulence. Right: The horizontal slice near the Kelvin-Helmholtz billow (along the white line in the left figure). The numbers at axes are the grid numbers. [2]

References

[1] G. D. Nastrom & K. S. Gage, *A Climatology of Atmospheric Wavenumber Spectra of Wind and Temperature Observed by Commercial Aircraft*, J. Atmos. Sci. **42** (1985), 950–960.

[2] Y. Kimura & J. R. Herring, *Energy spectra of stably stratified turbulence*, J. Fluid Mech. **698** (2012), 19–51.

Stochastic partial differential equations in mathematical fluid dynamics

WILHELM STANNAT

We consider the stochastic Navier-Stokes equations

$$\frac{\partial u}{\partial t} - \nu \Delta u + (u \cdot \nabla)\, u + \nabla p = \dot{\eta}(t,x)\,, \ \text{div}\; u = 0 \tag{1}$$

on the d-dimensional torus $\mathbb{T}^d = [0, 2\pi]^d$, $d = 2, 3$. The stochastic forcing term $\eta(t,x)$ is a Wiener process taking values in the Hilbert space $H = L^2_\sigma$ of square integrable divergence free vector fields $u : \mathbb{T}^d \to R^d$ having mean zero $\int u\, dx = 0$. Consequently, $\eta(t,x)$ can be represented as

$$\eta(t,x) = \sum_k \alpha_k \beta_k(t) e_k(x)$$

where $\{e_j\}$ is an orthonormal system of H, $\{\beta_k\}$ are independent 1-dimensional standard Brownian motions (defined on an underlying probability space $(\Omega, \mathcal{F}, P)$) and $\{\alpha_k\} \subset \mathbb{R}$. W.l.o.g. we assume in the following that $\{e_j\}$ is the complete ornonormal system of eigenvectors corresponding to the Stokes operator $A = \nu\Pi\Delta$ on H. For later reference, let us introduce the interpolation spaces

$$H^\alpha := \{u = \sum_k u_k e_k \mid \sum_k |k|^{2\alpha} u_k^2 < \infty\}, \alpha \in \mathbb{R}$$

and observe that independence of β_k, $k = 1, 2, \ldots$, implies $E\left[\|\eta(t,\cdot)\|_{H^\alpha}^2\right] = \sum_k |k|^{2\alpha}\alpha_k^2\, t \leq \infty$.

Existence and uniqueness of martingale solutions

Given an initial distribution μ_0 on H, a probability measure P on $\Omega = C([0,T]; D(\Delta)')$ is called a martingale solution of (1) if the *canonical process* $\xi_t : \Omega \to D(\Delta)'$, $\omega \mapsto \omega(t)$ satisfies the following three conditions:
(i) $P\left[\sup_{t\in[0,T]} |\xi_t|_H + \int_0^T |\xi_s|_{H^1}^2\, ds < \infty\right] = 1$,
(ii) for all smooth divergence-free vector fields φ having zero mean

$$M_t^\varphi := \langle \xi_t, \varphi\rangle_H + \nu \int_0^t \langle \xi_s, \Delta\varphi\rangle_H\, ds - \int_0^t \langle (\xi_s \nabla)\, \varphi, \xi_s\rangle_H\, ds$$

is a continuous square-integrable martingale with quadratic variation $\langle M^\varphi\rangle_t = \sum_k \alpha_k^2 \langle e_k, \varphi\rangle_H^2 \cdot t$
(iii) $P_{\xi_0} = \mu_0$.

For a thorough discussion of the properties of martingale solutions as well as its interrelation with the notion of weak solutions of (1) we refer to [1].

In the 2D-case existence and uniqueness of martingale solutions for smooth noise satisfying $E\left[\|\eta(t,\cdot)\|_{H^1}^2\right] < \infty$ goes back to Viot (1975). Existence of martingale solutions in the general case $E\left[\|\eta(t,\cdot)\|_H^2\right] < \infty$ both in 2D and 3D was shown by Flandoli and Gatarek (1995), leaving the question of uniqueness open. Existence of martingale solutions could be extended also to rough noise satisfying $E\left[\|\eta(t,\cdot)\|_{H^\alpha}^2\right] < \infty$ for $\alpha > -\frac{1}{2}$ by Flandoli (1995). Particular interest has been put in the literature on the case of space-time white noise $\alpha_k \equiv 1$: existence of martingale solutions has been proven by Albeverio and Cruzeiro (1990) and existence and uniqueness of strong solutions (a.s. w.r.t. the invariant measure μ_ν to be introduced below) by Da Prato and Debussche (2001).

An alternative approach to solving (1) directly, is to construct the semigroup of transition probabilities of a full Markov process associated with (1) (if it exists). To this end note that if (1) has a unique martingale solution for all deterministic initial conditions $u_0 \in H$, it follows that

$$T_t F(u_0) := E\left(F(u(t)) \mid u(0) = u_0\right)$$

defines a semigroup of linear operators ($T_0 = Id$ and $T_t \circ T_s = T_{t+s}$, $t, s \geq 0$). Its infinitesimal generator L can be represented as

$$\begin{aligned} LF(u_0) &:= \frac{d}{dt} T_t F(u_0)_{|t=0} \\ &= \frac{1}{2} \sum_k \alpha_k^2 \langle F''(u_0) e_k, e_k \rangle_H + \langle \nu \Delta u_0 - (u_0 \cdot \nabla) u_0, F'(u_0) \rangle_H \end{aligned}$$

for sufficiently smooth functions F. L is called the Kolmogorov operator associated with (1) and it can be analyzed on suitable function spaces, in particular on the space $L^p(H, \mu)$ w.r.t. an invariant measure μ of (1). Having the Kolmogorov operator we can also weaken the notion of an invariant measure as follows: a probability measure μ on $(H, \mathcal{B}(H))$ is called infinitesimally invariant if $\int LF \, d\mu = 0$ for all F contained in a suitable domain of smooth functions.

Existence and uniqueness of invariant measures

Existence of invariant measures for (1) has been obtained in the 2D-case for η satisfying $E\left[\|\eta(t,\cdot)\|_{H^\alpha}^2\right] < \infty$ and $\alpha > -\frac{1}{2}$ by Flandoli (1995) and in the 3D-case for η satisfying $E\left[\|\eta(t,\cdot)\|_{H}^2\right] < \infty$ and ν sufficiently large by Flandoli and Gatarek (1995). Uniqueness has been proven for η satisfying $E\left[\|\eta(t,\cdot)\|_{H^\alpha}^2\right] < \infty$ with $\alpha > \frac{3}{4}$ by Flandoli and Maslowski (1995), for $\alpha = 1$ by Kuksin (2002) and by Mattingly and Hairer (2006). Up to now, no uniqueness results are known in the 3D-case.

The existence of the invariant measures is mostly obtained employing a compactness argument and only little is known about their properties apart from their support and some moment estimates. There is however one remarkable exception: in the 2D-case with space-time white noise the Gaussian measure

$$\mu_\nu(du) = N\left(0, \frac{1}{\nu} \int_{\mathbb{T}^2} \left|(-\Delta)^{-\frac{1}{2}} u\right|^2 \, dx\right)(du)$$

is infinitesimally invariant for the associated Kolmogorov operator.

As already mentioned above, using invariant measures one can construct an extension of the Kolmogorov operator in the space $L^p(H, \mu)$ generating a strongly continuous semigroup that can then be considered as the transition semigroup of a Markov process associated with the stochastic partial differential equation (1). If there is only one such extension the Kolmogorov operator is called L^p-unique. For the precise definition of the domain of the Kolmogorov operator as well as implications of L^p-uniqueness on the uniqueness of the martingale problem we refer to [3, 4].

For the case of the stochastic Navier-Stokes equations (1) L^2-uniqueness for the Kolmogorov operator for η satisfying $E\left[\|\eta(t,\cdot)\|_{H^1}^2\right] < \infty$ has been obtained first by Barbu, Da Prato and Debussche (2004), L^p-uniqueness for all finite p in [4], under the same assumption on the noise, also for the stochastic Navier-Stokes equations in a rotational setting. For space-time white noise L^1-uniqueness has been obtained in the paper [3] for sufficiently large viscosity ν. [2] extends the latter result to the stochastic Navier-Stokes equations in a rotational setting.

Stochastic power law fluids

Many of the results obtained for the stochastic Navier-Stokes equations can be generalized to stochastic power law fluids

$$\partial_t u - \text{ div } \nu\left(|Eu|\right) Eu + (u \cdot \nabla)\, u + \nabla p = \dot{\eta}(t,x)\,, \text{ div } u = 0 \tag{2}$$

that have been introduced by Terasawa and Yoshida in [5]. Here, $Eu = \frac{1}{2}\nabla u + \frac{1}{2}\nabla^T u$ and $\nu(x) = \nu_0\left(1+x^2\right)^{\frac{p-2}{2}}$.

Existence of martingale solutions have been obtained by Terasawa and Yoshida (2011) for η satisfying $E\left[\|\eta(t,\cdot)\|^2_{H^1}\right] < \infty$ for $p > \frac{3d}{d+2}$ and pathwise uniqueness under the more restrictive assumption $p \geq 1 + \frac{d}{2}$. Sauer (2012) could relax the smoothness assumption on the noise to $E\left[\|\eta(t,\cdot)\|^2_{H}\right] < \infty$ and obtained existence of a strong solution (in the probabilistic sense) for $p \geq 1+\frac{d}{2}$. Existence of invariant measures has been obtained by Sauer (2012) under the same assumptions. For smoother noise satisfying $E\left[\|\eta(t,\cdot)\|^2_{H^1}\right] < \infty$ existence of invariant measures has been obtained in the 2D-case for $p > 1$ and in the 3D-case for $p > 1 + \frac{2d}{d+2}$. In the 2D-case Sauer (2012) also obtained L^r-uniqueness, $r \in [1,p]$, of the associated Kolmogorov operator for smooth noise $E\left[\|\eta(t,\cdot)\|^2_{H^1}\right] < \infty$ and $p \in (p_2, 2]$, where p_2 is the second root of $p^3 - 8p^2 + 14p - 6$ (approx. 1.61).

References

[1] Flandoli, F., *Random Perturbation of PDEs and Fluid Dynamic Models*, Lecture Notes in Mathematics, **2015**, Springer, 2011.

[2] Sauer, M., *Existence and Uniqueness Results for randomly forced generalized Newtonian Fluids*, PhD-thesis, TU Darmstadt, (2012).

[3] Stannat, W., *A new a priori estimate for the Kolmogorov operator of a 2D-Stochastic Navier-Stokes equation*, Infin. Dimens. Anal. Quantum Probab. Relat. Top., **10** (2007), 483–497.

[4] Stannat, W., *L^p-Uniqueness of Kolmogorov operators associated with 2D-stochastic Navier-Stokes-Coriolis Equations*, Math. Nachr., **284** (2011), 2287–2296.

[5] Terasawa, Y., Yoshida, N., *Stochastic Power Law Fluids: Existence and uniqueness of weak solutions*, Ann. Appl. Probab., **21** (2011), 1827–1859.

The relative entropy method for geophysical flows

Yann Brenier

The "relative entropy method" is used in several fields of mathematical physics, PDEs and probabilities (kinetic theory, systems of particles, systems of hyperbolic conservation laws...). A striking example is the derivation of the Euler equations of incompressible fluids from the Boltzmann equation some years ago by Laure Saint-Raymond. Another example is E. Feireisl's contribution to the workshop. Here, we report on two examples of geophysical fluid dynamics for which the relative entropy method applies. The first example has been treated by several authors,

mainly E. Grenier, N. Masmoudi-T.K. Wong, and the author (without Coriolis force). The second by M. Cullen and the author.
The first example is the 2d Euler equation in a very thin domain, with a variable Coriolis force involving the so-called $\beta-$plane approximation, namely

$$D_t = \partial_t + u\partial_x + v\partial_y, \quad \partial_x u + \partial_y v = 0,$$
$$D_t u + \partial_x p = -\beta y v, \quad D_t v + \partial_y p = \beta y u,$$
$$x \in R/Z, \quad -\varepsilon < y < +\varepsilon, \quad v = 0 \text{ at } |y| = \varepsilon.$$

After rescaling $(y, v) \to (y, v)\varepsilon$, $\beta \to \beta/\varepsilon^2$ and setting $\Omega = \partial_y u - \varepsilon^2 \partial_x v + \beta y$, we get the "rescaled equations in vorticity form" (REVF)

$$D_t = \partial_t + u\partial_x + v\partial_y, \quad u = \partial_y \psi, \quad v = -\partial_x \psi,$$
$$D_t \Omega = 0, \quad (\partial_y^2 + \varepsilon^2 \partial_x^2)\psi + \beta y = \Omega$$
$$x \in R/Z, \quad 1 < y < +1, \quad v = 0 \text{ at } |y| = 1.$$

We want to compare the solutions (ψ, Ω) of these equations to those $(\overline{\psi}, \overline{\Omega})$ of the formal limit equations (FLE) obtained by letting ε go to zero. Here is a positive answer:
Let $T > 0$ and a smooth solution $(\overline{\psi}, \overline{\Omega})$ of the FLE. Assume there is a constant $r > 0$ such that

$$(1) \qquad r \leq \partial_y^2 \overline{u}(t, x, y) + \beta \leq 1/r, \quad \forall (t, x, y) \in [0, T] \times R/Z \times]-1, 1[.$$

Then, for each $c_0 > 0$, there is a constant C depending only on $(c_0, T, r, \overline{\psi}, \overline{\Omega})$ with the following property: let (ψ, Ω) be any solution of the REVF for which

$$e(t) = ||\partial_y \psi - \partial_y \overline{\psi}||^2 + \varepsilon^2 ||\partial_x \psi - \partial_x \overline{\psi}||^2 + ||\Omega - \overline{\Omega}||^2$$

(where $||\cdot||$ denotes the L^2 norm in $(x, y) \in R/Z \times]-1, +1[)$ is smaller than $c_o \varepsilon^2$ at time $t = 0$, then $e(t)$ stays smaller than $C\varepsilon^2$ for all $t \in [0, T]$. In addition, there are examples of no-convergence when condition (1) is violated. The error term $e(t)$ is hard to control (a naive estimate leads to $e'(t)/e(t) \leq c/\varepsilon$, with is untractable). The convergence is established by substituting for $||\overline{\Omega} - \Omega||^2$ (in the definition of $e(t)$) the "relative entropy"

$$\int (F(t, x, \Omega) - F(t, x, \overline{\Omega}) - \partial_3 F(t, x, \overline{\Omega})(\Omega - \overline{\Omega}))dxdy$$

where F is constructed out of $(\overline{\psi}, \overline{\Omega})$ and chosen, for some constant γ, so that

$$\partial_3 F(t, x, \overline{\Omega}(t, x, y)) = \frac{\partial_y \overline{\psi}(t, x, y) - \gamma}{\partial_y \overline{\Omega}(t, x, y)} > 0.$$

The second example is the Boussinesq equation (with constant Coriolis f frequency and buoyancy frequency N) with fields (u, v, w, p, θ) depending only on (t, x, z) (but not y) and periodic boundary conditions. We also include given source terms $(g, h)(t, x, z)$:

$$D_t = \partial_t + u\partial_x + w\partial_z, \quad \partial_x u + \partial_z w = 0,$$
$$D_t u + \partial_x p = fv, \quad D_t v = -fu + g, \quad D_t w + \partial_z p = N\theta, \quad D_t \theta = -Nw + h$$

Then, we rescale the source terms, the velocity field and the time variable: $(g,h) \to (g,h)\varepsilon$, $(u,w) \to (u,w)\varepsilon$, $t \to 1/\varepsilon$, to see the long term effect of small source terms. This does not affect the equations, except

$$\varepsilon^2 D_t u + \partial_x p = fv, \quad \varepsilon^2 D_t w + \partial_z p = N\theta.$$

The limit equations are obtained by setting $\varepsilon = 0$. Then, we get the same kind of convergence result as for the first example, under the following convexity condition ("Cullen-Purser's stability")

$$0 < r \leq \text{eigenvalues}(D^2_{x,z}(\frac{fx^2 + Nz^2}{2} + \overline{p}(t,x,z))) \leq 1/r,$$

for some constant $r > 0$. In this case, the error between the solutions (u,v,w,p,θ) of the original equations and those $(\overline{u},\overline{v},\overline{w},\overline{p},\overline{\theta})$ of the limit equations is measured with the "relative entropy"

$$\int dxdz(\Phi(t,v,\theta) - \Phi(t,\overline{v},\overline{\theta}) - \nabla\Phi(t,\overline{v},\overline{\theta}) \cdot (v - \overline{v}, \theta - \overline{\theta})),$$

where we have set up $f = N = 1$ (for simplicity) and

$$\Phi(t,\xi,\zeta) = \sup_{x,z} \;\; x\xi + z\zeta - \frac{x^2+z^2}{2} - \overline{p}(t,x,z).$$

References

[1] Y. Brenier, Nonlinearity 12, 495-512 (1999), Bull. Sci. Math. 127, 585-595 (2003), Rearrangement, convection and entropy (submitted, 2012).
[2] Y. Brenier, M. Cullen, Commun. Math. Sci. 7 (2009) 779-784.
[3] E. Grenier, M2AN Math. Model. Numer. Anal. 33 (1999) 965-970.
[4] N. Masmoudi, T.K. Wong, Arch. Ration. Mech. Anal. 204 (2012) 231-271.
[5] L. Saint-Raymond, Ann. Inst. H. Poincaré ANL 26 (2009) 705-744.

