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Topologie
(hybrid meeting)

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Abstract. The Oberwolfach conference "Topologie" is one of only a few opportunities for researchers from many different areas in algebraic and geometric topology to meet and exchange ideas. On this occasion, because of the Corona pandemic, only about 20 participants attended in person, but another ~ 25 attended online. Speakers were selected from both groups. A topic of special interest emphasized at the workshop was the rational homotopy theory of embedding spaces and relations to graph complexes and formality. Two 50 minute lectures on this theme were given by Thomas Willwacher, and one by Victor Turchin. The rest of the program covered a wide range of topics, among them: homotopy properties of diffeomorphism groups of high dimensional manifolds, advances in the classification of high-dimensional highly connected smooth manifolds, parametrized algebraic surgery in relation to hermitian algebraic K-theory, other advances in and geometric applications of algebraic K-theory, stable homotopy interpretation of link invariants, geometry of surface bundles and cohomology of mapping class groups, boundary concepts in geometric group theory, and Koszul duality for operads.

Mathematics Subject Classification (2010): 55-xx, 57-xx.

Introduction by the Organizers

This *Topologie* conference in Oberwolfach was organized by a committee consisting of Mark Behrens, Ruth Charney, Soren Galatius and Michael Weiss. Because of the Corona pandemic, only about 20 mathematicians from many different areas of algebraic and geometric topology attended in person and another 25 attended (many of) the lectures online. Similarly, two of the organizers were able to participate in person (Galatius and Weiss), the other two participated online. We are indebted to Nathalie Wahl who, as one of the in-person participants, agreed at short notice to help with the local organization. We are also indebted to Fabian Hebestreit who took on the job of a "technical assistant" throughout the meeting.

The talks came in several formats. There were 11 regular 50 minute talks given in the afternoon to allow "virtual" participation from America; 4 morning talks at 30 minutes each and 2 morning talks at 50 minutes; and the 3 keynote talks by Willwacher and Turchin at 50 minutes each (in the afternoon).

Keynote speaker Thomas Willwacher (participating "virtually") gave two talks on joint work with B. Fresse and V. Turchin describing new developments in the rational homotopy theory of spaces of smooth embeddings, especially embeddings of manifolds into euclidean space $\mathbb{R}^n$ subject to boundary conditions (and codimension conditions). The topic has a close connection to operad theory via manifold calculus, but in Willwacher's talks this was combined with formality results which constitute a highlight in the theory of operads (associated with the name of Kontsevich). Graph complexes are the machinery of choice to state, prove and exploit such formality theorems. It was striking to see how traditional (algebraic) models for rational homotopy theory, broadly speaking Quillen's Lie algebra formulation, unfold their potential here in combination with graph complexes. In Victor Turchin's talk, it was shown that some of the most important formulas of the theory have classical precursors, often at the level of π_0 (embeddings up to isotopy).

The remaining talks of the conference covered a wide variety of topics including homotopy properties of diffeomorphism groups of high dimensional manifolds, advances in the classification of $2n$-dimensional $(n-1)$-connected smooth manifolds, parametrized algebraic surgery in relation to hermitian algebraic K-theory, vanishing results for chromatic localizations in algebraic K-theory, applications of algebraic K-theory to scissors congruence and cutting and pasting, stable homotopy interpretation of link invariants, functor calculus methods applied to knot theory, operads in QFT, arc complexes and finite generation properties of groups, geometry of surface bundles and surface homeomorphisms and cohomology of mapping class groups, boundary concepts in geometric group theory, and Koszul duality for operads.

Speakers were instructed to give talks that could be appreciated by an audience of topologists of many different kinds, and they were generally very successful in doing so. However, there was a noticeable language barrier between (a) participants coming from geometric group theory and very low-dimensional topology and (b) those coming from algebraic topology and high-dimensional manifold theory. We hope to make this less pronounced in future meetings of this kind by inviting more speakers working in the geometric theory of 3-manifolds and 4-manifolds. (This was also our policy in previous meetings, e.g. the 2018 meeting.)

We now briefly describe the themes of the remaining regular talks (both 50 minute talks and 30 minute talks). For more detailed information, also information on coauthors, see the talk summaries.

Manuel Krannich explained to us how cobordism categories and parametrized surgery can be used to re-prove, and improve on, Igusa's stability theorem for smooth concordances and smooth h-cobordism spaces in many cases. Somewhat similar ideas were advertised in Alexander Kupers' talk which described new results on the rational homotopy type of the space of diffeomorphisms of an even-dimensional disk D^{2n} for $n \geq 3$. Cobordism categories and parametrized surgery were also prominent, but in an algebraic setting, in the talks by Markus Land and Fabian Hebestreit. One of the major outcomes was the resolution of an old problem in hermitian algebraic K-theory. Jeremy Hahn gave an overview of recent work on the classification of $(n-1)$-connected $2n$-manifolds, based on an improved understanding of patterns in the classical Adams spectral sequence converging to the homotopy of the sphere spectrum. Georg Tamme spoke on chromatic versions of classical theorems stating that, under mild conditions, a map of rings or ring spectra which is a local weak equivalence induces a local weak equivalence of the algebraic K-theory spectra. Gijs Heuts gave a talk on Koszul duality for topological operads (and co-operads), resolving the question as to which operads (and co-operads) are Koszul dualizable. PROPs, which are a generalization of operads, made an appearance in the talk by Marcy Robertson. The main point was that specific examples of "wheeled" PROPs defined in geometric terms encode solutions to the the Kashiwara-Vergne problem in the theory of Lie algebras. Nitu Kitchloo described a new categorification of well-known link invariants using Markov's presentation of links by braids and a construction making spectra (in the sense of stable homotopy theory) out of braids. Danica Kosanovic's talk was on the usefulness of manifold calculus in classical knot theory and the finite type invariants for knots due to Vassiliev. One of the main points was a surjectivity result stating that elements in a finite Taylor approximation to a space of knots can be realized up to isotopy. Jing Tao spoke on the Thurston classification of surface (self-)homeomorphisms up to isotopy. She gave a new proof relying on hyperbolic structures, new representatives for the pseudo-Anosov classes and a new description of the three types in terms of the Thurston metric on Teichmüller space. Sam Payne's talk was on the rational homology of the moduli space of genus g (complex) curves and relations to moduli spaces of stable tropical curves of genus g. Ursula Hamenstädt talked about a new inequality for surface bundles over surfaces, relating signature and Euler characteristic of the total space. In Yulan Qing's talk, a new concept of *boundary* for (Caley graphs of) certain groups was introduced, generalizing the Gromov boundary for hyperbolic groups. Kai-Uwe Bux talked on a combinatorial notion closely related to arc complexes on surfaces and also to matching complexes associated with graphs. He used this to obtain new results on higher finiteness properties of groups. Corey Bregman reported on the development of a concept of outer space for right-angled Artin groups, analogous to Teichmüller space for a surface and the Culler-Vogtmann outer space for a finitely generated free group.

Our thanks go to the institute for making the conference possible in these difficult times. The dedication and resourcefulness of the Oberwolfach staff was noted and pointed out by many participants.

Workshop (hybrid meeting): Topologie

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Abstracts

Moduli spaces of h-cobordisms of discs

Manuel Krannich

The classical s-cobordism theorem identifies the set of isomorphism classes of h-cobordisms on a compact connected smooth manifold M of dimension $d \geq 5$ with a quotient of the first algebraic K-group of $\mathbf{Z}[\pi_1 M]$—the *Whitehead group*

$$\mathrm{Wh}(\mathbf{Z}[\pi_1 M]) := K_1(\mathbf{Z}[\pi_1 M])/\pm\pi_1 M.$$

This identification can be seen as a computation of the components of the *moduli space* $H(M)$ of h-cobordisms on M, and it turns out that the higher homotopy type of $H(M)$ is related to algebraic K-theory in a similar way: foundational work of Waldhausen [4] provides a canonical map

$$H(M) \to \Omega^\infty \mathrm{Wh}^{\mathrm{Diff}}(M)$$

to the infinite loop space of a spectral refinement of the Whitehead group—the *smooth Whitehead spectrum*

$$\mathrm{Wh}^{\mathrm{Diff}}(M) := K(\mathbf{S}[\Omega M])/(\Sigma^\infty_+ M).$$

On path-components, this induces the isomorphism provided by the s-cobordism theorem, but this map is known to be more highly connected by a combination of two major results in the parametrised study of high-dimensional manifolds:

(1) Taking cylinders induces a stabilisation map $H(M) \to H(M \times [0,1])$ which is compatible with the map above and has been shown by Igusa [1] to be approximately $(d/3)$-connected, based on parametrised Morse theory.
(2) The map $\mathrm{hocolim}_k H(M \times [0,1]^k) \to \mathrm{Wh}^{\mathrm{Diff}}(M)$ is an equivalence by Waldhausen–Jahren–Rognes' *stable parametrised h-cobordism theorem* [5].

In my talk, I explained a new approach to study the relation between $H(M)$ and $\mathrm{Wh}^{\mathrm{Diff}}(M)$ in the case $M = D^{2n}$. So far, it has led to the following.

Theorem 1. *There exists a $(n-2)$-connected map $H(D^{2n}) \to \Omega^\infty \mathrm{Wh}^{\mathrm{Diff}}(*)$.*

Remark 2. *In* [2]*, based on a different strategy, it is shown that, as long as one is willing to invert primes that are large with respect to the dimension and the degree, then the map in Theorem 1 becomes twice as connected (see also* [3]*).*

In contrast to (1) and (2)—which Theorem 1 recovers in the case $M = D^{2n}$(with an improved range)—the proof of Theorem 1 does not involve stabilising the dimension, but instead relates $H(D^{2n})$ directly to algebraic K-theory. It relies on several ingredients of which some might be of independent interest, such as an analysis of the *moduli space of block-thickenings* of a finite complex, which is largely geometric, or a general homological vanishing result for the stable twisted homology of $\mathrm{BGL}(\mathbf{S}[\Omega M])$. I will explain the latter in a special case.

