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Non-Archimedean Geometry and Applications

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Abstract. The workshop focused on recent developments in non-Archimedean analytic geometry with various applications to other fields. The topics of the talks included applications to complex geometry, mirror symmetry, p-adic Hodge theory, tropical geometry, resolution of singularities, p-adic dynamical systems and diophantine geometry.

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Introduction by the Organizers

The workshop on Non-Archimedean Analytic Geometry and Applications was organized by Vladimir Berkovich (Rehovot), Walter Gubler (Regensburg), Peter Schneider (Münster) and Annette Werner (Frankfurt). Non-Archimedean analytic geometry is a central area of arithmetic geometry. The first analytic spaces over fields with a non-Archimedean absolute value were introduced by John Tate and explored by many other mathematicians. They have found numerous applications to problems in number theory and algebraic geometry. In the 1990s, Vladimir Berkovich initiated a different approach to non-Archimedean analytic geometry, providing spaces with good topological properties which behave similarly as complex analytic spaces. Independently, Roland Huber developed a similar theory of adic spaces. Recently, fields medalist Peter Scholze has introduced perfectoid spaces as a ground breaking new tool to attack deep problems in p-adic Hodge theory and representation theory.

Non-archimedean spaces have played an important in number theory and algebraic geometry for decades. Recent years have seen new connections to other fields and successful applications to the solution of celebrated problems not only in arithmetic geometry. Since 2015, when the last workshop on the same topic took place in Oberwolfach, exciting new developments have served both to enlarge the foundations of the field and to embark to new horizons. Many of these recent developments have been discussed in the workshop which brought together researchers form different areas.

The workshop had 53 participants and we had 19 one hour talks. A summary of the topics can be found below. All talks were followed by lively discussions, in the form of plenary questions and also in the form of blackboard discussions in smaller groups. Several participants explained work in progress or new conjectures or promising techniques to attack open conjectures. The workshop provided a lively platform to discuss these new ideas with other experts.

During the workshop, we saw how the reduction of not necessarily strict affinoid spaces behave in families (Ducros). A canonical compactification of complex analytic varieties was presented (Poineau) in the hybrid setting which is a mixture of complex spaces and non-archimedean spaces suitable for degenerations.

A skeleton is a polyhedral substructure of a Berkovich space which is a deformation retract and which is induced by a mildly singular model over the ring of integers of the non-archimedean field. Mazzon explained a non-archimedean approach to the famous $P = W$ conjecture in non-abelian Hodge theory, and she used properties of the essential skeleton to prove it in special cases. The construction of the non-archimedean SYZ-fibration from mirror symmetry was explained and it was shown that it is an affinoid torus fibration away from a codimension 2 locus (Nicaise). Mirror symmetry inspired also the talk of Sustretov where he described the Gromov–Hausdorff limits of curves with flat metrics.

A ground-breaking new approach to Hironaka's celebrated theorem was exposed in Temkin's lecture on a canonical functorial algorithm for resolution of singularities.

An incidence compactification of strata of abelian differentials was descibed (Tyomkin) in terms of non-archimedean geometry which was known previously only in terms of complex geometry.

A degeneration result of p-adic volume forms induced by base extensions to a Lebesgue measure of the skeleton was presented (Jonsson). In characteristic $p > 0$, Canton gave an interpretation of anticanonical metrics as operator norms of Cartier operator.

The unexpected behavior of tropical Dolbeault cohomology of non-archimedean curves was explained (Jell). Loeser introduced a structure of a non-archimedean field on a non-standard model of the field of complex numbers, and he used this to show that one-parameter families of complex integrals behave asymptotically as integrals of Chambert-Loir–Ducros forms on the Berkovich space. A tropical approach to classical Prym–Brill–Noether theory was given which leads to new upper bounds for the dimension of the Brill–Noether locus (Ulirsch). An application of non-archimedean methods to diophantine geometry was presented in Shokrieh's talk in which he described the stable Faltings height of a principally polarized abelian variety over a number field in terms of local invariants.

The theory of equivariant D-modules on rigid spaces was used to show that certain representations of $GL(2)$ of a local field associated to the first Drinfeld covering are irreducible and admissible (Ardakov). In the talk of Huyghes, she described a category of coherent D-modules which is analogous to the classical BGG-category.

A tame étale site of an adic space was introduced by Hübner who showed that its cohomology groups with p-torsion coefficients behave better than those of the whole étale site. The Poincaré duality was established for Zariski open subsets of proper smooth rigid analytic varieties (Liu); the proof relies very much on the use of perfectoid spaces.

A conjectural analogue of the Chebotarëv's density theorem for convergent F-isocrystals was proposed and proven in special cases (Hartl). Formal groups over p-adic rings were described in terms of torsion points and related to p-adic dynamical systems and the theory of (ϕ, Γ)-modules (Berger).

The atmosphere during the workshop was very good and the participants continued to work in small groups after the plenary talks. During the breaks and in the evenings many informal mathematical discussions took place, in which the young participants played an active role. The organizers made a specific effort to invite PhD students and Postdocs. Altogether we had 15 participants from this group. For most of them it was the first Oberwolfach workshop they ever attended. The stimulating Oberwolfach atmosphere provided a unique opportunity of meeting the international leaders of the subject and of keeping track with current developments. The organizers also identified possible female invitees, thus ensuring that among the participants of the workshop were 11 women mathematicians. An informal concert took place on Thursday evening where 7 participants played for an audience of about 30 people in the music room.

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Workshop: Non-Archimedean Geometry and Applications

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Abstracts

Reduction of affinoid spaces in family

Antoine Ducros

We fix a complete non-Archimedean field k and a divisible subgroup Γ of $\mathbf{R}_{>0}$; we only assume that $\Gamma \neq \{1\}$ if $|k^{\times}| = \{1\}$. A k-affinoid algebra A (in the sense of Berkovich) is said to be Γ-*strict* if it can be written as a quotient $k\{T_1/r_1, \ldots, T_n/r_n\}/I$ where the radii r_i's all belong to Γ. This class of affinoid algebras gives rise by glueing to the class of Γ-strict analytic spaces (see [5], chapter 3 for more details). The reader should keep the following extreme examples in mind:

- if $\Gamma = \{1\}$ (which can occur only if k is non-trivially valued), then the class of Γ-strict k-analytic spaces is nothing but that of strict k-analytic spaces, that is, the Berkovich version of rigid-analytic spaces over the ground field k;
- if $\Gamma = \mathbf{R}_{>0}$ then the class of Γ-strict k-analytic spaces is that of *all* k-analytic spaces.

Let $X = \mathscr{M}(A)$ be a Γ-strict k-affinoid space. Following Temkin [8], one associates to A its Γ-*graded* reduction

$$\widetilde{A} := \bigoplus_{r\in\Gamma} \{a \in A, \|a\| \leqslant r\}/\{a \in A, \|a\| < r\}$$

where $\|\cdot\|$ is the spectral semi-norm on A. This is a graded algebra over the "graded residue field" $\widetilde{k}$ (which is a graded field, in the sense that every non-zero *homogeneous* element $\widetilde{k}$ is invertible); note that if $\Gamma = \{1\}$ all these reductions are the usual ones. If a is an element of A such that $\|a\| \in \Gamma$ we shall denote by $\widetilde{a}$ its image in the $\|a\|$-th graded component of $\widetilde{A}$.

By adding the words "graded" or "homogeneous" almost everywhere, one can mimic in the graded setting all classical constructions of commutative algebra and even algebraic geometry; for instance one can define homogeneous prime and maximal ideals of a Γ-graded ring, and then its spectrum (which is the set of its homogeneous prime ideals), and so forth. Going back to $X = \mathscr{M}(A)$ we shall denote by $\widetilde{X}$ the spectrum of $\widetilde{A}$. This is an "affine graded scheme of finite type over $\widetilde{k}$", and it comes with an anti-continuous reduction map $X \to \widetilde{X}$.

The assignment $X \mapsto \widetilde{X}$ is functorial, but it does not commute in general to fiber products nor ground field extension. The aim of this talk was to present a recent work [4] in which we remedy this problem in some specific situations. In order to describe our main result, it will be convenient to introduce the following definition. Let $Y \to X$ be a morphism between two Γ-strict k-affinoid spaces $Y = \mathscr{M}(B)$ and $X = \mathscr{M}(A)$. A Γ-*nice presentation* of B over A (or of Y over X) is a presentation $B \simeq A\{T_1/r_1, \ldots, T_n/r_n\}/(a_1, \ldots, a_m)$ that fulfills the following conditions:

(1) the r_i's all belong to Γ; for all j, the spectral norm ρ_j of a_j also belongs to Γ;
(2) for every point x of X, the following holds:
 - for all j, the norm of the image $a_j(x)$ of a_j in $\mathscr{H}(x)\{T_1/r_1,\dots,T_n/r_n\}$ is equal to ρ_j ;
 - every element b of the ideal $(a_1(x),\dots,a_m(x))$ of $\mathscr{H}(x)\{T_1/r_1,\dots,T_n/r_n\}$, can be written $\sum b_j a_j(x)$ with $\|b_j\|\cdot\rho_j \leqslant \|b\|$ for all j;
(3) the natural map

$$p\colon \mathrm{Spec}\ \widetilde{A}[T_1/r_1,\dots,T_n/r_n]/(\widetilde{a_1},\dots,\widetilde{a_m}) \to \mathrm{Spec}\ \widetilde{A}$$

is flat, its fibers are geometrically reduced, and their irreducible components are geometrically irreducible.

Let us make some comments

- By $\widetilde{A}[T_1/r_1,\dots,T_n/r_n]$ we denote the graded algebra of polynomials with coefficients in $\widetilde{A}$ in indeterminates $T_1,\dots,T_n$, with T_i being of degree r_i for all i; this is the graded reduction of $A\{T_1/r_1,\dots,T_n/r_n\}$.
- The conditions required for being a nice presentation might look quite technical. The crucial point is the following: they imply that for every $x \in X$ whose image in $\widetilde{X}$ is denoted by $\widetilde{x}$, the graded reduction $\widetilde{Y_x}$ is equal to

$$p^{-1}(\widetilde{x})_{\widetilde{\mathscr{H}(x)}} = \mathrm{Spec}\ \widetilde{\mathscr{H}(x)}[T_1/r_1,\dots,T_n/r_n]/(\widetilde{a_1(x)},\dots,\widetilde{a_m(x)}).$$

Thus the family of reductions $(\widetilde{Y_x})$ is induced by the fibers of a flat family over $\widetilde{X}$. Our main theorem ([4], Th. 3.5) then asserts the following.

Theorem. Let $Y \to X$ be a morphism between Γ-strict k-affinoid spaces. Assume that $Y \to X$ is flat with geometrically reduced fibers. Hence there exists a finite family of morphisms $f_i\colon X_i \to X$ satisfying the following:

(1) each X_i is affinoid and Γ-strict, and $X = \bigcup f_i(X_i)$;
(2) if $|k^\times| \neq \{1\}$ then each f_i is quasi-étale; if $|k^\times| = \{1\}$ then each f_i is the composition of a finite, flat and radicial map followed by a quasi-étale one;
(3) for each i the map $Y \times_X X_i \to X_i$ admits a Γ-nice presentation.

Remark. We keep the notation and the assumptions of the theorem. Assume moreover that $\Gamma = \{1\}$ and the map $Y \to X$ is equidimensional. Then the so-called *reduced fiber theorme* of Bosch, Lütkebohmert and Raynaud ([3], Th. 2.1) asserts that $Y \to X$ admits after a strictly affinoid quasi-étale base-change a formal model which is flat with geometrically reduced fibers. Once such a model exists, it is not difficult, up to performing another strictly affinoid quasi-étale base-change, to ensure that the irreducible components of its fibers are geometrically irreducible. Thus we see that when $\Gamma = \{1\}$ and the map $Y \to X$ is equidimensional, our theorem is a consequence of the reduced fiber theorem.

But we do not use this theorem, and in fact we even recover it under extra-assumptions, for instance if X is reduced and k algebraically closed.

A few words about our proof. As said above, we do not use the reduced fiber theorem; in fact, we do not use any formal geometry. Instead we work with Temkin's theory of (graded) reduction of germs of analytic spaces.

More precisely, we use as a key tool a theorem by Grauert and Remmert (see [2], 6.2.4, Thm. 1), which can be seen as an absolute version (that is, over $\mathrm{Spf}(k^\circ)$) of the reduced fiber theorem. This theorem in some sense enables us to prove our result for every fiber of the map $Y \to X$, and we need then Temkins's method for spreading out. Note that we also use (while working with graded reductions) a reduced fiber theorems for schemes of finite type over an arbitrary valuation ring, but this theorem can de deduced from the result by Grauert and Remmert alluded to above (see [7], section 6).

Motivation Given a Γ-strict affinoid space X, the anti-continuous reduction map $X \to \widetilde{X}$ induces a bijection $\pi_0(X) \simeq \pi_0(\widetilde{X})$. Therefore our result enables us to reduce the study of the variation of geometric connected components of the fibers of flat map with reduced fibers in the analytic setting to the analogue problem in scheme theory, which is adressed in EGA IV. This plays a crucial role in our work in progress on flattening techniques for Berkovich spaces. (The use of reduction to describe the behaviour of connected components in families is not new: in the strictly k-analytic context and with the help of the reduced fiber theorem, this strategy was followed by Abbes and Saito in [1] and by Poineau in [6]).

A question. This is natural to asl whether they shoud exist a Γ-strict version of the reduced fiber theorem (our main result being a substitute for it). The answer is likely positive, but this would first require to develop a theory of Γ-graded or Γ-filtered formal schemes, if only for stating the theorem.

References

[1] , A. Abbes and T. Saito, *Ramification of local fields with imperfect residue fields*, Amer. J. Math. **124** (2002), 879-920.

[2] , S. Bosch, U; Güntzer and R. Remmert, *Non-Archimedean analysis. A systematic approach to rigid analytic geometry*, Grundlehren der Mathematischen Wissenschaften **261** (1984), Springer-Verlag, Berlin, pp. xii+436.

[3] S. Bosch, W. Lütkebohmert and M. Raynaud, *Formal and rigid geometry. IV. The reduced fiber theorem*, Invent. Math. **119** (1995), 361-398.

[4] A. Ducros, *Réduction en famille d'espaces affinoïdes*, preprint, `ArXiv: 1903.00942`

[5] A. Ducros, *Families of Berkovich spaces*, volume **400** of the Astérisque collection (2018), vii+262 pp.

[6] , J. Poineau, *Un résultat de connexité pour les variétés analytiques p-adiques: privilège et noethérianité.*, Compos. Math. **144** (2008), 107-133.

[7] , M. Temkin, *Stable modification of relative curves*, J. Algebraic Geom. **19** (2010), 603-677.

[8] M. Temkin, *On local properties of non-Archimedean analytic spaces.II*, Israel J. Math. **140** (2004), 1-27.

The first Drinfeld covering and equivariant $\mathcal{D}$-modules on rigid analytic spaces

Konstantin Ardakov
(joint work with Simon J. Wadsley)

1. Background and main result

Let p be a prime number, let $\mathbb{Q}_p$ be the field of p-adic numbers and let F be a finite extension of $\mathbb{Q}_p$. Fix a uniformiser $\pi \in \mathcal{O}_F \subset F$ and let $\breve{F}$ be the completion of the maximal unramified extension of F. Let

$$\Omega := (\mathbb{P}^{1,\mathrm{an}} - \mathbb{P}^1(F)) \times_F \breve{F}$$

denote the Drinfeld upper half plane. In [6], Drinfeld defined a tower

$$\cdots \to \mathcal{M}_2 \to \mathcal{M}_1 \to \mathcal{M}_0 = \Omega \times \mathbb{Z}$$

of rigid analytic varieties such that

- the natural $G := \mathrm{GL}_2(F)$-action on $\mathcal{M}_0$ lifts to each $\mathcal{M}_n$,
- each map $\mathcal{M}_n \to \mathcal{M}_{n-1}$ is G-equivariant, finite étale and Galois,
- $\mathrm{Gal}(\mathcal{M}_n/\mathcal{M}_0) = \mathcal{O}_D^\times/(1 + \Pi^n \mathcal{O}_D)$ for each $n \geqslant 0$, where D is the quaternion division algebra over F and Π is a generator of the unique maximal ideal of the maximal order $\mathcal{O}_D$ of D,
- the actions of $\mathrm{Gal}(\mathcal{M}_n/\mathcal{M}_0)$ and G on $\mathcal{M}_n$ commute.

This tower is known to realise both the Jacquet-Langlands and local Langlands correspondences in (compactly supported) ℓ-adic etale cohomology. We give a more precise version of this statement in Theorem 1.1 below, after establishing the necessary notation.

Fix a prime number ℓ different from p. For any p-adic Lie group H, let $\mathrm{Irr}(H)$ denote the set of isomorphism classes of $\overline{\mathbb{Q}}_\ell$-linear, irreducible, *smooth* representations of $D^\times$. The *Jacquet-Langlands correspondence* [4], [14], [10] is an injection $JL : \mathrm{Irr}(D^\times) \hookrightarrow \mathrm{Irr}(G)$, whose image consists of those G-representations whose matrix coefficients are square-integrable. Let W_F denote the Weil group [1] of F, and define $\Sigma_n := \mathcal{M}_n / \begin{pmatrix} \pi & 0 \\ 0 & \pi \end{pmatrix}^{\mathbb{Z}}$ for each $n \geqslant 0$.

Theorem 1.1 (Faltings, Harris-Taylor, Fargues, Mieda)**.** *For each $\rho \in \mathrm{Irr}(D^\times)$ there is a two-dimensional $\overline{\mathbb{Q}}_\ell$-linear representation V_ρ of W_F, and an isomorphism of $\overline{\mathbb{Q}}_\ell$-linear $D^\times \times G \times W_F$-representations*

$$\varinjlim_n H^1_{\acute{e}t,c}(\Sigma_n \times_{\breve{F}} \overline{F}, \overline{\mathbb{Q}}_\ell) \quad \cong \quad \bigoplus_{\rho \in \mathrm{Irr}(D^\times)} \rho \otimes JL(\rho) \otimes V_\rho.$$

Question 1.2. *What can one say about other cohomology groups of the tower $\mathcal{M}_\infty$, such as the p-adic étale cohomology, or the coherent cohomology?*

[1] For some background on Galois representations, we refer the reader to [13].

We say nothing about the p-adic étale cohomology, except to point out that when $F = \mathbb{Q}_p$, the p-adic étale cohomology of the tower is known [3] to realise Colmez's p-adic local Langlands correspondence [2]. The coherent cohomology is expected to produce *admissible locally analytic* representations of G, in the sense of Schneider and Teitelbaum [15]. More precisely, we have the following conjecture; it can be viewed as a version of the (unpublished) Breuil-Strauch conjecture.

Conjecture 1.3.

(a) For all $n \geqslant 0$, $\mathcal{O}(\Sigma_n)^*$ is an admissible locally analytic representation of G,
(b) $\mathcal{O}(\Sigma_n)^*$ always has finite length.

The evidence for part (a) consists of the following two statements.

Theorem 1.4 (Dospinescu-Le Bras, 2017)**.**
(a) holds if $F = \mathbb{Q}_p$ and $n \geqslant 0$ is arbitrary.

The proof can be found at [5, Remarque 1.3(b)], and uses the full strength of the p-adic local Langlands correspondence. On the other hand, [11, Theorem 7.2.1(iv)] states the following

Theorem 1.5 (Patel-Schmidt-Strauch 2019)**.**
(a) holds if $n = 1$ and F is arbitrary.

Let $f_n : \Sigma_n \to \Omega$ denote the structure map of the étale covering Σ_n of Ω. Then the $\mathcal{O}_\Omega$-module $\mathcal{V}_n := f_{n,*}\mathcal{O}_{\Sigma_n}$ is locally free of finite rank and carries the Gauss-Manin connection. It is therefore naturally a G-equivariant $\mathcal{D}$-module on Ω. Furthermore it carries an action of the Galois group $\mathrm{Gal}(\Sigma_n/\Sigma_0)$ which commutes with both the G-action and the $\mathcal{D}$-action, and for any irreducible representation ρ of this finite group, the ρ-isotypic component $\mathcal{V}_n^\rho$ of $\mathcal{V}_n$ is again a G-equivariant $\mathcal{D}$-module. The group $\mathrm{Gal}(\Sigma_n/\Sigma_0)$ is abelian when $n = 1$, so each ρ is a one dimensional character and an easy Kummer-theory argument shows that $\mathcal{V}_1^\rho$ is in fact an invertible $\mathcal{O}_\Omega$-module which has finite order in an appropriate Grothendieck group of G-equivariant line bundles with connection.

Let $D(G)$ be the locally F-analytic distribution algebra of G, and recall from [15, §6] that $\mathcal{C}_G$ denotes the category of *coadmissible $D(G)$-modules*. We can now state our main result.

Theorem A. *Let $\mathscr{L}$ be a G-equivariant $\mathcal{D}$-module on Ω which is invertible as an $\mathcal{O}_\Omega$-module. Suppose that there exists a positive integer d such that $\mathscr{L}^{\otimes d} \cong \mathcal{O}_\Omega$ as a G-equivariant $\mathcal{D}$-module. Suppose that d is least possible and $p \nmid d$. Then $\mathscr{L}(\Omega) \in \mathcal{C}_G$, and $\mathscr{L}(\Omega)$ is an irreducible $D(G)$-module whenever $d > 1$.*

The argument sketched above shows that Theorem A implies Theorem 1.5, and also gives additional evidence to part (b) of the Conjecture for arbitrary F.

2. Sketch of the proof

Let K be a complete non-Archmedean field extension of F and let G be a p-adic Lie group acting continuously on a smooth rigid K-analytic space X. The paper [1] introduced the abelian category $\mathcal{C}_{X/G}$ of *co-admissible G-equivariant $\mathcal{D}$-modules on X*, in order to prove a Beilinson-Bernstein style Localisation Theorem for the coadmissible $D(G)$-modules.

Theorem 2.1. [1, Theorem C] *Let $\mathbb{G}$ be a connected affine algebraic group of finite type over F such that $\mathbb{G}_K := \mathbb{G} \otimes_F K$ is split semisimple. Let G be an open subgroup of $\mathbb{G}(F)$ and let $X = (\mathbb{G}_K/\mathbb{B})^{\mathrm{an}}$ be the rigid-analytic flag variety. Then $\Gamma(X, -)$ is an equivalence of categories between $\mathcal{C}_{X/G}$ and the category of co-admissible $D(G)$-modules with trivial infinitesimal central character.*

Note that if $\mathbb{G} = \mathrm{SL}_2$ then $X = \mathbb{P}^{1,\mathrm{an}}$. Now let $j : \Omega \to \mathbb{P}^{1,\mathrm{an}}$ be the open embedding and let $\mathscr{L}$ satisfy the hypotheses of Theorem A. Theorem A follows easily from Theorem 2.1 and

Theorem 2.2. $j_*\mathscr{L} \in \mathcal{C}_{\mathbb{P}^{1,\mathrm{an}}/G}$.

Choose a coordinate x on $\mathbb{P}^{1,\mathrm{an}}$ and let $\mathbb{D} := \operatorname{Sp} K\langle x\rangle \subset \mathbb{P}^{1,\mathrm{an}}$ be a closed disk. If $w := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then $\{\mathbb{D}, w\mathbb{D}\}$ is an admissible affinoid covering of $\mathbb{P}^{1,\mathrm{an}}$. The definition of the category $\mathcal{C}_{\mathbb{P}^{1,\mathrm{an}}/G}$ found at [1, Definition 3.6.7] shows that to prove Theorem 2.2, it is enough to show that $j_*\mathscr{L}$ is $\{\mathbb{D}, w\mathbb{D}\}$-coadmissible. The symmetry of the situation quickly reduces Theorem 2.2 to the following statements.

Theorem 2.3. *Let G_1 denote the first congruence kernel of $\mathrm{GL}_2(\mathcal{O}_F)$.*

(a) $\mathscr{L}(\mathbb{D} \cap \Omega)$ is a coadmissible $\widehat{\mathcal{D}}(\mathbb{D}, G_1)$-module.