Finite-time blow up solutions for the inviscid primitive equation in two and three space dimension

Slim Ibrahim

(joint work with Chongsheng Cao, Kenji Nakanishi, and Edriss S. Titi)

1. Introduction

In large oceanic and atmospheric dynamic models, the viscous Primitive equations can be derived from Boussinesq equations using the so called *hydrostatic* *balance approximation.* We refer the interested reader to e.g., [8, 12, 14] for the details of such a derivation. The $3D$ primitive equations are given by the system:

$$\frac{\partial u}{\partial t} + uu_x + vu_y + wu_z + p_x - Rv = \nu\Delta u \tag{1}$$

$$\frac{\partial v}{\partial t} + uv_x + vv_y + wv_z + p_y + Ru = \nu\Delta v \tag{2}$$

$$\partial_z p + T = 0, \tag{3}$$

$$\frac{\partial T}{\partial t} + uT_x + vT_y + wT_z = Q + \kappa\Delta T, \tag{4}$$

$$u_x + v_y + w_z = 0, \tag{5}$$

where the unknowns are the velocity vector field (u, v, w) of the fluid, its pressure p and its temperature T. Here, Q is a heat source, $R > 0$ is the rotation parameter due to the Coriolis force, $\nu > 0$ is the viscosity of the fluid, and $\kappa > 0$ is the diffusion coefficient.

Equations (1) and (2) represent the horizontal components of the momentum equation while (3) is the vertical motion under the hydrostatic balance. Supplementing the above system with initial value (u_0, v_0, T_0) and the relevant geophysical boundary conditions, the global well-posedness (for all time and for all initial data) of strong solutions in 3D has been proven first in [1] in the viscous diffusive case. This result has been improved recently in [2] to the case of only partial anisotropic vertical diffusion.

As in geophysical situations the viscosity coefficients are very small, then it becomes interesting to know whether the non-viscous primitive equations are globally regular or that they develop finite time singularity. Moreover, since the rotation term did not play any role in establishing the global regularity in the viscous cases, we also assume there is no rotation by taking $R = 0$. In contrast, notice that for the three dimensional Navier-Stokes, Euler and Boussinesq equations, fast rotation avoids resonannces at the limit equations and leads to strong dispersion and averaging mechanism that weakens the nonlinear effects. This of course allows for establishing the global regularity result in the viscous Navier-Stokes case, and prolongs the life-space of the solution in the Euler case (see also and [4, 3] and references therein.

Hence, in the horizontal channel $\Omega = \{(x, y, z) : 0 \le z \le H,\ (x, y) \in \mathbb{R}^2\}$, the inviscid primitive equations without the Coriolis force reads:

$$u_t + uu_x + vu_y + wu_z + p_x = 0, \tag{6}$$

$$v_t + uv_x + vv_y + wv_z + p_y = 0, \tag{7}$$

$$p_z + T = 0, \tag{8}$$

$$T_t + uT_x + vT_y + wT_z = \kappa T, \tag{9}$$

$$u_x + v_y + w_z = 0, \tag{10}$$

in the horizontal channel $\Omega = \{(x, y, z) : 0 \le z \le H,\ (x, y) \in \mathbb{R}^2\}$; subject to the boundary conditions: no-normal flow and no heat flux in the vertical direction at the physical solid (top $z = H$ and bottom $z = 0$) boundaries, and periodic boundary conditions, say with period L, in horizontal directions. For results concerning the short time existence and uniqueness of the inviscid primitive equations see, for example, [7, 9, 13] and references therein. Our main Theorem is the following

Theorem 1. *There exists a smooth initial data for which the corresponding smooth solution of* (6)-(10) *blows-up in finite time.*

In order to establish the Theorem, we assume that we are given a smooth solution to the inviscid primitive equation. First we derive a reduced equation that this smooth solution will satisfy. Then we follow [5] (see also [10]) to show that for certain class of initial data the corresponding solutions to this reduced equation blow up in finite time. Finally, we provide a family of initial data whose corresponding smooth solutions to the inviscid primitive equations blow up in finite time.

2. Sketch of the Proof

Since our goal is to establish the blowup for certain class of smooth solutions and initial data, we restrict ourselves to smooth solutions with constant temperature T, zero v component, that are y-independent, and periodic with respect to x with the following symmetry

$$u(x,z,t) = -u(-x,z,t);\ p(x,z,t) = p(-x,z,t);\ w(x,z,t) = w(-x,z,t),$$

subject to the boundary condition

$$w|_{z=H} = w|_{z=0} = 0, \tag{11}$$

It is not difficult to see that the solution map preserves these symmetries. Taking into account these symmetries, one can solve for the pressure term and find that

$$p_x = -\frac{2}{H}\int_0^H u u_x\, dz := -2\overline{uu_x}., \tag{12}$$

and therefore, u satisfies the following non-local closed system

$$\frac{\partial u}{\partial t} + uu_x + wu_z - 2\overline{uu_x} = 0, \tag{13}$$

Differentiating the above equation with respect to x, using the divergence free condition and setting $W(z,t) := w(0,z,t)$, we end up with the reduced system

$$W_{tz} - (W_z)^2 + WW_{zz} + \frac{2}{H}\int_0^H (W_z)^2\, dz = 0, \tag{14}$$

$$W(0,t) = W(H,t) = 0. \tag{15}$$

As equation (14) is invariant under the scaling $W(z,t) \mapsto \lambda W(z, \lambda t)$, then we look for a self-similar solution in the form

$$W(z,t) = \frac{\varphi(z)}{1-t}, \text{ with } \varphi(0) = \varphi(H) = 0.$$

That is, for $m^2 = \frac{2}{H}\int_0^H (\varphi_m'(z))^2 dz$, the blow-up profile φ satisfies

$$\varphi' - (\varphi')^2 + \varphi\varphi'' + m^2 = 0, \quad \varphi(0) = \varphi(H) = 0. \tag{16}$$

An explicit phase-portrait analysis of the above nonlinear eigenvalue problem insures the existence of nontrivial solution φ, and consequently the finite time blow-up of the unique solution of the reduced problem .

References

[1] C. Cao and E.S. Titi, *Global well–posedness of the three–dimensional viscous primitive equations of large scale ocean and atmosphere dynamics,* Ann. of Math., **166** (2007), 245–267.

[2] C. Cao and E.S. Titi, *Global well-posedness of the three-dimensional stratified primitive equations with partial vertical mixing turbulence diffusion*, Comm. Math. Phys. **310** (2012), 537–568.

[3] P.F. Embid and A. J. Majda, *Averaging over fast gravity waves for geophysical flows with arbitrary potential vorticity,* Comm. Partial Diff. Eqs. **21** (1996), 619–658.

[4] J.-Y. Chemin, B. Desjardines, I. Gallagher and E. Grenier, *Mathematical Geophysics. An introduction to rotating fluids and the Navier-Stokes Equations,* Oxford lecture series in Mathematics and Its Applications, Vol. 32. Clarendon Press, Oxford, 2006.

[5] S. Childress, G.R. Ierley, E.A. Spiegel and W.R. Young, *Blow-up of unsteady two-dimensional Euler and Navier-Stokes solutions having stagnation-point form,* J. Fluid Mech. **203** (1989), 1–22.

[6] W. E and B. Engquist, *Blowup of solutions of the unsteady Prandtl's equation,* Comm. Pure Appl. Math. **50** (1997), 1287–1293.

[7] I. Kukavica, R. Temam, V.C. Vicol, M. Ziane, *Local existence and uniqueness for the hydrostatic Euler equations on a bounded domain,* J. Diff. Equ. **250(3)** (2011), 1719–1746.

[8] A. Majda, *Introduction to PDEs and Waves for the Atmosphere and Ocean,* Courant Institute Lecture Notes No. 9 (American Mathematical Society, Providence, RI, 2003).

[9] N. Masmoudi and T. Kwong Wong, *On the H^s theory of hydrostatic Euler equations*, Arch. Rational Mech. Anal. **204** (2012), 231–271.

[10] H. Okamoto, *Well-posedness of the generalized Proudman-Johnson equation without viscosity*, Journal of Mathematical Fluid Mechanics **11** (2009), 46–59.

[11] H. Okamoto and J. Zhu, *Some similarity solutions of the Navier-Stokes equations and related topics* , Taiwanese Journal of Mathematics **4:1** (2000), 65–103.

[12] J. Pedlosky, *Geophysical Fluid Dynamics,* Springer-Verlag, New York, 1987.

[13] M. Renardy, *Ill-posedness of the hydrostatic Euler and Navier-Stokes equations,* Arch. Rational Mech. Anal. **194(3)** (2009), 877–886.

[14] R. Salmon, *Lectures on Geophysical Fluid Dynamics,* Oxford University Press, New York, Oxford, 1998.

Boundary layers for non-linear flows in pipes and channels

Anna L. Mazzucato

We present recent results on the analysis of viscous boundary layers for certain classes of non-linear, 3D incompressible flows in pipes and channels. We use both effective equations for flow correctors, and singular perturbation analysis for a heat equation with variable drift in the small diffusion limit.

Weak Neumann implies H^∞ calculus of the Stokes operator

Matthias Geissert

We show that the Stokes operator admits an $\mathcal{H}^\infty$-calculus on $L^q_\sigma(\Omega)$ provided the Helmholtz decomposition exists in $L^q(\Omega)$ and the boundary of $\Omega \subset \mathbb{R}^n$ is smooth enough. The proof is based on the properties of the Dirichlet-Laplacian and an abstract result by Kalton, Kunstmann and Weis (see [1]). We also discuss some related results. In particular, we discuss maximal L^p-regularity estimates for the Stokes equations.

References

[1] N. Kalton, P. Kunstmann, and L. Weis, *Perturbation and interpolation theorems for the H^∞-calculus with applications to differential operators, Math. Ann.* ***336*** *(2006), 747–801.*

On two approaches to Navier-Stokes problems: frequency analysis and geometric analysis

Tsuyoshi Yoneda

I gave two topics in my talk but I only give one abstract (geometric analysis). The other one (frequency analysis) can be found in [5]. Ohya and Karasudani [4] developed a new wind turbine system that consists of a diffuser shroud with a broad-ring at the exit periphery and a wind turbine inside it. Their experiments show that a diffuser-shaped (not nozzle-shaped) structure can accelerate the wind at the entrance of the body. A strong vortex formation with a low-pressure region is created behind the broad brim. Accordingly, the wind flows into a low-pressure region, the wind velocity is accelerated further near the entrance of the diffuser. We would like to analyze this "wind-lens phenomena" in pure mathematical approach. For the first step, we need to figure out why the diffuser shroud creates vortices easier than the nozzle shroud. In general, creation of a vortex needs separation phenomena near a boundary, and before separating from the boundary, the flow moves toward reverse direction near the boundary against the laminar flow direction. There are several results related to the separation in pure mathematics. Maekawa [3] considered the two-dimensional Navier-Stokes equations in a half plane under the no-slip boundary condition. He established a solution formula for the vorticity equations and got a sufficient condition on the initial data for the vorticity to blow up to the inviscid limit. Ma and Wang [2] provided a characterization of the boundary layer separation of 2-D incompressible viscous fluids. They considered a separation equation linking a separation location and a time with the Reynolds number, the external forcing and the initial velocity field. However, none of the above studies has shown the mechanism behind the reverse flow phenomena rigorously. In this talk we show that a diffuser-shaped boundary induces the reverse flow. Let us be more precise. We consider the two-dimensional Navier-Stokes equation in $\Omega \subset \mathbb{R}^2$ (define Ω later) with mixed no-slip and inflow-outflow conditions on $\partial\Omega$. We need to handle a shape of the boundary $\partial\Omega$ precisely, thus we set parametrized smooth lower and upper boundaries $\varphi = (\varphi_1, \varphi_2), \varphi^* = (\varphi_1^*, \varphi_2^*) : (0, \delta\pi) \mapsto \mathbb{R}^2$ as $|\partial_s \varphi(s)| = |\partial_s \varphi^*(s)| = 1$, $|\partial_s^2 \varphi(s)| = |\partial_s^2 \varphi^*(s)| = 1/\delta$, $\varphi(0) = (0, 0)$, $\varphi^*(0) = (0, 1)$, $\partial_s \varphi(0) = \partial_s \varphi^*(0) = (1, 0)$, $\partial_s^2 \varphi(0) = (0, -1/\delta)$ and $\partial_s^2 \varphi^*(0) = (0, 1/\delta)$. We define the domain Ω as follows:

$$\Omega := \{(x_1, x_2) \in \mathbb{R}^2 : \varphi_2(s) < x_2 < \varphi_2^*(s),\ x_1 = \varphi_1(s) \quad \text{for} \quad 0 < s < \delta\pi\}.$$

We see that $\partial\Omega$ is composed by

$$\partial\Omega = \cup_{0<s<\delta\pi}\varphi(s) \bigcup \cup_{0<s<\delta\pi}\varphi^*(s) \bigcup \cup_{0<x_1<1}\{(0, x)\} \bigcup \cup_{\varphi_2(\delta\pi)<x_2<\varphi_2^*(\delta\pi)}\{(\varphi_1(\delta\pi), x_2)\}.$$

The non-stationary two-dimensional Navier-Stokes equation is expressed as

$$\text{(1)} \qquad \begin{cases} \partial_t u - \nu\Delta u + (u \cdot \nabla)u = -\nabla p, \quad \div u = 0 \quad in \quad \Omega \subset \mathbb{R}^2, \\ u|_{\cup_{0<s<\delta\pi}\varphi(s)} = 0, u|_{\cup_{0<s<\delta\pi}\varphi^*(s)} = 0, \quad \text{and} \quad u|_{t=0} = u_0, \end{cases}$$

where $u = u(x) = u(x, t) = (u^1(x_1, x_2, t), u^2(x_1, x_2, t))$. In this talk we sometimes abbreviate the time t not x. Let $\alpha_1, \alpha_2 > 0$ be coefficients of the inflow profile (Poiseuille flow profile) such that (we do not need to assume outflow profile in this problem):

$$u^1(0, x_2, t) = \alpha_1(t)x_2 - \frac{\alpha_2(t)}{2}x_2^2$$

with $\alpha_1(t) = \alpha_2(t)/2$. Also assume $u^2(0, x_2) = 0$.

Definition 1. (Normal coordinate.) For $s \in [0, \delta\pi]$, let

$$\Phi(s, r) = \Phi_\varphi(s, r) := (\partial_s \varphi(s))^\perp r + \varphi(s).$$

We define $\perp$ as the upward direction.

We assume that the solution is smooth near the lower boundary and the inflow. More precisely, let us choose sufficiently small S and R, and let $\Omega_{S,R} := \cup_{0<s<S, 0<r<R}\Phi(s, r)$. Assume that the solution (u, p) to (1) is in

$$C^\infty([0, T] \times D) \cap C^\infty((0, T) \times \Omega_{S,R}) \quad \text{for any} \quad D \Subset \Omega_{S,R}.$$

In this setting, we can avoid interior blow-up by taking sufficiently small R. Thus we only need to care boundary regularity not interior regularity (our method is applicable to the 3D case). For the boundary regularity, see [1] for example. Boundary layers appear on the surface of bodies in viscous flow because the fluid seems to stick to the boundary $\partial\Omega$. Right at the surface the flow has zero relative speed and this fluid transfers momentum to adjacent layers through the action of viscosity. To handle such physical phenomena in mathematics, we need to define "laminar flow"

Definition 2. (Laminar flow.) u has "laminar flow" iff u is smooth, $|u| \neq 0$ and the flow u is to the rightward direction (laminar flow direction), namely,

$$0 < \left\langle \partial_s \varphi(s), \frac{u}{|u|}(\Phi(s, r)) \right\rangle \leq 1,$$

for sufficiently small s and r, where $\langle \cdot, \cdot \rangle$ means usual inner product.

We mainly consider a topological shape of the laminar flow near the lower boundary. In this case, one of the three situations only occur (for fixed time t): diffusing-parallel laminar flow, concentrating laminar flow and topologically changing flow (inducing the reverse flow, limiting case).

Definition 3. For $s, r > 0$, let

$$u^s(s,r) := \langle u(\Phi(s,r)), \partial_s\varphi(s)\rangle, \quad u^r(s,r) := \langle u(\Phi(s,r)), (\partial_s\varphi(s))^\perp\rangle$$

and

$$\mathcal{L}(s,r) := \frac{u^r(s,r,t)}{u^s(s,r,t)} \cdot \frac{r+\delta}{\delta}.$$

- Diffusing-parallel laminar flow: We call diffusing-parallel laminar flow iff
$$\lim_{r\to 0} \partial_r \mathcal{L}(s,r) \geq 0 \quad \text{for sufficiently small} \quad s.$$
- Concentrating laminar flow: We call concentrating laminar flow iff
$$\lim_{r\to 0} \partial_r \mathcal{L}(s,r) < 0. \quad \text{for sufficiently small} \quad s.$$
- Topologically changing flow (inducing the reverse flow, limiting case): We say topologically changing flow iff there are sufficiently small $s > 0$ and $r > 0$ such that $u^s(\Phi(s,r)) = 0$.

We note that if the initial datum has diffusing laminar flow, the solution does no have concentrating laminar flow near the initial time, more precisely, there is no $\{t_j\}_j$ such that $t_j \to 0$ and each $u(t_j)$ has concentrating laminar flow. Thus it is reasonable to assume that u persists diffusing-parallel laminar flow for $[0, T_d)$ $(T_d > 0)$. The following is the main theorem.

Theorem 4. *Assume u persists diffusing-parallel laminar flow for $[0, T_d)$ $(T_d > 0)$. Then the following three assertions are hold:*

- *u cannot keep diffusing-parallel laminar flow for infinite time, namely $T_d \neq \infty$.*
- *u cannot create concentrating laminar flow from diffusing-parallel laminar flow. More precisely, assume that u has concentrating laminar flow at time T_c. Then $T_d \neq T_c$.*
- *Topologically changing flow occurs at finite time, namely, there are $s > 0$ and $r > 0$ (near the boundary) such that $\lim_{t\to T_d} u^s(\Phi(s,r),t) = 0$.*

Remark 5. Our result should be closely related to the Ma and Wang's work [2]. They gave an example to demonstrate how the external forcing with reverse orientation to the initial velocity field leads to structural bifurcation and boundary layer separation. In other words, our work should be closely related to their "reverse orientation".

