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Mini-Workshop: Kähler Groups

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Abstract. We reviewed recent advances in the study of fundamental groups of compact Kähler manifolds, involving an interesting mix of complex geometry, harmonic maps and non-Abelian Hodge theory, and geometric group theory. There were also extensive discussions of open problems in the area.

Mathematics Subject Classification (2010): 14C30, 14F35, 20F65, 32J25, 32Q15.

Introduction by the Organisers

Kähler groups are groups that occur as fundamental groups of closed Kähler manifolds. They include all the fundamental groups of smooth complex projective varieties, and an interesting open problem is to find out whether this inclusion is strict. Kähler groups form a very restricted subclass of the class of all finitely presentable groups. Traditionally, restrictions arose mostly from Hodge theory and from rational homotopy theory, but over the years many other techniques – such as harmonic maps and geometric group theory – have been brought to bear on the subject.

The workshop brought together 16 participants from Europe, the United States and India, all of whom are actively working in the subject, approaching it from different directions: some from complex algebraic geometry, and others from geometric group theory, or from differential geometry and analysis. The purpose was to review the progress made in the area in the last 10 years or so, and to discuss open problems. The form of the discussions evolved during the week, with many more formal lectures in the first half, and more informal discussions and problem sessions during the second half of the week. Many of the formal lectures gave rise to followup discussions during the informal sessions. There were 13 formal lectures altogether. On Wednesday a lengthy informal session was held, that also included the first problem session. There was another problem session on Thursday evening, and final wrap-up discussion on Friday.

It is much easier to prove restrictions on Kähler groups, than it is to exhibit interesting, non-obvious, examples. During the workshop, the lectures by Dimca, Kapovich and Panov were devoted to methods and attempts at constructions, and this was also in important topic of discussion during the informal and problem sessions. Linear representations of Kähler groups, and, more generally, the question of how close to linear a general Kähler group might be, appeared in one form or another in the lectures of Klingler, Brunbarbe, Maubon and Eyssidieux. The main problem arising from these lectures and subsequent discussions is: are there infinite Kähler groups with no linear representation with infinite image? The natural conjecture is that such groups should exist, and the challenge is to construct them. One interesting suggestion for a construction would be the search for a complex hyperbolic analogue of the work of Panov and Petrunin in real hyperbolic geometry.

While many lectures had an algebro-geometric and/or Hodge theoretic flavour, there were other directions. Methods of D-modules applied to the topology of varieties, notably cohomology, appeared in the lectures of Schnell and Wang. Methods of geometric group theory and coarse geometry featured prominently in several lectures, particularly those of Delzant. The methods of geometric group theory quickly lead to the class of "fibered" Kähler groups: those that map to surface groups, and the corresponding Kähler manifolds map homolomorphically to complex curves. The lecture by Mahan also used geometric group theory methods. The lecture by Py was devoted to other factorization theorems through complex curves proved by applications of harmonic maps.

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Mini-Workshop: Kähler Groups

Table of Contents

Abstracts

Cuts in Kähler groups

Thomas Delzant

The aim of these two lectures is to explain our joint paper [1] with Misha Gromov. The main result can be stated as follows :

Let X be a compact Kähler manifold. Assume that its fundamental group G has a subgroup H such that G/H is stable at infinity, and H cuts G at infinity in at least 3 relative ends, then there exists a finite (unramified) cover of X, say X_1, which admits a holomorphic map to a hyperbolic compact Riemann surface S so that $H \supset K$, the kernel of $G_1 = \pi_1(G_1) \to \pi_1(S)$. In particular if G is hyperbolic and Kähler it does not admits a convex subgroup H such that the number of relative ends is greater than 2, unless it is a virtual surface group.

According to Napier and Ramachandran (GAFA, 2009) the hypothesis of stability at infinity is not useful; the necessity of at least three ends is however fundamental, as one can see for example in the case of abelian groups.

In the first lecture, we explain some elementary facts about the topology at infinity of a group; we recall Freudenthal's definition of ends, and extend this definition to relative ends or "cuts". Let G be the fundamental group of a compact manifold X. We prove that a subgroup H cuts G at infinity if and only if there exists a $H-$equivariant, $H-$proper map from the universl cover of X to a H-tree, called the Freudenthal map.

In the second lecture we recall Gromov-Schoen theory. Such a proper map to a tree can be (under certain conditions) promoted as a H-equivariant harmonic map to the tree of finite energy. This map is proved to be pluriharmonic and therefore admits a complexification by a codimension one complex foliation of X, whose leaves are defined by a quadratic differential, and contained in the compact levels of the Freudenthal map. A Tischler-like theorem for such foliations on Kähler manifolds is stated: if one leaf is compact, then all leaves are. The branching of the tree forces the existence of a multiple leaf. One prove that all leaves are compact, and a finite cover is constructed by geometric arguments.

Some examples are discussed. The main corollary is that a "small cancellation" group cannot be Kähler unless it is a surface group. Better, a morphism from a Kähler group to a small cancelation group factorizes through a virtual surface group.