(b) The canonical map $\widehat{\mathcal{D}}(\mathbb{D} \cap w\mathbb{D}, G_1) \underset{\widehat{\mathcal{D}}(\mathbb{D},G_1)}{\widehat{\otimes}} \mathscr{L}(\mathbb{D} \cap \Omega) \longrightarrow \mathscr{L}(\mathbb{D} \cap w\mathbb{D} \cap \Omega)$ is an isomorphism.

We will focus on the proof of Theorem 2.3(a) only in what follows. Recall from [12, §5] that for any rigid analytic space X there is an associated topological space $\tilde{X}$ that comes with an equivalence $\mathcal{F} \mapsto \tilde{\mathcal{F}}$ between the abelian sheaves on X and the abelian sheaves on $\tilde{X}$.

Definition 2.4. *Let U be an admissible open subspace of the rigid analytic space X, let $\mathcal{F}$ be an abelian sheaf on X and let $i : \overline{\tilde{U}} \hookrightarrow \tilde{X}$ be the embedding of the closure of $\tilde{U}$. Define another abelian sheaf $i_U^\dagger \mathcal{F}$ on X by*

$$\widetilde{i_U^\dagger \mathcal{F}} = i_* i^{-1} \tilde{\mathcal{F}}.$$

Example 2.5. *Suppose that $U = X(\frac{g_1}{g_0}, \cdots, \frac{g_n}{g_0})$ is a rational subdomain of the affinoid variety X. For each $s \in K$ with $|s| > 1$, let $U(s)$ denote the slightly larger affinoid subdomain $U(s) = X(\frac{g_1}{sg_0}, \cdots, \frac{g_n}{sg_0})$ of X. Then the sections of $i_U^\dagger \mathcal{F}$ on* *affinoid subdomains* Y *of* X *are given by*

$$(i_U^\dagger \mathcal{F})(Y) = \varinjlim_{|s|>1} \mathcal{F}(Y \cap U(s)).$$

Definition 2.6. *Let* $n \geqslant 0$ *be an integer.*

(a) $U_n := \mathbb{D} \backslash \bigcup_{a \in \mathcal{O}_F} \{|z-a| < |\pi|^n\}$, *and* $i_{U_n} : U_n \hookrightarrow \mathbb{D}$ *is the embedding.*

(b) For each affinoid subdomain Y *of* $\mathbb{D}$ *and each* $r \in K^\times$ *define*

$$\mathcal{D}_r(Y) := \mathcal{O}(Y)\langle \partial_x / r \rangle \quad \text{and} \quad \mathcal{D}_r^\dagger(Y) := \bigcup_{|s|>|r|} \mathcal{D}_s(Y).$$

(c) Let $\varpi := p^{\frac{1}{p-1}}$, *and set* $D_n := \mathcal{D}^\dagger_{\varpi/\pi^n}(\mathbb{D})$.

The Banach $\mathcal{O}(Y)$-modules $\mathcal{D}_r(Y)$ are in fact associative Banach algebra completions of the algebra $\mathcal{D}(Y) = \mathcal{O}(Y)[\partial_x]$ of finite order differential operators on Y, whenever $|r|$ is sufficiently large relative to Y. Morally, they are quantisations of certain 'boxes' inside the cotangent bundle T^*Y of Y, and in fact we have

$$\widehat{\mathcal{D}}(Y) = \varprojlim \mathcal{D}_r(Y) = \varprojlim \mathcal{D}_r^\dagger(Y).$$

We now introduce the linear differential operators $R(z)$ that play an important role in our proof of Theorem 2.3(a).

Notation 2.7.

(a) $\alpha_1, \ldots, \alpha_m \in K$ *are pairwise distinct, and* $k_1, \ldots, k_m \in \mathbb{Z}$.

(b) $u = \prod_{i=1}^m (x - \alpha_i)^{k_i} \in K(x)$ *and* $d \geqslant 1$ *is an integer.*

(c) $z := u^{\frac{1}{d}}$ *and* $\Delta_z := \prod_{i=1}^m (x - \alpha_i)$.

(d) $R(z) := \Delta_z z \partial_x z^{-1} = \Delta_z(\partial_x - \frac{1}{d}\frac{\partial_x(u)}{u}) \in K[x, \partial_x]$.

Our main technical tool is an explicit presentation of the sections of $(j_*\mathscr{L})_{|\mathbb{D}}$ that overconverge along the interior annuli of U_n as a module over a particular completion of $\mathcal{D}(\mathbb{D})$, namely the ring D_n from Definition 2.6(c).

Theorem 2.8. *Let* $\psi : \mathscr{L}^{\otimes d} \xrightarrow{\cong} \mathcal{O}_\Omega$ *be a* G-$\mathcal{D}$*-linear isomorphism and let* $n \in \mathbb{N}$.

(a) The natural $\mathcal{D}(\mathbb{D})$*-action on* $M_n := i_{U_n}^\dagger (j_*\mathscr{L})_{|\mathbb{D}}(\mathbb{D})$ *extends to* D_n.

(b) There exists $u_n \in K(x) \cap \mathcal{O}(U_{n+\frac{1}{2}})^\times$ *and* $z_n \in \mathscr{L}(U_{n+\frac{1}{2}})$, *such that*

$$\mathscr{L}_{|U_{n+\frac{1}{2}}} = \mathcal{O}_{U_{n+\frac{1}{2}}} \cdot z_n \quad \text{and} \quad \psi(z_n^{\otimes d}) = u_n.$$

(c) $M_n = D_n \cdot z_n \cong D_n / D_n R(z_n)$.

Since $\mathscr{L}(\mathbb{D} \cap \Omega) = \varprojlim M_n$, $\widehat{\mathcal{D}}(\mathbb{D}) = \varprojlim D_n$, and each M_n is a finitely presented D_n-module by Theorem 2.8(c), one might be tempted to conclude that in fact $\mathscr{L}(\mathbb{D} \cap \Omega)$ is already coadmissible as a $\widehat{\mathcal{D}}(\mathbb{D})$-module. However, this is not the case: the connecting maps $D_n \otimes_{D_{n+1}} M_{n+1} \to M_n$ fail to be injective, and one needs to 'switch on the group action' in order to force them to be isomorphisms.

Definition 2.9. *Let* $n \geqslant 0$ *be an integer. Define*

(a) $G_n := \ker(\mathrm{GL}_2(\mathcal{O}_F) \to \mathrm{GL}_2(\mathcal{O}_F/\pi^n\mathcal{O}_F))$,

(b) $\beta : G_{n+1} \to D_n^\times$ *by* $g \mapsto \sum_{n=0}^{\infty} (g \cdot x - x)^n \frac{\partial_x^n}{n!}$, *and*

(c) $S_n := D_n \rtimes_{G_{n+1}} G_1 := (D_n \rtimes G_1)/\langle \beta(g) - g : g \in G_{n+1} \rangle$.

It is not hard to show that $(S_n)_{n=0}^{\infty}$ forms a weak Fréchet-Stein structure on $\widehat{\mathcal{D}}(\mathbb{D}, G_1)$ in the sense of [7, Definition 1.2.6]. Theorem 2.3(a) follows from

Theorem 2.10. *Let $n \geqslant 0$ be an integer.*

(a) The D_n and G_1-actions on M_n extend to S_n.

(b) M_n is a finitely presented S_n-module.

(c) The connecting map $S_n \otimes_{S_{n+1}} M_{n+1} \to M_n$ is an isomorphism.

Theorem 2.10(c) is the hardest part of the entire proof of Theorem A. After some microlocal analysis, its proof relies on showing that the power series

$$\sum_{n=0}^{\infty} \left(\sum_{r=0}^{n} \frac{\binom{\frac{k}{d}}{r} \binom{-\frac{k}{d} - (q-1)r}{(n-r)(q-1)}}{(n-r)(q-1)+1} \right) t^n \in K[[t]]$$

does not have bounded coefficients, which in turn requires a precise calculation of the p-adic valuation of the binomial coefficients appearing in this series. Here q denotes the order of the residue field of $\mathcal{O}_F$ and $1 \leqslant k < d$ is an integer. We perform this calculation using Kummer's classical result from [9].

References

[1] Ardakov, K. *Equivariant $\mathcal{D}$-modules on rigid analytic spaces*, (2017), preprint.

[2] Colmez, P. *Représentations de $GL_2(\mathbb{Q}_p)$ et (ϕ, Γ)-modules*, Astérisque **330** (2010), 281-509.

[3] Colmez, P., Dospinescu, G., Niziol, W. *Cohomologie p-adique de la tour de Drinfeld: le cas de la dimension* 1, preprint.

[4] Deligne, P., Kazhdan, D. and Vignéras M.-F. *Représentations des algébres centrales simples p-adiques*, Representations of reductive groups over a local field, Travaux en Cours, Hermann, Paris, (1984), 33-117.

[5] Dospinescu, G. and Le Bras, A. *Revêtements du demi-plan de Drinfeld et correspondance de Langlands locale p-adique*, Ann. of Math. **186** (2017), 321-411.

[6] Drinfeld, V. *Coverings of p-adic symmetric regions*, Func. Anal. Appl. **10**(2) (1976), 29-40.

[7] Emerton, M. *Locally analytic vectors in representations of locally p-adic analytic groups*, Mem. Amer. Math. Soc. **248**(1175), (2017), 1-158.

[8] Harris, M. and Taylor, R. *The geometry and cohomology of some simple Shimura varieties*, with an Appendix by V. G. Berkovich. Ann. of Math. Stud. **151** Princeton University Press, Princeton, NJ, (2001).

[9] Kummer, E. *Über die Ergänzungssätze zu den allgemeinen Reciprocitätsgesetzen*, J. Reine Angew. Math. **44** (1852), 93-146.

[10] Mieda, Y. *Geometric approach to the local Jacquet-Langlands correspondence*, American Journal of Mathematics **136**(4) (2014), 1067-1091.

[11] Patel, D., Schmidt, T. and Strauch, M. *Locally analytic representations of $GL(2, L)$ via semistable models of* $\mathbb{P}^1$ J. Inst. Math. Jussieu **18**(1) (2019), 125-187.

[12] Van der Put, M., and Schneider, P. *Points and topologies in rigid geometry* Math. Ann. **302**(1) (1995) 81-103.

[13] Taylor, R. *Galois representations*, Proceedings of the ICM, Vol **I**, (2002), 449-474.

[14] Rogawsky, J. *Representations of $GL(n)$ and division algebras over a p-adic field*, Duke Math. J., **50**(1) (1983), 161-196.

[15] P. Schneider and J. Teitelbaum, *Algebras of p-adic distributions and admissible representations*, Invent. Math. **153** (2003), 145-196.

The adic tame site

Katharina Hübner

For a smooth variety over the complex numbers and a prime number ℓ, étale and analytic cohomology with coefficients $\mathbb{Z}/\ell\mathbb{Z}$ coincide. If the characteristic p of the base field is positive but different from ℓ, the étale cohomology groups with coefficients $\mathbb{Z}/\ell\mathbb{Z}$ retain the same good properties as over $\mathbb{C}$. For instance, there are finiteness theorems, cohomological purity, and a smooth base change theorem. The cohomology groups are homotopy invariant and the Künneth formula holds (not only for cohomology with compact support). All this breaks down, however, if base field and coefficient ring have the same characteristic. There is overwhelming evidence that these problems are due to the existence of wild ramification "at the boundary of X". To give an example consider the first cohomology group $H^1_{ét}(\mathbb{A}^1_{\bar{\mathbb{F}}_p}, \mathbb{Z}/p\mathbb{Z})$, which classifies finite étale coverings of X. It is infinite dimensional because of the huge amount of étale coverings of $\mathbb{A}^1_{\bar{\mathbb{F}}_p}$ wildly ramified in ∞.

For assertions concerning the fundamental group this problem has been adressed by introducing the tame fundamental group (see [2]) Under suitable regularity assumptions the tame fundamental group is topologically finitely generated and the specialization map of the tame fundamental group is at least surjective ([7], VIII 2.11). Moreover, the tame fundamental group satisfies the Künneth formula ([3] and there is a Lefschetz-Theorem ([4]). So the question arises whether one can modify the étale site to obtain a tame site whose fundamental group coincides with the tame fundamental group. Tame cohomology groups with torsion coefficients away from the characteristic p should coincide with the corresponding étale cohomology groups and should be better behaved than étale cohomology groups for p-torsion coefficients. One would furthermore expect that the tame cohomology groups satisfy finiteness theorems and a version of cohomological purity and smooth base change. The Künneth formula should be true without restrictions and the tame cohomology groups should be homotopy invariant.

Since tameness is a valuation theoretic concept, it turns out to be natural to work with adic spaces instead of schemes. We say that an étale morphism of adic spaces $f : X \to Y$ is tame if for every $x \in X$ the residue field extension $k(x)|k(f(x))$ is tamely ramified with respect to the valuation of $k(x)$ corresponding to x. For every adic space X this defines a site X_t (the tame site of X). Associating with a scheme Y over a base scheme S the discretely ringed adic space $Spa(Y, S)$ (an easy generalization of $Spa(A, A^+)$ for a Huber pair (A, A^+) with discrete toplogy), we obtain a tame site also for schemes.

As expected, the tame fundamental group of $Spa(Y, S)_t$ is naturally isomorphic to the curve-tame fundamental group of Y/S ([1], section 7). Moreover, tame cohomology coincides with étale cohomology in the required cases ([1], section 6):

For every adic space the tame cohomology groups of torsion sheaves with torsion coprime to the characteristic are isomorphic to the corresponding étale cohomology groups. If $Y \to S$ is a proper morphism of schemes, tame and étale cohomology groups of $Spa(Y,S)_t$ are isomorphic. Moreover, there is a first version of cohomological purity:

Theorem 1 ([1],Cor. 12.5)**.** *Let X be a quasi-compact, quasi-separated, quasi-excellent scheme of characteristic $p > 0$ and X a regular scheme which is separated and essentially of finite type over S. Assume resolution of singularities holds over S. Then for every pro-open dense subscheme $U \subseteq X$ there is a natural isomorphism*

$$H^i(Spa(U,S)_t, \mathbb{Z}/p\mathbb{Z}) \cong H^i(Spa(X,S)_t, \mathbb{Z}/p\mathbb{Z}).$$

Via the excision sequence for a closed subscheme $Z \subseteq X$ we conclude that the tame cohomology groups with support in $Spa(Z,S)$ and coefficients $\mathbb{Z}/p\mathbb{Z}$ are trivial. This is different from the case of coefficients $\mathbb{Z}/\ell\mathbb{Z}$ with $\ell \neq p$, where for regular pairs (X,Z) of codimension c we have a canonical isomorphism (see [6])

$$H^i_Z(X, \mathbb{Z}/\ell\mathbb{Z}) \cong H^{i-2c}(Z, \mathbb{Z}/\ell\mathbb{Z}(-c)).$$

However, this is the expected outcome in characteristic $p > 0$. Evidence for this is provided by cohomological purity for the logarithmic deRham Witt sheaves $\nu_n(s)$ ([5], Prop. 2.1): Let Z be a smooth closed subscheme of codimension c of a smooth, quasi-projective scheme X over a perfect field k of characteristic p and $s \geqslant c$. Then $H^i_Z(X_{ét}, \nu_n(s)) = 0$ for $i < c$ and there is a Gysin isomorphism

$$H^0(Z_{ét}, \nu_n(s-c)) \cong H^c_Z(X_{ét}, \nu_n(s)).$$

However, there is no good description of the cohomology groups $H^r_Z(X_{ét}, \nu_n(s))$ for $r > c$. My hope is that replacing étale with tame cohomology we have purity in all dimensions. If $s = 0$, the sheaf $\nu_n(s)$ is isomorphic to $\mathbb{Z}/p^n\mathbb{Z}$ and for $s < 0$ it is zero. Hence, Theorem 1 is the expected purity result for $s = 0$.

A direct consequence of cohomological purity is homotopy invariance of the tame cohomology groups with torsion coefficients ([1], Cor. 12.6). This is a considerable advantage compared to étale cohomology in positive characteristic and hints to the motivic nature of tame cohomology.

References

[1] K. Hübner, *The adic tame site*, arXiv:1801.04776 [math.AG] (2018).

[2] M. Kerz and A. Schmidt, *On different notions of tameness in arithmetic geometry*, Mathematische Annalen **346** (2010), 641–668.

[3] Y. Hoshi, *The exactness of the log homotopy sequence*, Hiroshima Mathematical Journal **39** (2009), 61–121.

[4] H. Esnault and L. Kindler *Lefschetz theorems for tamely ramified coverings*, Proceedings of the Americal Mathematical Society **144** (2016), 5071–5080.

[5] J. Milne *Values of zeta functions of varieties over finite fields*, American Journal of Mathematics **108** (1986), 297–360.

[6] K. Fujiwara *A proof of the absolute purity conjecture (after Gabber)*, Algebraic geometry 2000, Azumino (Hotaka), Advanced Studies in Pure Mathematics **36** (2002), 153–183

[7] A. Grothendieck *Revêtements étales et groupe fondamental (SGA 1)*, Séminaire de géometrie algébrique du Bois Marie - 1960-61 (1971).

Non-archimedean compactifications of complex analytic spaces

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(joint work with Marco Maculan)

Let X be a complex algebraic variety, *i.e.* a separated scheme of finite type over $\mathbf{C}$, and let X^{h} be its analytification. We would like to construct a compactification of X^{h} that is canonical in some sense. This is not possible to achieve in the category of complex analytic spaces, so our aim will be to find a compact locally ringed space $X^{\daleth}$ with an open embedding $X^{\mathrm{h}} \hookrightarrow X^{\daleth}$.

1. A valuative boundary

We denote by $\mathbf{C}_0$ the field $\mathbf{C}$ endowed with the trivial absolute value $|\cdot|_0$. We will work in the category of analytic spaces over $\mathbf{C}_0$ in the sense of V. Berkovich (see [2, 3]). Recall that we have an analytification functor $X \mapsto X_0^{\mathrm{an}}$ from algebraic varieties over $\mathbf{C}$ to analytic spaces over $\mathbf{C}_0$.

In the affine case $X = \mathrm{Spec}(A)$, where A is an algebra of finite type over $\mathbf{C}$, X_0^{an} may be defined as the set of multiplicative seminorms on A that induce the trivial absolute value $|\cdot|_0$ on $\mathbf{C}$ endowed with the weak topology. It is also endowed with a sheaf of analytic functions. The general case may be obtained from the affine case by glueing.

Starting with an algebraic variety X over $\mathbf{C}$, there is another natural way to associate an analytic space over $\mathbf{C}_0$. Endowing $\mathbf{C}$ with the discrete topology, one may consider X as a formal scheme and consider its generic fiber in the sense of Raynaud. Following [6], we will denote it by $X^{\beth}$. It is a compact subset of X_0^{an}. In the affine case $X = \mathrm{Spec}(A)$, we have

$$X^{\beth} = \{x \in X_0^{\mathrm{an}} : |f(x)| \leqslant 1, f \in A\}.$$

We may now define the non-archimedean boundary of X by

$$X_\infty := X_0^{\mathrm{an}} - X^{\beth}.$$

It may be identified with the set of seminorms that have no center on X.

It is interesting to remark that, if X is embedded as an open subset in a proper algebraic variety Y over $\mathbf{C}$ with complement Z, then X_∞ may be identified with the generic fiber (in the sense of Raynaud–Berthelot) of the formal completion $\hat{Y}_Z$ of Y along Z deprived of (the analytication of) its special fiber Z. In particular, the latter construction does not depend on the choice of Y.

This set was first defined by Berkovich in a letter to V. Drinfeld and subsequentely used by O. Ben–Bassat and M. Temkin in [1] to prove some descent results (reconstructing coherent sheaves on Y from coherent sheaves on $\hat{Y}_Z$ and X). It was also independently defined by A. Thuillier in [6], where he proved that if Y is regular and Z has normal crossings, then the dual complex of Z is homotopy equivalent to X_∞. As a consequence, the homotopy type of the dual complex of the boundary depends only on X and not on the chosen compactification.

2. Hybrid spaces

In order to put together the spaces X^h and X_∞, we need a "hybrid" space that contains both usual complex analytic spaces and analytic spaces over $\mathbf{C}_0$.

Denote by $\mathbf{C}_{\mathrm{hyb}}$ the field $\mathbf{C}$ endowed with the norm $\|\cdot\|_{\mathrm{hyb}} := \max(|\cdot|_0, |\cdot|_\infty)$, where $|\cdot|_\infty$ is the usual absolute value on $\mathbf{C}$. It is a Banach ring. As a consequence, the theory developed in [2] provides us with a definition of analytic space over $\mathbf{C}_{\mathrm{hyb}}$ and an analyfication functor $X \mapsto X^{\mathrm{hyb}}$.

In the affine case $X = \mathrm{Spec}(A)$, the definition is close to the usual one: X^{hyb} may be defined as the set of multiplicative seminorms on A that are bounded by the norm $\|\cdot\|_{\mathrm{hyb}}$ on $\mathbf{C}$ endowed with the weak topology. It is also endowed with a sheaf of analytic functions.

The basic example is the analytification of $\mathrm{Spec}(\mathbf{C})$, which may be explicitly described as

$$\mathrm{Spec}(\mathbf{C})^{\mathrm{hyb}} = \{|\cdot|_\infty^\varepsilon, 0 \leqslant \varepsilon \leqslant 1\},$$

where $|\cdot|_\infty^0 := |\cdot|_0$.

Let X be a complex algebraic variety. By functoriality, the structure morphism $\pi\colon X \to \mathrm{Spec}(\mathbf{C})$ gives rise to a morphism $\pi^{\mathrm{hyb}}\colon X^{\mathrm{hyb}} \to \mathrm{Spec}(\mathbf{C})^{\mathrm{hyb}}$ whose fibers we can describe: we have $(\pi^{\mathrm{hyb}})^{-1}(|\cdot|_0) = X_0^{\mathrm{an}}$ and, for each $\varepsilon \in (0,1]$, we have $(\pi^{\mathrm{hyb}})^{-1}(|\cdot|_\infty^\varepsilon) \simeq X^{\mathrm{h}}$.

To sum up, we obtain a locally ringed space with complex analytic fibers that seem to "degenerate" on a non-archimedean fiber. Such spaces have been used by V. Berkovich in [4] to give a topological interpretation (in an analytic space over $\mathbf{C}_0$) of the weight zero part of the limit mixed Hodge structure of a degenerating family of compact complex manifolds. They can also be found in the work [7] of S. Boucksom and M. Jonsson about the asymptotic behavior of volume forms in the same setting.

3. The compactification

Let X be a complex algebraic variety. We set

$$X^+ := X^{\mathrm{hyb}} - X^{\beth}.$$

Since $X^{\beth}$ is a closed subset of X_0^{an}, which is itself closed in X^{hyb}, X^+ is an open subset of X^{hyb}. In particular, it inherits a structure of locally ringed space. Denote by π^+ the restriction of π^{hyb} to X^+. We have

$$(\pi^+)^{-1}(|\cdot|_0) = X_\infty$$

and, for each $\varepsilon \in (0,1]$,

$$(\pi^+)^{-1}(|\cdot|_\infty^\varepsilon) = (\pi^{\mathrm{hyb}})^{-1}(|\cdot|_\infty^\varepsilon) \simeq X^{\mathrm{h}}.$$

The resulting space is not compact in general and contains several copies of X^{h}. To solve this issue, we will identify the points in the space X^+ that correspond to equivalent seminorms, *i.e.* seminorms that can be obtained one form the other by raising to some power $\lambda \in \mathbf{R}_{>0}$. Denote by $X^{\urcorner}$ the quotient space. We turn it into a locally ringed space by endowing it with the push-forward of the structure sheaf on X^+.

The archimedean part of the space $X^{\urcorner}$ now consists in exactly one copy of X^{h}. The non-archimedean part, which is the quotient of X_∞ by the equivalence of seminorms, is a so-called normalized space, as introduced by L. Fantini in [5].