References

[1] S. Gustafson, K. Kang and T-P Tsai, *Regularity criteria for suitable weak solutions of the Navier-Stokes equations near the boundary.* J. Diff. Eq. 226 (2006), 594–618.
[2] T. Ma and S. Wang, *Boundary layer separation and structural bifurcation for 2-D incompressible fluid flows. Partial differential equations and applications.* Discrete Contin. Dyn. Syst. 10 (2004), 459–472.
[3] Y. Maekawa, *Solution formula for the vorticity equations in the half plane with application to high vorticity creation at zero viscosity limit.* Dept. Math, Hokkaido Univ. EPrints Server, no. 992.
[4] Y. Ohya and T. Karasudani, *A Shrouded Wind Turbine Generating High Output Power with Wind-lens Technology.* Energies 3 (2010), 634–649.
[5] M. Yamada and T. Yoneda, *Resonant interaction of Rossby waves in two-dimensional flow on a beta plane.* to appear in Physica D. Hokkaido Univ. Preprint Series in Math., no. 999

New developments in geostrophic turbulence and its implications for climate modeling and weather predictability

Joseph Tribbia

One of the many areas in geophysical fluid dynamics that is reflected in the theory of how we model dissipation in the climate system is the theory of two-dimensional and quasi-geostrophic turbulence and its consequent impact on atmospheric flow. Upscale energy and and downscale enstrophy cascades have been observed in the atmosphere along with the -3 power law predicted in two-dimensional turbulence theory developed by Batchelor and Kraichnan in the late 1960s. A consequence of this observational finding is the fact that, unlike three-dimensional turbulence in which the eddy turnover time decreases with eddy length scale, in two-dimensional and quasi-geostrophic turbulence the eddy turnover time is constant independent of eddy length scale in the enstrophy cascading range. A further consequence of this is that the Rossby number is constant through the enstrophy cascade. This implies that instabilities which depend on ageostrophic processes are restricted because the scaling laws which imply balanced, quasi-geostrophic dynamics are valid at all length scales. Recent results show, however, even given that all of the above statements are true and maintained in the dynamics, there is a mechanism through which quasi-geostrophic turbulence becomes inconsistent and develops the seeds of its own destruction at small scales.

Stochastic Three-Dimensional Rotating Navier-Stokes Equations: Averaging, Convergence and Regularity

Alex Mahalov

We consider stochastic three-dimensional rotating Navier-Stokes equations and prove averaging theorems for stochastic problems in the case of strong rotation. Regularity results are established by bootstrapping from global regularity of the limit stochastic equations and convergence theorems. The energy injected in the system by the noise is large, the initial condition has large energy, and the regularization time horizon is long. Regularization is the consequence of a precise mechanism of relevant three-dimensional nonlinear dynamics. We establish multi-scale averaging and convergence theorems for the stochastic dynamics.

References

[1] F. Flandoli and A. Mahalov, *Stochastic 3D Rotating Navier-Stokes equations: averaging, convergence and regularity*, Archive for Rational Mechanics and Analysis, **205**, Issue 1 (2012), p. 195–237.

Rayleigh-Bénard convection: Bounds on the Nusselt number

Felix Otto
(joint work with Christian Seis and Camilla Nobili)

This is about Rayleigh-Bénard convection, which arises when a viscous fluid is heated from the bottom and cooled from the top. More precisely, it is about turbulent Rayleigh-Bénard convection, when there are temperature boundary layers and plumes in the bulk. In the infinite Prandtl Number limit and with help of the Boussinesq equation it is modeled by the coupling of an advection diffusion equation for the temperature T to the Stokes equations for the fluid velocity u:

$$\partial_t T + \nabla \cdot (Tu) - \triangle T = 0, \quad -\triangle u + \nabla p = T\begin{pmatrix}0\\1\end{pmatrix}, \quad \nabla \cdot u = 0.$$

The Nusselt number Nu is the time-space averaged upwards heat flux. Experiments and numerical simulations show that the Nusselt number is independent of the container height H, where we impose the temperature and no-flux boundary conditions:

$$T = 1 \quad \text{for } z = 0, \quad T = 0 \quad \text{for } z = H, \quad u = 0 \quad \text{for } z = 0, H.$$

Constantin & Doering [1] showed by a (logarithmically failing) maximal regularity estimate in $H^{2,\infty}$ for the Stokes equation that $Nu \lesssim \log^{2/3} H$ (which can be easily improved to $Nu \lesssim \log^{1/3} H$, see [4, Lemma 4]). Doering & O.& Reznikoff [2] showed by the "background field method" that $Nu \lesssim \log^{1/3} H$ (which we improved to $Nu \lesssim \log^{1/15} H$ [4, Theorem 1]). The background field method is appealing, since it yields an inequality (believed to be an equality) for Nu by the solution of a saddle point problem that encapsulates the idea of marginal stability of the boundary layer. However, we show that

- $Nu \lesssim \log^{1/15} H$ is optimal within the background field method, see [3]
- A combination of the methods yields the stronger bound $Nu \lesssim \log^{1/3} \log H$, see [4, Theorem 2]

Hence the background field method has *no physical meaning* (in particular the optimal background temperature profile is unrelated to the horizontally averaged temperature profile) because it does not even yield an optimal bound on Nu.

This is joint work with Christian Seis and Camilla Nobili.

References

[1] Constantin, Peter; Doering, Charles R. Infinite Prandtl number convection. *J. Statist. Phys.* **94** (1999), no. 1-2, 159–172.
[2] Doering, Charles R.; Otto, Felix; Reznikoff, Maria G. Bounds on vertical heat transport for infinite-Prandtl-number Rayleigh-Bnard convection. (English summary) *J. Fluid Mech.* **560** (2006), 229–241.
[3] Nobili, Camilla; Otto, Felix, in preparation
[4] Otto, Felix; Seis, Christian. Rayleigh-Bénard convection: improved bounds on the Nusselt number. *J. Math. Phys.* **52** (2011), no. 8

Rossby waves trapped by quantum mechanics

Thierry Paul

Rossby and Poincaré waves appear naturally in the study of large scale oceanography. Poincaré waves (PW), of period of the order of a day, are fast dispersive waves and are due to the rotation of the Earth through the Coriolis force. Much slower, Rossby waves (RW) are sensitive to the variations of the Coriolis parameter, propagate only eastwards and remain localized for long period of times. We would like here to report on some new results, obtained in collaboration with C. Cheverry, I. Gallagher and L. Saint-Raymond [1, 2, 3] studying this phenomenon, dispersivity of PW and trapping of RW, as a consequence of the study of the oceanic waves in a shallow water flow subject to strong wind forcing and rotation, linearized around a inhomogeneous (non zonal) stationary profile. The main feature of our results, compared to earlier ones, [5, 7, 4] to quote only very few of them, consists in the fact that we abandon both the betaplane approximation (constant Coriolis force) and the zonal aspect (non dependence w.r.t. the latitude) of the convection term (coupling with the wind).

After some scalings and dimensional homogenizations, the Saint-Venant system of equations for the variations η, u near a constant value of the height $\bar{h}$ and divergence free stationary profile of velocity $\bar{u}$ takes the form (see [2, 3] for details)

$$\begin{aligned} \partial_t \eta + \tfrac{1}{\varepsilon}\nabla\cdot u + \bar{u}\cdot\nabla\eta + \varepsilon^2\nabla\cdot(\eta u) &= 0 \\ \partial_t u + \tfrac{1}{\varepsilon^2} b u^{\perp} + \tfrac{1}{\varepsilon}\nabla\eta + \bar{u}\cdot\nabla u + u\cdot\nabla\bar{u} + \varepsilon^2 u\cdot\nabla u &= 0 \end{aligned} \tag{1}$$

where b is the horizontal component of the Earth rotation vector normalized to one and ε^{-1} measures the Coriolis force.

The linear version of (1) reads (here $D := \frac{1}{i}\partial$ and $x = (x_1, x_2) \in \mathbb{R}^2$):

$$\varepsilon^2 i\partial_t \mathbf{v} + A(x, \varepsilon D, \varepsilon)\mathbf{v} = 0, \qquad \mathbf{v} = (v_0, v_1, v_2) = (\eta, u_1, u_2), \tag{2}$$

with the linear propagator

$$A(x, \varepsilon D, \varepsilon) := i\begin{pmatrix} \varepsilon\bar{u}\cdot\varepsilon\nabla & \varepsilon\partial_1 & \varepsilon\partial_2 \\ \varepsilon\partial_1 & \varepsilon\bar{u}\cdot\varepsilon\nabla + \varepsilon^2\partial_1\bar{u}_1 & -b + \varepsilon^2\partial_2\bar{u}_1 \\ \varepsilon\partial_2 & b + \varepsilon^2\partial_1\bar{u}_2 & \varepsilon\bar{u}\cdot\varepsilon\nabla + \varepsilon^2\partial_2\bar{u}_2 \end{pmatrix}. \tag{3}$$

We will concentrate on (2) with the condition that, essentially, b is increasing at infinity with all derivatives bounded in module by $|b|$ and only non degenerate critical points. Moreover $\bar{u}$ will have to be smooth with compact support.

A simplified version of our main result reads as follows (see [3] for details).

Theorem 1. *Under certain microlocalization properties of the initial condition, the solution $\mathbf{v}_\varepsilon(t) = \mathbf{v}_\varepsilon(t,.)$ of (2) decomposes on two Rossby and Poincaré vector fields $\mathbf{v}_\varepsilon(t) = \mathbf{v}_\varepsilon^R(t) + \mathbf{v}_\varepsilon^P(t)$ satisfying*

- $\forall t > 0,\ \forall \Omega$ *compact set of* $\mathbb{R}^2$,

$$\|\mathbf{v}_\varepsilon^P(t)\|_{L^2(\Omega)} = O(\varepsilon^\infty) \tag{4}$$

- $\exists \Omega$ *bounded set of* $\mathbb{R}$ *such that,* $\forall t > 0$

$$\|\mathbf{v}_\varepsilon^R(t)\|_{L^2(\mathbb{R}_{x_1}\times(\mathbb{R}/\Omega)_{x_2})} = O(\varepsilon^\infty). \tag{5}$$

Theorem 1 shows clearly the different nature of the two type of waves: dispersion for Poincaré and confining in x_2 for Rossby. The method of proving Theorem 1 will consist in diagonalizing the "matrix" $A(x, \varepsilon D, \varepsilon)$. Such diagonalization, if possible, would immediately solve (2) by reducing it to the form $\varepsilon^2 i\partial_t \mathbf{u} + D(x, \varepsilon D, \varepsilon)\mathbf{u} = 0$ with $D(x, \varepsilon D, \varepsilon)$ diagonal and solving it component by component. Diagonalizing matrices with operator valued entries is not a simple task, but our next result will show how to achieve it modulo ε^∞ in the case of matrices with ε-semiclassical type operators entries.

To any (regular enough) function $\mathcal{A}_\varepsilon \sim \sum_0^\infty \varepsilon^l \mathcal{A}_l$ on $\mathbb{R}^{2n} = T^*\mathbb{R}^n$, possibly matrix valued, we associate the operator A_ε (densely defined) on $L^2(\mathbb{R}^n)$ defined by:

$$f \to A_\varepsilon f,\ (A_\varepsilon f)(x) = \int \mathcal{A}_\varepsilon(\tfrac{x+y}{2}, \xi) e^{i\frac{\xi(x-y)}{\varepsilon}} f(y) \tfrac{dy d\xi}{\varepsilon^n}.$$

$\mathcal{A}_\varepsilon$ is called the symbol of A_ε and A_ε the (Weyl) quantization of $\mathcal{A}_\varepsilon$.

Let A_ε be such $N \times N$ operator valued matrix of symbol $\mathcal{A}_\varepsilon \sim \sum_0^\infty \varepsilon^l \mathcal{A}_l$. We will suppose that $\mathcal{A}_0(x,\xi)$ is Hermitian and therefore is diagonalizable (at each point) by $\mathcal{U} = \mathcal{U}(x,\xi)$, $\mathcal{U}^*\mathcal{A}_0\mathcal{U} = \operatorname{diag}(\lambda_1, \dots, \lambda_N) := \mathcal{D}$. We will suppose moreover that

$$\forall (x,\xi),\ \forall i \neq j,\ |\lambda_i(x,\xi) - \lambda_i(x,\xi)| \geq C > 0. \tag{6}$$

Theorem 2 ([3])**.** *There exist V_ε semiclassical operator and D_ε diagonal (w.r.t. the $N \times N$ structure) such that*

$$V_\varepsilon^{-1} A_\varepsilon V_\varepsilon = D_\varepsilon + O(\varepsilon^\infty) \quad \textit{and} \quad V_\varepsilon^* V_\varepsilon = Id_{L^2(\mathbb{R}^n, \mathbb{C}^N)} + O(\varepsilon^\infty) = V_\varepsilon V_\varepsilon^* + O(\varepsilon^\infty).$$

Moreover $D_\varepsilon = D + \varepsilon D_1 + O(\varepsilon^2)$, where D is the Weyl quantization of $\mathcal{D}$ and D_1 is the diagonal part of $(\Delta_1 - \frac{DI_1 + I_1 D}{2})$ with (U being the Weyl quantization of $\mathcal{U}$)

$$\Delta_1 = \frac{U^* A_\varepsilon U - D}{\varepsilon}\Big|_{\varepsilon=0}, \qquad I_1 = \frac{U^*U - Id_{L^2(\mathbb{R}^n, \mathbb{C}^N)}}{\varepsilon}\Big|_{\varepsilon=0}. \tag{7}$$

Let us go back now to the case given by (3). One checks easily that $A(x, \varepsilon D, \varepsilon)$ is of semiclassical type. Its symbol is

(8)

$$\mathcal{A}(x,\xi,\varepsilon) = \begin{pmatrix} \varepsilon\bar{u}\cdot\xi & \xi_1 & \xi_2 \\ \xi_1 & \varepsilon\bar{u}\cdot\xi + \varepsilon^2\partial_1\bar{u}_1 & -b + \varepsilon^2\partial_2\bar{u}_1 \\ \xi_2 & b + \varepsilon^2\partial_1\bar{u}_2 & \varepsilon\bar{u}\cdot\xi + \varepsilon^2\partial_2\bar{u}_2 \end{pmatrix} = \begin{pmatrix} 0 & \xi_1 & \xi_2 \\ \xi_1 & 0 & -b \\ \xi_2 & b & 0 \end{pmatrix} + O(\varepsilon).$$

The spectrum of the leading order $\mathcal{A}(x,\xi,0)$ is $\{-\sqrt{\xi^2+b^2(x_2)},0,+\sqrt{\xi^2+b^2(x_2)}\}$. Therefore Condition (6) is satisfied only if $\xi^2+b^2(x)\geq C>0$ which correspond to the microlocalization condition in Theorem 1. Theorem 2 gives, after a tedious computation, that $A(x,\varepsilon D,\varepsilon)$ is unitary equivalent (modulo ε^2) to the diagonal matrix $\operatorname{diag}(T^+,T^R,T^-)$ where $T^\pm$ is the Weyl quantization of $\tau^\pm(x,\xi):=\pm\sqrt{\xi^2+b^2(x_2)}$ and T^R is the quantization of the Rossby Hamiltonian $\tau^R(x,\xi):=\varepsilon(\frac{\xi_1 b'(x_2)}{\xi^2+b^2(x)}+\bar{u}(x)\cdot\xi)$.

Under the betaplane approximation, $b(x_2)=\beta x_2$, the Hamiltonians $T^\pm$ are exactly solvable and one shows by hand the dispersive effect for the Poincaré waves. In our situation this doesn't work, and because of the ε^2 term in the r.h.s. of (2) the method of characteristics does not apply. A general argument, inherited form quantum mechanics will provide us the solution. First we remark that the Poisson bracket $\{\tau^\pm,x_1\}=\xi_1/\tau^\pm$. This indicates, at a classical level, that $\dot{x}_1$ has a sign for each Poincaré polarization, leading to no return travel. The following theory, due to Eric Mourre, gives the "quantum" equivalent of this argument.

Let H and A be two self-adjoint operators on a Hilbert space $\mathcal{H}$ such that: the intersection of the domains of H and A is dense in the domain of H, $t\mapsto e^{itA}$ maps the domain of H to itself and $\sup_{[0,1]}\|He^{itA}\varphi\|<\infty$ for φ in the domain of H, and $i[H,A]$ is bounded from below, closable and the domain of its closure contains the domain of H. Finally let us suppose the following

Positivity condition: there exist $\theta>0$ and an open interval Δ of $\mathbb{R}$ such that if E_Δ is the corresponding spectral projection of H, then

$$E_\Delta i[H,A]E_\Delta\geq\theta E_\Delta, \tag{9}$$

namely $i[H,A]>0$ on any spectral interval of H contained in Δ.

Theorem 3 (E. Mourre '80, [6]). *For any integer $m\in\mathbb{N}$ and for any $\theta'\in]0,\theta[$, there is a constant C such that*

$$\|\chi_-(A-a-\theta' t)e^{-iHt}g(H)\chi_+(A-a)\|\leq Ct^{-m}$$

where $\chi_\pm$ is the characteristic function of $\mathbb{R}^\pm$, g is any smooth compactly supported function in Δ, and the above bound is uniform in $a\in\mathbb{R}$.