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Non-finiteness properties of the fundamental groups of smooth projective varieties

Alexandru Dimca
(joint work with S. Papadima, A. Suciu)

Since a smooth connected projective variety X is homotopy equivalent to a finite simplicial complex, its fundamental group $G = \pi_1(X, x_0)$ is finitely presentable. One may ask which additional finiteness properties such a group may enjoy in general, or if one imposses some extra conditions on X, e.g. by asking that the universal cover $\tilde{X}$ of X is a Stein manifold.

As an example, if X is a product of smooth projective curves of genus $g > 1$, then X is a classifying space $K(G,1)$ and the universal cover $\tilde{X}$ is a Stein manifold, being a product of discs. Note that it is very exceptional for a group G to have a classifying space $K(G,1)$ which is a finite complex.

In the paper [2] we have constructed smooth hypersurfaces Y in a product of curves as above, such that the universal cover $\tilde{Y}$ of Y is a Stein manifold but Y has very bad finiteness properties, reflected by the fact that $\tilde{Y}$ is $(d-1)$-connected but has an infinite homotopy group $\pi_d(\tilde{Y})$, where d is the dimension of Y. In particular, such a fundamental group does not have a classifying space $K(G,1)$ which is a finite complex.

It is even easier to construct affine smooth varieties with such properties, see the paper [1], where we show that such a 'bad' hypersurface can be constructed in the complex affine space C^n for $n \geq 3$ by choosing a sequence of integers $(k_1, ..., k_n)$ with $k_j > 0$ and setting

$$Y : x_1(x_1+1)\cdots(x_1+k_1)x_2(x_2+1)\cdots(x_2+k_2)\cdots x_n(x_n+1)\cdots(x_n+k_n) = a,$$

for a generic $a \in C$.

To prove such results, one relates the usual finiteness properties F_n and FP_n considered in geometric group theory to the theory of twisted cohomology jumping loci, for which we refer to [3]. Similar techniques have been used in our recent paper [4] to treat finiteness properties of Torelli groups.

In addition, one uses the complex Morse theory which studies the change in the topology when we pass from the generic fiber of a Lefschetz pencil to the total space (whose topology in our cases is very simple). The new ingredient is that our Lefschetz pencils involve infinitely many critical points, which lead in the end to infinitely many cells to be attached.

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Symmetric differentials I

Bruno Klingler

I gave two talks on symmetric differentials. The first one was devoted to my recent joint work with Brunebarbe and Totaro [4] (and was continued by Yohan Brunebarbe). In the second one I tried to explain the results obtained by Bogomolov and De Oliveira in [2] and [3].

1. Symmetric differentials and linear representations of the fundamental group

Let X be a compact connected Kähler manifold. The Hodge decomposition

$$H^i(X, \mathbf{C}) = \bigoplus_{i=p+q} H^p(X, \Lambda^q \Omega^1_X)$$

tells us that the exterior algebra $\Lambda^\bullet \Omega^1_X$ (which is a purely holomorphic object) completely controls the Betti cohomology of X. Our work with Brunebarbe and Totaro started with the following question: what is the link between the *symmetric* algebra $S^\bullet \Omega^1_X$ and the topology of X?

Notice that if one denotes by $\mathcal{O}(1)$ the relatively ample line bundle on the projective bundle $\pi : \mathbb{P}\Omega^1_X \to X$ of hyperplanes of Ω^1_X, then $\pi_* \mathcal{O}(i) = S^i \Omega^1_X$ and $H^0(X, S^i \Omega^1_X) = H^0(\mathbb{P}\Omega^1_X, \mathcal{O}(i))$. Hence global symmetric differentials on X are controlled by the positivity properties of $\mathcal{O}(1)$ on $\mathbb{P}\Omega^1_X$.

I spent my first talk describing the relation between global symmetric differentials on X and the finite dimensional representation theory of the fundamental group $\pi_1(X)$. I first recalled the following theorem and its proof:

Theorem 1.1. *(Hitchin, Simpson, Arapura, Katzarkov, Zuo) Let X be a compact connected manifold and r a positive integer. Suppose $H^0(X, S^i \Omega^1_X) = 0$ for $0 < i \leq r$. Then:*

(1) *the character variety* $\mathrm{Hom}(\pi_1(X), GL(r, \mathbf{C})//GL(r, \mathbf{C})$ *is a finite set of points.*
(2) *Any semi-simple representation $\rho : \pi_1(X) \to GL(r, F)$, F a non-archimedean local field, has bounded image.*

Then I stated our main result and explained its proof for k a field of positive characteristic:

Theorem 1.2. *(Brunebarbe, Klingler, Totaro) Let X be a compact connected Kähler manifold. Suppose $H^0(X, S^{>0} \Omega^1_X) = 0$. Then any representation*

$$\rho : \pi_1(X) \to GL(r, k)$$

(k any field, r any integer) has finite image.