Stable twisted homology of $\mathrm{GL}_g(\mathbf{S})$. As $\pi_1 \mathrm{BGL}_g(\mathbf{S}) \cong \mathrm{GL}_g(\mathbf{Z})$, a local system M_g over $\mathrm{BGL}_g(\mathbf{S})$ is a $\mathbf{Z}[\mathrm{GL}_g(\mathbf{Z})]$-module. Given a sequence $M_g \to M_{g+1}$ of compatible local systems, we can form the colimit

$$\mathrm{H}_*(\mathrm{BGL}(\mathbf{S}); M_\infty) = \mathrm{colim}_g \mathrm{H}_*(\mathrm{BGL}_g(\mathbf{S}); M_g) \tag{1}$$

and ask for conditions on the sequence M_g that ensure vanishing of (1). If M_g is constant, then (1) agrees with the homology of $\Omega_0^\infty K(\mathbf{S})$, so it can be very nontrivial. Another example is $M_g = \mathrm{Hom}_{\mathbf{Z}}(\mathbf{Z}^g, \mathbf{Z}^g)$ in which case (1) does not vanish either; for instance $\mathrm{H}_0(\mathrm{BGL}(\mathbf{S}); M_\infty) = \mathrm{colim}_g \mathrm{Hom}_{\mathbf{Z}}(\mathbf{Z}^g, \mathbf{Z}^g)_{\mathrm{GL}_g(\mathbf{Z})}$ does not. Note however that for $M_g = \mathbf{Z}^g$, the analogous group $\mathrm{H}_0(\mathrm{BGL}(\mathbf{S}); M_\infty) = \mathrm{colim}_g (\mathbf{Z}^g)_{\mathrm{GL}_g(\mathbf{Z})}$ does vanish and this is no coincidence: the sequence $M_g = \mathbf{Z}^g$ is part of a class of sequences M_g that are induced by a an abelian-group valued functor M on the category $\mathrm{P}(\mathbf{Z})$ of finitely generated projective modules via

$$M_g := M(\mathbf{Z}^g) \xrightarrow{M(\mathbf{Z}^g \subset \mathbf{Z}^g \oplus \mathbf{Z})} M(\mathbf{Z}^{g+1}) =: M_{g+1},$$

where $\mathrm{GL}_g(\mathbf{Z})$ acts by functoriality. Such a functor is called *reduced* if $M(0) = 0$ and it is called *analytic* if it a colimit of polynomial functors. A simplified version of the homological vanishing result that goes in the proof of Theorem 1 shows that *any* sequence M_g that extends to such a functor has vanishing stable homology.

Theorem 3. *For an abelian-group valued functor M on $\mathrm{P}(\mathbf{Z})$ that is reduced and analytic, the stable homology $\mathrm{H}_*(\mathrm{BGL}(\mathbf{S}); M_\infty)$ vanishes.*

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Top weight cohomology of M_g

Sam Payne

(joint work with Melody Chan and Søren Galatius)

The weight filtration on the cohomology of an algebraic variety (or Deligne-Mumford stack) X of dimension d is an increasing filtration of rational vector spaces,

$$0 \subset W_0 H^*(X; \mathbb{Q}) \subset \cdots \subset W_{2d} H^*(X; \mathbb{Q}) = H^*(X; \mathbb{Q}).$$

This is functorial for all natural maps between cohomology groups that are induced by algebraic morphisms. Diffeomorphic complex manifolds (or orbifolds) may have many different realizations as the analytifications of algebraic varieties or stacks. Each choice of an algebraic structure induces a weight filtration on the singular cohomology of the manifold, and in some cases this turns out to be useful, e.g., for proving vanishing or non-vanishing results by separately studying different graded pieces of the weight filtration.

In this talk, I focused on the cohomology of M_g, the moduli space of smooth complex algebraic curves of genus g, which is a classifying space for the mapping class group Mod_g. This algebraic realization of BMod_g induces a weight filtration on the cohomology of the mapping class group, and we focus, in particular, on the top graded piece of the weight filtration, i.e., $\mathrm{Gr}^W_{2d} H^*(X;\mathbb{Q})$, in the case where $X = M_g$ and $d = 3g-3$ is its complex dimension.

Standard arguments from Hodge theory allow us to identify the top weight cohomology $\mathrm{Gr}^W_{2d} H^*(X;\mathbb{Q})$ with the singular homology of the dual complex of the boundary divisor in some (or any) simple normal crossing compactification. (Such compactifications exist, by famous classical algebraic geometry theorems of Nagata and Hironaka.) With somewhat more care, one can define the dual complex of the boundary divisor in a normal crossing compactification, where the boundary components may be singular, and self-intersect, with nontrivial monodromy, as is the case for the Deligne-Mumford stable curves compactification $\overline{M}_g$ of M_g.

Having set up this technical construction, we study the dual complex of the boundary divisor $D_g = \overline{M}_g \setminus M_g$. By previous joint work with Abramovich and Caporaso [1], this is naturally identified with a moduli space of stable tropical curves of genus g and volume 1. This is also the quotient of the simplicial completion of Culler-Vogtmann Outer Space by the action of $\mathrm{Out}(F_g)$ [6].

By examining the cellular chain complex of this tropical moduli space, and by proving contractibility of the image of the boundary of the simplicial completion of Outer Space (the locus of stable tropical curves with vertices of positive weight), we identify its rational homology with the homology of Kontsevich's graph complex $K^{(g)}$. Grading conventions on this graph complex vary in the literature. If we grade the graph complex so that the degree of a graph is its number of edges, then a deep theorem of Willwacher [7], identifies $\prod_g (H_{2g}(K^{(g)}))^\vee$ with the Grothendieck-Teichmüller Lie algebra. Another deep theorem of Brown from Grothendieck-Teichmüller theory [2] shows that this Lie algebra contains a free Lie subalgebra generated by the Soulé classes $\sigma_g \in H_{2g}(K^g)^\vee$, for odd $g \geq 3$. One then deduces that $\dim H_{2g}(K^{(g)})$ grows exponentially with g, and hence so does $H^{4g-6}(M_g;\mathbb{Q})$. This disproves the conjecture of Kontsevich [5, Conjecture 7C] and of Church, Farb, and Putman [4, Conjecture 9] that, for fixed k, the cohomology groups $H^{4g-4-k}(M_g;\mathbb{Q})$ should vanish for $g \gg 0$.

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Polygons, Horogons, and the Thurston Classification of Surface Homeomorphisms

Jing Tao
(joint work with Camille Horbez)

Let S be a closed surface of genus $g \geq 2$. The *mapping class group* $\Gamma(S)$ of S is the group of isotopy classes of orientation-preserving homeomorphisms of S. In the 1970s, Thurston [2, 5] gave a characterization of the elements in $\Gamma(S)$, completing the work first initiated by Nielsen. This is known as the Thurston (or Nielsen-Thurston) Classification Theorem, which goes as follows.

Theorem 1 (Thurston Classification). *For any element* $\phi \in \Gamma(S)$*, there is a representative* $f \in Homeo^+(S)$ *of* ϕ *such that*

- f *is periodic, i.e. some power of* f *is the identity;*
- f *is reducible, i.e.* f *preserves a closed* 1*–manifold on* S*; or*
- f *is pseudo-Anosov, i.e. there exist a pair of transverse (singular) measured foliations* F_+ *and* F_- *on* S*, and* $K > 1$*, such that* $f(F_\pm) = K^{\pm 1} F_\pm$*.*

An element $\phi \in \Gamma(S)$ is called periodic, reducible, and pseudo-Anosov accordingly. Note that the three types are not mutually exclusive: ϕ can be periodic and reducible. However, a pseudo-Anosov mapping class is never periodic or reducible.

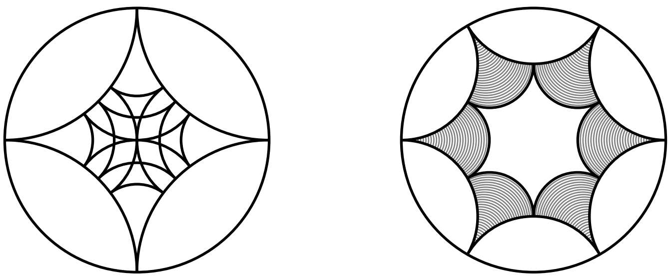
A well-known proof of the Thurston classification is due to Bers who rephrased the problem in terms of extremal quasi-conformal maps between complex structures on S [1]. Bers' version of the classification can be stated as follows.

Theorem 2 ([1]). *Suppose* $\phi \in \Gamma(S)$ *is not periodic or reducible, then there exist a complex structure* X *on* S*, a quadratic differential* q *on* X*, a constant* $K > 1$*, and a representative* $f \in Homeo^+(S)$ *of* ϕ*, such that the following statements hold.*

- *The map* $f\colon X \to X$ *has quasi-conformal constant* K^2*, which is the minimal quasi-conformal constant among all maps of* X *representing* ϕ*.*
- *The map* f *preserves the leaves of the vertical and horizontal foliations* V_q *and* H_q *of* q*, acting by* $f(V_q) = KV_q$ *and* $f(H_q) = K^{-1}H_q$*.*

By considering extremal Lipschitz maps between hyperbolic structures on S, we derive a new proof of the Thurston classification as well as new representatives for pseudo-Anosov mapping classes. In the following, we will set up the background needed to state our main result.

Polygons and horogons. Let $\mathbb{H}$ be the Poincaré disc model of the hyperbolic plane. An ideal polygon P in $\mathbb{H}$ with n vertices is called *regular* if P is isometric to the ideal n–gon in $\mathbb{H}$ with vertices lying on the n–th roots of unity in $S^1 = \partial\mathbb{H}$. By a *horogon* we will mean a polygon h in $\mathbb{H}$ whose sides are made up horocyclic segments with angle 0 at all vertices. Non-adjacent sides of h are allowed to meet but must be tangent when this happens. A horogon h determines an ideal polygon P in which it is *inscribed*, whose ideal vertices are the horocyclic centers of the sides of h. We also say P *circumscribes* h. An ideal polygon is called *circumscribing* if it circumscribes some horogon. Not all ideal polygons are circumscribing and an inscribed horogon may not be unique. Some examples: (1) Every regular ideal polygon circumscribes a unique equilateral horogon. (2) Every ideal triangle is regular, and the only inscribed horogon is the equilateral one. (3) The only circumscribing ideal quadrilateral is regular, which circumscribes a family of horogon. For each inscribed h, if a and b are two adjacent side lengths, then $ab = 2$ and $1 \leq a, b \leq 2$. See the left side of the figure below.



Given a horogon h inscribed in P, then h induces a measured foliation $F_P(h)$ with support on $P \setminus \mathrm{int}(h)$, whose leaves are horocycles, and whose measure on transverse arcs coincides with the Lebesque measure along the sides of P. Call $F_P(h)$ the *horocyclic foliation dual to* (P, h). See the right side of the figure above.

Filling geodesic laminations. A *geodesic lamination* on a hyperbolic surface X is a nonempty closed subset λ of X foliated by simple complete geodesics, called the *leaves* of λ. A geodesic lamination λ is *filling* if each complementary region of $X \setminus \lambda$ is an ideal polygon, and *maximal* if each complementary region is an ideal triangle. Given a filling geodesic lamination λ on X, enumerate the complementary polygons of $X \setminus \lambda$ by $P_1, \ldots, P_k$. We say a sequence $H = (h_1, \ldots, h_k)$ of horogons is *inscribable* in X if each h_i is inscribable in P_i. We say X is λ*–circumscribing* if each P_i is circumscribing, and X is λ*–symmetric* if each P_i is regular. When λ is maximal, then X is always λ–circumscribing and λ–symmetric.