In other words, to talk in the quantum langage, if one starts with an initial condition φ such that "$A\geq a$" and the positivity condition (9) holds, after any time t the "probability" that "$A\leq\theta' t$" is of order t^{-m}. In particular, as $t\to\infty$ the solution $e^{-iHt}\varphi$ escape from any compact spectral region of A.

Taking $A=x_1$, Theorem 3 gives, after verification that it applies, exactly the "Poincaré" part of Theorem 1. The "Rossby part" is given by using the bicharacteristic method and a small computation done in [3] which shows that bicharacteristics are trapped in finite regions in the latitude (x_2) direction.

Let us mention to finish that the nonlinear terms can be handled by using a "L^∞" Gronwall Lemma and working in some anisotropic and semiclasical Sobolev spaces, so that the solution of (1) is close to the one of (2) as $\varepsilon\to 0$.

References

[1] C. Cheverry, I. Gallagher, T. Paul & L. Saint-Raymond, Trapping Rossby waves, *Notes aux Comptes-Rendus de l'Académie des Sciences de Paris*, **347** (2009), no. 2, pages 879-884.
[2] C. Cheverry, I. Gallagher, T. Paul & L. Saint-Raymond, Semiclassical and spectral analysis of oceanic waves, *Duke Math. Journal*, **161** (2012), pages 845-892.
[3] I. Gallagher, T. Paul & L. Saint-Raymond, On the propagation of oceanic waves driven by a macroscopic current, "Nonlinear Partial Differential Equations-The Abel Symposium 2010" H.Holden, K.Karlsen (Eds.), Abel Symposia, **7** (2012), Springer .
[4] I. Gallagher & L. Saint-Raymond, Mathematical study of the betaplane model: equatorial waves and convergence results. Mém. Soc. Math. Fr. (N.S.) **107** (2006).
[5] E. Grenier: Oscillatory Perturbations of the Navier–Stokes Equations, *Journal de Mathématiques Pures et Appliquées*, **76** (1997), pages 477-498.
[6] A. Jensen, E. Mourre & P. Perry, Multiple commutator estimates and resolvent smoothness in quantum scattering theory, *Annales de l'IHP, section A*, **41** 2 (1984), pages 297–225.
[7] S. Schochet: Fast Singular Limits of Hyperbolic PDEs, *Journal of Differential Equations*, **114** (1994), pages 476-512.

On the inviscid limit problem for viscous incompressible flows in the half plane

Yasunori Maekawa

We consider the Navier-Stokes equations for viscous incompressible flows in the half plane under the no-slip boundary conditions:

$$(\mathrm{NS}_\nu) \qquad \begin{cases} \partial_t u - \nu\Delta u + u\cdot\nabla u + \nabla p = 0 \qquad t>0,\ x\in\mathbb{R}^2_+, \\ \mathrm{div}\ u = 0 \qquad t\geq 0,\ x\in\mathbb{R}^2_+, \\ u = 0 \qquad t\geq 0,\ x\in\partial\mathbb{R}^2_+, \\ u|_{t=0} = a \qquad x\in\mathbb{R}^2_+. \end{cases}$$

Here $\mathbb{R}^2_+ = \{(x_1,x_2)\in\mathbb{R}^2 \mid x_2>0\}$ and ν is the kinematic viscosity which is assumed to be a positive constant, while $u = u(t,x) = (u_1(t,x), u_2(t,x))$ and $p = p(t,x)$ denote the velocity field and the pressure field, respectively. We will use the standard notations for derivatives; $\partial_t = \partial/\partial t$, $\partial_j = \partial/\partial x_j$, $\Delta = \sum_{j=1}^2 \partial_j^2$, div $u = \sum_{j=1}^2 \partial_j u_j$, and $u\cdot\nabla u = \sum_{j=1}^2 u_j\partial_j u$.

The behavior of viscous incompressible flows at the inviscid limit is a classical issue in the fluid dynamics. When the fluid domain has no boundary it is well known that the solution of the Navier-Stokes equations converges to the one of the Euler equations. However, in the presence of nontrivial boundary one is faced with a serious difficulty in this problem even in the two-dimensional case if the no-slip boundary condition is imposed on the velocity field. This is due to the appearance of the boundary layer, whose formation is formally explained by Prandtl's theory and the thickness of the boundary layer is estimated as the square root of the viscosity. So far the rigorous verification of the formal asymptotic expansion proposed by Prandtl is achieved only within the analytic framework [1, 7, 8]. More precisely, in [1, 7, 8] it is proved that for analytic initial data the solution of (NS_ν) converges to the one of the Euler equations outside the boundary layer and to the one of the Prandtl equations in the boundary layer. However, it is still an open problem in the Sobolev framework even for a short time period.

In the fluid dynamics the vorticity field, i.e., the curl of the velocity field, is also an important quantity and useful in understanding various phenomena. At the inviscid limit it is recognized that the vorticity is highly produced in the boundary layer and forms a vortex sheet (or line in the two dimension) along the boundary. However, under the no-slip boundary condition on the velocity field the study of the vorticity field is still less developed mathematically, since the vorticity is subject to a nonlocal and nonlinear boundary condition from which it is not easy to derive useful informations. This is contrasting with the case of the whole plane, where the detailed analysis has been established; e.g. [5, 2]. In the case of the half plane the situation is relaxed a little, since the solution formula is available for the linearized problem [3].

The object of research in this talk is the inviscid limit behavior of solutions to (NS_ν) by using the vorticity formulation when the initial vorticity is located away from the boundary. In particular, we do not impose the analyticity of initial data in the whole fluid domain. This class of initial data includes a dipole-type localized vortex, which is often used in numerical works as a benchmark to investigate the interaction between the vorticity created on the boundary and the vorticity away from the boundary which is originated from the initial one; cf. [6]. It should be noted here that, even if there is no vorticity near the boundary at the initial time, the vorticity is immediately created there and forms a vortex line along the boundary in positive time. In particular, we have to deal with the boundary layer also in this case. We give a rigorous description of the asymptotic expansion at $\nu \to 0$, which is verified at least for a time period $0 \le t \le \mathcal{O}(\min\{d_0, 1\})$, where $d_0 > 0$ is the distance between the support of the initial vorticity and the boundary. The details of the results are available at [4].

References

[1] K. Asano, Zero viscosity limit of incompressible Navier-Stokes equations II, Surikaisekikenkyusho Kokyuroku, Mathematical analysis of fluid and plasma dynamics I (Kyoto, 1986) **656** (1988) 105-128.

[2] M.-H. Giga, Y. Giga, and J. Saal, *Nonlinear Partial Differential Equations: Asymptotic Behavior of Solutions and Self-Similar Solutions*, Birkhäuser, Boston-Basel-Berlin, 2010.

[3] Y. Maekawa, Solution formula for the vorticity equations in the half plane with application to high vorticity creation at zero viscosity limit, Advances in Differential Equations, **18** (2013) 101-146.

[4] Y. Maekawa, On the inviscid limit problem of the vorticity equations for viscous incompressible flows in the half plane. Preprint.

[5] A. J. Majda, A. L. Bertozzi, *Vorticity and incompressible flows*, Cambridge University Press, Cambridge, 2002.

[6] R. Nguyen van yen, M. Farge, K. Schneider, Energy dissipating structures produced by walls in two-dimensional flows at vanishing viscosity, Phys. Rev. Lett. **106** (2011) 184502.

[7] M. Sammartino, R. E. Caflisch, Zero viscosity limit for analytic solutions of the Navier-Stokes equation on a half-space I, Existence for Euler and Prandtl equations. Comm. Math. Phys. **192** (1998) 433-461.

[8] M. Sammartino, R. E. Caflisch, Zero viscosity limit for analytic solutions of the Navier-Stokes equation on a half-space II, Construction of the Navier-Stokes solution. Comm. Math. Phys. **192** (1998) 463-491.

Optimal mixing and optimal stirring for fixed energy, fixed power, and fixed palenstrophy flows

Evelyn Lunasin

The advection of a substance by an incompressible flow is important in many physical settings. This process often involves complex evolving structures of wide range of space and time scales. We concentrate on the case of scalar advection where the transported quantity is passive, so has negligible feedback on the flow. We address the following question: Given an initial tracer distribution what incompressible flow field, satisfying certain reasonable amplitude constraints, should be imposed that will stir the scalar quantity in an optimal manner. We will discuss how one can quantify the degree of mixedness of the passive scalar field, what we mean by optimal stirring and what is the quantity that needs to be optimized in the stirring process. We will also discuss what are the relevant constraints on the flows.

We focus on the optimal stirring strategy recently proposed by Lin,Thiffeault and Doering (2011). We then show an explicit example demonstrating finite-time perfect mixing with a finite energy constraint on the stirring flow. On the other hand, if the two-dimensional incompressible flow is constrained to have a particular smoothness property finite-time perfect mixing is ruled out. The case of finite power constraint is an open problem. We discuss some partial results and discuss related problems from other areas of analysis.

The Navier-Stokes equations in spaces of bounded functions

Ken Abe

We give a local existence theorem for the Navier-Stokes equations on L^∞. This is known for $\mathbf{R}^n$ and $\mathbf{R}^n_+$ but unknown for domains with non-trivial boundaries. We focus on bounded domains and also estimate the maximum of solutions near blow-up time.

References

[1] K. Abe, *The Navier-Stokes equations in spaces of bounded functions, in preparation*

Dynamics of Nematic Liquid Crystal Flows: The Quasilinear Approach

Katharina Schade
(joint work with Matthias Hieber, Manuel Nesensohn and Jan Prüss)

Consider the (simplified) Leslie-Ericksen model for the flow of nematic liquid crystals in a bounded domain $\Omega \subset \mathbb{R}^n$ for $n > 1$. We develop a complete dynamic theory for these equations, analyzing the system as a quasilinear parabolic evolution equation in an L_p-L_q-setting. First, the existence of a unique local strong solution is proved. This solution extends to a global strong solution, provided the initial data are close to an equilibrium or the solution is eventually bounded in the natural norm of the underlying state space. In this case the solution converges exponentially to an equilibrium. Moreover, the solution is shown to be real analytic, jointly in time and space.

References

[1] Leslie, F.M., *Some constitutive equations for liquid crystals*, Arch. Rational Mech. Anal. **28** (1968), no. 4, 265–283.
[2] Lin, F.-H. and Liu, C., *Nonparabolic dissipative systems modeling the flow of liquid crystals*, Comm. Pure Appl. Math. **48** (2009), no. 5, 501–537.
[3] Köhne, M., Prüss, J. and Wilke, M, *On quasilinear parabolic evolution equations in L_p-spaces*, J. Evol. Eqns **9** (2009), no. 10.
[4] Prüss, J., Simonett, G. and Zacher, R., *On convergence of solutions to equilibria for quasilinear parabolic problems*, J. Differential Equations **246** (2009), no. 10, 3902–3931.
[5] Hieber, M., Nesensohn, M., Prüss, J. and Schade, K., *Dynamics of Nematic Liquid Crystal Flows: the Quasilinear Approach*, preprint.

Global solutions for the Navier-Stokes equations in the rotational framework

Tsukasa Iwabuchi
(joint work with Ryo Takada)

We consider the initial value problem for the Navier-Stokes equations with the Coriolis force

$$(\text{NSC}) \qquad \begin{cases} \dfrac{\partial u}{\partial t} - \Delta u + \Omega e_3 \times u + (u \cdot \nabla)u + \nabla p = 0 & \text{in } \mathbb{R}^3 \times (0,\infty), \\ \text{div } u = 0 & \text{in } \mathbb{R}^3 \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in } \mathbb{R}^3, \end{cases}$$

where $\Omega \in \mathbb{R}$, $e_3 = (0,0,1)$ and $u = u(x,t) = (u_1(x,t), u_2(x,t), u_3(x,t))$ and $p = p(x,t)$ denote the unknown velocity field and the unknown pressure of the fluid at the point $(x,t) \in \mathbb{R}^3 \times (0,\infty)$, respectively. The purpose of this talk is to show the existence and the uniqueness of the global solutions to (NSC) in the homogeneous Sobolev spaces $\dot{H}^s(\mathbb{R}^3)$ $(s \geq 1/2)$.

For the existence of solutions to (NSC), Babin-Mahalov-Nicolaenko [1, 2, 3] showed the existence of global solutions and the regularity of the solutions to

(NSC) for the periodic initial data with large $|\Omega|$. Chemin-Desjardins-Gallagher-Grenier [5] proved that for any initial data $u_0 \in H^{\frac{1}{2}}(\mathbb{R}^3)^3$, there exists a positive parameter Ω_0 such that for every $\Omega \in \mathbb{R}$ with $|\Omega| \geq \Omega_0$ there exists a unique global solution. On the other hand, Giga-Inui-Mahalov-Saal [7] showed the existence of global solutions for small initial data $u_0 \in FM_0^{-1}(\mathbb{R}^3)^3$, where the condition of smallness is independent of the speed of the rotation Ω, and $FM_0^{-1}(\mathbb{R}^3)$ is scaling invariant to (NSC) with $\Omega = 0$. Indeed, for the solution u to (NSC) with $\Omega = 0$, let $u_\lambda(x,t) := \lambda u(\lambda x, \lambda^2 t)$ for $\lambda > 0$. Then, u_λ is also a solution to (NSC) with $\Omega = 0$ and we have $\|u_\lambda(\cdot,0)\|_{FM_0^{-1}} = \|u(\cdot,0)\|_{FM_0^{-1}}$ for all $\lambda > 0$. On such other results of global solutions for small initial data, Hieber-Shibata [8] studied in the Sobolev space $H^{\frac{1}{2}}(\mathbb{R}^3)$, Konieczny-Yoneda [11] studied in the Fourier-Besov space $F\dot{B}^{2-\frac{3}{p}}_{p,\infty}(\mathbb{R}^3)$ with $1 < p \leq \infty$. On the well-posedness for (NSC) with $\Omega = 0$ in the scaling invarint spaces, we refer to Fujita-Kato [6], Kato [9], Kozono-Yamazaki [12], Koch-Tataru [10].

In this talk, we would like to show that it is possible to characterize the sufficient speed of rotation Ω with the initial data. The sufficient speed of the rotation is characterized with the norm of initial data $u_0 \in \dot{H}^s(\mathbb{R}^3)$ for the case $s > 1/2$ and each precompact set of $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$, which the initial data belong to, for the case $s = 1/2$.

References

[1] A. Babin, A. Mahalov, B. Nicolaenko, *Regularity and integrability of 3D Euler and Navier-Stokes equations for rotating fluids*, Asymptot. Anal., **15** (1997), 103–150.

[2] A. Babin, A. Mahalov, B. Nicolaenko, *Global regularity of 3D rotating Navier-Stokes equations for resonant domains*, Indiana Univ. Math. J., **48** (1999), 1133–1176.

[3] A. Babin, A. Mahalov, B. Nicolaenko, *3D Navier-Stokes and Euler equations with initial data characterized by uniformly large vorticity*, Indiana Univ. Math. J., **50** (2001), 1–35.

[4] H. Brezis, *Remarks on the preceding paper by M. Ben-Artzi, "Global solutions of two-dimensional Navier-Stokes and Euler equations*", Arch. Rational Mech. Anal., **128** (1994), no. 4, 359–360.

[5] J.-Y. Chemin, B. Desjardins, I. Gallagher, E. Grenier, "Mathematical geophysics," Oxford Lecture Series in Mathematics and its Applications, vol. 32, The Clarendon Press Oxford University Press, Oxford, 2006.

[6] H. Fujita, T. Kato, *On the Navier-Stokes initial value problem I*, Arch. Rational Mech. Anal., **16** (1964), 269–315.

[7] Y. Giga, K. Inui, A. Mahalov, J. Saal, *Uniform global solvability of the rotating Navier-Stokes equations for nondecaying initial data*, Indiana Univ. Math. J., **57** (2008), 2775–2791.

[8] M. Hieber, Y. Shibata, *The Fujita-Kato approach to the Navier-Stokes equations in the rotational framework*, Math. Z., **265** (2010), 481–491.

[9] T. Kato, *Strong L^p-solutions of the Navier-Stokes equation in $\mathbb{R}^m$, with applications to weak solutions*, Math. Z., **187** (1984), 471–480.

[10] H. Koch, D. Tataru, *Well-posedness for the Navier-Stokes equations*, Adv. Math., **157** (2001), 22–35.

[11] P. Konieczny, T. Yoneda, *On dispersive effect of the Coriolis force for the stationary Navier-Stokes equations*, J. Differential Equations, **250** (2011), 3859–3873.

[12] H. Kozono, M. Yamazaki, *Semilinear heat equations and the Navier-Stokes equation with distributions in new function spaces as initial data*, Comm. Partial Differential Equations, **19** (1994), 959–1014.

Global well-posedness of an inviscid 3D pseudo-Hasegawa-Mima model

Aseel Farhat
(joint work with Chongsheng Cao and Edriss S. Titi)

In plasma physics, the 3D Hasegawa-Mima model is one of the most fundamental models that describe the electrostatic drift waves. In the context of geophysical fluid dynamics, the 3D Hasegawa-Mima model appears as a simplified model of a reduced Rayleigh-Bénard convection model that describes the motion of a fluid heated from below. Investigating the inviscid 3D Hasegawa-Mima model is challenging even though the equations look simpler than the 3D Euler equations. Inspired by these models, we introduce and study an analytical model that has a nicer mathematical structure which we call the pseudo-Hasegawa-Mima model. We prove a global well-posedness result for the inviscid 3D pseudo-Hasegawa-Mima model. These results are one of the first results related to the 3D Hasegawa-Mima equations.

References

[1] C. Cao, A. Farhat, and E. S. Titi, *Global well-posedness of an inviscid three-dimensional pseudo-Hasegawa-Mima model*, Commun. Math. Phys., (2012); doi: 10.1007/s00220-012-1626-5.