This theorem can be seen as creating symmetric differentials from topology. As any such differential vanishes on the rational curves of X it might have applications for studying hyperbolicity of compact Kähler manifolds. On the other hand notice that many algebraic varieties do satisfy the assumptions of theorem 1.2: a classical result of Schneider states that any smooth subvariety of $\mathbb{P}^N\mathbb{C}$ of dimension at least $N/2$ has no symmetric differential. All these examples have finite fundamental group. I expect that any X as in theorem 1.2 should have a finite étale fundamental group.

2. The work of Bogomolov and De Oliveira

I first explained the following result [1]:

Proposition 2.1. *(Bogomolov-De Oliveira) For $m \geq 2$, $H^0(X, S^m\Omega^1_X)$ is not a topological invariant.*

This motivates the definition by Bogomolov and De Oliveira of the subclass of *closed* symmetric differentials (generalizing the classical notion in degree 1). In some special cases they show that their existence forces strong topological restrictions on X.

Definition 2.2. *A symmetric form $\omega \in H^0(X, S^m\Omega^1_X)$ is said to be:*

(1) exact *on an open set U of X if $\omega_{|U} = (df_1)^{m_1} \cdots (df_k)^{m_k}$ where $f_i \in \mathcal{O}(U)$ and $\sum_i m_i = m$.*
(2) closed *on X if is exact in a neighborhood of a general point of X.*
(3) closed of the first kind *on X if there exists a covering (U_i) of X such that $\omega_{|U_i}$ is exact for all i.*

I explained the proof of the following result [3]:

Theorem 2.3. *(Bogomolov-De Oliveira) Let X be a smooth projective complex variety. Let $\omega \in H^0(X, S^2\Omega^1_X)$ of rank 2 and closed of the first kind. Then:*

(a) *There exists a finite unramified Galois covering $f : X' \to X$ such that $f^*\omega$ comes from a symmetric two-form on the Albanese of X'.*
(b) *The group $\pi_1(X)$ is infinite. More precisely there exists a finite index subgroup of $\pi_1(X)$ with infinite abelianisation.*

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Cohomology jump loci

Christian Schnell

Let X be a topological space, and denote by $\mathrm{Char}(X) = \mathrm{Hom}\big(\pi_1(X), \mathbb{C}^*\big)$ the space of characters of the fundamental group. Each character $\rho \in \mathrm{Char}(X)$ determines a rank-one local system $\mathbb{C}_\rho$ on X. The *cohomology jump loci* of X are the sets

$$\Sigma_m^k(X) = \{\, \rho \in \mathrm{Char}(X) \mid \dim H^k(X, \mathbb{C}_\rho) \geq m \,\}.$$

When X is sufficiently nice – for example, homotopy-equivalent to a finite CW-complex – $\mathrm{Char}(X)$ is an affine algebraic variety, and every $\Sigma_m^k(X)$ is a closed algebraic subvariety. For $k = 1$, the cohomology jump loci only depend on the fundamental group of X; for $k \geq 2$, they only depend on X up to homotopy.

In the late 1980s, Beauville and Catanese conjectured that when X is a compact Kähler manifold, every irreducible component of $\Sigma_m^k(X)$ should be a translate of an affine torus by a point of finite order. The proof of the conjecture was recently completed by Wang [8], who also spoke at the workshop. Let me briefly outline the history of this result. Beauville [2] showed that positive-dimensional components of $\Sigma_1^1(X)$ correspond (more or less bijectively) to morphisms from X to compact Riemann surfaces of genus ≥ 2. Arapura [1] showed that every irreducible component of $\Sigma_m^k(X)$ is a translate of an affine torus, but left open the question of whether the translates are by points of finite order. Simpson [7] proved the conjecture for smooth projective X, with the help of the Schneider-Lang criterion from transcendence theory. Campana [3] deduced from Simpson's result that the conjecture is true for $\Sigma_1^1(X)$; an alternative argument, using methods from geometric group theory, was later given by Delzant [4]. Pink and Roessler [5] reproved Simpson's theorem by using reduction to positive characteristic. More recently, I found a third proof [6] that uses some results about Hodge modules on abelian varieties; this argument was extended by Wang to prove the conjecture in all cases.

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Symmetric differentials II

Yohan Brunebarbe

(joint work with Bruno Klingler, Burt Totaro)

In my talk I explained the proof of the following result:

Theorem 0.1. (see [1], theorem 0.1) Let X be a compact Kähler manifold. Suppose that there is a finite-dimensional complex representation of its fundamental group $\rho : \pi_1(X) \to GL_n(\mathbf{C})$ with infinite image. Then X has a nonzero symmetric holomorphic differential form, i.e. a nonzero element of some $H^0(X, S^k\Omega^1_X)$, $k \geq 1$.

First, using classical Hodge theory and the results that Bruno explained in his talk, I showed how to reduce the proof of theorem 0.1 to the special case where ρ has discrete image and is the monodromy of a polarizable complex variation of Hodge structures. Theorem 0.1 is then a consequence of the following result:

Theorem 0.2. (see [1], theorem 3.1) A compact complex manifold which supports a polarizable complex variation of Hodge structures with infinite and discrete monodromy has a nonzero symmetric holomorphic differential form, i.e. a nonzero element of some $H^0(X, S^k\Omega^1_X)$, $k \geq 1$. Moreover, if the corresponding period map is immersive in at least one point of X, then the cotangent bundle Ω^1_X of X is big.