In [4], Thurston defined the *horocyclic foliation* dual to a maximal geodesic lamination. We generalize this to a filling geodesic lamination λ on X for which X is λ–circumscribing. In this case, let $H = (h_1, \ldots, h_k)$ be inscribable in X, and let $F_i = F_{P_i}(h_i)$ be the horocyclic foliation dual to (P_i, h_i). The tangent line field of the union of F_i has a continuous extension to the leaves λ which is Lipschitz. Integrating then yields a measured foliation $F_X(\lambda, H)$ with support on $X - \bigcup_i \mathrm{int}(h_i)$ that extends each F_i. Call $F_X(\lambda, H)$ the horocyclic foliation on X dual to (λ, H). When X is λ–symmetric and each h_i is the equilateral horogon in

P_i, then $F_X(\lambda) = F_X(\lambda, H)$ is the *symmetric horocyclic foliation* dual to λ. When λ is maximal, then $F_X(\lambda)$ coincides with Thurston's construction in [4].

Main results. We now state our version of the Thurston Classification Theorem.

Theorem 3 ([3]). *Suppose $\phi \in \Gamma(S)$ is not periodic or reducible, then there exist a hyperbolic structure X on S, a perfect (no isolated leaves) and filling geodesic lamination λ on X for which X is λ–symmetric, a constant $K > 1$, and a representative $f \in Homeo^+(S)$ of ϕ, such that the following statements hold.*

- *The map $f\colon X \to X$ has Lipschitz constant K, which is the minimal Lipschitz constant among all maps of X representing ϕ.*
- *The map f preserves the leaves of λ and the symmetric horocyclic foliation $F = F_X(\lambda)$ dual to λ, acting by $f(F) = KF$.*

Our proof of the Thurston classification is inspired by Bers' proof. The main tool is the Teichmüller space $\mathcal{T}(S)$ of S. While Bers used the Teichmüller metric on $\mathcal{T}(S)$, which is suited for comparing complex structures on S, we use the Thurston metric [4] which is better suited for comparing hyperbolic structures on S. The mapping class group $\Gamma(S)$ acts on $\mathcal{T}(S)$ by isometries with respect to either metric. The starting point of Bers' and our proof is to show that a non-periodic irreducible $\phi \in \Gamma(S)$ must act on $\mathcal{T}(S)$ as a hyperbolic isometry. From here the details of the two proofs diverge due to the diverging behavior of the two metrics.

We end this abstract with the classification of isometries of the Thurston metric which is one application of Theorem 3. The best way to state our result is to contrast it with the known result for the Teichmüller metric.

Theorem 4. *Consider $\mathcal{T}(S)$ equipped with the either the Teichmüller metric or the Thurston metric. Then for any $\phi \in \Gamma(S)$,*

Teichmüller metric [1]	*Thurston Metric* [3]
ϕ is elliptic iff ϕ is periodic.	*Same.*
ϕ is parabolic iff ϕ is reducible but not periodic.	*ϕ is parabolic iff some power of ϕ is a multi-twist.*
ϕ is hyperbolic iff ϕ is pseudo-Ansoso.	*ϕ is hyperbolic iff ϕ has a pseudo-Anosov component.*

When ϕ is reducible but has a pseudo-Anosov component, then it has positive translation length in either metric, but its Teichmüller translation length is not realized (hence ϕ is parabolic on the left), while its Thurston translation length is (hence hyperbolic on the right). The realization of the Thurston translation length is the heart of our theorem which requires the representatives that we find for pseudo-Anosov mapping classes from Theorem 3.

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Koszul duality and a conjecture of Francis–Gaitsgory

Gijs Heuts

Moore [6] constructed a duality between the homotopy theories of augmented associative differential graded algebras and of coaugmented coassociative differential graded coalgebras, using the bar and cobar constructions. In his work on rational homotopy theory, Quillen [7] constructed a different (but similar) duality between the homotopy theories of differential graded Lie algebras and of cocommutative coaugmented coalgebras over $\mathbb{Q}$, which can be phrased as an adjoint pair of functors

$$\mathrm{Lie}(\mathrm{Ch}_{\mathbb{Q}}) \underset{\mathrm{prim}}{\overset{\mathrm{CE}}{\rightleftarrows}} \mathrm{coCAlg}^{\mathrm{aug}}(\mathrm{Ch}_{\mathbb{Q}}).$$

Here the left adjoint is the Chevalley–Eilenberg complex, whereas the right adjoint takes the 'derived primitives' of a coalgebra; the latter can also be explicitly defined in terms of a certain cobar construction. In the work of Ginzburg–Kapranov [5] and Getzler–Jones [4], both of these algebraic dualities were recognized as special instances of a general phenomenon, now often referred to as *Koszul duality.* Let us review a rather general formulation of it (cf. [3]).

Let $\mathcal{C}$ be a presentable, stable, symmetric monoidal ∞-category (such as that of spectra, or of chain complexes over a commutative ring k, or of modules over a commutative ring spectrum, etc.) and let $\mathcal{O}$ be an operad in $\mathcal{C}$ with $\mathcal{O}(1) = \mathbf{1}$ (with $\mathbf{1}$ the monoidal unit) and $\mathcal{O}(0)$ (so $\mathcal{O}$ is *nonunital*). Thinking of $\mathcal{O}$ as a monoid in the ∞-category of symmetric sequences in $\mathcal{C}$, one can form its *bar construction* $B\mathcal{O}$, which is a cooperad in $\mathcal{C}$. Then there is an adjoint pair of functors

$$\mathrm{Alg}_{\mathcal{O}}(\mathcal{C}) \underset{\mathrm{prim}_{B\mathcal{O}}}{\overset{\mathrm{indec}_{\mathcal{O}}}{\rightleftarrows}} \mathrm{coAlg}^{\mathrm{dp}}_{B\mathcal{O}}(\mathcal{C}),$$

where:

(1) The functor $\mathrm{indec}_{\mathcal{O}}$ (resp. $\mathrm{prim}_{B\mathcal{O}}$) takes the *derived indecomposables* of an $\mathcal{O}$-algebra (resp. the derived primitives of a $B\mathcal{O}$-coalgebra). These indecomposables can be constructed as the derived pushforward of an $\mathcal{O}$-algebra along the augmentation $\mathcal{O} \to \mathbf{1}$.

(2) The ∞-category $\mathrm{coAlg}^{\mathrm{dp}}_{B\mathcal{O}}(\mathcal{C})$ is that of *divided power $B\mathcal{O}$-coalgebras* in $\mathcal{C}$. Roughly, a $B\mathcal{O}$-coalgebra is an object X of $\mathcal{C}$ equipped with comultiplication maps

$$X \xrightarrow{\delta_n} (B\mathcal{O}(n) \otimes X^{\otimes n})^{h\Sigma_n}$$

and a coherent system of homotopies recording the compatibilities between these. A *divided power structure* on such a coalgebra is the data of factorizations of the maps δ_n through the norm maps

$$(B\mathcal{O}(n) \otimes X^{\otimes n})_{h\Sigma_n} \xrightarrow{\mathrm{Nm}_{\Sigma_n}} (B\mathcal{O}(n) \otimes X^{\otimes n})^{h\Sigma_n}$$

and a further coherent system of homotopies relating these.

The evident question to ask is the following: on what subcategories of $\mathrm{Alg}_{\mathcal{O}}(\mathcal{C})$ and of $\mathrm{coAlg}^{\mathrm{dp}}_{B\mathcal{O}}(\mathcal{C})$ does this adjoint pair restrict to an equivalence? In a 2012 paper [3], Francis and Gaitsgory conjecture the following:

Conjecture 1 (Francis–Gaitsgory)**.** *Koszul duality restricts to an equivalence between the subcategories of pro-nilpotent $\mathcal{O}$-algebras and ind-conilpotent $B\mathcal{O}$-coalgebras.*

Here an $\mathcal{O}$-algebra is *pro-nilpotent* if it can be written as a limit of trivial (i.e., square-zero) $\mathcal{O}$-algebras. Dually, a coalgebra is *ind-conilpotent* if it can be built as a colimit of trivial coalgebras. Several cases of this conjecture have been established in the literature. For example, Ching–Harper [2] prove special cases when $\mathcal{C}$ is the ∞-category of modules over a commutative ring spectrum and the algebras under consideration are connected. Brantner–Mathew [1] prove a result under slightly weaker connectivity assumptions: they allow a nontrivial π_0, but impose a finiteness condition on it.

The main novelty in this talk is the following, which makes no reference to connectivity:

Theorem 2. *The unit of the adjunction*

$$\mathrm{Alg}_{\mathcal{O}}(\mathcal{C}) \underset{\mathrm{prim}_{B\mathcal{O}}}{\overset{\mathrm{indec}_{\mathcal{O}}}{\rightleftarrows}} \mathrm{coAlg}^{\mathrm{dp}}_{B\mathcal{O}}(\mathcal{C}),$$

is the derived completion *of $\mathcal{O}$-algebras. Dually, the counit is the* derived cocompletion *of divided power $B\mathcal{O}$-coalgebras. In particular, Koszul duality restricts to an equivalence between the full subcategories of complete $\mathcal{O}$-algebras an cocomplete $B\mathcal{O}$-coalgebras with divided powers (and these subcategories are the largest on which this works).*

Remark 3. *Completeness (to be discussed below) is usually straightforward to verify under appropriate connectivity assumptions, which allows one to deduce the Koszul duality results of Ching–Harper and Brantner–Mathew from the theorem above.*

Remark 4. *A complete algebra is always pro-nilpotent, but the converse need not be true. This can be leveraged to find counterexamples to the conjecture of Francis–Gaitsgory. For example, take $\mathcal{C}$ to be the ∞-category of chain complexes* *over $\mathbb{Q}$ and $\mathcal{O}$ to be the commutative operad. Then a power series ring in infinitely many variables is an example of an algebra that is pro-nilpotent, but not (derived) complete at its augmentation ideal. In light of this observation, the conjecture of Francis–Gaitsgory cannot be quite true as stated. We view the theorem above as an adequate replacement.*

We briefly comment on completeness. Define the *n-truncation* of the operad $\mathcal{O}$ by

$$\tau_n\mathcal{O}(k) = \begin{cases} \mathcal{O}(k) & \text{if } k \leq n, \\ 0 & \text{if } k > n. \end{cases}$$

The evident map of operads $\mathcal{O} \to \tau_n\mathcal{O}$ defines an adjoint pair

$$\mathrm{Alg}_{\mathcal{O}}(\mathcal{C}) \rightleftarrows \mathrm{Alg}_{\tau_n\mathcal{O}}(\mathcal{C}).$$

For $X \in \mathrm{Alg}_{\mathcal{O}}(\mathcal{C})$, write $X \to t_nX$ for the unit of this adjunction. Then the *derived completion* of X is the map

$$X \to \varprojlim_n t_nX.$$

The algebra X is called *complete* if this map is an equivalence. Cocompletion of coalgebras can be defined in dual fashion.