Climate dynamics of a coupled Aquaplanet

Josiane Salameh
(joint work with Peter Korn)

The idea behind an Aquaplanet, an idealized configuration of the current earth with all the landmasses removed, is not recent. However, most of the research is conducted with stand-alone atmospheric models. Thus, the originality behind considering the coupled Aquaplanet setup, highlights the impacts of the ocean and allows us to directly interpret the fundamental processes and feedbacks between ocean and atmosphere without any land interference.

The first coupled Aquaplanet experiments, were conducted by two research groups using different general circulation models (GCM) of coarse resolution. Compared to the current climate on Earth, the global climate was respectively warm in [1] with no sea-ice formation and cold in [2] with ice caps reaching down to 55° of latitude. Major contrasts appeared in the meridional temperature profile, ocean heat transport and the strength of Hadley cells. However, the direct comparison of the results between the two models is limited, because of their diverse model properties.

Ferreira et al. [3] raised the question if the coupled Aquaplanet is a deterministic system by verifying the existence of multiple equilibrium states, through integrating the same model (identical external forcing and parameters) from just different random initial conditions. The three stable states were: a cold state with sea-ice extending to midlatitudes, a warm state totally free of ice and a "snowball" state completely covered with sea-ice.

The extreme disparity in the final climate states cited above drives us to further investigate the Aquaplanet climate with more complex models. In particular, we aim to analyze the atmosphere-ocean interactions via capturing the global temperature profile, the wind and ocean currents pattern, the heat transport, the atmospheric and oceanic circulation and others. In order to achieve a higher physical understanding of the Earth rotation and its effect on the global circulation of the Aquaplanet, we consider two extreme cases with a slow and fast rotation rate. Some preliminary atmospheric aspects were already discussed in the case of a tidally locked Aquaplanet [4]. However, our coupled Aquaplanet setup presents a broader description of the global climatic features affected, not only atmospheric ones.

References

[1] R. Smith and C. Dubois and J. Marotzke, *Global climate and ocean circulation on an Aquaplanet ocean-atmosphere general circulation model*, Journal of climate **19** (2006), 4719–4737.

[2] J. Marshall and D. Ferreira and J.-M. Campin and D. Enderton, *Mean climate and variability of the atmosphere and ocean on an Aquaplanet*, Journal of atmospheric sciences **64** (2007), 4270–4286.

[3] D. Ferreira and J. Marshall and B. Rose, *Climate determinism revisited: multiple equilibria in a complex climate model*, Journal of climate **24** (2011), 992–1012.

[4] T. Merlis and T. Schneider, *Atmospheric dynamics of Earth-like tidally locked aquaplanets*, Journal of advances in modeling earth systems **2** (2010), 13.

Free fall of a rigid body in a viscoelastic fluid

Karoline Götze

We consider a coupled system of equations describing the movement of a rigid body immersed in a viscoelastic fluid. It is shown that under natural assumptions on the smoothness and compatibility of the data and for general geometries of the rigid body, a unique local-in-time strong solution exists.

More precisely, in this model, the fluid viscous stress $\mathbf{S} = 2\alpha\mu D(v) + \tau$ is constituted by a Newtonian part, $D(v)$ denotes the symmetric part of the fluid velocity gradient, and an elastic part τ which is given by the transport equation

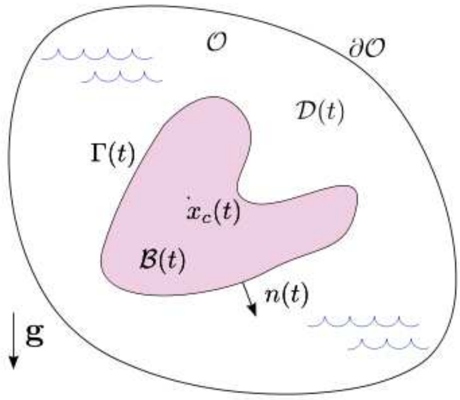
$$\lambda_1(\partial_t\tau + (v\cdot\nabla)\tau + g_a(\tau,\nabla v)) + \tau = 2\mu(1-\alpha)D(v). \tag{1}$$

The model can be specified through relaxation time $\lambda_1 > 0$, $\alpha \sim \frac{1}{\lambda_1} > 0$, a viscosity parameter μ and through the objective function

$$g_a(\tau,\nabla v) := \tau W(v) - W(v)\tau - a(D(v)\tau + \tau D(v)),\ a \in [-1,1],\ W(v) = \nabla v - D(v),$$

where $a = 1$ gives the Oldroyd-B model.

We assume that a container given as the bounded domain $\mathcal{O}$ holds the rigid body in a compact domain $\mathcal{B}(t)$ with outer normal $n(t)$ such that the fluid fills a



domain $\mathcal{D}(t) = \mathcal{O}\backslash\mathcal{B}(t)$. Since $\mathcal{D}(t)$ will change with time, we have to consider the fluid equations and equation (1) on the space-time domain

$$Q_{\mathcal{D}} := \{(t,x) \in \mathbb{R}^4 : t \in \mathbb{R}_+, x \in \mathcal{D}(t)\},$$

so the velocity v and pressure q satisfy

$$(2) \qquad \begin{cases} \partial_t v + (v\cdot\nabla)v - \operatorname{div}\mathbf{S} + \nabla q \;=\; f_0, & \text{in } \mathcal{Q}_{\mathcal{D}}, \\ \operatorname{div} v \;=\; 0, & \text{in } \mathcal{Q}_{\mathcal{D}}, \end{cases}$$

where f_0 may be some external force. We assume that no-slip conditions hold at the boundary of the fluid domain, so $v = 0$ on $\partial\mathcal{O}$ and

$$v(t,x) = \eta(t) + \theta(t) \times (x - x_c(t)), \qquad \text{on } Q_{\partial\mathcal{B}},$$

where fluid and rigid body meet. The velocity of the rigid body is given by a translational velocity η and an angular velocity θ, calculated with respect to the position of the center of mass x_c. They satisfy the equations for balance of momentum and angular momentum,

$$(3) \qquad \begin{cases} \mathrm{m}\eta' + \int_{\Gamma(t)} (\mathbf{S} - q\mathrm{Id})n \,\mathrm{d}\sigma \;=\; f_1, \\ (J\theta)' + \int_{\Gamma(t)} (x - x_c(t)) \times (\mathbf{S} - q\mathrm{Id})n \,\mathrm{d}\sigma \;=\; f_2, \end{cases}$$

which contain the drag force and torque exerted by the fluid. The constant $\mathrm{m} > 0$ is the body's mass and J is its inertia tensor and f_1 and f_2 denote external forces and torques. In order to model free fall, we can set $f_0 = \mathbf{g}$, $f_1 = \mathrm{m}\mathbf{g}$ and $f_2 = 0$ for some constant vector $\mathbf{g}$.

Adding initial conditions, we combine (1), (2) and (3) into one coupled system of equations in the unknowns v, q, η, θ and implicitly, $\mathcal{D}(t)$, and show that for sufficiently smooth compatible data, a unique local-in-time solution exists which strongly satisfies the system and depends continuously on the data [1].

The proof relies mainly on two previous results regarding the linearized equations. The first is on maximal L^s-regularity estimates in L^r for the Newtonian coupled problem, i.e. $1 - \alpha = 0$ and only equations (2) and (3) are relevant. This was shown recently in [2]. The second result concerns the local well-posedness of viscoelastic flows of this type, equations (1) and (2) without the rigid body, shown in [3] for $s > 1$, $r > 3$. Using higher-regularity a-priori estimates, the coupling of parabolic and hyperbolic parts of the system can be treated via a modified Schauder fixed point argument.

The main motivation for studying the local well-posedness of this system were open questions regarding the sedimentation of particles in a viscoelastic fluid. It is known from experiments and mathematical analysis [4, 5] that the stable orientations of particles falling through a viscoelastic liquid may be the opposite of those obtained in a Newtonian liquid, due to normal stress effects. In order to extend the mathematical treatment of this phenomenon, it may be a first step to construct local-in-time regular solutions and global solutions for small data.

References

[1] K. Götze *Strong solutions for the interaction of a rigid body and a viscoelastic fluid*, J. Math. Fluid Mech. (2013), DOI 10.1007/s00021-012-0131-0

[2] M. Geissert, K. Götze, and M. Hieber, *L^p-theory for strong solutions to fluid rigid-body interaction in Newtonian and generalized Newtonian fluids.*, Trans. Amer. Math. Soc. **365** (2013), no. 3, 1393–1439.

[3] E. Fernández-Cara, F. Guillén, and R. R. Ortega, *Some theoretical results concerning non Newtonian fluids of the Oldroyd kind*, Annali della Scuola Normale Superiore di Pisa **26** (1998), no. 1, 1–29.

[4] G. P. Galdi and A. Vaidya, *Translational steady fall of symmetric bodies in a Navier-Stokes liquid, with application to particle sedimentation*, J. Math. Fluid Mech. **3** (2001), no. 2, 183–211.

[5] G. P. Galdi, A. Vaidya, M. Pokorný, D. D. Joseph, and J. Feng, *Orientation of symmetric bodies falling in a second-order liquid at nonzero Reynolds number*, Math. Models Methods Appl. Sci. **12** (2002), no. 11, 1653–1690.

On the spontaneous generation of inertia-gravity waves

Leslie M. Smith

(joint work with Gerardo Hernandez-Duenas and Sam Stechmann)

The spontaneous generation of inertia-gravity waves is investigated for the three-dimensional (3D) rotating Boussinesq equations in a triply periodic domain. We compare quasi-geostrophic (QG) dynamics without waves to the dynamics of a hierarchy of models that are constructed to include more and more nonlinear interactions involving inertia-gravity waves [1]. Two case studies are presented, both starting with initial data that projects only onto the vortical-mode (balanced) component of the flow.

The 3D rotating Boussinesq equations are given by

$$\frac{D\mathbf{u}}{Dt} + f\hat{\mathbf{z}} \times \mathbf{u} = -\nabla P - N\theta\hat{\mathbf{z}} + \nu\nabla^2\mathbf{u}, \quad \nabla \cdot \mathbf{u} = 0$$

$$\frac{D\theta}{Dt} - Nw = \kappa\nabla^2\theta, \quad \theta = \frac{g}{N\rho_o}\rho' \tag{1}$$

where the $\hat{\mathbf{z}}$-axis is the axis of rotation and stratification. The frame rotation rate is $f/2$ and the density is given by $\rho = \rho_o - bz + \rho', \rho' \ll \rho_o, |bz|$ with $N^2 = gb/\rho_o$. Nondimensional Rossby Ro and Froude Fr numbers may be defined as $Ro = U/(fL), Fr = U/(NH)$, where L/U is a time scale associated with the initial data, and here $H = L$ is the length of the domain. We work mainly in the regime $Ro \approx Fr \approx 0.1$ typical of a range of scales in the oceans and the atmospheric mid-latitudes. Solutions in the unforced, linear, inviscid limit are $\mathbf{v}(\mathbf{x}, t; \mathbf{k}) = \boldsymbol{\phi}(\mathbf{k}) \exp\left[i\left(\mathbf{k}\cdot\mathbf{x} - \sigma(\mathbf{k})t\right)\right] +$ c.c. where $\mathbf{v}$ is the state vector with eigenmodes $\boldsymbol{\phi}_s(\mathbf{k})$ and eigenvalues $\sigma_s(\mathbf{k})$, $s = 0, +, -$. The wave modes $\boldsymbol{\phi}_\pm(\mathbf{k})$ have frequencies $\sigma_\pm(\mathbf{k}) = \pm(N^2 k_h^2 + f^2 k_z^2)^{1/2}/k$ and the vortical mode $\boldsymbol{\phi}_0(\mathbf{k})$ has zero frequency $\sigma_0(\mathbf{k}) = 0$. Since $\boldsymbol{\phi}_s(\mathbf{k})$, $s = \pm, 0$ form an orthogonal basis, (1) may be written as

$$\frac{\partial}{\partial t} b_{s_\mathbf{k}} = \sum_{\triangle} \sum_{s_\mathbf{p}, s_\mathbf{q}} C^{s_\mathbf{k} s_\mathbf{p} s_\mathbf{q}}_{\mathbf{kpq}} \; b^*_{s_\mathbf{p}} \, b^*_{s_\mathbf{q}} \exp\left[i\left(\sigma_{s_\mathbf{k}} + \sigma_{s_\mathbf{p}} + \sigma_{s_\mathbf{q}}\right)t\right] \tag{2}$$

where $u(\mathbf{x}, t) = \sum_\mathbf{k} \sum_s b_s(t; \mathbf{k}) \boldsymbol{\phi}_s(\mathbf{k}) \exp\left[i\left(\mathbf{k}\cdot\mathbf{x} - \sigma_s(\mathbf{k})t\right)\right]$, and there is a sum over triads $\mathbf{k} + \mathbf{p} + \mathbf{q} = 0$ and a sum over modes types $s = 0, \pm$. Symbolically, (2) is given by

$$\begin{aligned} 0 &\leftarrow [00] \oplus [0+] \oplus [0-] \oplus [++] \oplus [+-] \oplus [--] \\ + &\leftarrow [00] \oplus [0+] \oplus [0-] \oplus [++] \oplus [+-] \oplus [--] \\ - &\leftarrow [00] \oplus [0+] \oplus [0-] \oplus [++] \oplus [+-] \oplus [--] \end{aligned} \tag{3}$$

where 0 indicates a vortical mode, $\pm$ indicates a wave mode, and the symbol $\oplus$ here means 'also including.' The QG approximation allows only interactions between vortical modes and is written symbolically as $0 \leftarrow [00]$ [2]. Intermediate models can be constructed by progressively including inertia-gravity waves, and two such models are PPG including interactions with exactly one inertia-gravity wave:

$$\begin{aligned} 0 &\leftarrow [00] \oplus [0+] \oplus [0-] \\ + &\leftarrow [00] \\ - &\leftarrow [00] \end{aligned} \tag{4}$$

and P2G including interactions with one and two inertia-gravity waves:

$$\begin{aligned} 0 &\leftarrow [00] \oplus [0+] \oplus [0-] \oplus [++] \oplus [+-] \oplus [--] \\ + &\leftarrow [00] \oplus [0+] \oplus [0-] \\ - &\leftarrow [00] \oplus [0+] \oplus [0-]. \end{aligned} \tag{5}$$

Both models PPG and P2G can be written as PDE systems in physical space, and both conserve energy. Furthermore, they incorporate two-way feedback between

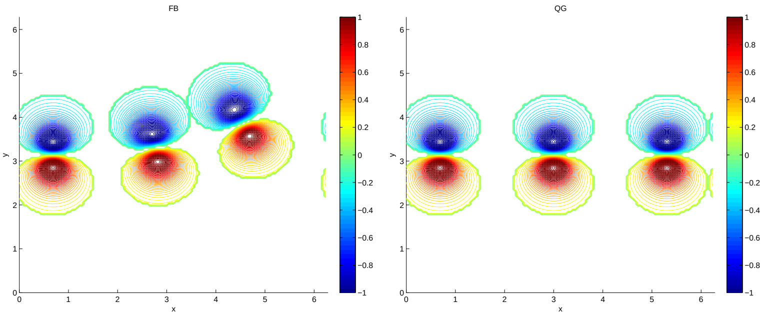


FIGURE 1. Full Boussinesq (left) and QG (right): Contours of streamfunction for $Ro = Fr = 0.2$; times $t = 0, 20, 40$ nonlinear turnover times.

waves and vortical modes. In contrast, forced linear models incorporate only one-way feedback from vortical modes to waves [3]. The model PPG can be understood as a generalization of higher-order PV inversion models [4], but where the slaving is eliminated.

For an initially balanced dipole, the generation of inertia-gravity waves increases the speed of the dipole and causes a cyclonic drift in the trajectory of the dipole (see Figure 1). For relatively small $Ro = Fr = 0.1$, PPG can capture the evolution of the dipole for roughly ten times longer than a forced linear model. However, for larger $Ro = Fr = 0.2$, neither PPG nor P2G can faithfully track the speed and trajectory of the dipole for long times, indicating that three-wave interactions are important for intermediate Ro and Fr (but still smaller than unity).

For random initial data, the spontaneous generation of inertia-gravity waves reduces the size of the emerging vortices. Figure 2 shows that PPG does not generate vortices. A third model P2SG containing vortical-vortical-vortical and vortical-wave-wave interactions behaves similarly to QG (with vortical-vortical-vortical interactions only). The model P2G can accurately reproduce the size of the vortices generated by the full Boussinesq dynamics. A qualitative understanding begins to emerge, whereby vortical-wave-wave interactions create radiating waves that may influence the evolution of balanced structures, but do not by themselves generate structures. By contrast, the vortical-wave-wave interactions take energy out of the wave modes and change the balanced state through feedback onto the vortical modes.

Acknowledgements: L.M. Smith gratefully acknowledges comments and suggestions made by Oberwolfach participants H. Bessaih, R. Klein, R. Samelson, and J. Tribbia.

References

[1] M. Remmel, M. & L.M. Smith, *New Intermediate Models for Rotating Shallow Water and an Investigation of the Preference for Anticyclones*, J. Fluid Mech. **635** (2009), 321-359.

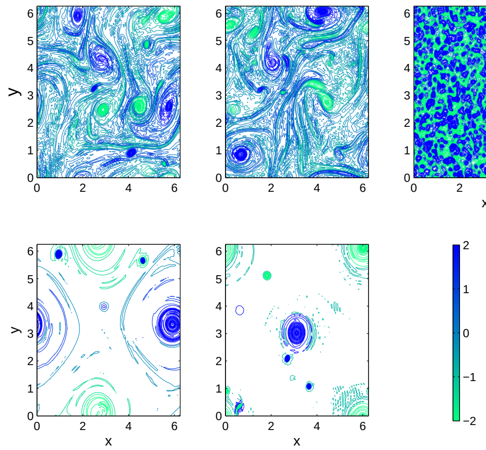


FIGURE 2. Vertical vorticity at $z = \pi$, $t = 200$ turnover times: Full (top left), P2G (top middle), PPG (top right), P2SG(bottom left) and QG (bottom right).