Recall that a holomorphic vector bundle on a compact complex manifold is nef (resp. big) if the tautological quotient line bundle $\mathscr{O}_E(1)$ on the projective bundle $\mathbb{P}(E)$ of hyperplanes in E has the corresponding property.

Let us give an idea of the proof of theorem 0.2 in the special case where the period map $\tilde{\phi} : \tilde{X} \to D$ is everywhere immersive. Recall that the period domain D is a complex manifold which is homogeneous under a semisimple real Lie group G of noncompact type, and that the period map is $\pi_1(X)$-equivariant with respect to the monodromy representation $\rho : \pi_1(X) \to G \subset GL_n(\mathbf{C})$. Griffiths and Schmid (cf. [3], theorem 9.1) defined a G-invariant hermitian metric on D. Because the period map is an immersion, one can pull back this metric to $\tilde{X}$. This defines a $\pi_1(X)$-equivariant hermitian metric on $\tilde{X}$, hence a hermitian metric on X. It turns out that this metric is Kähler, has nonpositive holomorphic bisectional curvature and negative holomorphic sectional curvature (here the horizontality of the period map plays a crucial role; see [1], section 3 for the details). It follows (cf. [1], theorem 1.1) that the cotangent bundle Ω^1_X is nef and big. In particular, X has a lot of symmetric differentials.

If the period is just generically immersive, then it is still possible by some trick to show the bigness of Ω^1_X (but of course Ω^1_X won't be nef in general). In the general case, as the image Γ of the monodromy is discrete, the period map induces a map $\phi : X \to D/\Gamma$. Suppose to simplify that Γ is torsion-free. Let Y be a resolution of singularities of the image of $\phi : X \to D/\Gamma$. As Γ is infinite, Y is positive dimensional. The period map defines on Y a polarizable complex variation of Hodge structures with infinite and discrete monodromy. Moreover, as now its period map is generically immersive, the precedent discussion shows that the cotangent bundle Ω^1_Y of Y is big. Finally, one obtains symmetric forms on X by pulling-back symmetric forms on Y along the rational map $X \dashrightarrow Y$.

At the end of the talk we said a few words about an extension of theorem 0.2 to non-compact varieties:

Theorem 0.3. (see [2]) Let U be a smooth complex algebraic variety which supports an integral variation of polarized Hodge structures with infinite monodromy. Then, for any smooth compactification X of U such that $D = X - U$ is a simple normal crossing divisor, there exists $k \geq 1$ such that $S^k\Omega^1_X(\log D)$ has a nonzero section. Moreover, if the corresponding period map is immersive in at least one point of U, then the logarithmic cotangent bundle $\Omega^1_X(\log D)$ of the pair (X, D) is big.

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Polyhedral complexes and topology of projective varieties

Michael Kapovich

This talk is based on our work with János Kollár [3] and my own paper [2]. This work is motivated by

Question 0.4. What are fundamental groups of (possibly) singular (complex) projective varieties.

It is well-known (and goes back to Serre) that singular reducible projective varieties can have arbitrary finitely-presented (fp) groups. The main question is what happens if one imposes some control on singularities and the irreducibility assumption. Simpson in [4] proved that every finitely-presented group appears as the fundamental group of an irreducible projective variety; he further asked if such variety can be chosen to have only normal crossing singularities. Our main results are:

Theorem 0.5. (Kapovich, Kollár, [3]) Every fp group appears as $\pi_1(Z)$ for some 2-dimensional projective variety Z with (simple) normal crossing singularities.

Corollary 0.6. (Kapovich, Kollár, [3]) Every fp group G appears as the fundamental group of the link of a 3-dimensional isolated (complex) singularity.

Theorem 0.7. (Kapovich, [2]) Every fp group appears as $\pi_1(Z)$ for some irreducible 2-dimensional projective variety Z whose singularities are normal crossings and Whitney umbrellas.

Eliminating Whitney umbrellas in this theorem remains an open problem. Proof of Theorem 0.7 is based on a combination of hyperbolic and algebraic geometry described in the following diagram:

$$\begin{array}{ccc} \boxed{\text{Group presentation}} & \longrightarrow & \boxed{\text{Hyperbolic orbifold}} \\ & & \downarrow \\ \boxed{\text{Projective variety}} & \longleftarrow & \boxed{\text{Generic Dirichlet domain}} \end{array}$$

The first arrow in this diagram is the following variation on a recent theorem of Panov and Petrunin [5]:

Theorem 0.8. (Kapovich, [2]) For every fp group G there exists a convex-cocompact subgroup $\Gamma < PO(3,1)$ where every finite order element is a Cartan involution, so that $G \cong \pi_1(\mathbb{H}^3/\Gamma)$.