To conclude, we summarize some of the ingredients of the proof of Theorem 2. We need 'approximations' of $\mathcal{O}$-algebras arising from maps of operads

$$\varphi_n\mathcal{O} \to \mathcal{O} \to \tau_n\mathcal{O}.$$

The second one was just discussed and can be characterized as the terminal map out of $\mathcal{O}$ that is an equivalence in arities $\leq n$. The first map can be characterized dually, as the initial map into $\mathcal{O}$ that is an equivalence in arities $\leq n$. The operad $\varphi_n\mathcal{O}$ is not obtained by 'extension by zero' as for $\tau_n\mathcal{O}$; rather, it is freely generated by the terms $\mathcal{O}(k)$ for $k \leq n$, subject to the relations existing in that portion of the operad $\mathcal{O}$. Explicit formulas for $\varphi_n\mathcal{O}(k)$ can be given in terms of certain partition complexes (or, alternatively, certain spaces of labelled trees). We write

$$f_nX \to X$$

for the counit of the adjoint pair between $\mathrm{Alg}_{\varphi_n\mathcal{O}}(\mathcal{C})$ and $\mathrm{Alg}_{\mathcal{O}}(\mathcal{C})$. Similarly, for a coalgebra Y there are dual 'approximations'

$$t^nY \to Y \to f^nY$$

resulting from maps of cooperads

$$\tau^nB\mathcal{O} \to B\mathcal{O} \to \varphi^nB\mathcal{O}.$$

The essential properties of these approximations are summarized in the following result:

Theorem 5. *(1) For an $\mathcal{O}$-algebra X, the tower $\cdots \to t_nX \to t_{n-1}X \to \cdots$ has associated graded*

$$\mathrm{fib}(t_nX \to t_{n-1}X) \cong \mathrm{triv}_{\mathcal{O}}\big(\mathcal{O}(n) \otimes (\mathrm{indec}_{\mathcal{O}}X)^{\otimes n}\big)_{h\Sigma_n}.$$

(2) The filtration $\cdots \to f_{n-1}X \to f_nX \to \cdots$ *has associated graded*

$$\mathrm{cof}(f_{n-1}X \to f_nX) \cong \mathrm{free}_{\mathcal{O}}(B\mathcal{O}(n) \otimes X^{\otimes n})_{h\Sigma_n}$$

and converges, in the sense that $\varinjlim_n f_nX \cong X$.

(3) For a divided power $B\mathcal{O}$*-coalgebra* Y*, the tower* $\cdots \to f^nY \to f^{n-1}Y \to \cdots$ *has associated graded*

$$\mathrm{fib}(f^nY \to f^{n-1}Y) \cong \mathrm{cofree}_{B\mathcal{O}}(\mathcal{O}(n) \otimes Y^{\otimes n})_{h\Sigma_n}$$

and converges, in the sense that $Y \cong \varprojlim_n f^nY$.

(4) The filtration $\cdots \to t^{n-1}Y \to t^nY \to \cdots$ *has associated graded*

$$\mathrm{cof}(t^{n-1}Y \to t^nY) \cong \mathrm{triv}_{B\mathcal{O}}\big(B\mathcal{O}(n) \otimes (\mathrm{prim}_{B\mathcal{O}}Y)^{\otimes n}\big)_{h\Sigma_n}.$$

One can think of (1) as describing the *Goodwillie tower* of the identity functor on the ∞-category $\mathrm{Alg}_{\mathcal{O}}(\mathcal{C})$. Item (2) can analogously be thought of as a 'dual Goodwillie filtration'; it consists of approximations by n-coexcisive functors. The special property of this filtration is that it always converges, in contrast with the Goodwillie tower (which only converges for complete algebras). Item (3) expresses the idea that $B\mathcal{O}$-coalgebras have an 'unconditionally convergent Goodwillie tower'. Koszul duality intertwines the various approximations listed above, in the following sense:

$$\begin{aligned} t_nX &\cong \mathrm{prim}_{B\mathcal{O}}(f^n\mathrm{indec}_{\mathcal{O}}X), \\ t^nY &\cong \mathrm{indec}_{\mathcal{O}}(f_n\mathrm{prim}_{B\mathcal{O}}Y). \end{aligned}$$

Theorem 2 is then not difficult to deduce. For example, for an $\mathcal{O}$-algebra X one calculates

$$\mathrm{prim}_{B\mathcal{O}}\mathrm{indec}_{\mathcal{O}}X \cong \varprojlim_n \mathrm{prim}_{B\mathcal{O}}(f^n\mathrm{indec}_{\mathcal{O}}X) \cong \varprojlim_n t_nX,$$

and dually for a coalgebra Y. Roughly speaking, one can thus summarize the proof idea as exploiting the fact that both $\mathrm{Alg}_{\mathcal{O}}(\mathcal{C})$ and $\mathrm{coAlg}^{\mathrm{dp}}_{B\mathcal{O}}(\mathcal{C})$ admit a good theory of Goodwillie calculus *and* of dual Goodwillie calculus, and that the two are related in a suitable way by Koszul duality.

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Cut and paste invariants of manifolds via algebraic K-theory

Julia Semikina

(joint work with Renee Hokzema, Mona Merling, Laura Murray, Carmen Rovi)

The classical scissors congruence problem asks whether given two polyhedra with the same volume P and Q in $\mathbb{R}^3$, one can cut P into a finite number of smaller polyhedra and reassemble these to form Q. There is an analogous definition of an SK (German "schneiden und kleben," cut and paste) relation for manifolds [3]. Given a closed smooth oriented manifold M, one can cut it along a separating codimension 1 submanifold Σ with trivial normal bundle and paste back the two pieces along an orientation preserving diffeomorphism $\Sigma \to \Sigma$ to obtain a new manifold, which we say is "cut and paste equivalent" to it.

Recent work of Zakharevich and Campbell [1] has focused on developing the K-theoretic machinery to study scissors congruence problems and applying these tools to the Grothendieck ring of varieties. In this talk we discuss a new application of their framework to study the cut and paste invariants of manifolds. Unfortunately, the tools used by Campbell and Zakharevich for varieties do not directly apply to the case of manifolds. However, work in progress of Campbell and Zakharevich on "K-theory with squares," $K^{\square}$, a further synthetization of scissors congruence relations as K-theory that generalizes Waldhausen K-theory, does give the right framework to construct the desired scissors congruence spectrum for manifolds.

The study of SK-invariants and SK-groups in [3] focuses on closed manifolds. However, in order for the $K^{\square}$-theoretic scissors congruence machinery to apply, we need to work in the category of manifolds with boundary, since the pieces in an SK-decomposition have boundary. This is not well-explored classically, as most of the existing work on SK-groups is for closed manifolds. We generalize the notion of SK-equivalence to the case of manifolds with boundary and denote the corresponding group by SK^{∂}_n. Our definition of SK^{∂}_n is different from the one mentioned in [3] in that we insist that every boundary along which we cut gets pasted, and this is crucial for the further application of the K-theoretic technology.

We formulate a suitable notion of a category with squares $\mathrm{Mfd}^{\partial}_n$, that fits into the framework of the K-theory with squares framework, and whose distinguished squares exactly encode the "cut and paste" relations for n-dimensional manifolds with boundary. The Ω-spectrum obtained from the construction of Campbell and Zakharevich, applied to $\mathrm{Mfd}^{\partial}_n$, which we denote by $K^{\square}(\mathrm{Mfd}^{\partial}_n)$, recovers the SK^{∂}_n as its zeroth homotopy group:

$$K^{\square}_0(\mathrm{Mfd}^{\partial}_n) \cong \mathrm{SK}^{\partial}_n,$$

where $K^{\square}_0(\mathrm{Mfd}^{\partial}_n)$ is π_0 of a scissors congruence K-theory spectrum $K^{\square}(\mathrm{Mfd}^{\partial}_n)$.

We also show that the Euler characteristic as a map to $\mathbb{Z}$, viewed as the zeroth K-theory group of $\mathbb{Z}$, is the π_0 level of a map of spectra from the scissors congruence spectrum for manifolds with boundary that we define. Namely, there is a map of K-theory spectra

$$K^{\square}(\mathrm{Mfd}^{\partial}) \to K(\mathbb{Z}),$$

which on π_0 agrees with the Euler characteristic for smooth compact manifolds with boundary.

Further, we also describe the connection of the spectrum $K^{\square}(\mathrm{Mfd}^{\partial})$ to the classical Madsen-Tillmann spectrum $MTSO(n)$.

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Rational homotopy theory of embedding spaces

Victor Turchin, Thomas Willwacher

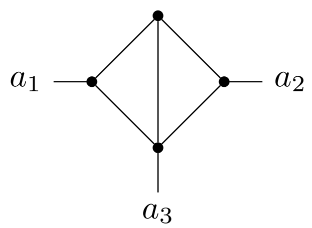
(joint work with Benoit Fresse)

The presentation is based on our collabaration work [4] with B. Fresse.

Let $M \subset \mathbb{R}^m$ be the complement of a compact submanifold possibly with boundary. We study the homotopy fiber of the space of long embeddings over long immersions

$$\overline{\mathrm{Emb}}_\partial(M, \mathbb{R}^n) = hofiber(\mathrm{Emb}_\partial(M, \mathbb{R}^n) \to \mathrm{Emb}_\partial(M, \mathbb{R}^n))$$

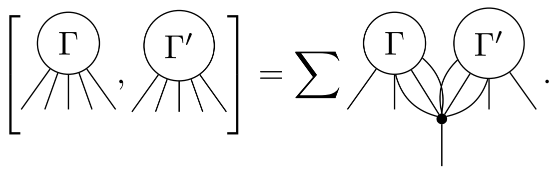
using a rational version of the Goodwillie-Weiss calculus. In the case that $n-m \geq 3$ we compute explicit rational models for the components of $\mathrm{Emb}_\partial(M, \mathbb{R}^n)$ through a combinatorial graph complex. More precisely, this graph complex has the following shape. For A some (possibly non-unital) differential graded commutative algebra we denote by $\mathrm{HGC}_{A,n}$ the complex of $\mathbb{Q}$-linear series of graphs with legs decorated by an element of our algebra. Here is an example:



, $\quad a_1, a_2, a_3 \in A.$

This graph complex can be equipped with a natural L_∞-algebra structure [4]. For example the Lie bracket is defined graphically by fusing hairs, multiplying the A-decorations:

(1)



Let now $M \subset \mathbb{R}^m$ be a complement to a compact submanifold (possibly with boundary). Let $\mathcal{F}_m$ be the Fulton-MacPherson operad homotopy equivalent to the little m-disks operad. Let $\mathcal{IF}_M$ be the Fulton-MacPherson version of the configuration space of points on M, on which one has an infinitesimal $\mathcal{F}_m$-bimodule structure [13]. The appropriate version of the Goodwillie-Weiss calculus (see [1, 13, 5, 6, 7]) then implies that, for $n - m \geq 3$ one has a weak equivalence

$$\overline{\mathrm{Emb}}_\partial(M, \mathbb{R}^n) \simeq \mathrm{IBimod}^h_{\mathcal{F}_m}(\mathcal{IF}_M, \mathcal{F}_n), \tag{2}$$

where $\mathrm{IBimod}^h_{\mathcal{F}_m}(\dots)$ is the derived mapping space of infinitesimal bimodules.