[2] R. Salmon, *Lectures on Geophysical Fluid Dynamics*, Oxford University Press, New York (1998).

[3] C. Snyder & R. Plougonven & D. Muraki, *Mechanisms for the Spontaneous Gravity Wave Generation within a Dipole Vortex*, J. Atmos. Sci. **66** (2009), 3464-3478.

[4] D. Muraki, C. Snyder & R. Rotunno, *The Next-Order Corrections to Quasi-Geostrophic Theory*, J. Atmos. Sci. **56** (1999), 1547-1560.

On Stability of Navier-Stokes-Boussinesq Type System and Ekman layers

HAJIME KOBA

We often consider the Boussinesq system when we treat fluids effected by heat convection. We refer to a system coupling among the fluid velocity, the temperature, and the pressure of the fluid as a Navier-Stokes-Boussinesq type system. We consider the stability for the spatial inhomogeneous Navier-Stokes-Boussinesq system. By stability we mean that if a solution of a system satisfies the asymptotic stability. Applying fractional powers of linear operators and maximal L^p-regularity, we show stability of energy solutions of the Navier-Stokes-Boussinesq type system. This approach is very important to show L^2-asymptotic stability for stationary solutions without knowing detailed information of the stationary solutions. We can easily prove stability of energy solutions of various fluid systems.

We also discuss weak nonlinear stability of Ekman boundary layers in rotating stratified fluids. A stationary solution of the rotating Navier-Stokes equations with a boundary condition is called an Ekman boundary layer. We construct stationary solutions of the rotating Navier-Stokes-Boussinesq equations with stratification effects (a geophysical fluid system) in the case when the rotating axis is not necessarily perpendicular to the horizon. We call such stationary solutions Ekman layers. Under some assumptions on the Ekman layers and the physical parameters, we show the existence of a weak solution to an Ekman perturbed system, which satisfies the asymptotic stability.

On Some Mathematical Aspects of High-Resolution Climate Modeling

Peter Korn

Numerical models of the global atmosphere and ocean circulation discretized the underlying Partial Differential Equations of the atmosphere and ocean, namely the compressible, non-hydrostatic Navier-Stokes equations for the atmosphere and the so-called Primitive equations (incompressible, hydrostatic, under the Boussinesq approximation) for the ocean. The domain is a rotating sphere. The discretization projects the continuous equations onto finite-dimensional subspaces. Usually these subspaces are chosen to be different for atmosphere and ocean, for example in the ocean they may be associated with a logically rectangular latitude-longitude grid, while in the atmosphere one uses spherical harmonics and a spectral grid. A new generation of atmosphere-ocean models is currently developed by several modeling centers[1] that deviates from this traditional practice by using the same type of grid in the atmosphere as well as in the ocean in order to facilitate the coupling of the two components. The choice of the common grid is delicate due to different requirements in both components of the coupled system and the best choice seems to be so-called "unstructured grids', i.e. non-orthogonal grids without any directional preference such as east-west and north-south. The grid that we use is a Delaunay triangulation of the sphere with a associated Voronoi grid of hexagons. On this grid we use a so-called C-type staggering.
The discretization on such unstructured grids requires new numerical approaches. Strong constraints on the numerical method are the computational efficiency and specific physical conditions from Geophysical Fluid Dynamics such as geostrophic balance or energetically neutral discrete Coriolis force. We describe in some detail a structure preserving discretization for the ocean primitive equations. The construction principle of this method is the use of vector calculus for the discretization of differential operators (divergence, gradient and curl) combined with a discrete weak-form of the equations and is oriented towards the conservation properties of the continuous equations. The resulting set of discrete equations for velocity v,

[1]The MPAS consortium of the National Center for Atmospheric Research and Los Alamos in the U.S and the ICON project of the Max Planck Institute for Meteorology and the German Weather Service in Germany

free surface elevation h and tracer C reads as follows

$$Velocity: \frac{d}{dt}P^T P v_{e,k} + \hat{P}^T((\omega + f)\hat{P}v)_{e,k} + P^T(\overline{w_{c,k+\frac{1}{2}}\partial_z P v_k}^z)$$
$$+ P^T P\nabla\big(\frac{|Pv|_k^2}{2} + \frac{p_k}{\rho_0}\big) - P^T div\mathcal{K}_v \nabla P v_{e,k} - \partial_z A_v \partial_z P^T P v_{e,k} = \mathcal{F}_v,$$
$$Hydrostatic: \partial_z p = -\bar{\rho} g, \qquad Incompress.: \ div\,(P^T P v) + \partial_z w = 0,$$
$$Freesurface: \frac{\partial h}{\partial t} + div\big[P^T\big(\int_{-H}^{h} Pv\,dz\big)\big] = \mathcal{F}_h,$$
$$Tracer: \frac{dC_{c,k}}{dt} + div\big[P^T(CPv)\big]_{c,k} - div\mathcal{K}_C \nabla C_{c,k} - \partial_z A_C \partial_z C_{c,k} = \mathcal{F}_C,$$

ω is the vorticity, f the Coriolis parameter, ρ the density and p the pressure, e, c denote edges and cells,k a vertical level,$\bar{\cdot}^z$ a vertical interpolation and $P, \hat{P}$ generic mappings from cell edges to cell centers and from cell edges to cell vertices, respectively. For the discrete scheme with continuous time one can for example show a discrete form of energy conservation in the inviscid and unforced case or the absence of spurious vorticity production in the absence of lateral boundaries. Results from numerical ocean simulations over 1000 years and phenomenological comparison with an established ocean model confirm the approach. One can finally state that it is in fact possible to develop an unstructured grid ocean model that shows essential features of a global ocean circulation on climate time scales. Preliminary results from a coupled atmosphere-ocean model with land completely removed (so-called coupled aquaplanet) are also promising.

For the majority of numerical atmosphere-ocean circulation models it is unclear what their relation to the continuous equations is. In the case of the Primitive Equations the global well-posedness was proven by Cao and Titi [1] for specific boundary conditions and constant parameters for viscosity and temperature diffusion. This theorem provides a mathematical foundation and discrete Primitive Equation models satisfying the conditions of the Cao-Titi theorem have to converge to this solution. Results on convergence or error estimates have not been established for the global Primitive Equation ocean models such as POP or MOM used in the Intergovernmental Panel on Climate Change (IPCC) scenarios.

Finally i suggest a new approach for combining coupled numerical circulation models with observations in Data Assimilation. This problem can be solved with variational methods that aim at minimizing the distance between observations and the model solution by controlling the initial conditions ψ_0. While the distance is usually based on the L^2-norm, i suggest to use a higher-order Sobolev norm in space. This controls via the Sobolev embedding theorem a control over the L^∞ norm while one still stays within the Hilbert-space framework. More specifically the cost functional

$$\mathcal{J}(\psi_0) := \mathcal{J}_b(\psi_0) + \mathcal{J}_o(\psi_0)$$

consists of a background term $\mathcal{J}_b(\psi_0)$ that measures the distance to the previous forecast, usually with a weighted L^2-norm according to the background error covariance matrix $\mathcal{J}_b(\psi_0) = ||\psi_0 - \psi_{back}||_{L^2(d\mu_\mathcal{B})}$ and an observational term $\mathcal{J}_o(\psi_0)$

that measures the deviation between model trajectory $\mathcal{M}[\psi_0]$ starting from initial state ψ_0 and evolving according to the model operator $\mathcal{M}$, where the distance is also a weighted L^2-norm, with a weight given by the observation error covariance, $\mathcal{J}_o(\psi_0) = \int_T ||\mathcal{M}[\psi_0] - \mathcal{H}\psi_{obs}||^2_{L^2(d\mu_{\mathcal{R}})}\, dt$. The *Data Assimilation Problem* is to determine initial condition ψ_0^* such that

$$\begin{aligned} \mathcal{J}(\psi_0^*) = \min_{\psi_0} \mathcal{J}(\psi_0) \\ \text{and } \psi(\psi_0^*) \text{ satisfies model equations} \end{aligned} \tag{1}$$

Our suggestion is to replace the L^2-norm in the background term $\mathcal{J}_b$ by

$$\mathcal{J}_b^s := (\psi_0) := ||\psi_0 - \psi_b||_{H^s(d\mu_{\mathcal{B}})} \tag{2}$$

and the L^2-norm observation term $\mathcal{J}_o$ by

$$\mathcal{J}_o^s := (\psi_0) := \int_T \sum_{|\alpha| \leq s} \int_T ||\mathcal{M}[\psi_0] - \mathcal{H}\psi_{obs}||^2_{H^{-\alpha}(d\mu_{\mathcal{R}})}\, dt \tag{3}$$

where s is a multi-index. We consider the following set of coupled Partial Differential Equations on a rotating plane with periodic boundary conditions that mimic in a very simple way an coupled Atmosphere-Ocean model

$$\begin{aligned} \text{Atmosphere:} \quad & \frac{\partial \mathbf{u}^a}{\partial t} + (\mathbf{u}^a \cdot \nabla)\mathbf{u}^a + f\mathbf{u}^{a\perp} + g^a \nabla \theta^a = \epsilon_u^a \triangle \mathbf{u}^a, \\ & \frac{\partial \theta^a}{\partial t} + div(h^a \mathbf{u}^a) = -\alpha\theta^o + \epsilon_h^a \triangle \theta^a. \\ \text{Ocean:} \quad & \frac{\partial \mathbf{u}^o}{\partial t} + (\mathbf{u}^o \cdot \nabla)\mathbf{u}^o + f\mathbf{u}^{o\perp} + g^o \nabla \theta^o = \beta \mathbf{u}^a + \epsilon_u^o \triangle \mathbf{u}^o, \\ & \frac{\partial \theta^o}{\partial t} + div(h^o \mathbf{u}^o) = \epsilon_h^o \triangle \theta^o. \end{aligned} \tag{4}$$

where $\mathbf{u}^{a/o}$ denotes velocity, $\theta^{a/o}$ height of the upper free surface, $h^{a/o} = \theta^{a/o} - \theta_b^{a/o}$ the total depth of atmosphere/ocean and $\alpha, \beta > 0$ are coupling constants. By choosing different fluid depths and using reduced gravity constants one can obtain a "fast" atmosphere coupled to a "slow" ocean model. One can now prove the following results[2]

Theorem 1. *Let observations $\psi_{obs} \in L^2(T; H)$ be given. Then there exist optimal initial conditions $\bar{\psi}_0 \in H^s$ for the coupled data assimilation problem (1) for the equations (4) using the cost functional (2) and (3).*

The minimizers can be characterized by a first-order necessary condition.

Theorem 2. *Let $\psi_0^* \in H^s$ be an optimal initial condition of the data assimilation problem and $\bar{\psi} = (\bar{\mathbf{u}}^a, \bar{h}^a, \bar{\mathbf{u}}^o, \bar{h}^o)$ the associated solution of the coupled model. Then ψ_0^* satisfies*

$$\psi_0^* = \psi_{back} - \mathcal{B}^{-1}\mathcal{S}_s \tilde{\Psi}_0,$$

where $\tilde{\Psi}$ is the solution of the adjoint equations with initial condition $\tilde{\psi}(t_1) = 0$ and forcing $\tilde{\mathcal{F}} := \sum_{|\alpha|\leq s}(-1)^{\alpha}\triangle^{-\alpha}\mathcal{R}\big(\psi(\psi_0) - \mathcal{H}\psi_{obs}\big)$

$$-\frac{\partial\tilde{\Psi}}{\partial t} + \mathcal{N}'^{*}[\bar{\psi}](\tilde{\Psi}) + L\tilde{\Psi} - \tilde{C}(\tilde{\Psi^{a}}, \tilde{\Psi^{o}}) = \tilde{\mathcal{F}}$$

where $\tilde{C}$ is the coupling operator, L the linear terms of the model, $\mathcal{N}'^{*}$ the adjoint of the derivative of the nonlinear operator and $\mathcal{S}_s$ is given by $\mathcal{S}_s := \big[I - \mathcal{D}^2 + \mathcal{D}^4 \dots \mathcal{D}^{2s}\big]^{-1}$.

References

[1] C. Cao, E. Titi *Global well-posedness of the three-dimensional viscous primitive equations of large scale ocean and atmosphere dynamics*, Annals of Mathematics, **165** (2007), 1–23

[2] P. Korn, *Analysis of a Coupled Data Assimilation Problem*, Preprint, Hamburg (2013)

The Navier-Stokes equations with Robin boundary conditions in bounded Lipschitz domains

Sylvie Monniaux

(joint work with Jürgen Saal)

In this work, we investigate the solvability of

$$\begin{cases} \partial_t u - \Delta u + \nabla\pi - u \times \operatorname{curl} u \;=\; 0, & \operatorname{div} u \;=\; 0 \qquad \text{in } (0,T)\times\Omega, \\ \nu\cdot u = 0, & \nu\times\operatorname{curl} u = \alpha u, \quad \text{on } (0,T)\times\partial\Omega, \\ u(0) \;=\; u_0, & \qquad\qquad \text{in } \Omega, \end{cases}$$

where $0 < T \leq \infty$, $\alpha \geq 0$, $\Omega \subseteq \mathbb{R}^3$ is a bounded Lipschitz domain, $\nu(x)$ denotes the outer unit normal vector at $x \in \partial\Omega$ and u_0 is an initial value in the critical space usually denoted by $\mathbb{L}^3_\sigma(\Omega)$, consisting of divergence-free vector fields in $L^3(\Omega;\mathbb{R}^3)$ with zero normal component at the boundary. Our strategy is to study the Laplace operator A_α associated to the problem, namely

$$\begin{aligned} D(A_\alpha) \;=\; & \big\{u \in L^2(\Omega;\mathbb{R}^3); u = u_1 + u_2 \text{ with } u_1, u_2 \in L^2(\Omega;\mathbb{R}^3), \\ & \operatorname{div} u_1 \in H^1(\Omega), \operatorname{curl} u_1, \operatorname{curl}\operatorname{curl} u_1 \in L^2(\Omega;\mathbb{R}^3) \text{ and } \Delta u_2 = 0, \\ & \mathcal{N}(u), \mathcal{N}(\operatorname{div} u), \mathcal{N}(\operatorname{curl} u) \in L^2(\partial\Omega) : \\ & \nu\cdot u_{1,2} = 0, \nu\times\operatorname{curl} u_2 = 0 \text{ and } \nu\times\operatorname{curl} u_2 = \alpha u \text{ on } \partial\Omega\big\}, \\ A_\alpha u \;=\; & -\Delta u, \end{aligned}$$

where $\mathcal{N}(f)(x) := \sup\big\{|f(y)|; y \in \Omega, |x-y| \leq (1+\kappa)\operatorname{dist}(y,\partial\Omega)\big\}$ denotes the maximal nontangential operator applied to f at $x \in \partial\Omega$. We first show how the boundary conditions $\nu\cdot u = 0$ and $\nu\times\operatorname{curl} u = \alpha u$ make sense. One of the difficulty is that in Lipschitz domains, the space of vector fields $u \in L^2(\Omega;\mathbb{R}^3)$ such that $\operatorname{div} u \in L^2(\Omega)$, $\operatorname{curl} u \in L^2(\Omega;\mathbb{R}^3)$ and $\nu\cdot u = 0$ on $\partial\Omega$ is not contained in $H^1(\Omega;\mathbb{R}^3)$. We prove that $-A_\alpha$ generates a bounded analytic semigroup on $L^2(\Omega;\mathbb{R}^3)$ which can be extended to $L^p(\Omega;\mathbb{R}^3)$ for $p \in \big[\frac{9}{7}, \frac{9}{2}\big]$. The case $\alpha = 0$ has been studied in

[1] and [2] where it has been noticed that the Helmoltz projection (the projection of vector fields onto divergence-free vector fields) commutes with the operator A_0. This is no more the case if $\alpha > 0$. However, the boundary conditions used here allow to treat the pressure term $\nabla\pi$ as a small perturbation of $A_\alpha u$ in the Navier-Stokes system. Note that the nonlinearity in the Navier-Stokes system used here is justified by the identity $(u \cdot \nabla)u = \frac{1}{2}\nabla|u|^2 - u \times \operatorname{curl} u$ for smooth vector fields u.

References

[1] Marius Mitrea and Sylvie Monniaux,, *On the analyticity of the semigroup generated by the Stokes operator with Neumann-type boundary conditions on Lipschitz subdomains of Riemannian manifolds*, Trans. Amer. Math. Soc. **361** (2009), no. 6, 3125–3157.
[2] Marius Mitrea and Sylvie Monniaux, *The nonlinear Hodge-Navier-Stokes equations in Lipschitz domains*, Differential Integral Equations **22** (2009), no. 3-4, 339–356.