Theorem 1. *The space $X^{\urcorner}$ is Hausdorff and compact and the map*

$$X^h = (\pi^+)^{-1}([|\cdot|_\infty]) \longrightarrow X^{\urcorner}$$

is an open embedding.

The map $X \mapsto X^{\urcorner}$ has additional properties. For instance, it is functorial with respect to proper morphisms.

Finally, to a coherent sheaf F on X, one may functorially associate a coherent sheaf $F^{\urcorner}$ on $X^{\urcorner}$. We have a GAGA theorem in this setting.

Theorem 2. *The functor*

$$F \in \mathrm{Coh}(X) \longmapsto F^{\urcorner} \in \mathrm{Coh}(X^{\urcorner})$$

is an equivalence of categories.

For each coherent sheaf F on X and each $q \geqslant 0$, we have a natural isomorphism

$$H^q(X, F) \xrightarrow{\sim} H^q(X^{\urcorner}, F^{\urcorner}).$$

Note that the space $X^{\urcorner}$ has an open subset isomorphic to X^{h}. As a consequence, the space $X^{\urcorner}$ may be used to relate the categories of coherent sheaves over X and X^{h}.

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References

[1] Oren Ben-Bassat and Michael Temkin, *Berkovich spaces and tubular descent*, Adv. Math. **234** (2013), 217–238.

[2] Vladimir G. Berkovich, *Spectral theory and analytic geometry over non-Archimedean fields*, Mathematical Surveys and Monographs, vol. 33, American Mathematical Society, Providence, RI, 1990.

[3] Vladimir G. Berkovich. Étale cohomology for non-Archimedean analytic spaces. *Inst. Hautes Études Sci. Publ. Math.*, (78):5–161 (1994), 1993.

[4] Vladimir G. Berkovich. A non-Archimedean interpretation of the weight zero subspaces of limit mixed Hodge structures. In *Algebra, arithmetic, and geometry: in honor of Yu. I. Manin. Vol. I*, volume 269 of *Progr. Math.*, pages 49–67. Birkhäuser Boston Inc., Boston, MA, 2009.

[5] Lorenzo Fantini, *Normalized Berkovich spaces and surface singularities*, Trans. Amer. Math. Soc. **370** (2018), no. 11, 7815–7859.

[6] Amaury Thuillier, *Géométrie toroïdale et géométrie analytique non archimédienne. Application au type d'homotopie de certains schémas formels*, Manuscripta Math. **123** (2007), no. 4, 381–451.

[7] Sébastien Boucksom and Mattias Jonsson, *Tropical and non-Archimedean limits of degenerating families of volume forms*, J. Éc. polytech. Math. **4** (2017), 87–139.

Non-standard analysis and non-archimedean geometry

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(joint work with Antoine Ducros, Ehud Hrushovski)

The aim of this work is to relate asymptotics of one-parameter families of complex integrals with the non-archimedean integrals introduced recently by Chambert-Loir and Ducros in [1]. This is performed by constructing a morphism of double complexes from a non-standard archimedean Dolbeault complex to a non-archimedean Chambert-Loir and Ducros complex which is compatible with integration.

1. Construction of the field C. We shall work over an algebraically closed field C containing $\mathbb{C}$ which is endowed with a norm $|\,| : C \to R_{\geqslant 0}$ with R a real closed field such that $C \simeq R(i)$, and also carries a non-archimedean norm $|\,|_\flat : C \to \mathbb{R}_{\geqslant 0}$.

The construction of the field C goes as follows. We fix a non principal ultrafilter $\mathcal{U}$ on $\mathbb{C}^\times$ containing all (punctured) neighbourhoods of the origin. We consider the ultrapowers ${}^*\mathbb{C} = \prod_{t \in \mathbb{C}^\times} \mathbb{C}/\mathcal{U}$ and ${}^*\mathbb{R} = \prod_{t \in \mathbb{C}^\times} \mathbb{R}/\mathcal{U}$. We say an element (a_t) in ${}^*\mathbb{C}$, resp. ${}^*\mathbb{R}$, is t-bounded if for some positive integer N, $|a_t| \leqslant |t|^{-N}$ along $\mathcal{U}$. Similarly, it is said to t-negligible if for every positive integer N, $|a_t| \leqslant |t|^N$ along $\mathcal{U}$. The set of t-bounded elements in ${}^*\mathbb{C}$, resp. ${}^*\mathbb{R}$, is a local ring which we denote by A, resp. A_r, with maximal ideal the subset of t-negligible elements which we denote by $\mathfrak{M}$, resp. $\mathfrak{M}_\mathrm{r}$. We now set $C := A/\mathfrak{M}$ and $R := A_\mathrm{r}/\mathfrak{M}_\mathrm{r}$. The field R is a real closed field and $C \simeq R(i)$ is algebraically closed. The norm $|\,| :{}^* \mathbb{C} \to{}^* \mathbb{R}_{\geqslant 0}$ induces an R-valued norm $|\,| : C \to R_{\geqslant 0}$. Furthermore, one can endow C with a real-valued non-archimedean norm $|\,|_\flat : C \to \mathbb{R}_{\geqslant 0}$ as follows. For any $z \in C^\times$, one checks that the norm of $\frac{\log|z|}{\log|t|}$ is bounded by some positive real number in $\mathbb{R}$. One can thus consider its standard part $\alpha = \mathrm{std}\left(\frac{\log|z|}{\log|t|}\right) \in \mathbb{R}$. Fixing $\tau \in (0,1) \subset \mathbb{R}$, one sets $|z|_\flat := \tau^\alpha$, so that $|z|_\flat = |t|_\flat^\alpha$.

2. The two complexes. If X is a smooth algebraic variety over R, it is possible to define a complex of sheaves of $\mathcal{C}^\infty$ differential forms on $X(R)$ and a well-behaved integration theory. In particular if ω is a top degree $\mathcal{C}^\infty$ differential form with support contained in a definably compact semi-algebraic subset of $X(R)$, $\int_{X(R)} \omega$ makes sense as an element of R and furthermore $|\int_{X(R)} \omega|$ is bounded by an element of $\mathbb{R}_{\geqslant 0}$.

Assume now X is a smooth algebraic variety over C and set $\lambda := -\log|t|$. We construct a Zariski-sheaf $\mathsf{A}^{p,q}$ of forms on X whose sections are locally of the form

$$\omega = \frac{1}{\lambda^p} \sum_{I,J} \phi_{I,J} \left(\frac{\log|f_1|}{\lambda}, \ldots, \frac{\log|f_m|}{\lambda} \right) d\log|f_I| \wedge d\arg f_J$$

with $(f_1, \ldots, f_m)$ regular invertible functions, for every pair (I, J) with I and J two subsets of $\{1, \ldots, m\}$ of respective cardinality p and q, a smooth function $\phi_{I,J}$ on $\mathbb{R}^m$ all whose derivatives are polynomially bounded, and $d\log|f|_I$ standing for

$d\log|f_{i_1}| \wedge \ldots \wedge d\log|f_{i_p}|$ if $i_1 < i_2 < \ldots < i_p$ are the elements of I, and similarly for $d\arg f_J$. There exist natural derivations $d : A^{p,q} \to A^{p+1,q}$ and $d^\sharp : A^{p,q} \to A^{p,q+1}$.

Similarly we set $\lambda_\flat := -\log|t|_\flat$. We denote by $\mathsf{B}^{p,q}$ the Zariski-sheaf on X whose sections are locally (on the analytification of X) (p,q)-smooth forms in the sense of Chambert-Loir and Ducros [1] of the form

$$\omega = \sum_{I,J} \phi_{I,J}\left(\log|f_1|_\flat, \ldots, \log|f_m|_\flat\right) d'\log|f_I|_\flat \wedge d''\log|f_J|_\flat$$

with $(f_1, \ldots, f_m)$ regular invertible functions, for every pair (I,J) with I and J two subsets of $\{1, \ldots, m\}$ of respective cardinality p and q, a smooth function $\phi_{I,J}$ on $\mathbb{R}^m$ all whose derivatives are polynomially bounded, and $d'\log|f_I|_\flat$ standing for $d'\log|f_{i_1}|_\flat \wedge \ldots \wedge d'\log|f_{i_p}|_\flat$ if $i_1 < i_2 < \ldots < i_p$ are the elements of I, and similarly for $d''\log|f_J|_\flat$.

3. The main result. We are now in position to state our main result:

Theorem. *Let X be a smooth algebraic variety over C. There exists a unique morphism of sheaves of bi-graded differential $\mathbb{R}$-algebras $\mathsf{A}^{*,*} \to \mathsf{B}^{*,*}$, sending a non-standard archimedean form ω to the non-archimedean form $\omega_\flat$, such that for every Zariski-open subset U of X, every finite family $(f_1, \ldots, f_m)$ of regular invertible functions on U, every smooth function ϕ on $\mathbb{R}^m$ all whose derivatives are polynomially bounded and every pair (I,J) of subsets of $\{1, \ldots, m\}$ one has*

$$\begin{aligned} &\left(\frac{1}{\lambda^{|I|}}\phi\left(\frac{\log|f_1|}{\lambda}, \ldots, \frac{\log|f_m|}{\lambda}\right) d\log|f_I| \wedge d\arg f_J\right)_\flat \\ = \quad &\frac{1}{\lambda_\flat^{|I|}}\phi\left(\frac{\log|f_1|_\flat}{\lambda_\flat}, \ldots, \frac{\log|f_m|_\flat}{\lambda_\flat}\right) d'\log|f_I|_\flat \wedge d''\log|f_J|_\flat. \end{aligned}$$

Furthermore, the mapping $\omega \mapsto \omega_\flat$ is compatible with integration: if one assumes that ω is defined on some Zariski open U and that its support is contained in a definably compact definable subset of $U(C)$, then $\omega_\flat$ is compactly supported, $\int_{U(C)}|\omega|$ is bounded by some positive real number in $\mathbb{R}$ and

$$\mathrm{std}\left(\int_{U(C)} \omega\right) = (2\pi)^n \int_{U^{\mathrm{an}}} \omega_\flat,$$

with U^{an} the non-archimedean analytification of U.

Note that the last statement can be interpreted as expressing asymptotics of one-parameter families of complex integrals as non-archimedean integrals in the sense of Chambert-Loir and Ducros.

References

[1] A. Chambert-Loir, A. Ducros, *Formes différentielles réelles et courants sur les espaces de Berkovich*, arXiv:1204.6277.

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Skeletons of Prym varieties and Brill-Noether theory

Martin Ulirsch
(joint work with Yoav Len)

1. Skeletons of Prym varieties

Let X be smooth projective curve over a non-Archimedean field K and let $\pi\colon \widetilde{X} \to X$ be an unramified double cover. The kernel of the *norm homomorphism*

$$\begin{aligned} \mathrm{Nm}_\pi\colon \ \mathrm{Pic}(\widetilde{X}) &\longrightarrow \mathrm{Pic}(X) \\ \mathcal{O}_{\widetilde{X}}(\widetilde{D}) &\longmapsto \mathcal{O}_X(\pi_\ast \widetilde{D}) \end{aligned}$$

has two components; the component containing the identity is known as the *Prym variety* $\mathrm{Pr}(X,\pi)$ associated to the unramified double cover $\pi\colon \widetilde{X} \to X$ and it carries a natural principal polarization.

Jensen and Len [JL18] gave us a tropical analogue of this construction: Let Γ be a tropical curve and an unramified double cover $\pi\colon \widetilde{\Gamma} \to \Gamma$. The kernel of the *tropical norm homomorphism*

$$\begin{aligned} \mathrm{Nm}_\pi\colon \ \mathrm{Pic}(\widetilde{\Gamma}) &\longrightarrow \mathrm{Pic}(\Gamma) \\ [\widetilde{D}] &\longmapsto \big[\pi_\ast \widetilde{D}\big] \end{aligned}$$

has one or two components; the component containing the identity is the *tropical Prym variety* $\mathrm{Pr}(\Gamma,\pi)$ associated to $\pi\colon \widetilde{\Gamma} \to \Gamma$. We show that $\mathrm{Pr}(\Gamma,\pi)$ also naturally carries a principal polarization.

Let Γ_X be the non-Archimedean skeleton of X^{an} and write $\rho_X\colon X^{an} \to \Gamma_X$ for the retraction map. There is a natural modular tropicalization map

$$\rho_{X,\ast}\colon \ \mathrm{Pr}(X,\pi)^{an} \longrightarrow \mathrm{Pr}(\Gamma_X, \pi^{trop})$$

from the Berkovich space $\mathrm{Pr}(X,\pi)^{an}$ to $\mathrm{Pr}(\Gamma_X,\pi^{trop})$ induced by pushing forward divisors along ρ_X. On the other hand, given an abelian variety A over K, by [Ber90], there is a natural strong deformation retraction $\rho_A\colon A^{an} \to \Sigma(A)$ from A^{an} onto a closed subset $\Sigma(A)$ of A^{an} that has the structure of a tropical abelian variety, the *non-Archimedean skeleton* of A^{an}. A fixed principal polarization on A hereby naturally induces principal polarization on $\Sigma(A)$. Expanding on the work of Baker-Rabinoff [BR15], we confirm [JL18, Conjecture 6.3].

Theorem 1. *There is a canonical isomorphism*

$$\mu_{X,\pi}\colon \ \mathrm{Pr}(\Gamma_X, \pi^{trop}) \xrightarrow{\simeq} \Sigma\big(\mathrm{Pr}(X,\pi)\big)$$

of principally polarized tropical abelian varieties that makes the natural diagram

$$\begin{array}{ccc} \mathrm{Pr}(X,\pi)^{an} & \xrightarrow{\rho_{X,*}} & \mathrm{Pr}(\Gamma_X,\pi^{trop}) \\ & \searrow^{\rho_{\mathrm{Pr}(X,\pi)}} & \downarrow{\scriptstyle \mu_{X,\pi}}\ \sim \\ & & \Sigma\big(\mathrm{Pr}(X,\pi)\big) \end{array}$$

commute.

Let $\mathcal{R}_g$ be the moduli space of unramified double covers, as e.g. introduced (and compactified) in [Bea77], and $\mathcal{A}_g$ the moduli space of principally polarized abelian varieties. There is a natural *Prym-Torelli morphism* $\mathrm{pr}\colon \mathcal{R}_g \to \mathcal{A}_g$ that associates to an unramified double cover $\pi\colon \widetilde{X} \to X$ its associated Prym variety $\mathrm{Pr}(X,\pi)$. Theorem 1 may be reinterpreted as saying that the diagram

$$\begin{array}{ccc} \mathcal{R}_g^{an} & \xrightarrow{\mathrm{trop}_{\mathcal{R}_g}} & R_g^{trop} \\ {\scriptstyle \mathrm{pr}^{an}}\downarrow & & \downarrow{\scriptstyle \mathrm{pr}^{trop}} \\ \mathcal{A}_g^{an} & \xrightarrow{\mathrm{trop}_{\mathcal{A}_g}} & A_g^{trop} \end{array}$$

commutes.

2. Tropical Prym-Brill-Noether theory

Let $\pi\colon \widetilde{X} \to X$. Let $r \geqslant 1$. In [Wel85], Welters has defined the *Prym-Brill-Noether locus* $V^r(X,\pi)$ to be the closed subset

$$\left\{L \in \mathrm{Pic}_{2g-2}(\widetilde{X}) \,\middle|\, \mathrm{Nm}_\pi(L) = \omega_X, h^0(L) \geqslant r+1 \text{ and } h^0(L) \equiv r+1 \pmod 2\right\}$$

in $\mathrm{Pic}_{2g-2}(\widetilde{X})$. Theorem 1 above, in combination with the Bieri-Groves theorem for maximally degenerate abelian varieties from [Gub07], allows us to apply tropical techniques, as in [CDPR12, Bak08, Pfl17], to find upper bounds on the dimension of $V^r(X,\pi)$.

Theorem 2. *Let $r \geqslant -1$ and write*

$$n = n(r,k) = \begin{cases} \frac{rk}{2} - \frac{k^2}{8} + \frac{k}{4} & \text{if } k \leqslant 2r-2 \\ \binom{r+1}{2} & \text{if } k \geqslant 2r-1. \end{cases}$$

Suppose $k \geqslant 2$ is either even or greater than $2r-2$. There is a non-empty open subset in the k-gonal locus of $\mathcal{R}_g$ such that for every unramified double cover $\pi\colon \widetilde{X} \to X$ in this open subset we have:

$$\dim V^r(X,\pi) \leqslant g - 1 - n(r,k)\ .$$

In particular, the Prym–Brill–Noether locus $V^r(X,\pi)$ is empty if $g-1 < n(r,k)$.

This, in particular, provides us with a tropical proof of the *Prym-Brill-Noether-Theorem*, which classically follows from Welters' Prym-Gieseker-Petri Theorem [Wel85, Theorem 1.11] and Bertram's existence theorem for Prym special divisors [Ber87].

Corollary 1. *There is a non-empty open subset of $\mathcal{R}_g$ such that for every unramified double cover $\pi\colon \widetilde{X} \to X$ in this set we have:*

$$\dim V^r(X,\pi) = g - 1 - \binom{r+1}{2}.$$

In particular, the Prym–Brill–Noether locus $V^r(X,\pi)$ is empty if and only if $g - 1 - \binom{r+1}{2} < 0$.

In general, we expect the inequality above to be an equality and we are currently investigating whether new techniques from logarithmic Gromov-Witten theory, as introduced in [JR17], can help us find a lower bound.

References

[Bak08] Matthew Baker, *Specialization of linear systems from curves to graphs*, Algebra Number Theory **2** (2008), no. 6, 613–653, With an appendix by Brian Conrad.

[Bea77] Arnaud Beauville, *Prym varieties and the Schottky problem*, Invent. Math. **41** (1977), no. 2, 149–196.

[Ber87] Aaron Bertram, *An existence theorem for Prym special divisors*, Invent. Math. **90** (1987), no. 3, 669–671.

[Ber90] Vladimir G. Berkovich, *Spectral theory and analytic geometry over non-Archimedean fields*, Mathematical Surveys and Monographs, vol. 33, American Mathematical Society, Providence, RI, 1990.

[BR15] Matthew Baker and Joseph Rabinoff, *The skeleton of the Jacobian, the Jacobian of the skeleton, and lifting meromorphic functions from tropical to algebraic curves*, Int. Math. Res. Not. IMRN (2015), no. 16, 7436–7472.

[CDPR12] Cools, Filip; Draisma, Jan; Payne, Sam; Robeva, Elina, *A tropical proof of the Brill-Noether theorem*, Adv. Math. 230 (2012), no. 2, 759–776.

[Gub07] Walter Gubler, *Tropical varieties for non-Archimedean analytic spaces*, Invent. Math. **169** (2007), no. 2, 321–376.

[JR17] David Jensen and Dhruv Ranganathan, *Brill-Noether theory for curves of a fixed gonality*, arXiv:1701.06579 [math] (2017).

[JL18] David Jensen and Yoav Len, *Tropicalization of theta characteristics, double covers, and Prym varieties*, Selecta Math. (N.S.) **24** (2018), no. 2, 1391–1410.

[Pfl17] Nathan Pflueger, *Brill-Noether varieties of k-gonal curves*, Adv. Math. **312** (2017), 46–63.

[Wel85] Gerald E. Welters, *A theorem of Gieseker-Petri type for Prym varieties*, Ann. Sci. École Norm. Sup. (4) **18** (1985), no. 4, 671–683.

The non-archimedean SYZ fibration

Johannes Nicaise

(joint work with Chenyang Xu, Tony Yue Yu)

1. Introduction

The main motivation for this talk is a tentative geometric explanation for the phenomenon of mirror symmetry between Calabi-Yau manifolds: the Strominger-Yau-Zaslow (SYZ) conjecture [5]. It has been finetuned over the years and is currently usually formulated in the following way. Let Δ be an open disk around the origin of the complex plane, and set $\Delta^* = \Delta\backslash\{0\}$. Let $X \to \Delta^*$ be a maximally degenerating projective family of Calabi-Yau manifolds of dimension n (the definition of a maximally degenerating family will be recalled below). Then the SYZ conjecture states that a general fiber X_t of the family admits a fibration $\rho\colon X_t \to S$ over a topological n-sphere $S \approx S^n$ such that, away from a codimension $\geqslant 2$ discriminant locus $D \subset S$, the map ρ is a smooth fibration in Lagrangian real tori of dimension n. Moreover, S can be realized as the Gromov-Hausdorff limit of the fibers X_t with their natural Ricci-flat metrics. The mirror partner of X_t can then be constructed by dualizing the torus fibration over $S\backslash D$ and compactifying the result after some subtle deformations (quantum corrections) of the complex structure.

In this formulation, the conjecture is still wide open. However, it has been quite influential for the development of the theory of mirror symmetry in algebraic geometry. It has inspired (at least) two powerful approaches: the Gross–Siebert programme (based on tropical and logarithmic geometry), and the Kontsevich–Soibelman programme (based on non-archimedean geometry). The starting point of the Kontsevich–Soibelman programme is the profound insight that the conjectural SYZ fibration ρ resembles a retraction map of a non-archimedean analytic space onto its skeleton, and that it should be possible to pass through the non-archimedean world to construct a mirror family for the degeneration X [1,2]. Kontsevich and Soibelman proposed a candidate for the base of the non-archimedean SYZ fibration: the essential skeleton.

2. The essential skeleton

We slightly generalize the set-up: let k be an algebraically closed field of characteristic zero, and set $R = k[[t]]$ and $K = k((t))$. We replace the disk Δ by its algebraic analog Spec$R$, and the punctured disk Δ^* by Spec$K$. We endow K with its t-adic absolute value $|\cdot| = \exp(-\mathrm{ord}_t(\cdot))$.

Let X be a geometrically connected smooth projective K-scheme of dimension n with trivial canonical bundle. We say that X is maximally degenerate if it has a semistable model $\mathcal{X}$ over R (that is, a regular projective R-model whose special fiber is a reduced divisor with strict normal crossings) and the monodromy action on $H^{n+1}(X \times_K K^a, \mathbb{Q}_\ell)$ has a Jordan block of rank $n+1$. The former condition can always be achieved by means of a finite extension of K, by the Semistable Reduction Theorem. The latter condition formalizes the intuitive idea that X is as far away from having good reduction as possible: the geometry of the irreducible components of the special fiber $\mathcal{X}_k$ is as simple as possible, and the geometric complexity of X has been transferred into the combinatorial structure of the dual intersection complex of $\mathcal{X}_k$. Good examples to keep in mind are abelian K-varieties with purely multiplicative reduction and type III degenerations of $K3$-surfaces.

Kontsevich and Soibelman identified in [2] a canonical subspace $\mathrm{Sk}(X)$ of the K-analytic space X^{an}. It is homeomorphic to a finite simplicial complex, and it is called the essential skeleton of X. If X is an abelian variety then $\mathrm{Sk}(X)$ is a real n-dimensional torus; if X is a $K3$-surface then $\mathrm{Sk}(X)$ is a 2-sphere. Kontsevich and

Soibelman proposed that $\mathrm{Sk}(X)$ should be the base of a non-archimedean analog of the SYZ fibration. What was missing from their construction was a candidate for the fibration itself.

3. The non-archimedean SYZ fibration

Chenyang Xu and the author have given in [3] a different interpretation of the essential skeleton $\mathrm{Sk}(X)$: it can be realized as the skeleton of any $\mathbb{Q}$-factorial minimal *dlt*-model $\mathcal{X}$ of X. These models appeared as natural generalizations of semistable models in the Minimal Model Programme. They are produced by running an MMP algorithm on a semistable model for X. The *dlt*-models are sufficiently close to semistable models to generalize Berkovich's construction of the skeleton $\mathrm{Sk}(\mathcal{X}) \subset X^{\mathrm{an}}$ and the retraction map

$$\rho_{\mathcal{X}} \colon X^{\mathrm{an}} \to \mathrm{Sk}(\mathcal{X}).$$

The skeleton $\mathrm{Sk}(\mathcal{X})$ is homeomorphic to the dual intersection complex of the special fiber $\mathcal{X}_k$. Chenyang Xu and the author have proven that $\mathrm{Sk}(\mathcal{X}) = \mathrm{Sk}(X)$; in particular, it is independent of the choice of $\mathcal{X}$. In this way, the retraction $\rho_{\mathcal{X}}$ (which does depend on the choice of $\mathcal{X}$) becomes a natural candidate for the non-archimedean SYZ fibration. This opens the prospect of using non-archimedean geometry as a bridge between mirror symmetry and birational geometry, translating mirror symmetry conjectures into statements about the geometry of minimal models and using MMP techniques to prove them.