To access the rational homotopy type one may further simplify the right-hand side of (2) using the following result using heavily previous work of Mienné [10, 11].

Theorem 1. *For $n - m \geq 3$ or $n - m \geq 2$ and $M \subsetneq \mathbb{R}^m$, the natural map*

$$\mathrm{IBimod}^h_{\mathcal{F}_m}(\mathcal{IF}_M, \mathcal{F}_n) \to \mathrm{IBimod}^h_{\mathcal{F}_m}(\mathcal{IF}_M, \mathcal{F}_n^{\mathbb{Q}}) \tag{3}$$

defines a rational equivalence of nilpotent spaces componentwise and is finite-to-one at the π_0-level. (Here $\mathcal{F}_n^{\mathbb{Q}}$ is the rationalization of $\mathcal{F}_n$, see [3]*.)*

Then our main technical result is then finally the following.

Theorem 2. *Given $M \subset \mathbb{R}^m$ as above let R be an augmented Sullivan model of the pointed space $M_* = M \cup \{\infty\}$ (with the base-point at infinity). For $n - m \geq 2$, we have a weak homotopy equivalence*

$$\mathrm{IBimod}^h_{\mathcal{F}_m}(\mathcal{IF}_M, \mathcal{F}_n^{\mathbb{Q}}) \simeq \mathsf{MC}_\bullet(\mathrm{HGC}_{\bar{R},n}),$$

where $\bar{R}$ denotes the augmentation ideal of R and $\mathsf{MC}_\bullet(\mathrm{HGC}_{\bar{R},n})$ denotes the simplicial set of Maurer–Cartan forms with values in the complete L_∞-algebra $\mathrm{HGC}_{\bar{R},n}$.

Overall this expresses the rational homotopy type of the components of $\overline{\mathrm{Emb}}_\partial(M, \mathbb{R}^n)$ through the hairy graph complex $\mathrm{HGC}_{\bar{R},n}$. For example, this also implies that the rational homotopy groups are computed as

$$\pi_k^{\mathbb{Q}} \overline{\mathrm{Emb}}_\partial(M, \mathbb{R}^n)_\psi \cong H_k(\mathrm{HGC}^{m(\psi)}_{\bar{R},n}),$$

where ψ is some point in the embedding space and $m(\psi) \in \mathrm{HGC}_{\bar{R},n}$ is the corresponding Maurer-Cartan element.

Examples. There are several interesting embedding spaces to which our approach applies.

(1) *String links.* Let $\sqcup_{i=1}^r R^{m_i}$, $m = max(m_i) + 1$, be a collection of non-intersecting planes in $\mathbb{R}^m$. By taking for M the complement of an m-ball of some big radius together with the disjoint union of tubular neighborhoods of each $\mathbb{R}^{m_i}$, we get $\overline{\mathrm{Emb}}_\partial(M, \mathbb{R}^n) \simeq \overline{\mathrm{Emb}}_\partial(\sqcup_{i=1}^r \mathbb{R}^{m_i}, \mathbb{R}^n)$.

(2) *Embeddings of a compact manifold.* Let L be a compact submanifold of $\mathbb{R}^m$. Define M to be the union of a tubular neighborhood $N(L)$ of L and the complement of a compact m-ball containing $N(L)$. We get $\overline{\mathrm{Emb}}_\partial(M, \mathbb{R}^n) \simeq \overline{\mathrm{Emb}}(L, \mathbb{R}^n)$. To be specific we can compute the rational type of the connected components of embedding spaces such as $\overline{\mathrm{Emb}}(S^1 \times S^2, \mathbb{R}^6)$ and $\overline{\mathrm{Emb}}(S^2 \times S^2, \mathbb{R}^7)$.

(3) *Spherical links.* In the example above, L can be disconnected with components of possibly different dimensions. In particular we can study $\overline{\mathrm{Emb}}\,(\sqcup_{i=1}^{r} S^{m_i}, \mathbb{R}^n)$, $n \geq max(m_i) + 3$.

As a consequence, our approach produces a natural finite-to-one map

$$\pi_0\, \overline{\mathrm{Emb}}_\partial(M, \mathbb{R}^n) \to \mathsf{MC}(\mathrm{HGC}_{\bar{R},n})/\sim,$$

where on the right we have the set of Maurer-Cartan elements modulo the gauge equivalence. The assigned Maurer-Cartan element completely determines the rational homotopy type of the corresponding connected component of the space of embeddings. Based on the computations of this set we conjecture that this map is related to many interesting geometrical invariants of embeddings studied in [2, 8, 9, 12], such as Boechat-Haefliger invariant of embeddings of 4-folds in $\mathbb{R}^7$, Whitney-Skopenkov invariant of embeddings of 3-folds in $\mathbb{R}^6$, linking number and higher linking of spherical or string links, in terms of Milnor invariants and unitrivalent trees.

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Topological invariants of surface bundles

Ursula Hamenstädt

A *surface bundle over a surface* $\Pi : E \to B$ is a smooth manifold which fibers over a closed oriented surface B, with fiber a closed oriented surface S_g of genus $g \geq 2$. Such a surface bundle is obtained as follows.

Let $\mathcal{M}_g$ be the *moduli space of complex curves of genus* g. The *universal curve* is the fiber bundle $\mathcal{C} \to \mathcal{M}_g$ whose fiber over a point X is just the Riemann surface X. There exists a classifying map $f : B \to \mathcal{M}_g$ such that $E = f^*\mathcal{C}$. Thus topological invariants of surface bundles over surfaces are related to topological properties of $\mathcal{M}_g$.

The lecture starts with describing the cohomology of such surface bundles. Namely, a spectral sequence argument of Morita states that if there exists a cohomology class $\alpha \in H^2(E, \mathbb{Z})$ which restricts to a generator of the cohomology of the fiber, then the map $\Pi^* : H^*(B, \mathbb{Z}) \to H^*(E, \mathbb{Z})$ is injective, and we have

$$H^p(E, \mathbb{Z}) \equiv H^p(B, \mathbb{Z}) \oplus H^{p-1}(B, H^1(S_g, \mathbb{Z})) \oplus \alpha H^{p-2}(B, \mathbb{Z}).$$

Denote by $\mathrm{Mod}(S_g)$ the *mapping class group* of S_g; this is the orbifold fundamental group of $\mathcal{M}_g$. The existence of such a class α can be guaranteed if the push forward group $f_*\pi_1(B) \subset \mathrm{Mod}(S_g)$ preserves a $(2g-2)$-spin structure. If $SS_g \to S_g$ is the unit tangent circle bundle, then such a spin structure is a cohomology class in $H^1(SS_g, \mathbb{Z}/(2g-2)\mathbb{Z})$ which restricts to a generator of the cohomology of the fiber, with coefficients $\mathbb{Z}/(2g-2)\mathbb{Z}$.

The unit circle bundles of the fibers of the universal curve $\mathcal{C} \to \mathcal{M}_g$ fit together to the circle subbundle of the tangent bundle ν of the fibers of $\mathcal{C}$. A circle bundle $\mathcal{S} \to Y$ over a simplicial complex Y is called *flat* if there exists a homomorphism $\rho : \pi_1(Y) \to \mathrm{Top}^+(S^1)$ into the group of orientation preserving homeomorphisms of the circle so that

$$\mathcal{S} = \tilde{Y} \times S^1/\pi_1(Y)$$

where $\tilde{Y}$ is the universal covering of Y and the action on S^1 is via the homomorphism ρ. The following result is due to Morita.

Theorem: [Morita] *The circle subbundle of the vertical tangent bundle of the universal curve is flat.*

The Euler class of a flat circle bundle is a *bounded* cohomology class, that is, it can be given by a bounded cocycle on the underlying fundamental group. Morita's theorem thus implies that the Euler class of the vertical tangent bundle of a surface bundle is a bounded cohomology class.

The *signature* $\sigma(E)$ of a surface bundle over a surface $E \to B$ can be represented by

$$3\sigma(E) = f^*\kappa_1([B])$$

where κ_1 is the first *Mumford Morita Miller class* of $\mathcal{M}_g$. Now $\kappa_1 = 12\lambda$ where λ is the first Chern class of the so-called *Hodge bundle* $\mathcal{H} \to \mathcal{M}_g$ and since λ is a bounded cohomology class, we know that κ_1 is bounded.

Using the theorem of Morita and nowhere vanishing sections of the Hodge bundle one can construct an explicit representation of κ_1 by a cocycle which is bounded in absolute value by $g-1$. The leads to the following main result, extending the *Miaoka inequality* for complex surfaces to all surface bundles over surfaces, where $\chi(E)$ denotes the Euler characteristic.

Theorem: *Let $E \to B$ be a surface bundle over a surface; then*

$$|3\sigma(E)| \leq |\chi(E)|.$$

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The classification of highly connected manifolds

Jeremy Hahn
(joint work with Robert Burklund, Andrew Senger)

The speaker reported on recent progress in the classification of $(n-1)$-connected, smooth, closed, oriented $(2n)$-manifolds.

Given such a manifold M, one can extract certain algebraic invariants. These include the middle homology group $H = H_n(M;\mathbb{Z})$, the bilinear intersection pairing $H \otimes H \to \mathbb{Z}$, and a certain *normal bundle invariant* $\alpha : H \to \pi_{n-1}SO(n)$. Explicitly, the α invariant takes a homology class x, represented by an embedded sphere $x : S^n \to M$, and records the normal bundle $\alpha(x) : S^n \to BSO(n)$ of that embedding.

The talk focused on new methods for constructing manifolds with prescribed algebraic invariants, building on 1960s work of C.T.C. Wall. A key new result is that, if $n > 124$, then any exotic sphere bounding an $(n-1)$-connected $(2n)$-manifold must also bound a parallelizable manifold. Interestingly, work of Burklund and Senger shows that this theorem may fail in lower dimensions, particularly in the study of 11-connected 24-manifolds.