Decay estimates of the Oseen semigroup in two-dimensional exterior domains

Toshiaki Hishida

Let Ω be an exterior domain in $\mathbb{R}^2$ with smooth boundary $\partial\Omega$. We consider the Navier-Stokes system

$$\begin{aligned} \partial_t u + u \cdot \nabla u = \Delta u - \nabla p, \qquad \text{div } u = 0, \\ u|_{\partial\Omega} = 0, \qquad u \to u_\infty \quad \text{as } |x| \to \infty \end{aligned} \tag{1}$$

which describes the motion of a viscous incompressible fluid past an obstacle $\mathbb{R}^2 \setminus \Omega$ (rigid body) that moves with translational velocity $-u_\infty$, where $u(x,t) = (u_1, u_2)$ and $p(x,t)$ respectively denote unknown velocity and pressure of the fluid, while $u_\infty \in \mathbb{R}^2 \setminus \{0\}$ is a given uniform velocity. Without loss of generality, one may take $u_\infty = -2\alpha e_1$ with $\alpha > 0$, where $e_1 = (1,0)$. By denoting $u - u_\infty$ by the same symbol u, (1) is reduced to

$$\begin{aligned} \partial_t u + u \cdot \nabla u = \Delta u + 2\alpha\partial_1 u - \nabla p, \qquad \text{div } u = 0, \\ u|_{\partial\Omega} = 2\alpha e_1, \qquad u \to 0 \quad \text{as } |x| \to \infty. \end{aligned} \tag{2}$$

It is an open question to clarify the large time behavior of solutions to the initial value problem for (2). Toward better understanding of this problem, it is important to study: (i) steady flows with fine decay/summability for $|x| \to \infty$; (ii) decay properties of solutions to the Oseen initial value problem, see (3) below, for $t \to \infty$. Concerning the first issue (i), it was proved by [5], [6], [11] and, later on, by [7] that if $\alpha > 0$ is sufficiently small, then (2) admits a steady flow (called a physically reasonable solution) that satisfies $u(x) = O(|x|^{-1/2})$ as $|x| \to \infty$ and exhibits a parabolic wake region behind the body like the Oseen fundamenatal solution. So far, the stability/instablity of this flow is unsolved, while we know the stability of small physically reasonable solutions in 3D exterior domains. The difficulty in 2D is due to less decay property for $|x| \to \infty$, that is not enough to show the stablity.

This presentation is concerned with the second issue (ii) above, that is, the large time behavior of solution $u(\cdot,t)=e^{-tL}f$ to the initial value problem for the Oseen system

$$\begin{aligned}&\partial_t u-\Delta u-2\alpha\partial_1 u+\nabla p=0, \qquad \text{div } u=0 \quad \text{in } \Omega\times(0,\infty)\\ &u|_{\partial\Omega}=0, \qquad u\to 0 \quad \text{as } |x|\to\infty, \qquad u(\cdot,0)=f\end{aligned} \tag{3}$$

where $L=L_\alpha$ denotes the Oseen operator defined in the space $L^q_\sigma(\Omega)$. Our aim is to show the L^q-L^r estimates (with $n=2$)

$$\|e^{-tL}f\|_r \le C\,t^{-(n/q-n/r)/2}\|f\|_q \qquad (1<q\le r\le\infty,\ q\ne\infty) \tag{4}$$

$$\|\nabla e^{-tL}f\|_r \le C\,t^{-(n/q-n/r)/2-1/2}\|f\|_q \qquad (1<q\le r\le n) \tag{5}$$

for $t>0$, where $n\ge 2$ is the space dimension and $\|\cdot\|_q$ stands for the norm of $L^q(\Omega)$. For the Stokes semigroup (case $\alpha=0$), these estimates were deduced by [8] ($n\ge3$), [1], [2] ($n=2$) and [10] ($n\ge2$). As for the Oseen semigroup (case $\alpha>0$), (4) and (5) were established by [9] ($n=3$) and [3], [4] ($n\ge3$), except the case of plane exterior domains, where the constant $C>0$ above can be taken uniformly with respect to small $\alpha>0$. This is important in the proof of stability of 3D steady flows as an application of (4)–(5). Unfortunately, our main result on the Oseen semigroup in 2D, see Theorem 1 below, does not provide such desirable situation, but I believe the theorem will have to be improved in the future. The most difficulty in 2D is to control both parameters λ (resolvent parameter) and α in asymptotic analysis of the Oseen resolvent. In fact, we have to deal with the logarithmic singularity such as $\log(\lambda+\alpha^2)$ for small (λ,α) unlike 3D case.

The main result on L^q-L^r estimate of the Oseen semigroup in 2D exterior domains reads as follows.

Theorem 1. *Let $\alpha>0$. Then*

$$\|e^{-tL}f\|_r \le C\,t^{-1/q+1/r}\|f\|_q \qquad (1<q\le r<\infty) \tag{6}$$

$$\|e^{-tL}f\|_\infty \le C\,t^{-1/q}(\log t)\|f\|_q \qquad (1<q<r=\infty) \tag{7}$$

$$\|\nabla e^{-tL}f\|_r \le C\,t^{-1/q+1/r-1/2}\|f\|_q \qquad (1<q\le r<2=n) \tag{8}$$

$$\|\nabla e^{-tL}f\|_2 \le C\,t^{-1/q}(\log t)\|f\|_q \qquad (1<q\le r=2=n) \tag{9}$$

for $t\ge2$ and $f\in L^q_\sigma(\Omega)$. Concerning the constant C, given arbitrary large $M>0$ and small $\varepsilon>0$, there is a constant $\widetilde{C}=\widetilde{C}(M,\varepsilon;\Omega,q,r)$ such that $C\le\widetilde{C}/\alpha^\rho$ provided $\alpha\in(0,M]$, where

$$\rho=\begin{cases}1+\varepsilon & 1/q-1/r\le 1/2\\ 2+\varepsilon & 1/q-1/r>1/2\end{cases} \qquad \text{for (6)}$$

$$\rho=\begin{cases}1+\varepsilon & q>2\\ 2+\varepsilon & q\le 2\end{cases} \qquad \text{for (7)}$$

$$\rho=\begin{cases}1+\varepsilon & q=r\\ 2+\varepsilon & q<r\end{cases} \qquad \text{for (8) and (9).}$$

As in [8] and [9], the essential step for the proof is to derive local energy decay properties of the semigroup in $\Omega_R = \Omega \cap B_R$ by means of spectral analysis. To do so, we analyze the regularity of the resolvent $(\lambda + L)^{-1}$ near $\lambda = 0$, whose parametrix is constructed by using the Oseen resolvents in the whole plane $\mathbb{R}^2$ and in a bounded domain near the obstacle $\mathbb{R}^2 \setminus \Omega$. The analysis of asymptotic structure as $\lambda \to 0$ (and $\alpha \to 0$ as well) of the fundamental solution of the Oseen resolvent in $\mathbb{R}^2$ plays a key role. A representation formula of the fundamental solution can be written in terms of the modified Bessel functions of the second kind.

References

[1] W. Dan and Y. Shibata, *On the L_q-L_r estimates of the Stokes semigroup in a two-dimensional exterior domain*, J. Math. Soc. Japan **51** (1999), 181–207.

[2] W. Dan and Y. Shibata, *Remark on the L_q-L_∞ estimate of the Stokes semigroup in a 2-dimensional exterior domain*, Pacific J. Math. **189** (1999), 223–239.

[3] Y. Enomoto and Y. Shibata, *Local energy decay of solutions to the Oseen equations in the exterior domains*, Indiana Univ. Math. J. **53** (2004), 1291–1330.

[4] Y. Enomoto and Y. Shibata, *On the rate of decay of the Oseen semigroup in exterior domains and its application to Navier-Stokes equation*, J. Math. Fluid Mech. **7** (2005), 339–367.

[5] R. Finn and D. R. Smith, *On the linearized hydrodynamical equations in two dimensions*, Arch. Rational Mech. Anal. **25** (1967), 1–25.

[6] R. Finn and D. R. Smith, *On the stationary solution of the Navier-Stokes equations in two dimensions*, Arch. Rational. Mech. Anal. **25** (1967), 26–39.

[7] G. P. Galdi, An Introduction to the Mathematical Theory of the Navier-Stokes Equations, Vol. I: Linerised Steady Problems, Vol. II: Nonlinear Steady Problems, Springer, New York, 1994.

[8] H. Iwashita, *L_q-L_r estimates for solutions of the nonstationary Stokes equations in an exterior domain and the Navier-Stokes initial value problem in L_q spaces*, Math. Ann. **285** (1989), 265–288.

[9] T. Kobayashi and Y. Shibata, *On the Oseen equation in the three dimensional exterior domains*, Math. Ann. **310** (1998), 1–45.

[10] P. Maremonti and V. A. Solonnikov, *On nonstationary Stokes problem in exterior domains*, Ann. Sc. Norm. Sup. Pisa **24** (1997), 395–449.

[11] D. R. Smith, *Estimates at infinity for stationary solutions of the Navier-Stokes equations in two dimensions*, Arch. Rational Mech. Anal. **20** (1965), 341–372.

The dark side of geophysical fluid dynamics: A report from behind the curtain

Sebastian Reich

My talk consisted of two parts with the first one summarizing some fundamental difficulties and challenges for mathematical modeling of atmospheric fluid dynamics. The second part gave an overview over ensemble based data assimilation schemes from the perspective of optimal transportation.

A goal of scientific research is to make skillful predictions, gain in understanding, and to close explanatory gaps. Mathematics has contributed to this goal through two main streams of mathematical research, namely learning from data and the derivation of statistical models on the one hand and the analysis and simulation of mechanistic, first principle-based models on the other. Furthermore, these techniques have been developed by statistics and applied mathematics to a large extent independently of each other. There are, of course, overlaps in areas such as inverse problems, Markov models, and approximation theory. But I believe that a much stronger integration of data-driven and first principle-driven modeling approaches is necessary in order to achieve fundamental advances in the understanding of weather and climate. This is largely due to the fact that, indeed, geophysical fluid dynamics rests upon first principles but uncontrolled approximations need to be made in order to get aggregated model hierarchies, which can be analysed and simulated. In order to formulate appropriate closure schemes in form of what are called parametrization in meteorology one needs to mimic the mean effect of the unresolved scales on the resolved ones in form of largely heuristically motivated and data tuned models. Furthermore, even if purely first principle-based models and their mathematically rigorous reduction were achievable, numerical approximations as well as uncertain initial and boundary conditions lead to uncertain predictions which will get further amplified by nonlinear dynamics. Hence again model forecasts need to be interfaced with data in order to constrain such a growth of uncertainty. This interfacing of models with data is called data assimilation in geophysics. While data assimilation could be viewed as a classic filtering or smoothing problem, novel aspects and challenges include the high dimensionality of phase space, strong nonlinearity of models, spatio-temporally correlated model errors due to uncontrolled approximations, and sparsity of data.

Current data assimilation algorithms used in practice are either variational (minimizing a cost function) or ensemble-based resting on the assumption of Gaussianity of uncertainties. There are currently many activities to make a link between those algorithms and consistent filtering algorithms such as sequential Monte Carlo methods. Consistency means that a filter algorithm is able to reproduce the analytic solution in the limit of ever increasing ensemble sizes. Such a limit is however of little practical relevance in atmospheric data assimilation. A novel approach, we have proposed, is to replace the resampling step of a sequential Monte Carlo method by a coupling between discrete random variables based on linear programming (optimal transport). This novel approach allows for a McKean interpretation of the Bayesian inference step, leads to a linear transformation of ensemble members and has the advantage of making localization possible. Here localization means that observation at a specific point in space should only affect the state variables near the observed point due to a limited spatial correlation of the data and forecast fields. This approximation has been shown in practice to effectively eliminate the curse of dimensionality for sufficiently well observed systems. Furthermore, our approach allows one to put different ensemble transform filtering algorithm within a unifying framework.

Some introductory notes on data assimilation and optimal transportation can be found in [1]. The sequential Monte Carlo method with an optimal transport based ensemble transform step is described in [2].

References

[1] C.J. Cotter and S. Reich, *Ensemble filter techniques for intermittent data assimilation*, to appear in Radon book series *Large-scale inverse problems and applications in the earth sciences* (2013).

[2] S. Reich, *A non-parametric ensemble transform method for Bayesian inference*, submitted (2013).

Scale interactions in compressible rotating fluids

Eduard Feireisl

The motion of a compressible viscous rotating fluid in the so-called f-plane approximation can be described by the Navier-Stokes system in the dimensionless variables as

$$\partial_t \varrho + \nabla \cdot (\varrho \vec{u}) = 0,$$

$$\partial_t(\varrho \vec{u}) + \nabla \cdot (\varrho \vec{u} \otimes \vec{u}) + \boxed{\frac{1}{\varepsilon}} \varrho \vec{f} \times \vec{u} + \boxed{\frac{1}{\varepsilon^{2m}}} \nabla p(\varrho) = \boxed{\varepsilon^{\alpha}} \nabla \cdot S(\nabla \vec{u}) + \boxed{\frac{1}{\varepsilon^{2n}}} \varrho \nabla G,$$

where ϱ is the fluid density, $\vec{u}$ is the velocity field, and $\varepsilon \to 0$ is a small parameter. The symbol $S(\nabla \vec{u})$ denotes the viscous stress, here given by Newton's rheological law

$$S(\nabla \vec{u}) = \mu \left(\nabla \vec{u} + \nabla^t \vec{u} - \frac{2}{3} \nabla \cdot \vec{u} I \right) + \eta \nabla \vec{u} I, \ \mu > 0, \ \eta \geq 0.$$

Finally, we fix the axis of rotation and the direction of the gravitational field as

$$\vec{f} = [0, 0, 1], \ \nabla \cdot G = [0, 0, -1].$$

The problem is considered in an infinite slab

$$\Omega = R^2 \times (0, 1)$$

and supplemented with the complete slip boundary conditions

$$\vec{u} \cdot \vec{n} = u_3|_{\partial\Omega} = 0, \ [S \cdot \vec{n}]_{\tan}|_{\partial\Omega} = 0,$$

and the far field conditions

$$\varrho \to \tilde{\varrho}_\varepsilon, \ \vec{u} \to 0 \text{ as } |x| \to \infty,$$

where $\tilde{\varrho}_\varepsilon$ is the static density distribution satisfying

$$\nabla p(\tilde{\varrho}_\varepsilon) = \varepsilon^{2(m-n)} \tilde{\varrho}_\varepsilon \nabla G, \ \tilde{\varrho}_\varepsilon \to 1 \text{ as } \varepsilon \to 0.$$

Our aim is to identify the singular limit system for $\varepsilon \to 0$. Note that this includes the following phenomena acting *simultaneously*:

- **Low Rossby number.**
  Rossby number $\approx \varepsilon$:

$$\text{3D flow } \to \text{2D flow},$$

  see [1], [2], [3].
- **Low Mach number.**
  Mach number $\approx \varepsilon^m$:

$$\text{compressible } \to \text{incompressible},$$

  see [5], [6], [7], [10], among others.
- **High Reynolds number.**
  Reynolds number $\approx \varepsilon^{-\alpha}$:

$$\text{viscous (Navier-Stokes) } \to \text{inviscid (Euler)},$$

  see Clopeau, Mikelić, Masmoudi [8], [9], [10], Swann [11], among others.

We discuss the asymptotic limit and identify the limit problem - the incompressible planar Euler system:

$$\nabla \cdot \vec{v} = 0,$$

$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} + \nabla \Pi = 0 \text{ in } (0,T) \times R^2.$$

References

[1] A. Babin, A. Mahalov, and B. Nicolaenko, *Global regularity of 3D rotating Navier-Stokes equations for resonant domains*, Indiana Univ. Math. J. **48**(1999), 1133–1176.

[2] A. Babin, A. Mahalov, and B. Nicolaenko, *3D Navier-Stokes and Euler equations with initial data characterized by uniformly large vorticity*, Indiana Univ. Math. J. **50** (Special Issue) (2001), 1–35.

[3] J.-Y. Chemin, B. Desjardins, I. Gallagher, and E. Grenier, *Mathematical geophysics*, volume **32** of *Oxford Lecture Series in Mathematics and its Applications*, The Clarendon Press Oxford University Press, Oxford, 2006.

[4] T. Clopeau, A. Mikelić, and R. Robert, *On the vanishing viscosity limit for the* 2D *incompressible Navier-Stokes equations with the friction type boundary conditions*, Nonlinearity **11** (1998), 1625–1636.

[5] D. B. Ebin, *The motion of slightly compressible fluids viewed as a motion with strong constraining force*, Ann. Math. **105** (1977), 141–200.

[6] S. Klainerman and A. Majda, *Singular limits of quasilinear hyperbolic systems with large parameters and the incompressible limit of compressible fluids*, Comm. Pure Appl. Math., **34** (1981), 481–524.

[7] P.-L. Lions and N. Masmoudi, *Incompressible limit for a viscous compressible fluid*, J. Math. Pures Appl. **77** (1988), 585–627.

[8] N. Masmoudi, *The Euler limit of the Navier-Stokes equations, and rotating fluids with boundary*, Arch. Rational Mech. Anal. **142** (1998), 375–394.

[9] N. Masmoudi, *Incompressible, inviscid limit of the compressible Navier-Stokes system*, Ann. Inst. Henri Poincaré, Anal. non linéaire **18**, 199–224.

[10] N. Masmoudi, *Examples of singular limits in hydrodynamics*, In Handbook of Differential Equations, III, C. Dafermos, E. Feireisl Eds., Elsevier, Amsterdam, 2006.

[11] H.S.G. Swann, *The convergence with vanishing viscosity of nonstationary Navier–Stokes flow to ideal flow in* R^3, Trans. Amer. Math. Soc. **157**, 373–397.

Is there Einstein's formula in fluid dynamics?

Anatoli Babin

In geophysical fluid dynamics there are two kinds of rotation: uniform rotation (the Earth rotation) and localized rotation (as in tornadoes, hurricanes, eddies). The subject of my talk is localized rotation.

A localized object can be characterized by its trajectory. The trajectory can be affected by external forces and by internal characteristics of the object, such as its energy. This type of relation is described by Einstein's formula $M = \frac{E}{c^2}$. In the Newtonian mechanics the inertial mass determines the response to an external force by the formula $\text{Mass} = \frac{\text{force}}{\text{acceleration}}$, and Einstein's formula relates the inertial mass with the internal energy of an object.