Our main result is an instance of this dictionary. Kontsevich and Soibelman have made a detailed list of conjectures about the properties that the non-archimedean SYZ fibration should satisfy. In particular, they predicted that it should be an affinoid torus fibration away from some codimension $\geqslant 2$ discriminant locus D in $\mathrm{Sk}(X)$, in accordance with the original statement of the SYZ conjecture. When applied to the retraction map $\rho_{\mathcal{X}}$, this means that, locally over $\mathrm{Sk}(X)\backslash D$, one should be able to identify $\rho_{\mathcal{X}}$ with the tropicalization map

$$\mathrm{trop} \colon \mathbb{G}_{m,K}^{\mathrm{an}} \to \mathbb{R}^n.$$

We have proven this conjecture in collaboration with Chenyang Xu and Tony Yue Yu [4]. Our main technical result is that any $\mathbb{Q}$-factorial minimal *dlt*-model $\mathcal{X}$ of X such that $\mathcal{X}$ has reduced special fiber is semistable locally around the one-dimensional strata of $\mathcal{X}_k$. We then proved that, after blowing up finitely many zero-dimensional strata in $\mathcal{X}_k$, the formal completion of $\mathcal{X}$ along each one-dimensional stratum is isomorphic to the formal completion of a toric R-scheme along a one-dimensional stratum of its special fiber. This finally implies the conjecture of Kontsevich and Soibelman, taking for D the union of codimension $\geqslant 2$ faces in $\mathrm{Sk}(\mathcal{X}) = \mathrm{Sk}(X)$.

References

[1] M. Kontsevich and Y. Soibelman. *Homological mirror symmetry and torus fibrations.* In: *Symplectic geometry and mirror symmetry (Seoul, 2000)*, pages 203–263. World Sci. Publ., River Edge, NJ, 2001.

[2] M. Kontsevich and Y. Soibelman, *Affine structures and non-archimedean analytic spaces.* In: P. Etingof, V. Retakh and I.M. Singer (eds). *The unity of mathematics. In honor of the ninetieth birthday of I. M. Gelfand.* Volume 244 of *Progress in Mathematics*, pages 312–385. Birkhäuser Boston, Inc., Boston, MA, 2006.

[3] J. Nicaise and C. Xu, *The essential skeleton of a degeneration of algebraic varieties*, Amer. Math. J., **138(6)** (2016), 1645–1667.

[4] J. Nicaise, C. Xu and T.Y. Yu, *The non-archimedean SYZ fibration*, to appear in Compositio Mathematica, 2019.

[5] A. Strominger, S.-T. Yau and E. Zaslow, *Mirror Symmetry is T-duality*, Nuclear Phys. B **479(1-2)** (1996), 243–259.

P-adic Riemann-Hilbert correspondence, de Rham comparison and periods on Shimura varieties

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(joint work with Kai-Wen Lan, Xinwen Zhu)

In previous works [2, 1], a p-adic Riemann–Hilbert functor was constructed as an analogue of Deligne's Riemann-Hilbert correspondence over $\mathbb{C}$ (see [1] for the general introduction and backgrounds). In the present work, we further investigate the properties of the p-adic Riemann–Hilbert functor. We establish the de Rham comparison isomorphisms for the cohomology with compact support under the p-adic Riemann–Hilbert correspondences, and show that they are compatible with duality. Precisely, we first obtain the following theorem:

Theorem 1. *Let U be a d-dimensional smooth algebraic variety over a finite extension k of $\mathbb{Q}_p$, and let $\mathbb{L}$ be a de Rham p-adic étale local system on U. Then there is a canonical comparison isomorphism*

$$H^i_{\text{ét,c}}(U_{\overline{k}}, \mathbb{L}) \otimes_{\mathbb{Q}_p} B_{\text{dR}} \cong H^i_{\text{dR,c}}(U, D^{\text{alg}}_{\text{dR}}(\mathbb{L})) \otimes_k B_{\text{dR}} \tag{1}$$

compatible with the canonical filtrations and the actions of $\mathrm{Gal}(\overline{k}/k)$ *on both sides. Here $D^{\text{alg}}_{\text{dR}}$ is the (above-mentioned) p-adic Riemann–Hilbert functor constructed in* [1], *and $H^i_{\text{ét,c}}$ (resp. $H^i_{\text{dR,c}}$) denotes the usual étale (resp. de Rham) cohomology with compact support.*

In addition, the above comparison isomorphism (1) *is compatible with the one in* [1, Theorem 1.1] (*for varying* $\mathbb{L}$) *in the following sense:*

(1) The following diagram

$$\begin{array}{ccc} H^i_{\text{ét,c}}(U_{\overline{k}}, \mathbb{L}) \otimes_{\mathbb{Q}_p} B_{\text{dR}} & \xrightarrow{\sim} & H^i_{\text{dR,c}}(U, D^{\text{alg}}_{\text{dR}}(\mathbb{L})) \otimes_k B_{\text{dR}} \\ \downarrow & & \downarrow \\ H^i_{\text{ét}}(U_{\overline{k}}, \mathbb{L}) \otimes_{\mathbb{Q}_p} B_{\text{dR}} & \xrightarrow{\sim} & H^i_{\text{dR}}(U, D^{\text{alg}}_{\text{dR}}(\mathbb{L})) \otimes_k B_{\text{dR}} \end{array}$$

is commutative, where the horizontal isomorphisms are the comparison isomorphisms, and where the vertical morphisms are the canonical ones.

(2) The following diagram

$$\begin{array}{ccc} H^i_{\text{ét,c}}\big(U_{\overline{k}}, \mathbb{L}\big) \otimes_{\mathbb{Q}_p} B_{\text{dR}} & \xrightarrow{\sim} & H^i_{\text{dR,c}}\big(U, D^{\text{alg}}_{\text{dR}}(\mathbb{L})\big) \otimes_k B_{\text{dR}} \\ \wr\big\downarrow & & \big\downarrow\wr \\ \Big(H^{2d-i}_{\text{ét}}\big(U_{\overline{k}}, \mathbb{L}^\vee(d)\big) \otimes_{\mathbb{Q}_p} B_{\text{dR}}\Big)^\vee & \xrightarrow{\sim} & \Big(H^{2d-i}_{\text{dR}}\big(U, D^{\text{alg}}_{\text{dR}}(\mathbb{L}^\vee(d))\big) \otimes_k B_{\text{dR}}\Big)^\vee \end{array}$$

is commutative, where the horizontal isomorphisms are given by the comparison isomorphisms (and where the duals are with respect to the base field B_{dR}), and where the vertical isomorphisms are given by the usual Poincaré duality of étale and de Rham cohomology.

As a byproduct, we obtain Poincaré duality for (rational) étale cohomology of smooth rigid analytic varieties that are complements of closed rigid analytic subvarieties in proper rigid analytic varieties. Recall that Scholze noted in [3] that it is interesting to prove Poincaré duality in the setup of *proper* smooth rigid analytic varieties there (in which case the cohomology with compact support coincides with the usual cohomology), and it is natural to ask whether the Poincaré duality is compatible with the de Rham comparison isomorphisms there. More precisely, we have the following theorem:

Theorem 2. *Let U be a d-dimensional smooth rigid analytic variety over k, which is of the form $U = X - Z$, where X is proper and $Z \subset X$ is a closed rigid analytic subvariety. Then there is a canonical trace morphism*

$$t_{\text{ét}} : H^{2d}_{\text{ét,c}}(U_{\overline{k}}, \mathbb{Q}_p(d)) \to \mathbb{Q}_p,$$

satisfying certain natural compatibility conditions. In addition, for each $\mathbb{Z}_p$-local system $\mathbb{L}$ on $U_{\text{ét}}$ (which is not necessarily de Rham), with $\mathbb{L}_{\mathbb{Q}_p} := \mathbb{L} \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$, we have a canonical perfect pairing

$$H^i_{\text{ét,c}}\big(U_{\overline{k}}, \mathbb{L}_{\mathbb{Q}_p}\big) \otimes_{\mathbb{Q}_p} H^{2d-i}_{\text{ét}}\big(U_{\overline{k}}, \mathbb{L}^\vee_{\mathbb{Q}_p}(d)\big) \to \mathbb{Q}_p,$$

which we call the Poincaré duality pairing*, defined by pre-composing $t_{\text{ét}}$ with the cup product pairing $H^i_{\text{ét,c}}\big(U_{\overline{k}}, \mathbb{L}_{\mathbb{Q}_p}\big) \otimes_{\mathbb{Q}_p} H^{2d-i}_{\text{ét}}\big(U_{\overline{k}}, \mathbb{L}^\vee_{\mathbb{Q}_p}(d)\big) \to H^{2d}_{\text{ét,c}}\big(U_{\overline{k}}, \mathbb{Q}_p(d)\big)$.*

We remark that our definition of compactly supported cohomology for rigid analytic varieties is different from the usual one (e.g. Huber's definition), and is based on the Kummer etale topology for log adic spaces developed in [1] . In fact, we will do a little more in this article. We can also study the cohomology with *partial compact support* and also some generalized interior cohomology, which is the image of a morphism between cohomology with different partial compact support conditions; and construct de Rham comparison isomorphisms for such cohomology that are also compatible with Poincaré duality.

References

[1] H. Diao, K. Lan, R. Liu, X. Zhu *Logarithmic Riemann–Hilbert correspondences for rigid varieties*, arXiv:1803.05786.

[2] R. Liu, X. Zhu *Rigidity and a Riemann-Hilbert correspondence for p-adic local systems*, Invent. Math. **207** (2017), 291–343.

[3] P. Scholze, *p-adic Hodge theory for rigid-analytic varieties*, Forum of Mathematics, Pi **1, e1** (2013).

Functorial semistable reduction and resolution of morphisms

Michael Temkin

(joint work with Dan Abramovich, Jarosław Włodarczyk)

1. Main results

In a joint project with Abramovich and Wlodarczyk, we construct a functorial resolution of morphisms in characteristic zero, see [2]. Already in the case of varieties, this leads to a new algorithm which is faster than the classical one and possesses better functorial properties, see [1].

1.1. Historical background.

1.1.1. *Classical desingularization.* Until our work there was known an essentially unique functorial (or canonical) resolution of singularities, to which we refer as the *classical algorithm*. It is based on (originally non-canonical) method of Hironaka developed further by Giraud, Bierstone-Millman, Villamayor, Wlodarczyk, Kollar, and other experts. They suggested different descriptions, of essentially the same algorithm with certain variations in combinatorial parts. In brief, the main result was

Theorem 1.1.2. *For any integral variety Z over a field k of characteristic zero there exists a modification $f\colon Z_{\rm res} \to Z$ such that $Z_{\rm res}$ is smooth. Moreover, the construction is smooth-functorial: if $Z' \to Z$ is smooth, then $Z'_{\rm res} = Z_{\rm res} \times_Z Z'$.*

The proof went by locally embedding Z into a manifold with a boundary (X, E) (i.e. X is smooth and E is an snc divisor) and principalizing the ideal $I_Z \subset \mathcal{O}_X$, i.e. finding a sequence of blow ups $g\colon (X', E') \to \cdots \to (X, E)$ such $g^{-1}(I_Z)$ is invertible with support on E'. Hironaka showed that principalization easily implies resolution, and also implies that on can resolve a closed subset $T \subsetneq Z$ to an snc divisor $T' = f^{-1}(T)$.

1.1.3. *Semistable reduction.* Kempf, Knudsen, Mumford and Saint-Donat proved the following theorem, which was the first instance of resolution of morphisms.

Theorem 1.1.4. *Let Z be an integral scheme of finite type over a trait $S =$ Spec(R) of residual characteristic zero such that the generic fiber Z_η is smooth.*

(i) There exists proper $Z_{\rm res} \to Z$ with $Z_{\rm res} \to S$ log smooth and $(Z_{\rm res})_\eta = Z_\eta$.

(ii) After a finite extension of R can even make $Z_{\rm res} \to S$ semistable.

Claim (i) follows by applying Hironaka's theorem to Z and the divisor $Z \setminus Z_\eta$, and claim (ii) is then deduced by a complicated combinatorics. In general, one can not make $Z \to S$ smooth, so this is the best one might hope for. On the other side this solution is rather non-canonical, e.g. it changes when one extends R.

1.2. **Resolution of morphisms.** It turns out that the theorem of KKMS can be extended to more general morphisms and made functorial, but this requires to work within the larger category of logarithmic DM stacks with finite diagonalizable stabilizers. For simplicity, we will stick to the case of stacks of finite type over a field k of characteristic zero.

Theorem 1.2.1 ([2]). *To dominant morphisms $f\colon X \to S$ of integral log varieties (or log DM stacks) over k one can associate either a non-representable modification $X_{\rm res} \to X$ or a "fail output" $X_{\rm res} = \varnothing$ such that $X_{\rm res} \to S$ is log smooth and*

(i) Non-failure up to refining the base: *for any f there exists a modification $S' \to S$ such that $(X \times_S S')_{\rm res}$ is non-empty.*

(ii) Log smooth functoriality: *if $X_{\rm res}$ is non-empty and $X' \to X$ is log smooth, then $X'_{\rm res} = X_{\rm res} \times_X X'$.*

(iii) Base change functoriality: *if $X_{\rm res} \neq \varnothing$, then $(X \times_S S')_{\rm res} = X_{\rm res} \times_S S'$ for any base change $S' \to S$.*

Furthermore, generalizing the polyhedral subdivision theorem of KKMS to maps of polyhedra Adiprasito, Liu and Temkin deduced the following refinement

Theorem 1.2.2 ([3]). *After replacing S by an alteration, one can even achieve that $X_{\rm res} \to S$ is semistable.*

2. The method

2.1. **Logarithmic geometry.** Logarithmic structures are important both for classical resolution, where they are encoded by the boundary, and semistable reduction. The starting idea of our project was that in order to construct log smooth resolution of morphisms one should work log-smooth functorially. Already doing this for varieties in [1] required to modify the algorithm tremendously, and in fact the same new algorithm was extended in [2] to morphisms. We suggest:

Principle 2.1.1. *If some aspects of the problem require to extend the notion of smoothness, it is preferable to run the whole algorithm in the extended setting.*

Implementing it in our case suggested to work with log varieties, log smoothness, etc. In particular, resolution is reduced to principalization of ideals on log smooth (or toroidal) varieties (X, E), without the assumption that X is smooth. In addition, we replaced all basic resolution tools, such as derivation of ideals, order of ideals, hypersurface of maximal contact, etc., by their logarithmic analogs.

2.2. **Stacks.** Surprisingly for us, the log smooth functoriality forced the new principalization algorithm to perform certain weighted blow ups that produced not log smooth varieties. However, working with stacks it is possible to realize these blow ups as coarse spaces of smooth non-representable modifications, which we call

Kummer blow ups. This suggested to extend our category further, in accordance with the above principle. Thus, our log smooth-functorial algorithm principalizes ideals on log smooth DM stacks even when the it starts with an ideal on a smooth variety. It is possible after that to return to log smooth or even smooth varieties by an additional modification, but the latter step can be only made smooth-functorial. Perhaps usage of stacks is unavoidable for getting a log smooth-functorial algorithm and resolution of morphisms. In the end, our algorithm operates with more complicated objects and modifications, but it is simpler and faster than its classical predecessor.

2.3. **Future works.** Our algorithm only performs weighted ideals of a special form, and we expect that there exists a much more efficient algorithm which also works with DM stacks and performs arbitrary weighted blow ups. This is the main topic of our current research.

References

[1] D. Abramovich, M. Temkin, J. Włodarczyk, *Principalization of ideals on toroidal orbifolds*, arXiv[1709.03185], to appear in JEMS.

[2] D. Abramovich, M. Temkin, J. Włodarczyk, *Relative desingularization and principalization of ideals*, in preparation.

[3] K. Adiprasito, G. Liu, M. Temkin, *Semistable reduction in characteristic 0*, arXiv[1810.03131].

Degenerations of *p*-adic volume forms

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(joint work with Johannes Nicaise)

Let K be a field and X a smooth proper variety over K of dimension $n \geqslant 1$. It is of general interest to understand the structure of the set $X(K)$ of K-rational points. For example:

(i) when K is a finite field, $X(K)$ is a finite set, whose cardinality is governed by the Lang–Weil estimates;

(ii) when $K = \mathbb{C}$, $X(K)$ is a complex manifold of dimension n;

(iii) more generally, when K is a local field, $X(K)$ is a compact K-analytic manifold, locally isomorphic to the open unit ball in K^n (and thus homeomorphic to a Cantor set);

(iv) when K is a general non-Archimedean field, $X(K)$ often fails to be (locally) compact, but embeds as a subset of the Berkovich analytification X^{an} of X, and this analytification is compact.

Now suppose we are given the additional data of a global regular n-form $\theta \in H^0(X, \omega_X)$. We allow θ to have zeros, but we assume that θ is not identically zero on any connected component of X. To the pair (X, θ) we can associate analytic data in cases (ii)-(iv) above. Namely, when $X = \mathbb{C}$, θ induces a volume form $|\theta|^2$ on the complex manifold $X(\mathbb{C})$ defined by $|\theta|^2 = i^{n^2}\theta \wedge \overline{\theta}$. Similarly, if K is a non-Archimedean local field K, then θ induces a smooth volume form $|\theta|$ (which we view as positive measure) on $X(K)$: if $\theta = f dz_1 \wedge dz_n$ in local coordinates $(z_1, \dots, z_n)$, then $|\theta| = |f| \mu_{\mathrm{Haar}}$, where μ_{Haar} is Haar measure on K^n, normalized so that the closed unit ball has mass 1.

Finally consider the case when K is a discretely valued non-Archimedean field. The work of Kontsevich–Soibelman [3], Mustaţă–Nicaise [4], and Temkin [5] allows us to associate to θ a function on (a dense subset of) X^{an}, called the *weight function* wt_θ. The (closure of the) locus where the weight function is minimal is the *Kontsevich–Soibelman skeleton* $\mathrm{Sk}(X, \theta) \subset X^{\mathrm{an}}$ of (X, θ). If X admits an snc model $\mathcal{X}$ over the valuation ring R of K, then $\mathrm{Sk}(X, \theta)$ is a subcomplex of the skeleton $\mathrm{Sk}(\mathcal{X})$, which in turn can be identified with the dual complex of the special fiber of $\mathcal{X}$. In this case, $\mathrm{Sk}(X, \theta)$ can further be endowed with a Lebesgue measure $\lambda_{\mathrm{Sk}(X,\theta)}$ induced by the integral (piecewise) linear structure on $\mathrm{Sk}(\mathcal{X})$.

Now suppose K is a local non-Archimedean field. We can then ask if there is any relation between the measure $|\theta|$ on $X(K) \subset X^{\mathrm{an}}$ and the Lebesgue measure $\lambda_{\mathrm{Sk}(X,\theta)}$ on the Kontsevich–Soibelman skeleton $\mathrm{Sk}(X, \theta)$. We prove that this is indeed the case. To explain the result, note that for any finite extension K'/K, we have a finite positive measure $|\theta \otimes_K K'|$ on $X(K')$. Now $X(K')$ embeds in the K'-analytic space $X^{\mathrm{an}}_{K'}$, and there is a natural continuous map $\pi_{K'} \colon X^{\mathrm{an}}_{K'} \to X^{\mathrm{an}}$, so the pushforward $(\pi_{K'})_* |\theta \otimes_K K'|$ is a finite positive (Radon) measure on the compact topological space X^{an}.

Main Theorem. Assume that X admits a semistable model over the valuation ring R. Then we have

$$\lim_{K'} \frac{q^{w[K':K]}}{e(K'/K)^d} (\pi_{K'})_* |\theta \otimes_K K'| = \lambda_{\mathrm{Sk}(X,\theta)},$$

in the weak sense of Radon measures on X^{an}. Here K' runs over the directed set of finite extensions of K contained in a fixed algebraic closure of K (the set of extensions is partially ordered by inclusion), q is the cardinality of the residue field of K, $e(K'/K)$ is the ramification index, w is the minimum of the weight function wt_θ, and d is the dimension of the Kontsevich–Soibelman skeleton.

We also obtain convergence results when restricting either to tame, or to unramified extensions K'/K. In these cases we need less assumptions on X: for the tame case, the existence of an snc model for X is enough, whereas the unramified case is unconditional. The limit measures are now Lebesgue measures on suitable subcomplexes of $\mathrm{Sk}(X, \theta)$, and d has to be replaced by the dimension of these.

The main ingredients in the proof of the main theorem are: the usage of log smooth (or log regular) models, the specialization map associated to a model of X, the Lang–Weil estimates, and the convergence of suitably normalized "lattice" measures to Lebesgue measure on a simplex.

The main theorem can be viewed as a p-adic version of the main result in [BJ17], which dealt with the case when $X \to \mathbb{D}^*$ is proper smooth family of complex manifolds, meromorphic at $0 \in \mathbb{D}$. In this case, given $\theta \in H^0(X, \omega_{X/\mathbb{D}})$, we have

$$\lim_{t \to 0} \frac{|t|^{-2w}}{(\log |t|^{-1})^d} |\theta|_{X_t}|^2 = \lambda_{\mathrm{Sk}(X,\theta)},$$

where the convergence now takes place in a *hybrid space* obtained by adding the Berkovich space $X^{\mathrm{an}}_{\mathbb{C}((t))}$ as a central fiber to X, and topologizing in a suitable way.

References

[1] S. Boucksom, M. Jonsson, *Tropical and non-Archimedean limits of degenerating families of volume forms*, J. Éc. polytech. Math., **4** (2017), 87–139.
[2] M. Jonsson, J. Nicaise, *Degenerations of p-adic volume forms*, in preparation.
[3] M. Kontsevich, Y. Soibelman. *Affine structures and non-Archimedean analytic spaces.* The unity of mathematics, 321–385. Progr. Math., 244. Birkhäuser, Boston, 2006.
[4] M. Mustaţă and J. Nicaise. *Weight functions on non-archimedean analytic spaces and the Kontsevich–Soibelman skeleton*, Algebr. Geom., **2** (2015), 365–404.
[5] M. Temkin, *Metrization of differential pluriforms on Berkovich analytic spaces.* In *Tropical and Non-Archimedean Geometry*, 195–285. Simons Symposia. Springer International Publishing, 2016.

Intermediate extensions and crystalline distribution algebras

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(joint work with Tobias Schmidt, Matthias Strauch)

1. Introduction

Let $\mathfrak{o}$ denote a complete discrete valuation ring with uniformizer ϖ and fraction field K. Let G be a connected reductive group over $\mathfrak{o}$ which is $\mathfrak{o}$-split. In [11] we have introduced and studied the crystalline algebra of arithmetic distributions $D^\dagger(\mathcal{G})_{\mathbf{Q}}$ associated to the p-adic completion $\mathcal{G}$ of G. It is a certain weak completion of the classical universal enveloping algebra $U(\mathfrak{g})$ associated to the Lie algebra $\mathfrak{g}$ of G (tensored with $\mathbf{Q}$). The interest in this algebra comes at least from two sources: on the one hand, $D^\dagger(\mathcal{G})_{\mathbf{Q}}$ acts as global arithmetic differential operators [3] on any formal $\mathfrak{o}$-scheme endowed with a $\mathcal{G}$-action. On the other hand, $D^\dagger(\mathcal{G})_{\mathbf{Q}}$ is canonically isomorphic to Emerton's analytic distribution algebra $D^{an}(G^\circ)$ of the rigid-analytic group G° (equal to the generic fibre of the completion of G at the unit section of its special fibre) which plays a crucial role in the representation theory of p-adic Lie groups [9].