The second arrow is related to the following conjecture (which appears as a theorem in the erroneous paper [1]):

Conjecture 0.9. Suppose that $\Gamma < PO(3,1)$ is a discrete torsion-free subgroup, then for a generic choice of a base-point $x \in \mathbb{H}^3$, the corresponding tiling $\mathcal{D}_x$ of $\mathbb{H}^3$ by Dirichlet fundamental domains of Γ (with respect to x) is *simple*.

Here simplicity of a tiling means that for every k-dimensional face c of the tiling, its residue Res_c (with respect to the tiling) is a simplicial complex isomorphic to the face-complex of the boundary of $3-k$-simplex. In other words, every edge of the tiling is shared by exactly 3 facets and every vertex is shared by exactly 4 facets. The partial result which suffices for our purposes is

Theorem 0.10. (Kapovich, [2]) Suppose that $\Gamma < PO(3,1)$ is a discrete subgroup without unipotent elements, such that each nontrivial finite order element is a Cartan involution. Then for a generic choice of x, the Dirichlet tiling $\mathcal{D}_x$ is simple away from its vertices.

The last arrow is a minor variation on the main construction in [3] which converts a finite hyperbolic polyhedral complex with a single facet, to an irreducible projective variety V. Normal crossings in V correspond to simplicity of edges in the Dirichlet tiling; Whitney umbrellas come from Cartan involutions in Γ. Irreducibility of V comes from transitivity of the action of Γ on facets of the Dirichlet tiling $\mathcal{D}_x$.

The following conjecture is motivated by Theorem 0.5 and our the current state of knowledge of Kähler groups:

Conjecture 0.11. Every finitely-presented group appears as a subgroup of some Kähler group. (In particular, the word problem for Kähler group is not solvable.)

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On the second cohomology of Kähler groups

Julien Maubon

(joint work with Bruno Klingler, Vincent Koziarz)

We are interested in the second cohomology of Kähler groups, i.e. fundamental groups of closed Kähler manifolds, and more precisely in the following question of Carlson and Toledo: *Is it true that any infinite Kähler group* Γ *has (virtually)* $H^2(\Gamma, \mathbb{R}) \neq 0$*?* A positive answer to this question would provide strong evidence to a conjecture of the same authors which says that the only cocompact lattices in semisimple Lie groups which are Kähler are the obvious ones, namely the lattices in Lie groups of Hermitian type.

Let M be a closed Kähler manifold and let Γ be its fundamental group. Assume Γ is infinite.

It is easily seen that $H^2(\Gamma, \mathbb{R}) \neq 0$ holds if $\pi_2(M)$ is trivial. By the Hard Lefschetz Theorem, its generalization by Simpson [5] to the case of local systems, and by a theorem of Reznikov [4], the result is also true if Γ admits a non locally rigid representation in $\mathrm{GL}(n, \mathbb{C})$ for some n, or in the isometry group of a Hilbert space.

We will assume that Γ admits a linear representation ρ in $\mathrm{GL}(n, \mathbb{C})$, and that this representation is unbounded, irreducible, and reductive. All known Kähler groups admit such a representation (although there exist non linear Kähler groups by [6]). This representation can be assumed to be locally rigid and by Simpson [5], it is therefore the monodromy of a polarized complex variation of Hodge structure ($\mathbb{C}$-VHS for short) on the Kähler manifold M.

This means that for some non degenerate Hermitian form h of signature (p,q) on $\mathbb{C}^n$, the representation ρ takes values in $\mathrm{U}(h)$, and that there is a $\mathrm{U}(h)$-period domain D and a holomorphic horizontal map f from the universal cover $\tilde{M}$ of M to D which is ρ-equivariant. The map f is called the period map. The period domain is an open $\mathrm{U}(h)$-orbit in a flag manifold in $\mathbb{C}^n$, and therefore a complex manifold. More precisely, for some positive integers $k \geq 2$ and $r_1, \ldots, r_k$, D is the space of flags $\mathbf{F} = (\{0\} = F^0 \subset F^1 \subset F^2 \subset \ldots \subset F^k = \mathbb{C}^n)$ such that for all $i = 1, \ldots, k$, $\dim F^i/F^{i-1} = r_i$ and $(-1)^i h$ is positive definite on the h-orthogonal complement of F^{i-1} in F^i. The horizontality of the period map f corresponds to Griffiths transversality condition on $\mathbb{C}$-VHS, see [2], and means that if for some $x \in \tilde{M}$, $f(x)$ is the flag $\mathbf{F} = (\{0\} = F^0 \subset F^1 \subset F^2 \subset \ldots \subset F^k = \mathbb{C}^n)$, then the differential of f maps the holomorphic tangent space of $\tilde{M}$ at x in the horizontal subspace $H_{\mathbf{F}} := \bigoplus_{i=1}^k \mathrm{Hom}(F^i/F^{i-1}, F^{i+1}/F^i)$ of the holomorphic tangent space of D at $\mathbf{F}$. The horizontal distribution $H := \{H_{\mathbf{F}}, \mathbf{F} \in D\}$ defines a holomorphic subbundle of the holomorphic tangent bundle of D. We call ϕ_i the composition of df with the projection $H_{\mathbf{F}} \to \mathrm{Hom}(F^i/F^{i-1}, F^{i+1}/F^i)$.