To understand the mechanics of extended localized objects we consider a simpler, but still nontrivial, model from electrodynamics. Namely we consider the nonlinear Klein-Gordon (KG) equation.

We have proven [1], [2] the following theorem: If solutions of the KG equation concentrate at a trajectory $\hat{\mathbf{r}}(t)$, then the trajectory satisfies the following equation:

$$\partial_t \left(\frac{1}{\mathrm{c}^2} \bar{\mathcal{E}}_\infty (t) \, \partial_t \hat{\mathbf{r}} \right) = \mathbf{f}_\infty (\hat{\mathbf{r}})$$

where $\bar{\mathcal{E}}_\infty (t)$ is the limit of the concentrating energy, $\mathbf{f}_\infty$ is the Lorentz force and c is the speed of light. Hence we obtain Einstein's formula $M = \frac{\bar{\mathcal{E}}_\infty(t)}{c^2}$ as a property of localized solutions of the KG equation. Examples show that the main contribution to the energy comes from the rotation in the complex plane described by the phase factor of a solution.

The concept of concentrating solutions used in the derivation of Einstein's formula is quite general and can be applied to problems in fluid mechanics. Now there is the question. Is it possible to derive an analog of Einstein's formula for dynamics of localized objects in fluid mechanics framework? In the talk I illustrate the method of concentrating solutions on the example of 2D Euler equation and 3D Euler equations.

References

[1] Babin A. and Figotin A., Relativistic dynamics of accelerating particles derived from field equations, Foundations of Physics, Volume 42, Issue 8, pp 996-1014 (2012).

[2] Babin A. and Figotin A., Relativistic point dynamics and Einstein formula as a property of localized solutions of a nonlinear Klein-Gordon equation, Comm. Math. Phys. (2013).

Models for nearly geostrophic shallow water with spatially varying Coriolis parameter

Marcel Oliver
(joint work with Mahmut Çalık, Sergiy Vasylkevych)

Large-scale flow in mid-latitude atmosphere and ocean dynamics is characterized by smallness of the Rossby number ε, which measures the relative importance of inertial vs. Coriolis forces. To leading order in ε, such flow is in geostrophic balance—the pressure gradient balances the Coriolis force exactly, and the flow is stationary. A balance model then describes the slow dynamics of small departures from a balanced state. In the simplest case, when the full flow is described by the rotating shallow water equations as we assume throughout this talk, there are two classical balance models, the semigeostrophic and the quasigeostrophic equations which differ in the assumed scaling of a second parameter, the Burger number, and in the scaling of the surface height variations. In this talk, we shall only be concerned with the semigeostrophic limit where Burger and Rossby numbers are of the same order and there are no restrictions on the magnitude of surface height variations except for a natural positivity condition on the layer depth.

Salmon [12] pioneered the derivation of balance models via the variational formulation of the fluid system and introduced two new models, the so-called L_1 model and the large-scale semigeostrophic (LSG) equations. (The term "large-scale semigeostrophic equations" was coined in [13], where the author implements similar ideas for a stratified flow.) His ideas were subsequently extended in a number of ways, see [4, 6, 7, 9, 15, 16, 17] and references therein. In this talk, we revisit the variational asymptotics introduced in [7] where the variational principle is written in a new coordinate system chosen precisely so that, when consistently truncated to a certain order in ε, the variational structure degenerates, thereby providing an implicit constraint on the dynamics.

When the Coriolis parameter is constant, this approach yields a one-parameter family of balance models, the *generalized LSG equations*. As a function of the model parameter, they "interpolate" between Salmon's L_1 model and the LSG equations. For a fixed value of the model parameter, an instance of the generalized LSG equations is, in many respects, similar to Hoskins' semigeostrophic equations. Both sets of equations are Hamiltonian (for the semigeostrophic equations, see [12], for the generalized LSG equations, see [8]), both coincide up to terms of order one in Rossby number, and, in the case of constant Coriolis parameter, both can be formulated as an advection equation for the potential vorticity in a transformed coordinate system coupled with a nonlinear potential vorticity inversion. In the semigeostrophic case, the transformation has been introduced by Hoskins [5] and is now known by his name; the associated potential vorticity inversion law is a nonlinear elliptic Monge–Ampère equation. Generalized LSG theory also employs separate semigeostrophic coordinate system in which advected potential vorticity is coupled to the velocity by a system of elliptic PDEs. The key difference is that the Hoskins transformation into semigeostrophic coordinates is explicit in the physical coordinate system and implicit in the new semigeostrophic coordinates. For the generalized LSG equations, the situation is reversed, which has an obvious benefit for the numerical implementation of the model. Advection of potential vorticity was used to prove well-posedness for the semigeostrophic equations [1] and for the generalized LSG equations [2]. Finally, for a constant Coriolis parameter, the semigeostrophic equations also possess a materially conserved potential vorticity in physical coordinates.

When the Coriolis parameter is spatially varying, there is no known conserved potential vorticity for the semigeostrophic equations in physical coordinates [10]. A conserved potential vorticity does exist in so-called vorticity coordinates [14, 10, 11], but computing the transformation to vorticity coordinates requires another prognostic equation [14]. More recently, Cullen *et al.* [3] use the theory of optimal transport to give a formal argument that the semigeostrophic equations on a sphere can be written in terms of potential vorticity advection and inversion, but in order to obtain a practical solution procedure, they continue to work in physical coordinates. Moreover, to our knowledge there are no known results on the mathematical well-posedness of the semigeostrophic equations in this general case.

In this talk, we show how the strategy of [7] extends to the case of the rotating shallow water equations with spatially varying Coriolis parameter. We assume that the Coriolis parameter f is a smooth function and that it remains bounded away from zero; however, no further restrictions are made. In our setting, the difficulties to semigeostrophic theory posed by spatial variations of f largely disappear. We find that the equations of motion can be derived in much the same way as for nonvarying f, and that they can be formulated as an advection equation for a transformed potential vorticity (PV) coupled with a nonlinear potential vorticity inversion relation. The transformation back to physical coordinates is explicit in the new coordinates and can be readily computed.

Invertibility of the potential vorticity relation across the family of generalized LSG models is guaranteed if either the Rossby number or the gradient of the Coriolis parameter is sufficiently small. However, the possible choices of semigeostrophic coordinates are more subtle than in the case of a constant Coriolis parameter. There are three distinguished cases.

First, we can derive a model for which invertibility hinges only on the positivity of the Coriolis parameter and of the initial potential vorticity. This condition is robust in the sense that it is satisfied for all times whenever it is satisfied at the initial time. The same condition already appears in the case of non-varying Coriolis parameter [2]. The condition appears to be sharp and physically reasonable; it unconditionally includes the β-plane approximation to the shallow water equations at mid-latitudes. However, we cannot deal with the degeneracy of the Coriolis parameter for the spherical shallow water equations at the equator—a fundamental difficulty for balance models in general. Although invertibility for this model is as good as for the L_1 model with constant f, the new models necessarily requires, unlike the L_1 model with constant f, an $O(\varepsilon)$ change of coordinates.

Second, we may choose a transformation which is negligible up to terms of $O(\varepsilon^2)$—terms beyond the formal order of accuracy of the model. In this case, we lose simplicity of the balance relation and we lose robust solvability. Third, there is a model where PV inversion gains the maximum possible three derivatives. It too has a non-robust solvability condition whenever the Coriolis parameter is spatially varying.

In conclusion, we believe that the new family of balance models characterized by robust solvability (and which also possess a relatively simple balance relation) is the most promising and warrants further numerical study.

References

[1] J.-D. Benamou and Y. Brenier, *Weak existence for the semigeostrophic equations formulated as a coupled Monge–Ampère/transport problem*, SIAM J. Appl. Math. **58** (1998), 1450–1461.

[2] M. Çalık, M. Oliver, and S. Vasylkevych, *Global well-posedness for models of rotating shallow water in semigeostrophic scaling*, Arch. Ration. Mech. An. **207** (2013), 969–990.

[3] M.J.P. Cullen, R.J. Douglas, I. Roulstone, and M.J. Sewell, *Generalized semi-geostrophic theory on a sphere*, J. Fluid Mech. **531** (2005), 123–157.

[4] D.D. Holm, *Hamiltonian balanced equations*, Physica D **98** (1996), 379–414.

[5] B.J. Hoskins, *The geostrophic momentum approximation and the semi-geostrophic equations*, J. Atmos. Sci. **32** (1975), 233–242.

[6] M.E. McIntyre and I. Roulstone, *Are there higher-accuracy analogues of semi-geostrophic theory*, Large-Scale Atmosphere-Ocean Dynamics II (ed. J. Norbury and I. Roulstone), Cambridge University Press, 2002, pp. 301–364.

[7] M. Oliver, *Variational asymptotics for rotating shallow water near geostrophy: A transformational approach*, J. Fluid Mech. **551** (2006), 197–234.

[8] M. Oliver and S. Vasylkevych, *Hamiltonian formalism for models of rotating shallow water in semigeostrophic scaling*, Discret. Contin. Dyn. S. **31** (2011), 827–846.

[9] R.J. Purser, *Legendre-transformable semigeostrophic theories*, J. Atmos. Sci. **56** (1999), 2522–2535.

[10] I. Roulstone and M.J. Sewell, *Potential vorticities in semi-geostrophic theory*, Q. J. R. Meteorolog. Soc. **122** (1996), 983–992.

[11] I. Roulstone and M.J. Sewell, *The mathematical structure of theories of semi-geostrophic type*, Phil. Trans. R. Soc. Lond. A **355** (1997), 2489–2517.

[12] R. Salmon, *New equations for nearly geostrophic flow*, J. Fluid Mech. **196** (1985), 345–358.

[13] R. Salmon, *Large-scale semi-geostrophic equations for use in ocean circulation models*, J. Fluid Mech. **318** (1996), 85–105.

[14] W.H. Schubert and G. Magnusdottir, *Vorticity coordinates and a canonical form for balanced models*, J. Atmos. Sci. **51** (1994), 3309–3319.

[15] G.J. Shutts, *Planetary semi-geostrophic equations derived from Hamilton's principle*, J. Fluid Mech. **208** (1989), 545–573.

[16] J. Vanneste and O. Bokhove, *Dirac-bracket approach to nearly geostrophic Hamiltonian balanced models*, Phys. D **164** (2002), 152–167.

[17] C. Wunderer, "Hamiltonian Models of Balanced Vortical Flow in the Atmosphere and Oceans," Ph.D. thesis, University of Cambridge, 2001.

Reporter: Katharina Schade

Participants

Ken Abe
Department of Mathematical Sciences
University of Tokyo
3-8-1 Komaba, Meguro-ku
Tokyo 153-8914
JAPAN

Prof. Dr. Anatoli V. Babin
Department of Mathematics
University of California, Irvine
Irvine, CA 92697-3875
UNITED STATES

Prof. Dr. Hakima Bessaih
Department of Mathematics
University of Wyoming
Ross Hall
P.O.Box 3036
Laramie WY 82071
UNITED STATES

Prof. Dr. Dieter Bothe
Center of Smart Interfaces
Technische Universität Darmstadt
Petersenstr. 32
64287 Darmstadt
GERMANY

Prof. Dr. Yann Brenier
CMLS
École Polytechnique
Plateau de Palaiseau
91128 Palaiseau Cedex
FRANCE

Dr. Aseel Farhat
Department of Mathematics
Indiana University at Bloomington
Bloomington, IN 47405
UNITED STATES

Prof. Dr. Reinhard Farwig
Fachbereich Mathematik
TU Darmstadt
Schloßgartenstr. 7
64289 Darmstadt
GERMANY

Prof. Dr. Eduard Feireisl
Institute of Mathematics of the AV CR
Zitna 25
115 67 Praha 1
CZECH REPUBLIC

PD Dr. Matthias Geißert
Fachbereich Mathematik
TU Darmstadt
Schloßgartenstr. 7
64289 Darmstadt
GERMANY

Prof. Dr. Yoshikazu Giga
Graduate School of Mathematical
Sciences
The University of Tokyo
3-8-1 Komaba, Meguro-ku
Tokyo 153-8914
JAPAN

Dr. Karoline Götze
Weierstraß-Institut für
Angewandte Analysis und Stochastik
Mohrenstr. 39
10117 Berlin
GERMANY

Yanqiu Guo
Department of Mathematics
Weizmann Institute
Rehovot 76100
ISRAEL

Prof. Dr. Matthias Hieber
Fachbereich Mathematik
TU Darmstadt
Schloßgartenstr. 7
64289 Darmstadt
GERMANY

Prof. Dr. Toshiaki Hishida
Graduate School of Mathematics
Nagoya University
Chikusa-ku, Furo-cho
Nagoya 464-8602
JAPAN

Prof. Dr. Slim Ibrahim
Dept. of Mathematics & Statistics
University of Victoria
P.O.Box 3060
Victoria, BC V8W 3R4
CANADA

Prof. Dr. Alexei A. Ilyin
M.V. Keldysh Institute of Applied Mathematics
Russian Academy of Sciences
Miusskaya pl. 4
125 047 Moscow
RUSSIAN FEDERATION

Dr. Tsukasa Iwabuchi
Department of Mathematics
Faculty of Science and Engineering
Chuo University
1-13-27, Kasuga, Bunkyo-ku
Tokyo 112-8551
JAPAN

Prof. Dr. Yoshifumi Kimura
Graduate School of Mathematics
Nagoya University
Chikusa-ku, Furo-cho
Nagoya 464-8602
JAPAN

Prof. Dr. Rupert Klein
Fachbereich Mathematik & Informatik
Freie Universität Berlin
Arnimallee 6
14195 Berlin
GERMANY

Hajime Koba
Department of Mathematical Sciences
University of Tokyo
3-8-1 Komaba, Meguro-ku
Tokyo 153-8914
JAPAN

Dr. Peter Korn
Max Planck Institut für Meteorologie
Universität Hamburg
Bundesstr. 55
20146 Hamburg
GERMANY

Prof. Dr. Igor Kukavica
Department of Mathematics
University of Southern California
3620 South Vermont Ave., KAP 108
Los Angeles, CA 90089-2532
UNITED STATES

Dr. Evelyn Lunasin
Department of Mathematics
University of Michigan
East Hall, 525 E. University
Ann Arbor, MI 48109-1109
UNITED STATES

Prof. Dr. Yasunori Maekawa
Department of Mathematics
Kobe University
Rokko-dai, 1-1 Nada-ku
Kobe 657-8501
JAPAN

Prof. Dr. Alex Mahalov
Dept. of Mathematics and Statistics
Arizona State University
Box 871804
Tempe, AZ 85287-1804
UNITED STATES

Prof. Dr. Anna Mazzucato
Department of Mathematics
Pennsylvania State University
324 McAllister Bldg.
University Park, PA 16802
UNITED STATES

Prof. Dr. Sylvie Monniaux
LATP (Mathématiques)
CMI-Technopôle de Château-Gombert
Université Aix-Marseille
39 rue F. Joliot-Curie
13397 Marseille Cedex 20
FRANCE

Prof. Dr. Thieu-Huy Nguyen
Fachbereich Mathematik
TU Darmstadt
64289 Darmstadt
GERMANY

Prof. Dr. Helena J. Nussenzveig-Lopes
Instituto de Matematica
Universidade Federal do Rio de Janeiro
Ilha do Fundao
P.O. Box 68530
Rio de Janeiro, RJ - 21941-909
BRAZIL

Prof. Dr. Marcel Oliver
School of Engineering and Science
Jacobs University Bremen
Campus Ring 1
28759 Bremen
GERMANY

Prof. Dr. Felix Otto
Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26
04103 Leipzig
GERMANY

Prof. Dr. Thierry Paul
Centre de Mathématiques Laurent
Schwartz
École Polytechnique
Route de Saclay
91128 Palaiseau
FRANCE

Prof. Dr. Jan Prüß
Institut für Mathematik
Fakultät NW II
Universität Halle-Wittenberg
Theodor-Lieser-Str. 5
06120 Halle
GERMANY

Prof. Dr. Sebastian Reich
Institut für Mathematik
Universität Potsdam
Am Neuen Palais 10
14469 Potsdam
GERMANY

Prof. Dr. Jürgen Saal
Center of Smart Interfaces
Technische Universität Darmstadt
Petersenstr. 32
64287 Darmstadt
GERMANY

Josiane Salameh
Max Planck Institut für Meteorologie
Universität Hamburg
Bundesstr. 55
20146 Hamburg
GERMANY

Prof. Dr. Roger M. Samelson
College of Earth, Ocean and
Atmospheric Sciences
Oregon State University
104 CEOAS Administration Bldg.
Corvallis, OR 97331-5503
UNITED STATES

Katharina Schade
Fachbereich Mathematik
TU Darmstadt
Schloßgartenstr. 7
64289 Darmstadt
GERMANY

Prof. Dr. Leslie Smith
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive
Madison, WI 53706-1388
UNITED STATES

Prof. Dr. Wilhelm Stannat
Fachbereich Mathematik
Technische Universität Berlin
Straße des 17. Juni 136
10623 Berlin
GERMANY

Dr. Ryo Takada
Department of Mathematics
Kyoto University
Kitashirakawa, Sakyo-ku
Kyoto 606-8502
JAPAN

Prof. Dr. Edriss S. Titi
Department of Mathematics
University of California, Irvine
Irvine, CA 92697-3875
UNITED STATES

Prof. Dr. Joe Tribbia
UCAR
Box 3000
Boulder, CO 80307
UNITED STATES

Prof. Dr. Tsuyoshi Yoneda
Department of Mathematics
Hokkaido University
Kita 8, Nishi 5, Kita-ku
Sapporo 060-0808
JAPAN

Prof. Dr. Mohammed Ziane
Dept. of Mathematics, KAP 104
University of Southern California
3620 Vermont Avenue
Los Angeles, CA 90089-1113
UNITED STATES