Due to its relation to representation theory, it is of considerable interest to have information about the $D^\dagger(\mathcal{G})_{\mathbf{Q}}$-modules of finite length. In the classical setting of $U(\mathfrak{g})$-modules, a geometric classification of many irreducible $U(\mathfrak{g})$-modules can be achieved through a combination of the Beilinson-Bernstein localization theorem over the flag variety [2,6], Kashiwara's theorem and the formalism of intermediate extensions [10]. A natural question is therefore whether a similar strategy works for $D^\dagger(\mathcal{G})_{\mathbf{Q}}$-modules.

In [12] we have completed the first step and established an analogue of the localization theorem for arithmetic differential operators. The aim of this note is to deal with the second step: we explain how the intermediate extension functor for arithmetic $\mathscr{D}$-modules, developed recently by Abe-Caro [1], together with our localization theorem can be used to geometrically construct interesting irreducible $D^\dagger(\mathcal{G})_{\mathbf{Q}}$-modules.

2. Complements on arithmetic differential operators

Let $\mathcal{P}$ be a smooth formal $\mathfrak{o}$-scheme, following [3], we introduce the ring of arithmetic differential operators $\mathscr{D}^\dagger_{\mathcal{P}}$ over $\mathcal{P}$. If $\mathcal{U} \subset \mathcal{P}$ is an affine formal scheme endowed with coordinates $x_1, \dots, x_M$, $|\cdot|$ be a Banach norm on the Tate algebra $\mathcal{O}_{\mathcal{P}}(\mathcal{U}) \otimes \mathbb{Q}$, an element $P \in \mathscr{D}^\dagger_{\mathcal{P}}(\mathcal{U})$ can be written

$$P = \sum_{\underline{\nu}} a_{\underline{\nu}} \underline{\partial}^{[\underline{\nu}]} = \sum_{\underline{\nu}} a_{\underline{\nu}} \underline{\partial}^{\underline{\nu}} / \underline{\nu}!,$$

where there exist $C > 0, \eta < 1$ (depending on P), such that $|a_{\underline{\nu}}| \leqslant C\eta^{|\underline{\nu}|}$.

We call $\mathscr{D}^\dagger_{\mathcal{P},\lambda}$ a sheaf of twisted arithmetic differential operators on $\mathcal{P}$, a sheaf of rings on $\mathcal{P}$, together with an injective ring homomorphism

$$\iota : \mathcal{O}_{\mathcal{P},\mathbb{Q}} \hookrightarrow \mathscr{D}^\dagger_{\mathcal{P},\lambda}$$

such that the pair $(\iota, \mathscr{D}^\dagger_{\mathcal{P},\lambda})$ is locally isomorphic to the usual pair $\mathcal{O}_{\mathcal{P},\mathbb{Q}} \hookrightarrow \mathscr{D}^\dagger_{\mathcal{P}}$ [2]. For example if $\mathcal{L}(\lambda)$ is an invertible sheaf of $\mathcal{O}_{\mathcal{P}}$-modules, the following sheaf is a sheaf of twisted arithmetic differential operators on $\mathcal{P}$

$$\mathscr{D}^\dagger_{\mathcal{P},\lambda} = \mathcal{L}(\lambda)^{-1} \otimes_{\mathcal{O}_{\mathcal{P}}} \mathscr{D}^\dagger_{\mathcal{P}} \otimes_{\mathcal{O}_{\mathcal{P}}} \mathcal{L}(\lambda).$$

We first state the Berthelot-Kashiwara theorem for twisted differential operators. Let $\mathcal{P}$ be a smooth formal $\mathfrak{o}$-scheme and let

$$i : \mathcal{Q} \hookrightarrow \mathcal{P}$$

be a smooth closed formal subscheme. We let $\mathscr{D}^\dagger_{\mathcal{P},\lambda}$ be a sheaf of twisted arithmetic differential operators on $\mathcal{P}$ (and similarly for $\mathcal{Q}$). Let us recall the direct image functor i_+ following [5]. So let $\mathscr{D}^\dagger_{\mathcal{P}\leftarrow\mathcal{Q},\lambda}$ be the $(i^{-1}\mathscr{D}^\dagger_{\mathcal{P},\lambda}, \mathscr{D}^\dagger_{\mathcal{Q},\lambda})$-bimodule equal to the inverse image $i^*\mathscr{D}^\dagger_{\mathcal{P},\lambda}$ followed by the side-changing operation. For a given coherent $\mathscr{D}^\dagger_{\mathcal{Q},\lambda}$-module $\mathscr{M}$ we let

$$i_+\mathscr{M} := i_*(\mathscr{D}^\dagger_{\mathcal{P}\leftarrow\mathcal{Q},\lambda} \otimes_{\mathscr{D}^\dagger_{\mathcal{Q},\lambda}} \mathscr{M}).$$

We also let

$$i^\natural \mathscr{M} = \mathcal{H}om_{i^{-1}\mathscr{D}^\dagger_{\mathcal{P},\lambda}}(\mathscr{D}^\dagger_{\mathcal{P}\leftarrow\mathcal{Q},\lambda}, i^{-1}\mathscr{M}).$$

This is a left-exact functor from $\mathscr{D}^\dagger_{\mathcal{P},\lambda}$-modules to $\mathscr{D}^\dagger_{\mathcal{Q},\lambda}$-modules and in [13], we state the following twisted analogous of Berthelot-Kashiwara theorem.

Proposition 2.1. *The functor i_+ induces an equivalence of categories between the category of coherent $\mathscr{D}^\dagger_{\mathcal{Q},\lambda}$-modules and the category of coherent $\mathscr{D}^\dagger_{\mathcal{P},\lambda}$-modules supported on $\mathcal{Q}$. The functor $i^\natural$ is a quasi-inverse functor.*

We now come to the intermediate extension as constructed be Abe-Caro [1, 1.4.1] in the non twisted case. Denote by $D^b_{\mathrm{ovhol}}(\mathscr{D}^\dagger_{\mathcal{P}})$ the triangulated category of complexes of overholonomic $\mathscr{D}^\dagger_{\mathcal{P}}$-modules as defined by Caro [8]. Let Y be a locally closed smooth subvariety of $\mathcal{P}_s$, the special fibre of $\mathcal{P}$. If X is the Zariski closure of Y in $\mathcal{P}_s$, then $\mathbb{Y} = (Y, X, \mathcal{P})$ is what Caro calls a frame, as well as $\mathbb{P} = (\mathcal{P}_s, \mathcal{P}_s, \mathcal{P})$ and we have a natural morphism of frames $u : \mathbb{Y} \to \mathbb{P}$. In loc. cit. Abe-Caro form the abelian category $\mathrm{Ovhol}(\mathbb{Y}/K)$ of overholonomic arithmetic $\mathscr{D}$-modules on $\mathbb{Y}$. It is a full subcategory of the derived category of bounded complexes of overholonomic modules $D^b_{\mathrm{ovhol}}(\mathscr{D}^\dagger_{\mathcal{P}})$, whose cohomology sheaves have support in X. A typical example is the following : let $Z \hookrightarrow \mathcal{P}_s$ be a divisor, $X = \mathcal{P}_s$, $Y = X \backslash Z$, and $\mathcal{E} = \mathcal{O}_{\mathcal{P}}(^\dagger Z) \in \mathrm{Ovhol}(\mathbb{Y}/K)$ defined in this way : take $\mathcal{U} \subset \mathcal{P}$, affine open, $t \in \mathcal{O}_{\mathcal{P}}(\mathcal{U})$, $\bar{t}$ its reduction in $\mathcal{O}_{\mathcal{P}_s}(\mathcal{U}_s)$ such that $\mathcal{U}_s \bigcap Z = V(\bar{t})$, then an element h of $\mathcal{O}_{\mathcal{P}}(^\dagger Z)(\mathcal{U})$ can be written $h = \sum_{n \geqslant 0} \frac{a_n}{t^n}$, with $a_n \in \mathcal{O}_{\mathcal{P},\mathbb{Q}}(\mathcal{U})$ and there exist $C > 0$, $\eta < 1$ (depending on h), such that $|a_n| < C\eta^n$, where $|\cdot|$ is any Banach norm on $\mathcal{O}_{\mathcal{P},\mathbb{Q}}(\mathcal{U})$. With our notations, this is a result of Berthelot [4], that $\mathcal{O}_{\mathcal{P}}(^\dagger Z) \in \mathrm{Ovhol}(\mathbb{Y}/K)$ for $\mathbb{Y} = (\mathcal{P}_s \backslash Z, \mathcal{P}_s, \mathcal{P})$. The sheaf $\mathcal{O}_{\mathcal{P}}(^\dagger Z)$ is the sheaf of overconvergent functions along the divisor Z. In this situation, objects of $\mathrm{Ovhol}(\mathbb{Y}/K)$ consist of single overholonomic modules $\mathscr{D}^\dagger_{\mathcal{P}}$, endowed with a structure of $\mathcal{O}_{\mathcal{P}}(^\dagger Z)$-module (compatible with the $\mathscr{D}^\dagger_{\mathcal{P}}$-module structure).

Let us come back now to the general case, for $\mathcal{E} \in \mathrm{Ovhol}(\mathbb{Y}/K)$, Abe-Caro define the functors [1, 1.2.9,1.4] $u^0_!$ and u^0_+, and the intermediate extension of $\mathcal{E}$

$$u_{!+}(\mathcal{E}) := \mathrm{Im}\big(\theta^0_{u,\mathcal{E}} : u^0_! \mathcal{E} \longrightarrow u^0_+ \mathcal{E}\big) \in \mathrm{Ovhol}(\mathbb{P}/K).$$

Considering the previous example when Z is a divisor of $\mathcal{P}_s$, we see that, if $\mathcal{P}_s \backslash Z$ is connected, $u_{!+}(\mathcal{O}_{\mathcal{P}}(^\dagger Z)) = \mathcal{O}_{\mathcal{P},\mathbb{Q}}$.

More generally, if Z is any closed subscheme of X, Caro defined in [7] the constant overconvergent F-isocrystal on the k-scheme Y, that belongs to $\mathrm{Ovhol}(\mathbb{Y}/K)$, and that we denote here $\mathcal{O}_{X,\mathbb{Q}}(^\dagger Z)$. Let $\mathcal{U} = \mathcal{P} \backslash Z$, when the closed immersion : $Y \hookrightarrow \mathcal{U}_s$ can be lifted to an immersion of smooth formal schemes $v : \mathcal{Y} \hookrightarrow \mathcal{U}$, then one has $v^\natural(\mathcal{O}_{X,\mathbb{Q}}(^\dagger Z))_{|\mathcal{U}} = \mathcal{O}_{\mathcal{Y}}$. Using this we prove that the sheaf $\mathcal{O}_{X,\mathbb{Q}}(^\dagger Z)$ is an irreducible $\mathscr{D}^\dagger_{\mathcal{P}}$-module when Y is connected. Thus, using [1, 1.4.7] we have the following

Proposition 2.2. *If Y is connected, the intermediate extension $u_{!+}\mathcal{O}_{X,\mathbb{Q}}(^\dagger Z)$ is an irreducible overholonomic $\mathscr{D}^\dagger_{\mathcal{P}}$-module.*

All this extends in a straightforward manner to twisted coefficients. Let $\mathscr{D}^\dagger_{\mathcal{P},\lambda}$ be a sheaf of twisted arithmetic differential operators on $\mathcal{P}$. A $\mathscr{D}^\dagger_{\mathcal{P},\lambda}$-module $\mathcal{M}$ is called *overholonomic* if, for every open $\mathcal{U} \subset \mathcal{P}$ trivializing the twist, the restriction $\mathcal{M}|_{\mathcal{U}}$ to $\mathcal{U}$ is an overholonomic $\mathscr{D}^\dagger_{\mathcal{U}}$-module in the usual sense. Imitating the construction of Abe-Caro, we obtain the abelian category $\mathrm{Ovhol}_\lambda(\mathbb{Y}/K)$ consisting of bounded complexes of overholonomic $\mathscr{D}^\dagger_{\mathcal{P},\lambda}$-modules with support on X. Moreover, the category $\mathrm{Ovhol}_\lambda(\mathbb{Y}/K)$ is noetherian and artinian. Analogously, there is a twisted intermediate extension functor

$$u_{!+} : \mathrm{Ovhol}_\lambda(\mathbb{Y}/K) \longrightarrow \mathrm{Ovhol}_\lambda(\mathbb{P}/K)$$

for any immersion of couples $u : \mathbb{Y} \to \mathbb{P}$ as above. It is locally isomorphic to the usual (untwisted) intermediate extension functor and hence, shares the analogous properties.

3. Compatibilities

Let us consider now the following situation: let $P_{\mathfrak{o}}$ be a smooth and proper scheme over $\mathfrak{o}$, $Y_{\mathfrak{o}}$ an open subscheme of $P_{\mathfrak{o}}$ and $X_{\mathfrak{o}}$ be the Zariski closure of $Y_{\mathfrak{o}}$ into $P_{\mathfrak{o}}$. Call $P_{\mathbb{Q}}$, $Y_{\mathbb{Q}}$ and $X_{\mathbb{Q}}$ the generic fibers of the previous schemes, and $\mathcal{P}_s$, Y and X their special fibers. We also denote $Z = X\backslash Y$, and we consider $\mathcal{P}$ the formal scheme obtained by p-adic completion of the scheme $P_{\mathfrak{o}}$, so that we have a frame $\mathbb{Y} = (Y, X, \mathcal{P})$ and a natural morphism of frames $u : \mathbb{Y} \to \mathbb{P}$.

Denote by j the open immersion : $P_{\mathbb{Q}} \hookrightarrow P_{\mathfrak{o}}$ and by α the closed immersion $\mathcal{P}_s \hookrightarrow P_{\mathfrak{o}}$. Over the scheme $P_{\mathbb{Q}}$ we have the usual algebraic sheaf of differential operators $\mathscr{D}_{P_{\mathbb{Q}}}$. Let $u_{\mathbb{Q}}$ be the open immersion $Y_{\mathbb{Q}} \hookrightarrow P_{\mathbb{Q}}$, $\mathcal{E}$ be an holonomic $\mathscr{D}_{Y_{\mathbb{Q}}}$-module, $u_{\mathbb{Q}!+}\mathcal{E}$ the intermediate extension, we can then consider

$$\mathscr{D}^{alg}_{P_{\mathbb{Q}}} := \alpha^{-1} j_* \mathscr{D}_{P_{\mathbb{Q}}}, \text{ and } u^{alg}_{!+}\mathcal{E} := \alpha^{-1} j_* u_{\mathbb{Q}!+}\mathcal{E},$$

that are sheaves over the topological space underlying the special fiber $|\mathcal{P}_s|$. Then the sheaf $\mathscr{D}^{alg}_{P_{\mathbb{Q}}}$ is a subsheaf of the sheaf $\mathscr{D}^\dagger_{\mathcal{P}}$ and using previous notations, we have the following isomorphism [13]

$$\mathscr{D}^\dagger_{\mathcal{P}} \otimes_{\mathscr{D}^{alg}_{P_{\mathbb{Q}}}} u^{alg}_{!+}\mathcal{O}_{Y_{\mathbb{Q}}} \simeq u_{!+}(\mathcal{O}_{X,\mathbb{Q}}(^\dagger Z)).$$

4. Crystalline distribution algebra and the $\mathcal{O}^\dagger$-category

Let G be a connected split reductive algebraic group over $\mathrm{Spec}\,\mathfrak{o}$. The crystalline distribution algebra $D^\dagger(\mathcal{G})$ is introduced in [11]. It contains $U(\mathfrak{g})$, the envelopping algebra tensored with $\mathbb{Q}$, of $\mathfrak{g} = Lie(G)$ and is naturally a locally convex algebra of compact type. In particular, it is Hausdorff, complete and barrelled. For θ a central character of $U(\mathfrak{g})$ we will always denote by $D^\dagger(\mathcal{G})_\theta$ the corresponding central reduction of $D^\dagger(\mathcal{G})$.

We now fix a central character θ and let $\lambda \in \mathfrak{t}^*$ be a character associated to θ via the Harish-Chandra morphism. Let Φ be the root system attached to (G, T) and let ρ be half the sum over the positive roots of Φ relative to B. One can associate to λ a twisted sheaf of differential operators $\mathscr{D}^\dagger_{\mathcal{P},\lambda}$ (which coincides with $\mathscr{D}^\dagger_{\mathcal{P}}$ when θ is trivial) and we have the following

Theorem 4.1. (a) *Suppose $\lambda + \rho$ is dominant and regular. The global section functor induces an equivalence of categories between coherent $\mathscr{D}^\dagger_{\mathcal{P},\lambda}$-modules and coherent $H^0(\mathcal{P}, \mathscr{D}^\dagger_{\mathcal{P},\lambda})$-modules.*

(b) *The $\mathcal{G}$-action on $\mathcal{P}$ induces an algebra isomorphism*

$$D^\dagger(\mathcal{G})_\theta \xrightarrow{\simeq} H^0(\mathcal{P}, \mathscr{D}^\dagger_{\mathcal{P},\lambda}).$$

Proof. For λ algebraic, this summarizes the main results of [12] and [14]. The general case is proved in [15]. Note that if θ is trivial, then (a) and (b) hold for $\mathscr{D}^\dagger_{\mathcal{P}}$. □

Definition 4.2. A $D^\dagger(\mathcal{G})_\theta$-module is called *overholonomic* if its associated $\mathscr{D}^\dagger_{\mathcal{P},\lambda}$-module is overholonomic.

Remark 4.3. The overholonomic $D^\dagger(\mathcal{G})_\theta$-modules form an abelian category which is noetherian and artinian. In particular, any object is of finite length. Conversely, if the semisimple rank of G is one (e.g. $G = \mathrm{GL}_2$) then *any* finite length $D^\dagger(\mathcal{G})_\theta$-module is overholonomic.

Let us define now the analytic highest weight representations. Let $\mathcal{O}$ be the classical BGG category. For any $M \in \mathcal{O}$ let

$$M^\dagger := D^\dagger(\mathcal{G}) \otimes_{U(\mathfrak{g})} M$$

and let $\mathcal{O}^\dagger$ be the full subcategory inside all $D^\dagger(\mathcal{G})$-modules generated by the $M^\dagger$. Applying a result of Schmidt [16], we observe that we have the following

Proposition 4.4. *The functor $M \rightsquigarrow M^\dagger$ induces an equivalence of categories*

$$\mathcal{O} \xrightarrow{\simeq} \mathcal{O}^\dagger.$$

For simplicity we now let θ be the trivial central character, and let $\mathcal{O}_0 \subset \mathcal{O}$ be the principal block, i.e. the block with trivial center character. Let W be the Weyl group of (G,T). For any $w \in W$ we let $\lambda_w = -w(\rho) - \rho$. Let $M_w \in \mathcal{O}_0$ be the Verma module of highest weight λ_w and let L_w be its irreducible quotient. The modules L_w exhaust all irreducible modules in $\mathcal{O}_0$. Consider the localization

$$\mathcal{L}^\dagger_w = \mathscr{D}^\dagger_{\mathcal{P}} \otimes_{D^\dagger(\mathcal{G})} L^\dagger_w.$$

Let $Y_w = B_s w B_s / B_s \subset \mathcal{P}_s$ be the w-th Bruhat cell in $\mathcal{P}_s$ and let $X_w = \bar{Y}_w$ be the corresponding Schubert variety. Let $Z_w = X_w \backslash Y_w$ and consider the constant overconvergent isocrystal $\mathcal{O}_{X_w,\mathbb{Q}}({}^\dagger Z_w)$ on the frame $\mathbb{Y}_w = (Y_w, X_w, \mathcal{P})$. Let $u : \mathbb{Y}_w \to \mathcal{P}$ be the inclusion. Then, applying the compatibility results of 3, Schmidt's result, as well as the analogous result in the classical algebraic setting, we prove in [13] the

Theorem 4.5. *One has an isomorphism of $\mathscr{D}^\dagger_{\mathcal{P}}$-modules*

$$\mathcal{L}^\dagger_w \xrightarrow{\simeq} u_{!+}(\mathcal{O}_{\mathcal{P},\mathbb{Q}}({}^\dagger Z_w)).$$

As a corollary, we see that the category $\mathcal{O}^\dagger$ consists of overholonomic $D^\dagger(\mathcal{G})$-modules.

References

[1] T. Abe and D. Caro. Theory of weights in p-adic cohomology. *American Journal of Mathematics*, 140(4), 2018.

[2] A. Beĭlinson and J. Bernstein. Localisation de $\mathfrak{g}$-modules. *C. R. Acad. Sci. Paris Sér. I Math.*, 292(1):15–18, 1981.

[3] P. Berthelot. $\mathcal{D}$-modules arithmétiques I. Opérateurs différentiels de niveau fini. *Ann. Sci. E.N.S*, 29:185–272, 1996.

[4] Pierre Berthelot. Cohérence différentielle des algèbres de fonctions surconvergentes. *C. R. Acad. Sci. Paris Sér. I Math.*, 323(1):35–40, 1996.

[5] Pierre Berthelot. Introduction à la théorie arithmétique des $\mathcal{D}$-modules. *Astérisque*, (279):1–80, 2002. Cohomologies p-adiques et applications arithmétiques, II.

[6] J.-L. Brylinski and M. Kashiwara. Kazhdan-Lusztig conjecture and holonomic systems. *Invent. Math.*, 64(3):387–410, 1981.

[7] D. Caro. F-isocristaux surconvergents et surcohérence différentielle. *Invent. Math.*, 170(3):507–539, 2007.

[8] Daniel Caro. $\mathcal{D}$-modules arithmétiques surholonomes. *Ann. Sci. Éc. Norm. Supér. (4)*, 42(1):141–192, 2009.

[9] M. Emerton. Locally analytic vectors in representations of locally p-adic analytic groups. *Preprint. To appear in: Memoirs of the AMS.*

[10] Ryoshi Hotta, Kiyoshi Takeuchi, and Toshiyuki Tanisaki. *D-modules, perverse sheaves, and representation theory*, volume 236 of *Progress in Mathematics*. Birkhäuser Boston Inc., Boston, MA, 2008. Translated from the 1995 Japanese edition by Takeuchi.

[11] C. Huyghe and T. Schmidt. $\mathcal{D}$-modules arithmétiques, distributions et localisation. To appear in Rend. Semin. Mat. Univ. Padova.

[12] C. Huyghe and T. Schmidt. $\mathcal{D}$-modules arithmétiques sur la varété de drapeaux. To appear in Journal für die reine und angewandte mathematik. https://??

[13] C. Huyghe, T. Schmidt, and M. Strauch. Intermediate extensions and crystalline distribution algebras. In Preparation 2019.

[14] Christine Noot-Huyghe. Un théorème de Beilinson-Bernstein pour les $\mathcal{D}$-modules arithmétiques. *Bull. Soc. Math. France*, 137(2):159–183, 2009.

[15] A. Sarrazola-Alzate. Twisted arithmetic differential operators on flag varieties. *PhD-thesis, Université de Rennes (in preparation).*

[16] T. Schmidt. Verma modules over p-adic Arens-Michael envelopes of reductive Lie algebras. *Journal of Algebra*, 390:160–180, 2013.

Crystalline Chebotarëv Density Theorems

Urs Hartl

(joint work with Ambrus Pál)

We formulate a conjectural analogue of Chebotarëv's density theorem for convergent F-isocrystals over a smooth geometrically irreducible curve defined over a finite field using the Tannakian formalism. We prove this analogue for several large classes, including direct sums of isoclinic convergent F-isocrystals and semisimple convergent F-isocrystals which have an overconvergent extension and such that their pull-back to a sufficient small non-empty open sub-curve has connected monodromy.