The proof of the new result makes contact with the classical Adams spectral sequence in homotopy theory. A helpful point, of relevance first recognized by Stephan Stolz, is that there are lines in the standard Adams charts above which all classes are v_1-periodic. Stolz's use of this line is improved upon via arguments with Toda brackets and $\mathbb{E}_\infty$-structures within Pstragowski's category of synthetic spectra.

Applications beyond the construction of manifolds include computations of mapping class groups and the classification of Stein fillable exotic spheres.

Stable equivariant homotopy types for link homologies

Nitu Kitchloo

Given a compact connected Lie group G endowed with root datum, and an element w in the corresponding Artin braid group for G, we describe a filtered G-equivariant stable homotopy type, up to a notion of quasi-equivalence. We call this homotopy type *Strict Broken Symmetries*, $s\mathscr{B}(w)$. As the name suggests, $s\mathscr{B}(w)$ is constructed from the stack of pincipal G-connections on a circle, whose holonomy is broken between consecutive sectors in a manner prescribed by a presentation of w. We show that $s\mathscr{B}(w)$ is independent of the choice of presentation of w, and also satisfies Markov type properties. Specializing to the case of the unitary group $G = \mathrm{U}(r)$, these properties imply that $s\mathscr{B}(w)$ is an invariant of the link L obtained by closing the r-stranded braid w. As such, we denote it by $s\mathscr{B}(L)$. The construction of strict broken symmetries also allows us to incorporate twistings. Applying suitable $\mathrm{U}(r)$-equivariant (possibly twisted) cohomology theories $\mathrm{E}_{\mathrm{U}(r)}$ to $s\mathscr{B}(L)$ gives rise to a spectral sequence of link invariants converging to $\mathrm{E}^*_{\mathrm{U}(r)}(s\mathscr{B}_\infty(L))$, where $s\mathscr{B}_\infty(L)$ is the direct limit of the filtration. In [2, 3], we offer two examples of such theories. In the first example, we study a universal twist of Borel-equivariant singular cohomology $\mathrm{H}_{\mathrm{U}(r)}$. The E_2-term in this case appears to recover $sl(n)$ link homologies for any value of n (depending on the choice of specialization of the universal twist). We also show that Triply-graded link homology corresponds to the trivial twist. In the next example, we apply a version of an equivariant K-theory ${}^n\mathscr{K}_{\mathrm{U}(r)}$ known as Dominant K-theory, which can be interpreted as twisted $\mathrm{U}(r)$-equivariant K-theory built from level n representations of the loop group of $\mathrm{U}(r)$. In this case, the E_2-term recovers a deformation of $sl(n)$-link homology, and has the property that its value on the unknot is the Grothendieck group of level n-representations of the loop group of $\mathrm{U}(1)$, given by $\mathbb{Z}[x^{\pm 1}]/(x^n - 1)$.

The main result presented in this talk is the construction of a filtered $\mathrm{U}(r)$-equivariant stable homotopy type $s\mathscr{B}(L)$ for links L that can be described as the closure of r-stranded braids, namely, elements of the braid group $\mathrm{Br}(r)$. We call this spectrum the spectrum of *strict broken symmetries* because it is built from the stack of principal $\mathrm{U}(r)$-connections on a circle with prescribed reductions of the structure group to the maximal torus at various points on the circle. Even though we have invoked the category of equivariant spectra, for links L that can be expressed as the closure of a positive braid, our spectrum $s\mathscr{B}(L)$ can be described entirely by the geometry of an underlying $\mathrm{U}(r)$-equivariant *space* of strict broken symmetries. For the convenience of non-experts, all the results in this abstract will be formulated for links given by the closure of a positive braid, with the general result for arbitrary braids described in later sections. We also point out that several results in this article will be shown to hold for arbitrary compact connected Lie groups G. We have chosen to highlight the case $G = \mathrm{U}(r)$ in the abstract for the purposes of exposition.

Before we proceed, let us say a few words about the category of G-spectra that will be used in this article. The main results of this article can be understood with very little background on equivariant spectra. It is helpful to bear in mind that G-spectra may be seen as a natural localization of the category of G-spaces where one is allowed to desuspend by arbitrary finite dimensional G-representations. As with G-spaces, one may evaluate G-spectra on G-equivariant cohomology theories. Given a subgroup $H < G$, one has restriction and induction functors defined respectively by considering a G-spectrum as an H-spectrum, or by inducing up an H-spectrum X to the G-spectrum $G_+ \wedge_H X$. As one would expect, the induction from from H-spectra to G-spectra is left adjoint to restriction. For those somewhat familiar with the language, by an equivariant G-spectrum we mean an equivariant spectrum indexed on a complete G-universe.

The spectra we study are filtered by a finite increasing filtration F_tX. The associated graded object $\mathrm{Gr}_t(X)$ of such a spectrum has a natural structure of a chain complex in the homotopy category of G-spectra. In particular, one may define an *acyclic filtered G-spectrum X* so that the associated graded object $\mathrm{Gr}_r(X)$ admits stable null homotopies. The notion of acyclicity allows us to define a notion of *quasi-equivalence* on our category of filtered G-spectra by demanding that two filtered G-spectra are equivalent if they are connected by a zig-zag of maps each of whose fiber (or cofiber) is acyclic.

Returning to the main application, we show that a braid w on r-strands gives rise to a filtered equivariant $\mathrm{U}(r)$-spectrum of strict broken symmetries, denoted by $s\mathscr{B}(w)$, which is well defined up to quasi-equivalence. Before we get to the definition of strict broken symmetries, let us first offer a geometric description of the $\mathrm{U}(r)$-spectrum of broken symmetries. Consider a braid element $w \in \mathrm{Br}(r)$, where $\mathrm{Br}(r)$ stands for the braid group on r-strands. For the sake of exposition, consider the case of a positive braid that can be expressed in terms of positive exponents of the elementary braids σ_i for $i < r$. Let $I = \{i_1, i_2, \ldots, i_k\}$ denote an indexing sequence with $i_j < r$, so that a positive braid w admits a presentation in terms of the fundamental generators of the braid group $\mathrm{Br}(r)$, $w = w_I := \sigma_{i_1}\sigma_{i_1}\ldots\sigma_{i_k}$. Let T, or T^r (if we need to specify rank), be the standard maximal torus, and let G_i denote the unitary (block-diagonal) form in reductive Levi subgroup having roots $\pm\alpha_i$. We consider G_i as a two-sided T-space under the left(resp. right) multiplication.

The equivariant $\mathrm{U}(r)$-spectrum of broken symmetries is defined as the (suspension) spectrum corresponding to the $\mathrm{U}(r)$-space $\mathscr{B}(w_I)$ that is induced up from a T-space $\mathscr{B}_T(w_I)$

$$\mathscr{B}(w_I) := \mathrm{U}(r) \times_T (G_{i_1} \times_T G_{i_2} \times_T \cdots \times_T G_{i_k}) = \mathrm{U}(r) \times_T \mathscr{B}_T(w_I),$$

with the T-action on $\mathscr{B}_T(w_I) := (G_{i_1} \times_T G_{i_2} \times_T \cdots \times_T G_{i_k})$ given by conjugation

$$t\,[(g_1, g_2, \cdots, g_{k-1}, g_k)] := [(tg_1, g_2, \cdots, g_{k-1}, g_k t^{-1})].$$

As mentioned above, the $\mathrm{U}(r)$-stack $\mathrm{U}(r) \times_T (G_{i_1} \times_T G_{i_2} \times_T \cdots \times_T G_{i_k})$ is equivalent to the stack of $\mathrm{U}(r)$-connections on a circle with k marked points, with the structure group being reduced to T at the points, and symmetry being broken to G_i along the i-th sector.

Definition 1. (Strict broken symmetries and their normalization)
Let L denote a link described by the closure of a positive braid $w \in \mathrm{Br}(r)$ with r-strands, and let w_I be a presentation of w as $w = \sigma_{i_1} \dots \sigma_{i_k}$. We first define the limiting $\mathrm{U}(r)$-spectrum $s\mathscr{B}_\infty(w_I)$ of strict broken symmetries as the space that fits into a cofiber sequence of $\mathrm{U}(r)$-spaces:

$$\mathrm{hocolim}_{J \in \mathcal{I}} \mathscr{B}(w_J) \longrightarrow \mathscr{B}(w_I) \longrightarrow s\mathscr{B}_\infty(w_I).$$

where $\mathcal{I}$ is the category of all proper subsets of $I = \{i_1, i_2, \dots, i_k\}$.

The spectrum $s\mathscr{B}_\infty(w_I)$ admits a natural increasing filtration by spaces $F_t\, s\mathscr{B}(w_I)$ defined as the cofiber on restricting the above homotopy colimit to the full subcategories $\mathcal{I}^t \subseteq \mathcal{I}$ generated by subsets of cardinality at least $(k-t)$, so that the lowest filtration is given by $F_0 s\mathscr{B}(w_I) = \mathscr{B}(w_I)$.

Define the spectrum of strict broken symmetries $s\mathscr{B}(w_I)$ to be the filtered spectrum $F_t\, s\mathscr{B}(w_I)$ above. The normalized spectrum of strict broken symmetries of the link L is defined as

$$s\mathscr{B}(L) := \Sigma^{-2k} s\mathscr{B}(w_I).$$

In order for the normalized definition to make sense, one would require proving that the construction of $s\mathscr{B}(L)$ is independent (up to quasi-equivalence) of the braid presentation w_I used to describe L. This comes down to checking the braid group relations, and the first and second Markov property. These results in fact admit a generalization to any compact connected Lie group G. We have chosen to highlight the case $G = \mathrm{U}(r)$ for the purposes of this abstract.

The second Markov property imposes a stability condition on the construction, requiring that it be invariant under the augmentation of w by the elementary braid σ_r (or its inverse) so as to be seen as a braid in $\mathrm{Br}(r+1)$. This is equivalent to the observation that the link L is unchanged on adding an extra strand that is braided with the previous one. In proving invariance under the second Markov property, we encounter a subtle point. Notice that $s\mathscr{B}(L)$ is induced up from a T^r-spectrum we shall denote by $s\mathscr{B}_{T^r}(L)$. Proving invariance under the second Markov property would therefore require showing that the $\mathrm{U}(r+1)$-spectrum obtained by considering L as the closure of $w\sigma_r^{\pm}$ is induced from $s\mathscr{B}_{T^r}(L)$ along the standard inclusion $T^r < \mathrm{U}(r+1)$. This requirement is *almost true* but for a small subtlety. We show that when L is seen as the closure of the $(r+1)$-stranded braid $w\sigma_r^{\pm}$, the corresponding $\mathrm{U}(r+1)$-spectrum, $s\mathscr{B}(L)$ is induced up from $s\mathscr{B}_{T^r}(L)$ along a *different inclusion* $\Delta_r : T^r \longrightarrow \mathrm{U}(r+1)$. This inclusion differs from the standard inclusion in the last entry. We proceed to resolve this issue by inducing up to a larger group. The upshot is that $s\mathscr{B}(L)$ is a link invariant.