The period domain projects to the symmetric space X associated to $U(h)$, but this projection is never holomorphic unless $D = X$. On the other hand, for each $i = 1, \ldots, n-1$, there is a holomorphic projection ν_i from D to an open $\mathrm{U}(h)$-orbit D_i in the Grassmaniann of $(r_1 + \ldots + r_i)$-dimensional subspaces in $\mathbb{C}^n$, which is given by $\nu_i(\mathbf{F}) = F^i$. Note that ϕ_i can be interpreted as the differential of $\nu_i \circ f$.

The idea is then to use line bundles over the period domain D or over the D_i's to produce, via their first Chern classes, the required non trivial element in $H^2(\Gamma, \mathbb{R})$. Because some of these line bundles, e.g. the canonical bundle, have good positivity properties in the horizontal directions, see [3] and [1], pulling them back by the period map we obtain non zero classes in $H^2(M, \mathbb{R})$. The question is then to decide if these classes belong to $H^2(\Gamma, \mathbb{R})$. This is true if the period map f or its post-composition with one of the projection ν_i kills $\pi_2(M)$.

Our first result is that under an hypothesis on the period map, namely that for some $1 \leq i \leq k$, ϕ_i has rank 1, this strategy indeed gives $H^2(\Gamma, \mathbb{R}) \neq 0$, because in this case $\nu_i \circ f$ factors through a complex curve ($\neq \mathbb{CP}^1$). We also prove that if the period domain satisfies that $r_i = 1$ for some $1 < i < k$, then our hypothesis on the period map is satisfied.

To understand the effect of the period map f on $\pi_2(M)$, one needs to know whether there are horizontal 2-spheres in the period domain D which are homotopically non trivial. It is known that if N is a fiber of the projection $D \to X$, $\pi_2(M)$ is naturally isomorphic to $\pi_2(N) \simeq \mathbb{Z}^{k-2}$. Using Gromov h-principle, we show that in many cases, there are elements in $\pi_2(N)$ which are homotopic to horizontal 2-spheres. In particular, when $r_i \geq 2$ for all $1 < i < k$, $\pi_2(D)$ can be generated by horizontal 2-spheres.

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Cohomology jump loci of compact Kähler manifolds

Botong Wang

Given a connected finite CW-complex X, the space of rank one local systems on X is canonically isomorphic to the space of rank one representations of the fundamental group of X. We denote the latter one by $\mathrm{Char}(X)$. The rank one cohomology jump loci of X are defined to be

$$\Sigma^k_l(X) = \{\rho \in \mathrm{Char}(X) | \dim H^k(X, L_\rho) \geq l\}$$

where L_ρ is the local system corresponding to representation ρ.

It is shown first by Simpson [3] that when X is a smooth complex projective variety, each $\Sigma^k_l(X)$ is a finite union of torsion translates of subtori. More recently, Schnell [2] gave a new proof based on the theory of D-modules. In the first part, I will give a generalization of Simpson's theorem to compact Kähler manifolds using Schnell's approach. The main step is to show that up to isogeny every simple polarizable Hodge module defined over $\mathbb{Z}$ on a compact complex torus is the pull-back of a polarizable Hodge module defined over $\mathbb{Z}$ via the algebraic reduction.

In the second part, we work on the opposite direction. We construct a compact Kähler manifold Y, and show that any other compact Kähler manifold having the same Σ^2_1 as Y is not a projective manifold. The example is essentially same as the one of Voisin. However, the argument is more natural, and the conclusion can be stronger. In fact, according to a recent result of Dimca-Papadima [1], any compact Kähler manifold that is of the same real 2-homotopy type as Y is not a projective manifold.

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A factorization theorem for harmonic maps of low rank

Pierre Py
(joint work with T. Delzant)

The purpose of this talk was to explain the proof of the following result:

Theorem 1. Let X be a compact Käbler manifold with universal cover $\widetilde{X}$. Let G be a simple Lie group with associated symmetric space G/K and let $\varrho : \pi_1(X) \to G$ be a Zariski dense representation of the fundamental group of X. Let

$$f : \widetilde{X} \to G/K$$

be the associated ϱ-equivariant pluriharmonic map. Assume that the complex rank of $df^{1,0}$ is 1.

Then, there exists a fibration $p : X \to \Sigma$ of X onto a hyperbolic 2-dimensional orbifold Σ, such that ϱ factors through the map $p_* : \pi_1(X) \to \pi_1^{orb}(\Sigma)$ induced by p.

Let us explain some of the hypothesis of the theorem.

- Here we assume that G is connected, has a simple Lie algebra, and has trivial center (so that G is the identity component of the isometry group of its symmetric space).
- Note that we are *not* assuming that G/K is a Hermitan symmetric space and that f is holomorphic. However it is known (thanks to classical results due to Siu and Sampson) that the pull-back by f of the complexified tangent bundle of G/K has the structure of a holomorphic vector bundle over $\widetilde{X}$. Hence one can decompose df by type: $df = df^{1,0} + df^{0,1}$ and it is known that $df^{1,0}$ is holomorphic for the previous holomorphic structure. Its generic rank is thus well-defined and we are assuming that this rank is 1.
- The conclusion of the theorem means that there exists a homomorphism $\psi : \pi_1^{orb}(\Sigma) \to G$ such that $\varrho = \psi \circ p_*$.