1. A Conjecture

Let U be a smooth, geometrically irreducible curve over a finite field $\mathbb{F}_q$ having q elements and characteristic p, and denote by $|U|$ the set of closed points of U. Let $F\colon U \to U$ be the (absolute) q-Frobenius which is the identity on $|U|$ and the q-power map on the structure sheaf. Let $K/\mathbb{Q}_q$ be a totally ramified finite field extension. We take $F = \mathrm{id}_K$ as a lift of the q-Frobenius $F = \mathrm{id}_{\mathbb{F}_q}$. For every $x \in |U|$ let $\mathbb{F}_x$, $\deg(x)$ and q_x denote the residue field of x, its degree over $\mathbb{F}_q$ and its cardinality, respectively. For $e \in \mathbb{N}$ let K_e be the unramified field extension of K of degree e. Then $\mathrm{Gal}(K_e/K) = \langle \mathrm{Frob}_q \rangle$.

Let $F\text{-}\mathrm{Isoc}_K(U)$ denote the K-linear rigid abelian tensor category of *K-linear convergent F-isocrystals on U*; see [Cre92, Chapter 1] for details. It is a Tannakian category with fiber functors ω_x for every $x \in |U|$ with $e := \deg(x)$ given by

$$\omega_x\colon\ F\text{-}\mathrm{Isoc}_K(U) \longrightarrow (K_e\text{-vector spaces})\,, \quad \mathcal{F} \mapsto x^*\mathcal{F}\,.$$

This fiber functor is non-neutral if $e > 1$. Actually, $x^*\mathcal{F}$ is an F-isocrystal over $\mathbb{F}_x$, that is an object of

$$F\text{-}\mathrm{Isoc}_K(\mathbb{F}_x) \ :=\ \big\{\, (W, F_W)\colon\ W \text{ a } K_e\text{-vector space}, \\ F_W : W \to W \text{ a } \mathrm{Frob}_q\text{-semilinear automorphisms} \,\big\}\,.$$

So $F_W^e : W \xrightarrow{\sim} W$ is a K_e-linear automorphism of W.

Now fix a base point $u \in U(\mathbb{F}_q)$. (We assume $\deg(u) = 1$ only for simplicity of the exposition.) For $\mathcal{F} \in F\text{-}\mathrm{Isoc}_K(U)$ let $\langle\!\langle \mathcal{F} \rangle\!\rangle \subset F\text{-}\mathrm{Isoc}_K(U)$ be the Tannakian subcategory generated by $\mathcal{F}$. Its *monodromy group* is defined as

$$\mathrm{Gr}(\mathcal{F}/U, u) \ :=\ \mathrm{Aut}^{\otimes}(\omega_u|\langle\!\langle \mathcal{F} \rangle\!\rangle)\,.$$

It is a linear algebraic group over K, *not necessarily connected.*

For every $x \in U(\mathbb{F}_{q^e})$ there is a non-canonical isomorphism of fiber functors $\omega_x \otimes_{K_e} \overline{K} \cong \omega_u \otimes_K \overline{K}$, where $\overline{K}$ is an algebraic closure of K. This induces a non-canonical isomorphism

$$\mathrm{Aut}^{\otimes}(\omega_x|\langle\!\langle \mathcal{F} \rangle\!\rangle) \times_{K_e} \overline{K} \ \cong\ \mathrm{Gr}(\mathcal{F}/U, u) \times_K \overline{K}\,. \tag{1}$$

We define $\mathrm{Frob}_x(\mathcal{F}) \subset \mathrm{Gr}(\mathcal{F}/U, u)(\overline{K})$ as the conjugacy class of the image under the isomorphism (1) of $F_w^e \in \mathrm{Aut}^{\otimes}(\omega_x|\langle\!\langle \mathcal{F} \rangle\!\rangle)$. This conjugacy class is independent of the choice of the isomorphism (1), and hence it is K-rational.

Conjecture A. *For every subset $S \subset |U|$ of Dirichlet density one the set*

$$\bigcup_{x \in S} \mathrm{Frob}_x(\mathcal{F}) \ \subset\ \mathrm{Gr}(\mathcal{F}/U, u)$$

is Zariski-dense.

2. Applications

Corollary. *Let $\mathcal{F}, \mathcal{G} \in F\text{-Isoc}_K(U)$ be convergent F-isocrystals on U of the same rank with $\operatorname{Tr}(\operatorname{Frob}_x(\mathcal{F})) = \operatorname{Tr}(\operatorname{Frob}_x(\mathcal{G}))$ for all points x in a subset $S \subset |U|$ of Dirichlet density one. If Conjecture* A *holds for the direct sum $\mathcal{F}^{ss} \oplus \mathcal{G}^{ss}$ of the semi-simplifications then $\mathcal{F}^{ss} \cong \mathcal{G}^{ss}$.*

Proof. Since $\mathcal{F}^{ss}$ lies in $\langle\langle \mathcal{F} \rangle\rangle$ there is an epimorphism of linear algebraic groups $\operatorname{Gr}(\mathcal{F}/U, u) \twoheadrightarrow \operatorname{Gr}(\mathcal{F}^{ss}/U, u)$ under which $\operatorname{Frob}_x(\mathcal{F})$ maps onto $\operatorname{Frob}_x(\mathcal{F}^{ss})$. The two spaces $\omega_u(\mathcal{F}^{ss})$ and $\omega_u(\mathcal{G}^{ss})$ are semi-simple representations of the group $\operatorname{Gr}\big((\mathcal{F}^{ss} \oplus \mathcal{G}^{ss})/U, u\big)$. By our hypothesis their trace functions coincide on the subset $\operatorname{Frob}_x(\mathcal{F}^{ss} \oplus \mathcal{G}^{ss})$. By Conjecture A the two trace functions coincide on all of $\operatorname{Gr}\big((\mathcal{F}^{ss} \oplus \mathcal{G}^{ss})/U, u\big)$. This implies that the two representations are isomorphic; see [Ser98, Lemma in § I.2.3 on p. I-11]. And therefore the convergent F-isocrystals $\mathcal{F}^{ss}$ and $\mathcal{G}^{ss}$ are isomorphic. □

Example. If A and B are abelian varieties over U with Dieudonné isocrystals $\mathcal{F} = D(A)$ and $\mathcal{G} = D(B)$, this gives an isogeny criterion for A and B.

3. Cases we can prove

Definition. Let $\mathcal{F} \in F\text{-Isoc}_K(U)$. We define

- the *slopes of $\mathcal{F}$ at $x \in |U|$* as
$$\tfrac{1}{\deg(x)} \cdot \operatorname{ord}_p\big(\text{eigenvalues of } \operatorname{Frob}_x(\mathcal{F}) \text{ on } \omega_u(\mathcal{F})\big)\,.$$
- $\mathcal{F}$ to be *isoclinic* if for all $x \in |U|$, $\mathcal{F}$ has a single slope at x (which is then the same for every x).
- $\mathcal{F}$ to be *unit-root* if it is isoclinic of slope zero.

Proposition. *Conjecture* A *holds for unit-root convergent F-isocrystals $\mathcal{F}$ on U.*

Proof. Choose a geometric base point $\bar{u}$ above u and let $\pi_1^{\text{ét}}(U, \bar{u})$ be the étale fundamental group of U. By a result of R. Crew [Cre87, Theorem 2.1 and Remark 2.2.4] the full subcategory of $F\text{-Isoc}_K(U)$ consisting of unit-root F-isocrystals is tensor equivalent to the category $\operatorname{Rep}_K \pi_1^{\text{ét}}(U, \bar{u})$ of continuous representations of $\pi_1^{\text{ét}}(U, \bar{u})$ on finite dimensional K-vector spaces. Moreover, under this equivalence the fiber functor $\omega_u \colon F\text{-Isoc}_K(U) \to (K\text{-vector spaces})$ and the forgetful fiber functor $\omega_{\text{forget}} \colon \operatorname{Rep}_K \pi_1^{\text{ét}}(U, \bar{u}) \to (K\text{-vector spaces})$ become isomorphic over $\overline{K}$.

Let $\rho_{\mathcal{F}} \colon \pi_1^{\text{ét}}(U, \bar{u}) \to \operatorname{GL}_r(K)$ be the representation corresponding to a unit-root F-isocrystal $\mathcal{F}$. Then $\operatorname{Gr}(\mathcal{F}/U, u)$ is a closed subgroup of $\operatorname{GL}_{r,K}$ such that $\operatorname{Gr}(\mathcal{F}/U, u) \times_K \overline{K}$ equals the Zariski-closure of the image of $\rho_{\mathcal{F}}$. Moreover, for all points $x \in |U|$ the $\operatorname{Gr}(\mathcal{F}/U, u)(\overline{K})$-conjugacy classes of $\rho(x_* \operatorname{Frob}_x^{-1})$ and $\operatorname{Frob}_x(\mathcal{F})$ coincide, where $\operatorname{Frob}_x^{-1} \in \operatorname{Gal}(\overline{\mathbb{F}}_x/\mathbb{F}_x)$ is the geometric Frobenius at x which maps $a \in \overline{\mathbb{F}}_x$ to a^{1/q_x} for $q_x = \#\mathbb{F}_x$.

If $S \subset |U|$ is a subset of Dirichlet density one, then by the classical Chebotarëv density theorem [Ser63, Theorem 7] the union of the Frobenius conjugacy classes

Frob_x^{-1} for the points $x \in S$ are dense in $\pi_1^{\text{ét}}(U, \bar{u})$ with respect to the pro-finite topology. Since this topology is finer than the restriction of the Zariski topology from $\mathrm{Gr}(\mathcal{F}/U, u)$, the set $\bigcup_{x \in S} \mathrm{Frob}_x(\mathcal{F})$ is Zariski-dense in $\mathrm{Gr}(\mathcal{F}/U, u)$. □

Example. Let $\mathcal{C}$ be the pullback to U of the constant F-isocrystal on $\mathbb{F}_q$ of rank one given by $(K, F = \pi^s)$ with $s \in \mathbb{Z}$, where $\pi \in K$ is a uniformizing parameter. If $s \neq 0$ then $\mathrm{Gr}(\mathcal{F}/U, u) = \mathbb{G}_{m,K}$. Indeed, $\mathrm{Gr}(\mathcal{F}/U, u)$ is a closed subgroup of $\mathrm{Aut}_K(u^*\mathcal{C}) = \mathbb{G}_{m,K}$ which contains $\mathrm{Frob}_x(\mathcal{C}) = \{\pi^{s \deg(x)}\}$. Since this set is infinite, the only such group is $\mathbb{G}_{m,K}$. The set $\bigcup_{x \in U} \mathrm{Frob}_x(\mathcal{F}) \subset \pi^{\mathbb{Z}s} \subset \mathbb{G}_{m,K}$ is Zariski-dense. However, this set is discrete in $\mathbb{G}_m(K)$ for the p-adic topology. For that reason we can only expect density for the Zariski-topology.

Theorem 1. *Conjecture* A *holds for the direct sum* $\mathcal{F} = \bigoplus_i \mathcal{F}_i$ *of isoclinic convergent* F*-isocrystals* $\mathcal{F}_i$ *on* U.

Idea of the proof. We twist away the slope of $\mathcal{F}_i$ by a constant rank one F-isocrystal $\mathcal{C}_i$ (after enlarging K). Then $\mathcal{G}_i := \mathcal{F}_i \otimes \mathcal{C}_i$ is unit-root. We set $\mathcal{G} := \bigoplus_i \mathcal{G}_i$ and $\mathcal{C} := \bigoplus_i \mathcal{C}_i$. Since $\mathcal{F} \in \langle\langle \mathcal{G} \oplus \mathcal{C} \rangle\rangle$ it is enough to prove Conjecture A for $\mathcal{G} \oplus \mathcal{C}$. In the diagram

$$\begin{array}{ccc} \mathrm{Gr}(\mathcal{G} \oplus \mathcal{C}) = \mathrm{Gr}(\mathcal{G}) & \underset{\mathrm{Gr}(\langle\langle \mathcal{G} \rangle\rangle \cap \langle\langle \mathcal{C} \rangle\rangle)}{\times} & \mathrm{Gr}(\mathcal{C}) \\ \uparrow & & \uparrow \\ C := \rho_{\mathcal{G}}\big(\pi_1^{\text{ét}}(U, \bar{u})\big) & & F_{\mathcal{C}}^{\mathbb{Z}} \end{array}$$

the subgroup C is compact, and hence a p-adic Lie group by [Ser92, Part II, § V.9, Corollary to Theorem 1 on page 155]. We now count the cardinality of

$$\left\{ c \in C \colon \exists\, x \in S \text{ with } (c, F_{\mathcal{C}}^{\deg(x)}) \in \mathrm{Frob}_x(\mathcal{G} \oplus \mathcal{C}) \right\}.$$

A lower bound is provided by the Chebotarëv density for $\mathcal{G}$. If $\bigcup_{x \in S} \mathrm{Frob}_x(\mathcal{G} \oplus \mathcal{C})$ was contained in a hyperplane we would obtain a contradicting upper bound by a result of Oesterlé [Oes82]. □

For the next result we let $\mathcal{F} \in F\text{-}\mathrm{Isoc}_K(U)$ be semi-simple. By the slope filtration theorem of Grothendieck and Katz [Kat79, Corollary 2.3.2] there is a non-empty open subcurve $f \colon V \hookrightarrow U$ such that $f^*\mathcal{F}$ has a slope filtration with isoclinic subquotients. It is always true that

$$\mathrm{Gr}(f^*\mathcal{F}/V, u) \hookrightarrow \mathrm{Gr}(\mathcal{F}/U, u)$$

is a closed immersion. Note, that in contrast to ℓ-adic and p-adic Galois representations this closed immersion can be strict for F-isocrystals.

Conjecture B. $\mathrm{Gr}(f^*\mathcal{F}/V, u) \hookrightarrow \mathrm{Gr}(\mathcal{F}/U, u)$ *is a parabolic subgroup.*

Theorem 2. *Conjecture* B *for* $\mathcal{F}$ *implies Conjecture* A *for* $\mathcal{F}$.

Theorem 3. *If* $\mathrm{Gr}(f^*\mathcal{F}/V, u)$ *is connected and* $\mathcal{F}$ *extends to an overconvergent* F*-isocrystal on* U*, then Conjecture* A *holds for* $\mathcal{F}$.

Idea of the proofs for both theorems. We use Theorem 1 for $(f^*\mathcal{F})^{ss}$ which is a direct sum of isoclinic convergent F-isocrystals on V. In the diagram

$$\begin{array}{ccc} \mathrm{Gr}(f^*\mathcal{F}/V,u) & \overset{}{\underset{\beta}{\hookrightarrow}} & \mathrm{Gr}(\mathcal{F}/U,u) \\ \big\downarrow\!\!\!\downarrow \alpha & & \\ \mathrm{Gr}\big((f^*\mathcal{F})^{ss}/V,u\big) & & \end{array} \qquad\qquad \begin{array}{ccc} \mathrm{Frob}_x(f^*\mathcal{F}) & \hookrightarrow & \mathrm{Frob}_x(\mathcal{F}) \\ \big\downarrow\!\!\!\downarrow & & \\ \mathrm{Frob}_x\big((f^*\mathcal{F})^{ss}\big) & & \end{array}$$

the vertical morphism α identifies $\mathrm{Gr}\big((f^*\mathcal{F})^{ss}/V,u\big)$ with the maximal reductive quotient of $\mathrm{Gr}(f^*\mathcal{F}/V,u)$. We then (develop and) use the theory of maximal quasi-tori as in the following

Definition. Let G be a linear algebraic group over an algebraically closed field L of characteristic zero, which is not necessarily connected. A closed subgroup $T \subset G$ is a *maximal quasi-torus* if the morphism $\alpha\colon G \twoheadrightarrow G/R_uG =: \widetilde{G}$ onto the maximal reductive quotient $\widetilde{G}$ of G induces an isomorphism $\alpha\colon T \xrightarrow{\sim} \alpha(T) \subset \widetilde{G}$ and there is a maximal torus and a Borel subgroup $\widetilde{T}^\circ \subset \widetilde{B} \subset \widetilde{G}^\circ$ such that $\alpha(T)$ equals the intersection $N_{\widetilde{G}}(\widetilde{B}) \cap N_{\widetilde{G}}(\widetilde{T}^\circ)$ of the normalizers. (Then $\alpha(T)^\circ = \widetilde{T}^\circ$).

Now Conjecture B (and likewise the hypotheses of Theorem 3) implies that β maps any maximal quasi-torus of $\mathrm{Gr}(f^*\mathcal{F}/V,u)$ onto a maximal quasi-torus T of $\mathrm{Gr}(\mathcal{F}/U,u)$. Then we (prove and) use that in the reductive group $\mathrm{Gr}(\mathcal{F}/U,u)$ the Zariski-density of the union $\bigcup_{x\in S}\mathrm{Frob}_x(\mathcal{F})$ is equivalent to the Zariski-density in T of $\bigcup_{x\in S} T \cap \overline{\mathrm{Frob}_x(\mathcal{F})}$. □

References

[Cre87] R. Crew, *F-isocrystals and p-adic representations*, Algebraic geometry, Bowdoin, 1985 (Brunswick, Maine, 1985), 111–138, Proc. Sympos. Pure Math., 46, Part 2, Amer. Math. Soc., Providence, RI, 1987.

[Cre92] R. Crew, *F-isocrystals and their monodromy groups*, Ann. Sci. École Norm. Sup. **25** (1992), 429–464.

[HP18] U. Hartl, A. Pál: *Crystalline Chebotarëv Density Theorems*, Preprint November 2018 on arXiv:1811.07084.

[Kat79] N. Katz, *Slope filtration of F-crystals*, Journées de Géométrie Algébrique de Rennes (Rennes, 1978), Vol. I, Astérisque, vol. 63, Soc. Math. France, Paris, 1979, pp. 113–163.

[Oes82] J. Oesterlé, *Réduction modulo p^n des sous-ensembles analytiques fermés de $\mathbb{Z}_p^N$*, Invent. Math. **66** (1982), 325–341.

[Ser63] J.-P. Serre: *Zeta and L-functions*, in Arithmetical Algebraic Geometry (Proc. Conf. Purdue Univ., 1963), Harper & Row, New York, pp. 82–92.

[Ser92] J.-P. Serre, *Lie algebras and Lie groups*, Lecture Notes in Mathematics **1500**, second edition, Springer-Verlag, Berlin (1992).

[Ser98] J.-P. Serre: *Abelian l-adic representations and elliptic curves*, Revised reprint of the 1968 original, Research Notes in Mathematics **7**, A K Peters, Ltd., Wellesley, MA, 1998.

The essential skeletons of pairs and the geometric P=W conjecture.

Enrica Mazzon
(joint work with Mirko Mauri and Matthew Stevenson)

The geometric P=W conjecture is a conjectural description of the asymptotic behavior of a celebrated correspondence in non-abelian Hodge theory. In particular, it is expected that the dual boundary complex of the compactification of character varieties has the homotopy type of a sphere. In the joint work [4] with Mirko Mauri and Matthew Stevenson, we compute the first non-trivial examples of these dual boundary complexes in the compact case. In this talk I will explain how the result is a combination of techniques from birational and non-archimedean geometry.

1. The geometric P=W conjecture

Let C be a complex projective smooth curve. The main object of study in non-abelian Hodge theory are the representations of the topological fundamental group of C in a reductive linear group G. The associated moduli space

$$M_B := \mathrm{Hom}(\pi_1(C), G) /\!\!/ G$$

is the GIT quotient by the conjugation action of G: it is usually called the *Betti moduli space*, or also the G-character variety associated to C.

A fundamental correspondence in non-abelian Hodgen theory relates M_B to another moduli space: the Hitchin's moduli space M_{Dol} of semistable principal Higgs G-bundles on X with vanishing Chern classes, also known as the *Dolbeault moduli space*. A distinctive feature of the moduli space M_{Dol} is that it comes equipped with the so-called Hitchin map

$$H : M_{\mathrm{Dol}} \to \mathbf{C}^N,$$

with $2N = \dim_{\mathbf{C}}(M_{\mathrm{Dol}})$.

The geometric P=W conjecture predicts the behaviour of the fundamental correspondence between M_B and M_{Dol} *at infinity*, in the following sense. Consider compactifications $\overline{M}_B$ of M_B, resp $\overline{M}_{\mathrm{Dol}}$ of M_{Dol}, with boundaries $\partial M_B := \overline{M}_B \backslash M_B$, resp $\partial M_{\mathrm{Dol}} := \overline{M}_{\mathrm{Dol}} \backslash M_{\mathrm{Dol}}$, and punctured neighbourhoods at infinity $N_B^* := N_B \backslash \partial M_B$, resp $N_{\mathrm{Dol}}^* := N_{\mathrm{Dol}} \backslash \partial M_{\mathrm{Dol}}$. The Hitchin map induces a map from N_{Dol}^* to a neighbourhood at infinity of $\mathbf{C}^N$, so up to homotopy we obtain a map

$$h : N_{\mathrm{Dol}}^* \xrightarrow{H} \mathbf{C}^N \backslash \{0\} \xrightarrow{\sim} \mathbb{S}^{2N-1}.$$

Assuming that the dual boundary complex $\mathcal{D}(\partial M_B)$ is well-defined, by means of a partition of unity one can construct a map from N_B^* to $\mathcal{D}(\partial M_B)$

$$\alpha : N_B^* \to \mathcal{D}(\partial M_B).$$

Note that if ∂M_B is an snc divisor, the homotopy type of $\mathcal{D}(\partial M_B)$ is independent of the choice of the snc compactification.

Stated by Katzarkov, Noll, Pandit and Simpson in [2], the geometric $P = W$ conjecture proposes a correspondence between the dual boundary complex of M_B and the sphere at infinity of the Hitchin base for M_{Dol}.

Conjecture 1 (Geometric $P = W$ conjecture)**.** *There exists a homotopy equivalence*

$$\mathcal{D}(\partial M_B) \sim \mathbb{S}^{2N-1}$$

such that the following diagram is homotopy commutative

$$\begin{array}{ccc} N^*_{\mathrm{Dol}} & \xrightarrow{\sim} & N^*_B \\ {\scriptstyle h}\big\downarrow & & \big\downarrow{\scriptstyle \alpha} \\ \mathbb{S}^{2N-1} & \xrightarrow{\sim} & \mathcal{D}(\partial M_B). \end{array}$$

A first evidence for the conjecture is due to Simpson: when M_B is the SL_2-character variety of local systems on a punctured sphere (such that conjugacy classes of the monodromies around the punctures are fixed), he proves in [6, Theorem 1.1] that the dual boundary complex $\mathcal{D}(\partial M_B)$ has the homotopy type of a sphere; see also [3, Theorem 1.4].

Our main result is the following:

Theorem 2. *Let C be a Riemann surface of genus 1. The dual boundary complex $\mathcal{D}(\partial M_B)$ of a dlt log Calabi–Yau compactification of M_G has the homeomorphism type of $\mathbb{S}^{2n-1}$ if $G = \mathrm{GL}_n$, and of $\mathbb{S}^{2n-3}$ if $G = \mathrm{SL}_n$.*

2. The dual boundary complex $\mathcal{D}(\partial M_G)$

The first part of my talk will be focused on the definition of the dual boundary complex $\mathcal{D}(\partial M_B)$. We identify a suitable class of compactifications of M_B such that the character varieties considered in Theorem 2 admit such a compactification, and the dual complex associated to the boundary is well-defined.

For $G = \mathrm{GL}_n$ or SL_n, the affine variety M_B is singular with canonical and factorial singularities. Hence, M_B does not allow an snc compactification. However, it admits dlt compactifications, and among all possible dlt compactifications of M_B we restrict to special ones, namely the dlt log Calabi–Yau compactifications.

In general, the advantage in considering a dlt log Calabi–Yau compactification of M_B is that its dual boundary complex identifies a distinguished homeomorphism class in the homotopy equivalence class of the dual complex of any dlt compactification. Moreover, this homotopy class actually coincides with that of the dual complex of any snc compactification of a resolution of M_B.

3. Non-archimedean approach

In the second part of my talk, I will introduce a characterization of the dual complex $\mathcal{D}(\partial M_B)$ in terms of non-archimedean geometry.