Theorem 2. *As a function of links L, the filtered* $\mathrm{U}(r)$*-spectrum of strict broken symmetries* $s\mathscr{B}(L)$ *is well-defined up to quasi-equivalence. In particular, the limiting equivariant stable homotopy type* $s\mathscr{B}_\infty(L)$ *is a well-defined link invariant in* $\mathrm{U}(r)$*-equivariant spectra. We discuss* $s\mathscr{B}_\infty(L)$ *below.*

An obvious way to obtain (group valued) link invariants from the filtered homotopy type $s\mathscr{B}(L)$ is to apply an equivariant cohomology and invoke the filtration to set up a spectral sequence. Let E_G denote a family of equivariant cohomology theories indexed by the collection $G = \mathrm{U}(r)$, with $r \geq 1$, and naturally compatible under restriction

$$\mathrm{E}_{\mathrm{U}(r)} \cong \iota^* \, \mathrm{E}_{\mathrm{U}(r+1)}, \quad \text{where} \quad \iota : \mathrm{U}(r) \longrightarrow \mathrm{U}(r+1).$$

Therefore, given a family of equivariant cohomology theories $\mathrm{E}_{\mathrm{U}(r)}$ as above that satisfies some algebraic conditions, the filtration of $s\mathscr{B}(L)$ described above does indeed give rise to a spectral sequence that converges to $\mathrm{E}^*_{\mathrm{U}(r)}(s\mathscr{B}_\infty(L))$. The E_2-term of this spectral sequence is itself a link invariant, and is given by the cohomology of the associated graded complex for the filtration of $s\mathscr{B}(L)$. We have

Theorem 3. *Assume that* $\mathrm{E}_{\mathrm{U}(r)}$ *is a family of* $\mathrm{U}(r)$*-equivariant cohomology theories as above that satisfy some algebraic conditions. Then, given a link L described as a closure of a positive braid presentation w_I on r-strands, one has a spectral sequence converging to* $\mathrm{E}^*_{\mathrm{U}(r)}(s\mathscr{B}_\infty(L))$ *and with E_1-term given by*

$$E_1^{t,s} = \bigoplus_{J \in \mathcal{I}^t/\mathcal{I}^{t-1}} \mathrm{E}^s_{\mathrm{U}(r)}(\mathscr{B}(w_J)) \;\Rightarrow\; \mathrm{E}^{s+t-2k}_{\mathrm{U}(r)}(s\mathscr{B}_\infty(L)).$$

The differential d_1 is the canonical simplicial differential. In addition, the terms $E_q(L)$ are invariants of the link L for all $q \geq 2$.

In [2, 3], we will relate special cases of the above spectral sequence to various well-known link homology theories. The limiting spectrum $s\mathscr{B}_\infty(L)$ can actually be described explicitly, and so we know exactly what the above spectral sequence converges to, yielding important information about each stage $E_q(L)$. To this point we prove a generalization of the following theorem for arbitrary compact connected Lie groups G, and for braid words that are not necessarily positive.

Theorem 4. *Given an indexing set $I = \{i_1, \ldots, i_k\}$, so that $w_I = \sigma_{i_1}\sigma_{i_2}\ldots\sigma_{i_k}$ is braid word that closes to the link L. Let V_I denote the representation of T given by a sum of root spaces*

$$V_I = \sum_{j \leq k} w_{I_{j-1}}(\alpha_{i_j}), \quad \textit{where} \quad w_{I_{j-1}} = \sigma_{i_1}\ldots\sigma_{i_{j-1}}, \quad w_{I_0} = id,$$

and where $w_{I_{j-1}}(\alpha_{i_j})$ denotes the root space for the root given by the $w_{I_{j-1}}$ translate of the simple root α_{i_j}. Then the $\mathrm{U}(r)$*-equivariant homotopy type of* $s\mathscr{B}_\infty(L)$ *is given by the equivariant Thom space (suitably desuspended)*

$$s\mathscr{B}_\infty(L) = \Sigma^{-2k}\, \mathrm{U}(r)_+ \wedge_T (S^{V_I} \wedge T(w)_+),$$

where S^{V_I} denotes the one-point compactification of the T-representation V_I, and $T(w)$ denotes the twisted conjugation action of T on itself

$$t(\lambda) := w^{-1}twt^{-1}\,\lambda \quad \textit{where} \quad w = \sigma_{i_1}\ldots\sigma_{i_k}, \quad t \in T, \quad \lambda \in T(w).$$

Remark 5. *Note that the structure group T of the above Thom spectrum can be reduced to a sub torus $T^w \subseteq T$ that is fixed by the Weyl element w that underlies w_I. The torus T^w is isomorphic to a product of rank one tori indexed by the components of L. More precisely, the factor corresponding to a particular component of L is the diagonal in the standard sub torus of T^r indexed by the strands belonging to that particular compoment. Since the cohomology of $s\mathscr{B}_\infty(L)$ (assuming Thom isomorphism) depends only on the number of components of L, we may think of $s\mathscr{B}_\infty(L)$ as a stable lift of the Lee homology.*

Let us point out an important piece of structure that is relevant to our framework. Notice that each space of broken symmetries $\mathscr{B}(w_I)$ admits a canonical map (given by composing the holonomies along the sectors) to the stack of principal connections on a circle, which is equivalent to the adjoint action of $\mathrm{U}(r)$-action on itself

$$\rho_I : \mathscr{B}(w_I) \longrightarrow \mathrm{U}(r), \qquad [(g, g_{i_1}, \ldots, g_{i_k})] \longmapsto g(g_{i_1}\ldots g_{i_k})g^{-1}.$$

These maps ρ_I are clearly compatible under inclusions $J \subseteq I$. In particular, the spectra $s\mathscr{B}(L)$ can be endowed with a $\mathrm{U}(r)$-equivariant "local system" by pulling back $\mathrm{U}(r)$-equivariant local systems on $\mathrm{U}(r)$. We will use this structure in [2] to twist the equivariant cohomology theories $\mathrm{E}_{\mathrm{U}(r)}$ considered above. More precisely, in [2, 3], we will study two examples of (twisted) cohomology theories and the corresponding spectral sequence. The first example is given by Borel-equivariant singular cohomology $\mathrm{H}_{\mathrm{U}(r)}$. The second example is given by a version of an equivariant K-theory ${}^n\mathscr{K}_{\mathrm{U}(r)}$ known as Dominant K-theory, built from level n representations of the loop group of $\mathrm{U}(r)$.

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The sub-linear(ly) Morse boundary

Yulan Qing

As a result of conceptualization of Gromov boundary of hyperbolic spaces, boundaries have been a topic of intense interest in geometric group theory. In joint work with Rafi, we define a family of boundaries called the *sublinearly Morse boundaries* of X, where X is a CAT(0) space. Fix a basepoint $\mathfrak{o} \in X$ and a monotone, concave

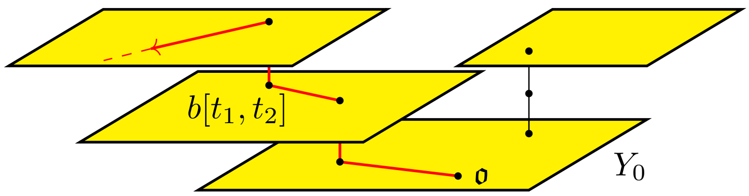

Figure 1. A geodesic ray in $\partial_{\log t} X$ is one whose projection to any flat $b[t_1, t_2]$ is bounded by $\mathsf{c} \log t_2$.

and sub-linear function κ once and for all. A geodesic ray γ is κ-*Morse* if there exists a function $\mathsf{m}(\mathsf{q}, \mathsf{Q})$ such that, any (q, Q)–quasi-geodesic segment $\eta\colon [0, s] \to X$ with endpoints on γ have the property

$$d(\eta(t), \gamma) \leq \mathsf{m}(\mathsf{q}, \mathsf{Q})\kappa(\|\eta(t)\|).$$

That is to say, all quasi-geodesics with endpoints on γ stay sublinearly close to γ. We denote the set of κ–Morse rays by $\partial_\kappa X$ and equip $\partial_\kappa X$ with the cone topology on quasi-geodesics. Roughly speaking, if two geodesic rays, and all their quasi-geodesic images, stay close for a period of time, then they are considered to be in an open set. We prove that $\partial_\kappa X$ is a quasi-isometry invariant. Equip this set with *coarse cone topology*, we show that this boundary is QI-invariant and metrizable.

In the current work discussed at the workshop, we expand this result to all proper metric spaces. Let $(X, \mathfrak{o})$ be a proper geodesic metric space with basepoint $\mathfrak{o}$ and fix a sublinear function κ. Let $\mathcal{N}_\kappa\big(\gamma, m_\gamma(q, Q)$ be the collection of points whose distance to γ is bounded above by $m_\gamma(q, Q)\kappa(||x||)$, where $||x|| = d(\mathfrak{o}, x)$.

Definition 1. A quasi-geodesic ray γ is κ–Morse if there exists a proper function $m_\gamma : \mathbb{R}^2 \to \mathbb{R}$ such that for any sublinear function κ' and for any $r > 0$, there exists R such that for any (q, Q)–quasi-geodesic β with $m_\gamma(q, Q)$ small compared to r, if

$$d_X(\beta_R, \gamma) \leq \kappa'(R) \qquad \text{then} \qquad \beta|_r \subset \mathcal{N}_\kappa\big(\gamma, m_\gamma(q, Q)\big)$$

The function m_γ will be called *Morse gauges* of γ.

A first example of a sublinearly Morse set is a unit speed, (quasi-)geodesic ray travelling in a "tree-of-flats" space and spending up to $\log t$ among of time in each flat where t is the time it leaves the flat.

We first show that these boundaries are group invariant.

Theorem 2 (Q-Rafi, Q-Rafi-Tiozzo)**.** *Let X be a proper, geodesic metric space and let κ be a sublinear function. Then $\partial_\kappa X$ is a topological space that is quasi-isometrically invariant, and metrizable.*

Our main application for the sublinear boundaries is that they serve as a topological model for for the Poisson boundaries of various groups. Let μ be...