This theorem was established in [2] and several applications of it were given there. Among these applications we obtained a generalization of a classical theorem of Carlson and Toledo [1] concerning representations of Kähler groups into the isometry groups of real hyperbolic spaces. In the case where the symmetric space G/K is the real hyperbolic space, the hypothesis on the rank of $df^{1,0}$ is always satisfied thanks to a result of Sampson. We also gave an application concerning representations of Kähler groups into the Cremona group. But the main purpose of the talk was to insist on the proof of Theorem 1, which, although it is technical, should be of interest since factorization theorems in the study of Kähler groups are always difficult to establish.

To prove the theorem, we make use of the following two facts:

(1) First, the harmonic map f is real analytic (being a solution of an elliptic equation with real-analytic coefficients),
(2) Second, we use the following theorem of Diederich and Mazzilli [3]: if A is a real-analytic set in $\mathbb{C}^n$ and if B denotes the subset of A made of all points p of A such that A contains a germ of holomorphic curve passing through p, then B is closed in A.

We then consider the maximal open set O of $\widetilde{X}$ on which the foliation defined by the kernel of $df^{1,0}$ extends. The set $\widetilde{X}-O$ has codimension at least 2. The key point to prove our theorem is to define an equivalence relation on O whose graph will be a subset of the real-analytic set defined by $A_0 = \{(x,y) \in O \times O, f(x) = f(y)\}$. We use the theorem of Diederich and Mazzilli to prove that this graph is closed. The idea of defining an equivalence relation whose graph is a complex analytic set contained in the real-analytic set A_0 already appears in the work of Mok [4].

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Linear representations of Kähler groups

Philippe Eyssidieux

(joint work with Frédéric Campana, Benoît Claudon)

I outlined a proof of the following partial positive result in the direction of the well-known open problem that any torsion free Kähler group is projective. It implies that a linear Kähler group is virtually projective.

Theorem: Let X be a compact Kähler manifold and $\rho : \pi_1(X) \to GL_n(\mathbb{C})$ a linear representation of its fundamental group. Then there exists a smooth projective variety Y and $\sigma : \pi_1(Y) \to GL_n(\mathbb{C})$ a representation whose image is a finite index subgroup of Γ. Moreover if ρ is injective, σ can be taken to be injective too.

The statement was in the first version of [1], with an erroneous proof, and I explained only the new ingredients developped in [2].

C. Voisin constructed compact Kähler manifolds non having the homotopy type of a complex projective manifold, however the fundamental group of her examples is projective.

This put an end to a problem attributed to Kodaira asking whether a compact Kähler manifold could deform to a complex projective manifold.

Our approach nevertheless begins with solving Kodaira's problem for smooth families of complex tori with a section over a projective base, then, after an étale cover of the base, for smooth families of complex tori with possibly no section.

One concludes using the results of [1]. The case when σ has Zariski dense image in a connected semisimple group is a theorem of K. Zuo [4] which we revisited in loc. cit. The general case proceeds by showing that the Shafarevich variety of ρ is (after some étale covering) bimeromorphic to a smooth family of complex tori over a general type base using results of [3].

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Line arrangements with aspherical complements

Dmitri Panov
(joint work with Anton Petrunin)

In this talk I am discussing a class of line arrangements in $\mathbb{C}P^2$, that have an aspherical complement. Arrangements with aspherical complement are quite rare and for the moment there is no idea how to classify them.

It is not hard to see that the complement to four generic lines in $\mathbb{C}P^2$ has non-trivial π_2. Indeed, the fundamental group of the complement to such an arrangement is $\mathbb{Z}^3$, at the same time the complement has homotopy type of a two-dimensional cell complex. One can deduce from this that a sufficiently generic line arrangement can not have an aspherical complement.

A large class of arrangements with aspherical complements is given by complexifications of real simplicial arrangements (see [1]), i.e., line arrangements in $\mathbb{R}P^2$ that cut it into triangles. Simplicial arrangements tend to have few double points and can be thought of as solutions to an extremal problem. Here is one more extremal problem.

Problem. For a line arrangement and a multiple point x on it define the *multiplicity* $\mu(x)$ as the number of lines going through x. Let the *total multiplicity* of an arrangement be the sum of multiplicities of its points. We ask the following question: for an arrangement of n lines what is the minimal possible multiplicity?

In order to rule out degenerate cases (such as when all lines pass through one point), we impose a stability condition. Call an arrangement *stable*, if the multiplicity of each point is less than $\frac{2n}{3}$.

The following result was proven in [3].

Theorem. The total multiplicity of a stable line arrangement of n lines is at least $\frac{(n+3)n}{3}$. In the case of equality each line intersect others in exactly $\frac{n+3}{3}$ points.