Building on the work of [5, 7, 1], we construct weight functions, Kontsevich–Soibelman skeletons, and essential skeletons associated to pairs (X, D) over a trivially-valued field of characteristic zero. In particular, we prove that the dual complex of a log canonical log Calabi–Yau pair (X, D) is homeomorphic to the link of the essential skeleton of (X, D).

We apply this result for the character variety associated to a Riemann surface of genus 1 when $G = \mathrm{GL}_n$ or SL_n: the dual complex $\mathcal{D}(\partial M_B)$ is homeomorphic to the essential skeleton of $(M_B, \partial M_B)$, and the explicit computation relies on the properties of the essential skeleton under the operations of taking products and finite quotients.

4. Degeneration approach

I will conclude by mentioning an alternative proof of Theorem 2. This approach adopts the notion of the essential skeleton in the discretely-valued field setting. It is technically more demanding, since it requires the construction of a degeneration of compact hyper-Kähler manifolds, but it suggests a relation of the geometric P=W conjecture with the conjecture below:

Conjecture 3. *Let $\mathscr{X}$ be a maximally unipotent good minimal dlt degeneration of compact hyper-Kähler manifolds over $\mathbf{C}((t))$. Then, the dual complex of the special fibre of $\mathscr{X}$ is homeomorphic to $\mathbf{P}^n(\mathbf{C})$.*

References

[1] M. Jonsson and M. Mustaţă. *Valuations and asymptotic invariants for sequences of ideals.* Ann. Inst. Fourier (Grenoble) 62.6 (2012), 2013.
[2] Katzarkov L., Noll A., Pandit P., and Simpson C., *Harmonic maps to buildings and singular perturbation theory*, Comm. Math. Phys., 2015.
[3] Komyo A., *On compactifications of character varieties of n-punctured projective line*, Ann. Inst. Fourier (Grenoble), 2015.
[4] Mauri M., Mazzon E., and Stevenson M., *Essential skeletons of pairs and the geometric P=W conjecture*, arXiv:1810.11837, 2018.
[5] M. Mustaţă and J. Nicaise. *Weight functions on non-Archimedean analytic spaces and the Kontsevich-Soibelman skeleton.* Algebr. Geom. 2.3, 2015.
[6] Simpson. C., *The dual boundary complex of the SL_2 character variety of a punctured sphere*, Ann. Fac. Sci. Toulouse Math., 2016.
[7] M. Temkin. *Metrization of differential pluriforms on Berkovich analytic spaces.* In: Nonarchimedean and Tropical Geometry. Ed. by M. Baker and S. Payne. Vol. Simons Symposia. 2016, 195–285.

Tropical Dolbeault cohomology of non-archimedean curves and harmonic tropicalizations

Philipp Jell

Let K be an algebraically closed complete non-archimedean field and X a variety over K of dimension n. We denote by X^{an} the Berkovich analytification of X and by $\mathcal{A}^{p,q}$ the sheaf of smooth real-valued differential forms on X^{an}, as introduced by Chambert–Loir and Ducros [1].

We state the following properties, all due to Chambert–Loir and Ducros.

- The $\mathcal{A}^{p,q}$ are fine sheaves of real vector spaces.
- The sheaf $\mathcal{A}^{0,0}$ is a subsheaf of the sheaf of continuous functions.

- There exist differential operators
$$d' \colon \mathcal{A}^{p,q} \to \mathcal{A}^{p+1,q} \text{ and } d'' \colon \mathcal{A}^{p,q} \to \mathcal{A}^{p,q+1}.$$
- There is integration of top dimensional forms with compact support that satisfies Stokes' theorem. It was shown by Gubler that Stokes' theorem for $\mathcal{A}_c^{n,n}$ is equivalent to the balancing condition in tropical geometry [2].

Definition 1. *We define the tropical Dolbeault cohomology of X^{an} to be*
$$\mathrm{H}^{p,q}(X^{\mathrm{an}}) := \mathrm{H}^q(\mathcal{A}^{p,\bullet}(X^{\mathrm{an}}), d'') = \frac{\ker\left(d'' \colon \mathcal{A}^{p,q}(X^{\mathrm{an}}) \to \mathcal{A}^{p,q+1}(X^{\mathrm{an}})\right)}{\mathrm{im}\,(d'' \colon \mathcal{A}^{p,q-1}(X^{\mathrm{an}}) \to \mathcal{A}^{p,q}(X^{\mathrm{an}}))}.$$

Question 2. *What can we say about $\mathrm{H}^{p,q}(X^{\mathrm{an}})$?*

The first result in this direction was the following Poincaré lemma type result.

Theorem 3. [4, Theorem 4.5] *The sequence of sheaves*
$$0 \to \mathcal{A}^{p,0} \xrightarrow{d''} \ldots \to \mathcal{A}^{p,n} \to 0$$
is exact at $\mathcal{A}^{p,q}$ for all $p \geqslant 0, q > 0$.

Note that this does not give a cover of X^{an} by acyclic domains. However, as a consequence we can show that there are sheaves which compute tropical Dolbeault cohomology. We write $\mathcal{F}^p := \ker(d'' \colon \mathcal{A}^{p,0} \to \mathcal{A}^{p,1})$.

Corollary 4. *We have*
$$\mathrm{H}^{p,q}(X^{\mathrm{an}}) = \mathrm{H}^q(X^{\mathrm{an}}, \mathcal{F}^p) \text{ and } \mathrm{H}^{0,q}(X^{\mathrm{an}}) = \mathrm{H}^q_{\mathrm{sing}}(X^{\mathrm{an}}, \mathbb{R}).$$

Note that it follows from a result by Hrushovsky and Loeser that $\mathrm{H}^q_{\mathrm{sing}}(X^{\mathrm{an}}, \mathbb{R})$ is finite dimensional [3].

As far as constructing non-trivial and interesting classes, Liu provided a lot of them with his construction of cycle class maps.

Theorem 5. [7, Theorem 1] *Let X be a smooth variety. Then for all k there is a homomorphism*
$$\mathrm{cl} \colon \mathrm{CH}^k(X) \to \mathrm{H}^{k,k}(X^{\mathrm{an}})$$
that is compatible with products on both sides.

From now on, let X be a proper variety. There is a natural pairing

$$(1) \qquad \mathrm{H}^{p,q}(X^{\mathrm{an}}) \times \mathrm{H}^{n-p,n-q}(X^{\mathrm{an}}) \to \mathbb{R}; \qquad ([\alpha],[\beta]) \mapsto \int_{X^{\mathrm{an}}} \alpha \wedge \beta.$$

Question 6. *When is (1) a perfect pairing?*

For curves, if (1) is a perfect pairing, we can compute all $\mathrm{H}^{p,q}(X^{\mathrm{an}})$ using identification with singular cohomology. Of course singular cohomology of Berkovich curves is well understood.

The next theorem provides a complete answer to Question 6 for curves. It turns out that the reduction behavior of the curve is crucial.

Definition 7. *Let X be a smooth projective curve. Let $\mathcal{X}$ be a strictly semistable model of X and let $C_1, \dots, C_k$ be the irreducible components of the special fiber of $\mathcal{X}$. Then*

$$S_X := \bigoplus_{i=1}^{k} \mathrm{Pic}^0(C_i) \otimes \mathbb{R}.$$

Theorem 8. [5, Theorem A] *Let X be a smooth projective curve. Then (1) is a perfect pairing for all p, q if and only if $S_X = 0$.*

There are two particular cases in which $S_X = 0$: When X is a Mumford curve, i.e all the C_i are isomorphic to the projective line and when the residue field of K is algebraic over a finite field. The fact that (1) is a perfect pairing for all p, q for Mumford curves was already known by joint work of Wanner and the author, using very different techniques [6].

We give a short idea of the proof of Theorem 8, since it will provide some insight where S_X comes into play.

Proof. The proof is based on two exact sequences of sheaves on X^{an}. We denote $\mathcal{K} := \ker(d'd'' \colon \mathcal{A}^{0,0} \to \mathcal{A}^{1,1})$. Then the following two sequences of sheaves on X^{an} are exact:

$$0 \to \underline{\mathbb{R}} \to \mathcal{K} \to \mathcal{F}^1 \to 0 \text{ and} \tag{2}$$

$$0 \to \mathcal{K} \to \mathcal{H} \to \mathcal{S}_X \to 0, \tag{3}$$

where $\mathcal{H}$ is Thuillier's sheaf of harmonic functions [9] and $\mathcal{S}_X$ is a sum of skyscraper sheaves that satisfies $\mathcal{S}_X(X^{\mathrm{an}}) = S_X$. Sequence (2) is a non-archimedean analogue of the tropical exponential sequence as introduced by Mikhalkin and Zharkov [8]. Exactness of sequence (3) was proved by Thuillier [9, Lemme 2.3.22], who used an explicit description of $\mathcal{K}$.

Writing down the two long exact sequences in cohomology, we find

$$0 \to \mathrm{H}^{1,0}(X^{\mathrm{an}}) \to \mathrm{H}^{0,1}(X^{\mathrm{an}}) \to \mathrm{H}^1(X^{\mathrm{an}}, \mathcal{K}) \to \mathrm{H}^{1,1}(X^{\mathrm{an}}) \to 0 \text{ and}$$
$$0 \to S_X \to \mathrm{H}^1(X^{\mathrm{an}}, \mathcal{K}) \to \mathrm{H}^1(X^{\mathrm{an}}, \mathcal{H}) \to 0.$$

Now using that $\mathrm{H}^{1,1}(X^{\mathrm{an}})$ is at least one-dimensional (which follows from Stokes' theorem) and $\mathrm{H}^1(X^{\mathrm{an}}, \mathcal{H})$ is one-dimensional (which can be shown using tools provided in Thuillier's thesis), after a short diagram chase we see that the theorem is true dimension-wise, and it is not difficult to prove the theorem from there. □

If the residue field of K is $\mathbb{C}$ and X is a curve of good reduction and positive genus, S_X is an infinite dimensional real vector space. The same diagram chase as above then shows that $\mathrm{H}^{1,1}(X^{\mathrm{an}})$ is infinite dimensional.

Problem. *"Not all harmonic functions are smooth"*

By this slogan we mean the problem that the sheaf $\mathcal{K}$ and the sheaf $\mathcal{H}$ do in general not agree and problems show up precisely when this happens.

Further evidence that this indeed poses a problem is a result by Wanner. She proves a version of a regularization theorem for plurisubharmonic functions on non-archimedean curves, again under the assumption that $S_X = 0$ (meaning that all harmonic functions are smooth) [10, Corollary 5.4].

Vague Solution. *Tweak the theory defined by Chambert–Loir and Ducros in a way that makes all harmonic functions smooth.*

The implementation of this solution is the subject of ongoing joint work with Joe Rabinoff, where we allow for a more flexible smooth functions via a notion of "harmonic tropicalization", which we also introduce and study in our work. As a short term goal this will provide a theory of forms in arbitrary dimension. This theory keeps the good properties of the theory by Chambert–Loir and Ducros and fixes the problems we already encountered for curves, such as the failure of duality and finite dimensionality. As a long term goal we plan to show finite dimensionality and duality also in higher dimension.

References

[1] A. Chambert–Loir and A. Ducros, *Formes différentielles et réelles sur les espaces de Berkovich*, arXiv:1204.6277

[2] W. Gubler, *Formes and currents on the analytification of an algebraic variety (after Chambert–Loir and Ducros)*, Non-archimedean and Tropical Geometry, Simons Symposia, 1–30, Switzerland, 2016. Spinger

[3] E. Hrushovski and F. Loeser, *Non-archimedean tame topology and stable dominated types*, volume 192 of Annals of Mathematics Studies. Princton University Press, Princton NJ, 2016.

[4] P. Jell, *A Poincaré lemma for smooth real-valued differential form on Berkovich spaces*, Mathematische Zeitschrift 282(3-4), 2016, 1149–1167.

[5] P. Jell, *Tropical Hodge numbers of non-archimedean curves*, Israel Journal of Mathematics 229(1), 2019, 287–305.

[6] P. Jell and V. Wanner, *Poincaré duality for the tropical Dolbeault cohomology of non-archimedean Mumford curves*, Journal of Number Theory 187, 2018, 334–371.

[7] Y. Liu, *Tropical cycle classes for non-Archimedean spaces and weight decomposition of de Rham cohomology sheaves*, Annales Scientifiques de l'École Normale Supérieure, to appear.

[8] G. Mikhalkin and I. Zharkov *Tropical curves, their Jacobians and theta functions*, In: Curves and abelian varieties, pp. 203–230, Contemp. Math., vol. 465, Amer. Math. Soc., Providence, RI,2008

[9] A. Thuillier, *Théorie du potentiel sur les courbes en géométrie analytique non archimédienne. Applications à la théorie d'Arakelov.*

[10] V. Wanner, *Comparison of two notions of subharmonicity on non-archimedean curves*, Mathematische Zeitschrift (2018). https://doi.org/10.1007/s00209-018-2205-z

Reduction and lifting of Berkovich curves with differentials

Ilya Tyomkin

(joint work with Michael Temkin)

In a recent paper [3], Bainbridge, Chen, Gendron, Grushevsky, and Möller studied what they called *Incidence compactification of strata of abelian differentials.* For a given pattern of zeroes (and poles) $\underline{\mu} \in \mathbb{N}^r$, they considered pairs $(C, \underline{p}; \omega)$ consisting of a smooth projective curve C with r marked points $\underline{p}$, and a (meromorphic) differential form ω up-to a multiplicative scalar, such that $div(\omega) = \sum \mu_i p_i$. The incidence compactification then is the closure of this locus in the projectivised Hodge bundle on $\overline{\mathcal{M}}_{g,r}$. The main result of [3] provides an explicit description of complex points of the incidence compactification in terms of level graphs (functions) and twisted differentials satisfying the usual compatibilities and a new striking condition introduced in [3] - the *global residue condition* for twisted differentials with respect to a level function.

The results of [3] have many important applications. In particular, Möller, Ulirsch, and Werner used [3] to provide a description of the liftable loci in the canonical systems on tropical curves [5]. More explicitly, given a tropical curve Γ and a divisor D in the canonical system on Γ, Möller, Ulirsch, and Werner provide a purely combinatorial necessary and sufficient condition for the pair (Γ, D) to be the tropicalization of a smooth curve X over a non-Archimedean field of zero equicharacterstic and an effective canonical divisor K on X.

In our work we studied meromorphic differential forms on *nice* k-analytic curves, i.e., quasi-smooth connected compact separated strictly k-analytic curves. One of our motivations was to find a Berkovich analytic proof of the main result of [3]. Starting with a nice curve X equipped with a non-zero meromrphic differential ω we describe a natural tropicalization datum associated to the pair. If (X, ω) is the analytification of an algebraic pair then the datum we associate to it almost coincides with the datum of [3] and [5], but in addition we associate a canonically defined residue function on the set of oriented edges of the skeleton Γ of $(X, div(\omega))$ with values in k. The residue function $\mathfrak{R}$ satisfies the very common in Berkovich geometry harmonicity condition: for any vertex x of Γ we have $\sum_{e \in Star(x)} \mathfrak{R}(e) = 0$. If (X, ω) is the analytification of an algebraic pair then the harmonicity condition of $\mathfrak{R}$ together with its compatibility with the residues of the associated twisted differential implies the global residue condition of [3].

Our main result is the lifting theorem asserting that given a tropical datum satisfying natural compatibility conditions and such that the residue function is harmonic, there exists a nice k-analytic curve X with a meromorphic differential ω, whose tropicalization coincides with the given datum. The proof of the theorem is based on the key lemma asserting that for any differential form $\omega_{\mathcal{A}}$ on an analytic annulus $\mathcal{A} = \mathcal{M}\{t, rt^{-1}\}$ that has neither zeroes nor poles, there exists a *good* analytic coordinate s on $\mathcal{A}$ such that $\omega_{\mathcal{A}} = ads^n + \mathfrak{R}\frac{ds}{s}$. The main conclusion from the key lemma is that a differential form on an annulus without zeroes and poles is determined by its norm and its residue uniquely up-to an orientation preserving automorphism. We shall emphasize that a similar lemma about the existence of good coordinates in the case of differential forms on small punctured complex discs was one of the ingredients also in the complex-analytic proof of the main theorem in [3]. Good coordinates allow us to patch local liftings along annuli similarly to the patchings of coverings of curves in the work of Amini, Baker, Brugallé, and Rabinoff [1, 2] in characteristic zero, and in the work of Brezner and Temkin [4] in positive characteristic. Also in the problem of patching of coverings of curves there were similar key lemmas providing explicit description of isomorphism classes of coverings of annuli, see e.g., [4, Thm. 4.3.8, Cor. 4.3.9]. To the best of our understanding, the patching technique we use in the Berkovich-analytic setting is close analogue of the plumbing technique used in [3].

We shall also mention, that our tropical reduction datum contains one more ingredient. Namely, for any oriented edge e of Γ with head x and tail y, consider an open annulus whose skeleton is the edge e. Then the set of good coordinates on the annulus induces a canonical identification of the torsors of good formal coordinates for the reduction (C_x, ω_x) and (C_y, ω_y) at the points corresponding to e. This extra "stacky" piece of reduction is not needed in the proof of the lifting theorem, but as it is absolutely canonical, we expect it to be useful for other applications. The situation here is analogues to the tropical and stacky tropical reductions introduced in [6]. In [6], one could prove the lifting result for regular non-stacky tropical reductions, but for a correspondence theorem one had to work with the stacky reductions.

References

[1] O. Amini, M. Baker, E. Brugallé, J, Rabinoff, *Lifting harmonic morphisms I: metrized complexes and Berkovich skeleta*, Res. Math. Sci. **2** (2015), Art. 7, 67 pp.

[2] O. Amini, M. Baker, E. Brugallé, J, Rabinoff, *Lifting harmonic morphisms II: Tropical curves and metrized complexes.* Algebra Number Theory **9** (2015), no. 2, 267–315.

[3] M. Bainbridge, D. Chen, Q. Gendron, S. Grushevsky, M. Möller, *Compactification of strata of Abelian differentials*, Duke Math. J. **167** (2018), no. 12, 2347–2416.

[4] U. Brezner, M. Temkin, *Lifting problem for minimally wild covers of Berkovich curves*, arXiv:1709.10416.

[5] M. Möller, M. Ulirsch, A. Werner, *Realizability of tropical canonical divisors*, arXiv:1710.06401.

[6] I. Tyomkin, *Tropical geometry and correspondence theorems via toric stacks*, Math. Ann. **353** (2012), no. 3, 945–995.

Formal groups and p-adic dynamical systems

Laurent Berger

I started my talk by explaining some results about formal groups that can be proved using ideas coming from Lubin's theory of p-adic dynamical systems. Let K be a finite extension of $\mathbf{Q}_p$, with integers $\mathcal{O}_K$, and let $F(X, Y) \in \mathcal{O}_K[\![X, Y]\!]$ be a formal group law over $\mathcal{O}_K$. Let $\mathrm{Tors}(F)$ denote the set of torsion points of F in $\mathfrak{m}_{\mathbf{C}_p}$. To what extent is F determined by its torsion points? The first result is that if two formal groups F and G have infinitely many torsion points in common, then $F = G$. The proof of this theorem rests on a rigidity result: if F is a formal group and if $h(X) \in X \cdot \mathcal{O}_K[\![X]\!]$ is such that $h(z) \in \mathrm{Tors}(F)$ for infinitely many $z \in \mathrm{Tors}(F)$, then h is an endomorphism of F. When $F = \mathbf{G}_m$, such a rigidity result had already been proved by Hida. The proofs of these theorems rest on (1) power series arguments inspired by Lubin's theory of p-adic dynamical systems and (2) the fact that if F is a formal group of finite height, then the image of the attached Galois representation contains an open subgroup of $\mathbf{Z}_p^\times \cdot \mathrm{Id}$. This fact follows from a theorem of Serre and Sen.

After discussing the proofs of these theorems, I gave a brief survey of some of Lubin's results on p-adic dynamical systems. I introduced the notion of a Lubin pair, namely a pair (f, u) of elements of $X \cdot \mathcal{O}_K[\![X]\!]$ that commute under composition, with f and u stable, and with f noninvertible and u invertible. I discussed Lubin's observation that given a Lubin pair, there must be a formal group somehow in the background. For example, if $K = \mathbf{Q}_p$ and if (f, u) is a Lubin pair in which f and all of its iterates have simple roots, and $f \not\equiv 0 \bmod p$, then f and u are endomorphisms of a formal group over $\mathbf{Z}_p$. In general, I conjectured that given a Lubin pair (f, u) with $f \not\equiv 0 \bmod \mathfrak{m}_K$, there is a formal group S such that f and u are semiconjugate to endomorphisms of S.

I finished by explaining my motivation for considering p-adic dynamical systems. They occur in the study of (φ, Γ)-modules. If K_∞/K is a sufficiently ramified (more precisely: strictly APF) Galois extension, and if $\Gamma = \mathrm{Gal}(K_\infty/K)$, then the field of norms of K_∞/K is a local field of characteristic p, endowed with a Frobenius map φ and an action of Γ. In order to have a theory of (φ, Γ)-modules for this Γ, we need to lift these actions to a ring of characteristic zero, such as $\mathcal{O}_K[\![X]\!]$. Such a lift gives rise to a p-adic dynamical system, and using Lubin's results we can prove that if such a lift exists, then K_∞/K is abelian. A recent result of Léo Poyeton then says that K_∞/K is generated by the torsion points of a relative Lubin-Tate group S, and that the power series that give the lifts of φ and of the elements of Γ are semiconjugate to endomorphisms of S.

References

[Ber14] L. Berger, *Lifting the field of norms*, J. École polytechnique - Math. 1 (2014), 29–38.

[Ber19a] L. Berger, *Nonarchimedean dynamical systems and formal groups*, Proc. Amer. Math. Soc. 147 (2019), no. 4, 1413–1419.

[Ber19b] L. Berger, *Rigidity and unlikely intersection for formal groups*, preprint, 2019.

[Fon90] J.-M. Fontaine, *Représentations p-adiques des corps locaux. I*, in The Grothendieck Festschrift, Vol. II, Progr. Math., vol. 87, Birkhäuser Boston, 1990, 249–309.

[Hid14] H. Hida, *Hecke fields of Hilbert modular analytic families*, in Automorphic forms and related geometry: assessing the legacy of I. I. Piatetski-Shapiro, Contemp. Math., vol. 614, Amer. Math. Soc., Providence, RI, 2014, 97–137.

[Lub94] J. Lubin, *Nonarchimedean dynamical systems*, Compositio Math. 94 (1994), no. 3, 321–346.

[Sen73] S. Sen, *Lie algebras of Galois groups arising from Hodge-Tate module*, Ann. of Math. (2) 97 (1973), 160–170.

[Ser67] J.-P. Serre, *Sur les groupes de Galois attachés aux groupes p-divisibles*, in Proc. Conf. Local Fields (Driebergen, 1966), Springer, Berlin, 1967, 118–131.

Heights and moments of abelian varieties

Farbod Shokrieh
(joint work with Robin de Jong)

Let (A, λ) be a principally polarized abelian variety over $\overline{\mathbb{Q}}$ of dimension $g \geqslant 1$. One has naturally associated to (A, λ) the *Néron-Tate height* $\mathrm{h}'_L(\Theta)$ of any symmetric effective divisor Θ defining λ on A, where $L = \mathcal{O}_A(\Theta)$. Another invariant attached to A is the *stable Faltings height* $\mathrm{h}_F(A)$ as introduced by Faltings [Fal83]. It is natural to ask how $\mathrm{h}'_L(\Theta)$ and $\mathrm{h}_F(A)$ are related. We prove a formula relating $\mathrm{h}'_L(\Theta)$ and $\mathrm{h}_F(A)$, completing earlier results due to Bost, Hindry, Autissier, and Wagener.