An example of such an arrangement (when each line intersect others in exactly $\frac{n+3}{3}$ points) is given by 6 lines that join 4 generic points in $\mathbb{C}P^2$. Hirzebruch asked in [2] if all such arrangements are related to complex reflection groups. This question is still open, but we proved in [4] the following result.

Theorem. Any stable arrangement of n lines, such that each line intersect others in $\frac{n+3}{3}$ points has an aspherical complement.

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Quasiprojective three-manifold groups and complexification of three-manifolds

Mahan Mj

(joint work with Indranil Biswas)

A group is called quasiprojective (respectively, Kähler) if it is the fundamental group of a smooth complex quasiprojective variety (respectively, compact Kähler manifold). Kähler and quasiprojective 3-manifold groups have attracted much attention of late. In this paper we characterize quasiprojective 3-manifold groups.

We shall follow the convention that our 3-manifolds have **no spherical boundary components.** Capping such boundary components off by 3-balls does not change the fundamental group, which is really what interests us here.

Theorem 2. Let N be a compact 3-manifold (with or without boundary). If $\pi_1(N)$ is a quasiprojective group, then N is either Seifert-fibered or $\pi_1(N)$ is one of the following

- virtually free, or
- virtually a surface group.

This characterization of quasiprojective 3-manifold groups answers Questions of Friedl-Suciu.

The following theorem provides an answer to a question of Friedl-Suciu under mild hypotheses.

Theorem 3. Suppose A and B are groups, such that the free product $G = A * B$ is a quasiprojective group. In addition suppose that both A and B admit nontrivial finite index subgroups, and at least one of A, B has a subgroup of index greater than 2. Then each of A, B are free products of cyclic groups. In particular both A and B are quasiprojective groups.

A good complexification of a closed smooth manifold M is defined by Totaro to be a smooth affine algebraic variety U over the real numbers such that M is diffeomorphic to $U(\mathbb{R})$ and the inclusion $U(\mathbb{R}) \longrightarrow U(\mathbb{C})$ is a homotopy equivalence. Totaro asks whether a closed smooth manifold M admits a good complexification if and only if M admits a metric of non-negative curvature. As an application of Theorem 2, we prove this in the following strong form for 3-manifolds.

Theorem 4. *A closed 3-manifold M admits a good complexification if and only if one of the following hold:*

(1) *M admits a flat metric,*
(2) *M admits a metric of constant positive curvature, and*
(3) *M is covered by the (metric) product of a round S^2 and $\mathbb{R}$.*

Curiously, the proof of Theorem 4 is direct and there is virtually no use of the method or results of earlier work on quasiprojective 3-manifold groups. Our main tools from recent developments in 3-manifolds are:

(1) The Geometrization Theorem and its consequences.
(2) Largeness of 3-manifold groups.

The basic complex geometric tool is a theorem of Bauer, regarding existence of irrational pencils for quasiprojective varieties. It is a useful existence result in the same genre as the classical Castelnuovo-de Franchis Theorem and a theorem of Gromov.

As a consequence of our results we deduce the restrictions on quasiprojective 3-manifold groups and Kähler 3-manifold groups obtained by previous authors on the subject, notably Dimca-Suciu, Kotschick, Dimca-Papadima-Suciu and Friedl-Suciu. and the restrictions on good complexifications of 3-manifolds deduced by Totaro.

Holomorphic families from the point of view of geometric group theory

Thomas Delzant

Let Xbe a Kähler manifold. A holomorphic family of Riemann surfaces of genus g over X is a pair (Y, π) where Y is a complex manifold and $\pi\colon Y \to X$ a holomorphic submersion with fibers Riemann surfaces of genus g. It is called non isotrivial if the family of Riemann surfaces $Y_s = \pi^{-1}(s)$ is not constant in the moduli space of Riemann surfaces. Such a family determines a monodromy, which is a homomorphism φ from the fundamental group $\pi_1(X, s_0)$ to the mapping class group $M(S)$of the topological surface underlying Y_{s_0}.

A fundamental result, (and in fact the basic result of the theory), due to Parshin and Arakelov and answering a question of Shafarevich asserts that given a Riemann surface B the set of families of given genus over B is finite. More generally, the number of non isotrivial families over a projective manifold X can be bounded in terms of this manifold.

A uniform result has even been described by L. Caporaso who proved that the Hilbert polynomial of a complex surface which is a non singular bundle of genus p over a base of genus g can only take a finite number of values. A consequence is that, given a surface Σ_g (of genus g), and of topological surface S (of genus p), the cardinal of the subset the set of homomorphisms from the fundamental group of Σ_g to the mapping class group of a surface S, which can be realized as a monodromy is finite modulo conjugacy at the target and automorphism at the source.

In this lecture, we explain (see [1] for details) how standard methods of geometric group theory (asymptotic cones) combine with the famous Gromov-Schoen theorem on Kähler groups acting on trees as well as Bestvina-Bromberg-Fujiwara's recent study on the asymptotic geometry of the mapping class group to get a bound in terms of the fundamental group rather than a bound in terms of the manifold.

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