Assume A and L are defined over a number field k. In the papers [Aut06] and [Hin93] by Autissier resp. Hindry one finds an identity relating $\mathrm{h}'_L(\Theta)$ and $\mathrm{h}_F(A)$ under the assumption that A has everywhere good reduction over k. Such an identity is also implicit in the paper [Bos96b] by Bost. Assume that an admissible adelic metric $(\|\cdot\|_v)_{v\in M(k)}$ has been chosen on L. Let s be any nonzero global section of L on A. For each $v \in M(k)_\infty$ (archimedean place) one defines

$$I(A_v, \lambda_v) = -\int_{A_v^{\mathrm{an}}} \log \|s\|_v \, \mathrm{d}\mu_v + \frac{1}{2} \log \int_{A_v^{\mathrm{an}}} \|s\|_v^2 \, \mathrm{d}\mu_v \,,$$

where μ_v denotes the probability Haar measure on the complex torus $A_v^{\mathrm{an}} = A_v(\mathbb{C})$. The real-valued local invariant $I(A_v, \lambda_v)$ is independent of the choice of s and L.

Autissier in [Aut06] proposed the following relation between $\mathrm{h}'_L(\Theta)$ and $\mathrm{h}_F(A)$: assume that A has semistable reduction over k. One should have an identity of the type

$$\mathrm{h}_F(A) + \kappa_0 g = 2g\, \mathrm{h}'_L(\Theta) + \frac{1}{[k:\mathbb{Q}]} \sum_{v\in M(k)_0} \alpha_v \log(Nv) + \frac{2}{[k:\mathbb{Q}]} \sum_{v\in M(k)_\infty} I(A_v, \lambda_v) \,,$$

where α_v, for each $v \in M(k)_0$ (non-archimedean place), is a non-negative rational number that can be calculated from the reduction of A at v, with $\alpha_v = 0$ if A has good reduction at v. Here $\kappa_0 = \log(\pi\sqrt{2})$. For non-archimedean places v, the size of the residue field at v is denoted by Nv.

As already mentioned, if A has everywhere good reduction, the equality was known, with $\alpha_v = 0$ for all $v \in M(k)_0$, by the works of Bost, Hindry, and Autissier. Autissier also proved such an identity in the case where (A, λ) is a principally polarized abelian variety of dimension one or two (or is a product of such). de Jong in [dJ18] exhibited natural α_v for all Jacobians; the necessary local invariants α_v are expressed in terms of the combinatorics of the dual graph of the underlying semistable curve at v.

We give a complete answer, relating $\mathrm{h}'_L(\Theta)$ and $\mathrm{h}_F(A)$, for general principally polarized abelian varieties. We show that the mysterious number α_v must be expressed in terms of *theta functions* in tropical and non-archimedean geometry.

Let v be a non-archimedean place and assume A has semistable reduction at v. Berkovich [Ber90] showed that the analytification A_v^{an} contains a canonical real torus Σ_v, called the *canonical skeleton*, onto which A_v^{an} deformation retracts. This skeleton is a *tropical abelian variety*: we have a finitely generated, free abelian group Λ (of rank at most g) equipped with a positive-definite, symmetric bilinear pairing $[\cdot,\cdot]_v \colon \Lambda \times \Lambda \to \mathbb{R}$ such that the real torus (skeleton) is $\Sigma_v = \Lambda_{\mathbb{R}}/\Lambda$.

We define our *modified tropical theta function* $\|\Psi_v\| \colon \Lambda_{\mathbb{R}} \to \mathbb{R}$ by

$$\|\Psi_v\|(x) = \frac{1}{2}\min_{\lambda\in\Lambda}\,[x-\lambda, x-\lambda]_v\,.$$

This is Λ-periodic and descends to a well-defined function on the real torus Σ_v, which we again denote by $\|\Psi_v\|$. This function is closely related to non-archimedean and tropical theta functions, see [FRSS18].

For each $v \in M(k)_0$, the *tropical moment* of (A_v, λ_v) (or of Σ_v) is the real number

$$I(A_v,\lambda_v) = 2\int_{\Sigma_v}\|\Psi_v\| d\mu_v\,,$$

where μ_v denotes the probability Haar measure on Σ_v. Clearly, $I(A_v,\lambda_v) \geqslant 0$. Also $I(A_v,\lambda_v) = 0$ if and only if A has good reduction at v (so Σ_v is just a point). In our setup, the associated bilinear map can be written in terms of the discrete valuation at v and, therefore, the tropical moment $I(A_v,\lambda_v)$ is a rational number.

Theorem 1 ([dJS18a]). *Let (A,λ) be a principally polarized abelian variety of dimension $g \geqslant 1$ with semistable reduction over a number field k. Let Θ be an effective symmetric ample divisor on A that defines the principal polarization λ, and put $L = \mathcal{O}_A(\Theta)$. For a finite place v of k let $I(A_v,\lambda_v)$ be the tropical moment of the skeleton of the Berkovich analytic space A_v^{an} of A at v. Then the following equality holds in $\mathbb{R}$:*

$$\mathrm{h}_F(A)+\kappa_0 g = 2g\,\mathrm{h}'_L(\Theta)+\frac{1}{[k:\mathbb{Q}]}\left(\sum_{v\in M(k)_0} I(A_v,\lambda_v)\log(Nv) + 2\sum_{v\in M(k)_\infty} I(A_v,\lambda_v)\right).$$

In other words, the mysterious number α_v of Autissier is the same as the associated tropical moment.

When $g = 1$ our equality in Theorem 1 boils down to the well-known Faltings-Silverman formula for Faltings heights of elliptic curves (see [Fal84, Theorem 7] and [Sil86, Proposition 1.1]). We also obtain the lower bounds

$$\mathrm{h}_F(A) \geqslant -\kappa_0\, g + \frac{2}{[k:\mathbb{Q}]}\sum_{v\in M(k)_\infty} I(A_v,\lambda_v) > -\kappa_0\, g$$

for $\mathrm{h}_F(A)$. These lower bounds were obtained in the 90s by Bost [Bos96a]. We also mention that Wagener, in his 2016 PhD thesis [Wag16, Thóréme A], has obtained the refined lower bound

$$\mathrm{h}_F(A) + \kappa_0 g \geqslant \frac{1}{[k:\mathbb{Q}]}\left(\sum_{v\in M(k)_0} I(A_v,\lambda_v)\log(Nv) + 2\sum_{v\in M(k)_\infty} I(A_v,\lambda_v)\right).$$

Motivated by Theorem 1, we also study (in [dJS18c, dJS18b]) the local invariants attached to Jacobians in more detail and relate Arakelov heights to the combinatorics and potential theory of metric graphs. Let C be a smooth projective geometrically connected curve over a number field k. Let $v \in M(k)_0$ and assume C has semistable reduction at v. Let Γ_v be a skeleton of the Berkovich curve C_v^{an}. The *tau invariant*, denoted by $\tau(\Gamma_v)$, is the 'capacity' associated to (1/2 times) the effective resistance function (see, e.g., [BR07, Corollary 14.2]). We prove the following remarkable relation. Let $\ell(\Gamma_v)$ denote the *total length* of Γ_v. Let J be the Jacobian of C endowed with its canonical principal polarization λ.

Theorem 2 ([dJS18c]). *$I(J_v, \lambda_v) = \frac{1}{8}\ell(\Gamma_v) - \frac{1}{2}\tau(\Gamma_v)$.*

This yields an efficient formula for computing the local terms in Theorem 1.

References

[Aut06] Pascal Autissier, *Hauteur de Faltings et hauteur de Néron-Tate du diviseur thêta*, Compos. Math. **142** (2006), no. 6, 1451–1458.

[BR07] Matt Baker and Robert Rumely, *Harmonic analysis on metrized graphs*, Canad. J. Math. **59** (2007), no. 2, 225–275.

[Ber90] Vladimir G. Berkovich, *Spectral theory and analytic geometry over non-Archimedean fields*, Mathematical Surveys and Monographs, vol. 33, American Mathematical Society, Providence, RI, 1990.

[Bos96a] J.-B. Bost, *Arakelov geometry of abelian varieties*, Proceedings of a conference on arithmetic geometry, technical report, Max-Planck Institut für Mat., Bonn, 1996.

[Bos96b] J.-B. Bost, *Intrinsic heights of stable varieties and abelian varieties*, Duke Math. J. **82** (1996), no. 1, 21–70.

[Fal83] Gerd Faltings, *Endlichkeitssätze für abelsche Varietäten über Zahlkörpern*, Invent. Math. **73** (1983), no. 3, 349–366.

[Fal84] Gerd Faltings, *Calculus on arithmetic surfaces*, Ann. of Math. (2) **119** (1984), no. 2, 387–424.

[FRSS18] Tyler Foster, Joseph Rabinoff, Farbod Shokrieh, and Alejandro Soto, *Non-Archimedean and tropical theta functions*, Math. Ann. **372** (2018), no. 3-4, 891–914.

[Hin93] Marc Hindry, *Sur les hauteurs locales de Néron sur les variétés abéliennes*, 1993. Unpublished manuscript, available at `https://webusers.imj-prg.fr/~marc.hindry/Neron.pdf`.

[dJ18] Robin de Jong, *Néron-Tate heights of cycles on Jacobians*, J. Algebraic Geom. **27** (2018), no. 2, 339–381.

[dJS18a] Robin de Jong and Farbod Shokrieh, *Faltings height and Néron-Tate height of a theta divisor*, 2018. Preprint available at `arXiv:1810.02637`.

[dJS18b] Robin de Jong and Farbod Shokrieh, *Metric graphs, cross ratios, and Rayleigh's laws*, 2018. Preprint available at `arXiv:1810.02638`.

[dJS18c] Robin de Jong and Farbod Shokrieh, *Tropical moments of tropical Jacobians*, 2018. Preprint available at `arXiv:1810.02639`.

[Sil86] Joseph H. Silverman, *Heights and elliptic curves*, Arithmetic geometry (Storrs, Conn., 1984), 1986, pp. 253–265.

[Wag16] Benjamin Wagener, *Géométrie arithmétique sur les variétés abéliennes: minoration explicite de la hauteur de Faltings et borne sur la torsion*, Ph.D. Thesis, 2016. Available at `https://benjamin-wagener.org/BW-Thesis-22112016.pdf`.

Gromov-Hausdorff limits of flat Riemannian surfaces and non-Archimedean geometry

Dmitry Sustretov

Let $\mathcal{X} \to \mathbb{C}^\times$ be a holomorphic family of smooth compact complex curves of genus $\geqslant 1$, and let Ω be a relative holomorphic 1-form on X. Assume that the action of the monodromy on $H^1(\mathcal{X}_t)$ for t close to 0 has a Jordan block of size 2. Consider the pseudo-Kahler metric on the fibres X_t with the Kahler form $i/2\Omega_t \wedge \bar{\Omega}_t$ and further rescale it so that the diameter of $\mathcal{X}_t$ is constantly 1. I describe the Gromov-Hausdorff limit of $\mathcal{X}_t$ as t tends to 0 in terms of the Berkovich analytification X^{an} of the variety X over $C((t))^{alg}$ associated to $\mathcal{X}$. In particular, the shape of the limit depends on the weight function wt_Ω on X^{an} associated to the form Ω. This weight function was introduced by Kontsevich and Soibelman and further studied by Mustata, Nicaise, Xu, and Temkin [KS04, MN15, NX16, Tem16].

There are two cases depending on the dimension of the limit: collapsed and non-collapsed. The limit is non-collapsed if and only if the weight function is never constant on an edge of the dual intersection complex of the special fibre of any semi-stable model of X. In the collapsed case the limit is a metric graph, the quotient of a dual intersection complex as above by an equivalence relation defined in terms of wt_Ω. In the non-collapsed case it is a union of flat surfaces corresponding to the components of the special fibre on which the function wt_Ω reaches its minimum, glued along finitely many points in a way determined by the dual intersection complex.

References

[KS04] Maxim Kontsevich and Yan Soibelman. Affine structures and non-Archimedean analytic spaces. In: P. Etingof, V. Retakh and I.M. Singer (eds). *The unity of mathematics. In honor of the ninetieth birthday of I. M. Gelfand.* Volume 244 of Progress in Mathematics. Birkhäuser Boston, Inc., Boston, MA, pages 312–385, 2006.

[MN15] Mircea Mustaţă and Johannes Nicaise. Weight functions on non-archimedean analytic spaces and the Kontsevich–Soibelman skeleton. *Algebraic Geometry*, 2(3):365–404, 2015.

[NX16] Johannes Nicaise and Chenyang Xu. Poles of maximal order of motivic zeta functions. *Duke Mathematical Journal*, 165(2):217–243, 2016.

[Sus18] Dmitry Sustretov. Gromov-Hausdorff limits of flat Riemannian surfaces and non-Archimedean geometry. *arXiv preprint arXiv:1802.03818*, 2018.

[Tem16] Michael Temkin. Metrization of differential pluriforms on Berkovich analytic spaces. In *Nonarchimedean and Tropical Geometry*, pages 195–285. Springer, 2016.

Anticanonical metrics as operator norms of Cartier operators

Eric Canton

A number of complications arise when attempting to work with singular varieties and Kähler differentials in positive characteristic due the existence of the Frobenius morphism. In this talk, I propose an approach to seminorms on canonical bundles that makes direct use of the Frobenius via Grothendieck duality.

1. Cartier operators and anti-pluricanonical forms

Suppose X is a normal variety over a field k of characteristic $p > 0$, assumed to be perfect. The aboslute Frobenius morphism $F : X \to X$ is finite, since X is of finite type over a perfect field. Consequently, we can apply Grothendieck duality to F, which has a very simple form in this case. The category of coherent sheaves of $\mathcal{O}_X$-modules and $F_*\mathcal{O}_X$-modules are equivalent [2], and so the coherent $F_*\mathcal{O}_X$-module $\mathcal{H}om_X(F_*\mathcal{O}_X, \mathcal{G})$ corresponds uniquely, up to isomorphism, to some coherent sheaf $F^!\mathcal{G}$. Said differently, $F_*F^!\mathcal{G} = \mathcal{H}om_X(F_*\mathcal{O}_X, \mathcal{G})$ uniquely specifies $F^!\mathcal{G}$. The two important examples we will need are:

(1) $F^!\mathcal{O}_X = \mathcal{H}om_X(F_*\mathcal{O}_X, \mathcal{O}_X)$. Note that the $\mathcal{O}_X$-module structure is in the "first coordinate" (i.e. as an $F_*\mathcal{O}_X$-module).

(2) $F^!\omega_X \cong \omega_X$, since $F_*F^!\omega_X = \mathcal{H}om_X(F_*\mathcal{O}_X, \omega_X)$ is well-known to be (isomorphic to) $F_*\omega_X$.

Putting these together, an easy consequence is the following.

Proposition 1. $F_*\omega_X^{\otimes(1-p)} \cong F^!\mathcal{O}_X$ *as* $F_*\mathcal{O}_X$*-modules.*

Consequently, we can interpret $\theta \in \Gamma(X, \omega_X^{\otimes(1-p)})$ as an $\mathcal{O}_X$-linear map $\psi_\theta : F_*\mathcal{O}_X \to \mathcal{O}_X$. Alternatively, to any nonzero $\psi \in \Gamma(X, F^!\mathcal{O}_X)$ one can associate an (essentially unique) $\theta_\psi \in \Gamma(X, \omega_X^{\otimes(1-p)})$, and thus also a effective divisor

$$\Delta_\psi := \frac{\mathrm{div}_X(\theta_\psi)}{p-1} \tag{$\star$}$$

with the property that $(p-1)(K_X + \Delta) \sim 0$; sometimes such an (X, Δ) would be described as a $\mathbb{Q}$*-log Calabi Yau pair* with index $p-1$. Summarizing, we have a bijection on normal varieties X over perfect fields:

$$\left\{ \begin{array}{c} \text{nonzero } \psi \in \Gamma(X, F^!\mathcal{O}_X) \\ \text{up to global units } u \in \Gamma(X, F_*\mathcal{O}_X^\times) \end{array} \right\} \leftrightarrow \left\{ \begin{array}{c} \text{Cartier divisors } \Delta \geqslant 0 \\ \text{such that } (p-1)(K_X + \Delta) \sim 0 \end{array} \right\}$$

2. Operator norms of Cartier operators

Suppose k is equipped with a norm, fix $\psi \in \Gamma(X, F^!\mathcal{O}_X)$ and $x \in X^{\mathrm{an}}$. Suppose $\ker(x) = Q \in X$, and set $R = \mathcal{O}_{X,Q}$. The unique way to equip F_*R with an R-module norm compatible with x is as $|F_*f|_x := |f|_x^{1/p}$. Since $\psi_Q : F_*R \to R$ is a map of R-seminormed modules, it has an operator norm in the sense of functional analysis.

Definition 1. *The* operator norm *of* ψ *at* $x \in X^{\mathrm{an}}$ *is*

$$\|\psi\|_{\mathrm{op},x} := \inf\{C > 0 \,:\, |\psi(F_*f)|_x \leqslant C|f|_x^{1/p} \text{ for all } f \in R\}.$$

In the remainder of the talk, we compare $\|\cdot\|_{\mathrm{op},\bullet}$ with other seminorms on canonical bundles seen in this workshop, namely Temkin's canonical (Kähler) seminorms [10] and the log discrepancy. Summarizing:

[2] which is a bit confusing in this case, since as sheaves of abelian groups $\mathcal{O}_X$ is the same as $F_*\mathcal{O}_X$; the only difference is how we view $F_*\mathcal{O}_X$ as coherent sheaf of $\mathcal{O}_X$-modules

Suppose X is smooth. Let $x \in X^{\mathrm{an}}$, and equip ω_X with a seminorm $\|\cdot\|'_x$ as follows. Let $\theta \in \Gamma(X, \omega_X)$; working locally near $\ker(x) \in X$, we can consider $F_*\theta^{\otimes(1-p)} \in F_*\omega_{X,\ker(x)}$. To this, we can associate $\psi_\theta \in F^!\mathcal{O}_{X,Q}$, and thus $\|\psi_\theta\|_{\mathrm{op},x}$. Define

$$\|\theta\|'_x := \|\psi_\theta\|_{\mathrm{op},x}^{\frac{p}{1-p}}.$$

Then $\|\theta\|'_x = \|\theta\|_{\omega,x}$ for all $x \in X^{\mathrm{an}}$, where $\|\theta\|_{\omega,x}$ is Temkin's seminorm.

Suppose k is trivially valued. For $\psi \in \Gamma(X, F^!\mathcal{O}_X)$, we have the divisor Δ_ψ from ($\star$). Now if $E \subset Y$ is a prime divisor on a normal variety admitting a proper birational morphism $\pi : Y \to X$, we have the divisorial point $\mathrm{ord}_E \in X^{\mathrm{an}}$. One defines a $\mathbb{Q}$-Weil divisor $\Delta_{\psi,Y}$ on Y via

$$K_Y + \Delta_{\psi,Y} = \pi^*(K_X + \Delta_\psi).$$

The *log discrepancy of* (X, Δ_ψ) *along* E is

$$A(E; X, \Delta_\psi) = 1 - \mathrm{ord}_E(\Delta_{\psi,Y}).$$

The key result here is

$$A(E; X, \Delta_\psi) = \frac{\log \|\psi\|_{\mathrm{op},\mathrm{ord}_E}}{p-1}$$

for all $E \subset Y$. This result is due to Cascini, Mustaţă, and Schwede in (something close to) this form; however, knowledge of results of this type goes back at least to [7, 9] and very explicitly appear in [5].

Continuing with k trivially valued, suppose also that log resolutions exist in characteristic p; then one has available the description of $X^{\mathrm{bir}} \cap X^{\sqsupset}$ (the space of valuations centered on X) used in [8] to extend log discrepancies to arbitrary valuations. One of the main theorems in the speaker's thesis [4] is that

$$(\star\star) \qquad A(x; X, \Delta_\psi) = \sup_{n \geqslant 1} \frac{\log \|\psi^n\|_{\mathrm{op},x}}{p^n - 1}$$

for all $x \in X^{\mathrm{bir}} \cap X^{\sqsupset}$. Here, $\psi^n = \psi \circ F_*\psi^{n-1}$ (so $\psi^2 = \psi \circ F_*\psi$). Thus, log discrepancy becomes the "spectral radius seminorm" of $\psi \in \Gamma(X, F^!\mathcal{O}_X)$.

Taking the right hand side of ($\star\star$) as the definition of $A(x; X, \Delta_\psi)$ (without assuming log resolutions exist) gives a function $A(-; X, \Delta_\psi) : X^{\sqsupset} \to [-\infty, \infty]$ with properties mirroring those known to experts on log discrepancy in characteristic zero. For example:

- $A(-; X, \Delta_\psi)$ is lower-semicontinuous [4, 8, 3, 1].
- When X is smooth, there exist valuations minimizing the log canonical threshold $\mathrm{lct}(-; X, \mathfrak{a}_\bullet)$ of any graded sequence of ideals $\mathfrak{a}_\bullet$ on X. (assuming there exists $x \in X^{\sqsupset}$ with $\mathrm{lct}(x; X, \mathfrak{a}_\bullet) < \infty$). See [4, 8].

The main point here is that these results rely on resolutions of singularities, and can be quite delicate and involved, over fields of characteristic zero. In positive characteristics, the proofs are significantly simpler, often amounting to elementary (if messy) real analysis.

(Brought to the speaker's attention by Temkin after this talk.) Brezner and Temkin use a construction nearly identical to the Cartier operator seminorms presented here in their study [2] of minimally wild covers of Berkovich curves. They use that the canonical (trace) morphism $\psi_f : f^*\Omega_X \to \Omega_Y$ is a non-zero morphism between seminormed sheaves (where $f : X \to Y$ is a finite morphism of Berkovich curves) and so naturally this morphism has an operator seminorm. In their setting, they realize this as the *different* δ_f. When $f = F : X \to X$ is the Frobenius, $F^*\Omega_X = \Omega_X^{\otimes p}$, and so ψ_F can be viewed as a section $\mathcal{O}_X \to \Omega_X^{\otimes(1-p)}$.

Questions:

(1) What can we say about the minimal locus of $\|\psi\|_{\mathrm{op},\bullet}$ as a function on X^{an}? This should be like a skeleton of ψ (in the sense of Kontsevich-Soibelman); in analogy with other settings, we expect combinatorial structures on this locus, and for there to be links with tropicalizations.

(2) When X is only normal, or when k is not perfect, how does Temkin's Kähler seminorm compare with the operator seminorms?

References

[1] H. Blum. *K-stability and Singularities.* Ph.D. thesis, U. of Michigan (2018).

[2] U. Brezner and M. Temkin. *Lifting problem for minimally wild covers of Berkovich curves.* arXiv: 1709.10416 (2016).

[3] S. Boucksom, T. de Fernex, C. Favre, and S. Urbinati. *Valuation spaces and multiplier ideals on singular varieties*, London Math. Soc. Lecture Note Ser., 417 (2015), 29–51.

[4] E. Canton. *Berkovich log discrepancies in positive characteristic*, arXiv: 1711.03002 (2018).

[5] N. Hara and K.-i. Watanabe. *F-regular and F-pure rings vs. log terminal and log canonical singularities*, Journal of Algebraic Geometry, **(2)** 11 (2002), 363–392.

[6] M. Hochster and C. Huneke. *Tight closure and strong F-regularity*, Mémoires de la Société Mathématique de France. Nouvelle Série, 38 (1989), 119–133.

[7] V. B. Mehta and A. Ramanathan. *Frobenius splitting and cohomology vanishing for Schubert varieties*, Annals of Mathematics, **(2)** 122.1 (1985), 27–40.

[8] M. Jonsson and M. Mustaţă. *Valuations and asymptotic invariants for sequences of ideals*, Ann. Inst. Fourier (Grenoble), **(6)** 62 (2013), 2145–2209.

[9] Ramanan, S. and Ramanathan, A.. *Projective normality of flag varieties and Schubert varieties*, Inventiones Mathematicae, **(2)** 79 (1985), 217–224.

[10] M. Temkin. *Metrization of Differential Pluriforms on Berkovich Analytic Spaces*, in the book "Nonarchimedean and Tropical Geometry" (Springer Intl. Publ.), (2016) 195–285. Editors: M. Baker and S. Payne.

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