Theorem 3 (Q-Rafi-Tiozzo)**.** *the Poisson boundaries (G, μ) can be identified with $(\partial_\kappa G, \nu)$ for the following groups.*

- *Right-angled Artin groups,* $\kappa(t) = \sqrt{t \log t}$.
- *Relative hyperbolic groups,* $\kappa(t) = \log t$
- *Mapping class groups,* $\kappa(t) = \log^d t$

Theorem 4 (Gekhtman-Qing-Rafi)**.** *Let X be a rank-1 CAT(0) space. A sublinearly Morse geodesic ray is generic with respect to*

- *Patterson Sullivan measure*
- *stationary measure associated with* (G, μ)

Corollary 5 (Gekhtman-Qing-Rafi)**.** *Let X be a rank-1 CAT(0) space. Let G acting properly and cocompactly on X, and let μ be any finitely supported measure on G, there exists a κ such that $(\partial_\kappa G, \nu)$ is a model for its Poisson boundary (G, μ) where ν is the associated hitting measure.*

Theorem 6 (Gekhtman-Qing-Rafi)**.** *There exists a κ such that the Poisson boundary of the mapping class group $(MCG(S), \mu)$ can be identified with $(\partial_\kappa X, \nu)$ where X is the associated Teichmüller space $T(S)$.*

Other interesting properties about sublinearly Morse boundaries have been proven as well. Let X be a CAT(0) space,

- $\partial_\kappa X$ is a strong visibility space.[Qing-Zalloum]
- a κ-Morse geodesics ray has at least quadratic κ-lower-divergence. [Qing-Murray-Zalloum]

We also define κ-contracting property for a set. we say a geodesic ray γ is *κ-contracting*, if there are constants $\mathsf{c}_\gamma > 0$ such that the diameter of the projection of a ball (disjoint from γ) to γ is bounded above by $\mathsf{c}_\gamma \kappa(t)$ where t measures the distance between $\mathfrak{o} := \gamma(0)$ and the center of the ball. That is to say, we consider geodesic rays whose contracting property weakens as they travel to infinity.

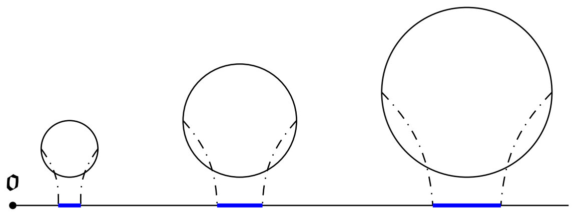

FIGURE 2. A κ–contracting geodesic ray

The connection between κ-contracting and κ-Morse are as follows:

- In CAT(0) spaces, κ-Morse is equivalent to κ-contracting. [Qing-Rafi]
- in proper metric spaces, κ-contracting is equivalence to sublinear Morse some function k'. [Qing-Rafi-Tiozzo]

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Poincaré ∞-categories and Grothendieck–Witt groups of Dedekind rings

Markus Land

(joint work with B. Calmès, E. Dotto, Y. Harpaz, F. Hebestreit, K. Moi, D. Nardin, T. Nikolaus, W. Steimle)

For a commutative ring R, we consider unimodular symmetric bilinear (respectively quadratic) forms over R. Such objects, together with their isomorphisms form a symmetric monoidal groupoid $(\mathrm{Unimod}^{\mathrm{x}}(R), \oplus)$ under the operation of orthogonal direct sum of forms; here $\mathrm{x} \in \{\mathrm{q}, \mathrm{s}\}$ denotes whether we consider quadratic or symmetric bilinear forms. Viewing a symmetric monoidal groupoid as a $\mathbb{E}_\infty$-space, we may define the Grothendieck–Witt spectrum

$$\mathcal{GW}^{\mathrm{x}}(R) = (\mathrm{Unimod}^{\mathrm{x}}(R), \oplus)^{\mathrm{grp}}$$

as its group-completion; this is then a grouplike $\mathbb{E}_\infty$-space, and we view it as a connective spectrum throughout. Its π_0 is the classical Grothendieck–Witt group $\mathrm{GW}_0^{\mathrm{x}}(R)$ of R and the higher homotopy groups of $\mathcal{GW}^{\mathrm{x}}(R)$ are by definition the higher Grothendieck–Witt groups of R. The classical Grothendieck–Witt group participates in the following exact sequence of abelian groups:

$$\mathrm{K}_0(R) \xrightarrow{\mathrm{hyp}} \mathrm{GW}_0^{\mathrm{x}}(R) \longrightarrow \mathrm{W}_0^{\mathrm{x}}(R) \longrightarrow 0$$

where $W_0^x(R)$ is the Witt group of R, which is the quotient of the monoid of isomorphism classes of unimodular forms by those which admit a Lagrangian. This quotient is indeed a group: the inverse of a form $[P, \varphi]$ is the form $[P, -\varphi]$. The map hyp sends a projective module to its hyperbolic form $(P \oplus P^*, \mathrm{ev})$. We notice that $\mathrm{hyp}(P)$ is canonically isomorphic to $\mathrm{hyp}(P^*)$, so that the hyperbolic map factors through the orbits of the C_2-action on $K_0(R)$ induced by sending a projective module to its dual. This construction in fact refines to a C_2-action on the connective K-theory spectrum $K(R)$.

We prove the following extension of the above exact sequence to higher Grothendieck–Witt groups:

Theorem 1. *There is a fibre sequence of spectra*

$$K(R)_{hC_2} \longrightarrow \mathcal{GW}^x(R) \longrightarrow \tau_{\geq 0} L^{gx}(R),$$

where $L^{gs}(R)$ *is a spectrum whose homotopy groups are Ranicki's original (non-periodic) symmetric L-groups. Furthermore there is an equivalence* $L^{gq}(R) \simeq \Sigma^4 L^{gs}(R)$.

The goal of the talk was to indicate how we prove this result, and to mention some applications to the Grothendieck–Witt groups of Dedekind rings.

Theorem 1 is proven by combining the following two results of which the first is a general result in the realm of Poincaré ∞-categories (I will define these momentarily) and the second says that the general results can recover the objects we are interested in.

Theorem 2. *For every Poincaré ∞-category $(\mathcal{C}, Q)$, there is a fibre sequence of spectra*

$$K(\mathcal{C})_{hC_2} \longrightarrow GW(\mathcal{C}, Q) \longrightarrow L(\mathcal{C}, Q)$$

Theorem 3. *There is a Poincaré ∞-category $(\mathcal{D}^p(R), Q^{gx})$ such that*

(1) *a canonical map $\mathcal{GW}^x(R) \longrightarrow \tau_{\geq 0} GW(\mathcal{D}^p(R), Q^{gx})$ is an equivalence, and*
(2) *the homotopy groups of $L(\mathcal{D}^p(R), Q^{gx})$ are, as described in Theorem 1: Ranicki's original (non-periodic) symmetric L-groups (for $x = s$).*

Remark 4.
(1) *Part (1) of Theorem 3 is a result of Hebestreit–Steimle, making use of parameterised algebraic surgery and was discussed in more detail in Hebestreit's talk.*
(2) *L-theory for Poincaré ∞-categories had previously been defined by Lurie in his lectures on Ranicki's theory of algebraic surgery and topological manifolds.*
(3) *GW-theory for Poincaré ∞-categories is defined in our work, using a hermitian Q-construction. Its underlying space can be described in terms of an algebraic cobordism category. Again, this perspective was discussed in more detail in Hebestreit's talk.*

Definition 5. A Poincaré ∞-category is a pair $(\mathcal{C}, Q)$ consisting of a small stable ∞-category $\mathcal{C}$ equipped with a Poincaré structure $Q\colon \mathcal{C}^{op} \to \mathrm{Sp}$, i.e. a reduced and 2-excisive functor Q satisfying a certain non-degeneracy condition.

The non-degeneracy condition allows to extract from Q (in fact from its symmetric cross effect) a duality equivalence $D\colon \mathcal{C}^{\mathrm{op}} \to \mathcal{C}$. Hence, a Poincaré ∞-category is a refinement of a category with duality: There are many Poincaré structures giving rise to the same duality, but whose Grothendieck–Witt theory will be generally different. Here is the most important example for the purpose of my talk:

Example 6. Let R be a commutative ring and let $\mathcal{D}^{\mathrm{p}}(R)$ be the perfect derived ∞-category of R. This category comes with a canonical functor $\mathrm{Proj}(R) \to \mathcal{D}^{\mathrm{p}}(R)$, and it turns out that a Poincaré structure is uniquely determined by its restriction to $\mathrm{Proj}(R)$. Having this, we consider the following Poincaré structures. Let $P \in \mathrm{Proj}(R)$, and let H denote the Eilenberg–Mac Lane functor.

(1) $Q^{\mathrm{s}}(P) = \big(H\left(\mathrm{Hom}_R(P \otimes_R P, R)\right)\big)^{hC_2}$
(2) $Q^{\mathrm{gs}}(P) = H\big(\mathrm{Hom}_R(P \otimes_R P, R)^{C_2}\big)$
(3) $Q^{\mathrm{gq}}(P) = H\big(\mathrm{Hom}_R(P \otimes_R P, R)_{C_2}\big)$
(4) $Q^{\mathrm{q}}(P) = \big(H\left(\mathrm{Hom}_R(P \otimes_R P, R)\right)\big)_{hC_2}$

Remark 7. (1) *There are canonical maps*

$$Q^{\mathrm{q}} \Rightarrow Q^{\mathrm{gq}} \Rightarrow Q^{\mathrm{gs}} \Rightarrow Q^{\mathrm{s}}$$

and these maps are equivalences if 2 *is invertible in* R*, but not in general (for instance for* $R = \mathbb{Z}$*).*

(2) *The Poincaré ∞-categories appearing in (2) and (3) are the ones that appear in Theorem 3.*

Having Theorem 1, one can prove results about GW-theory by proving results for K and L-theory. In fact, the K-theory side of Theorem 1 only depends on the underlying category with duality, which is the same for all of the Poincaré structures described in the above example. We then use Ranicki's method of algebraic surgery to prove the following comparison results for L-theory, and consequently for GW-theory:

Theorem 8. *Let R be a commutative ring. Then*

(1) *the maps* $\mathrm{L}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{q}}) \to \mathrm{L}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{gq}})$ *and* $\mathrm{GW}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{q}}) \to \mathrm{GW}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{gq}})$ *are isomorphisms on* π_k *for* $k \leq 1$ *and surjective for* $k = 2$.
(2) *The maps* $\mathrm{L}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{q}}) \to \mathrm{L}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{gs}})$ *and* $\mathrm{GW}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{q}}) \to \mathrm{GW}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{gs}})$ *are isomorphisms on* π_k *for* $k \leq -3$ *and surjective for* $k = -2$.

If R is in addition Noetherian and of finite global dimension d, then

(1) *the maps* $\mathrm{L}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{gq}}) \to \mathrm{L}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{s}})$ *and* $\mathrm{GW}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{gq}}) \to \mathrm{GW}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{s}})$ *are isomorphisms on* π_k *for* $k \geq d+3$ *and injective for* $k = d+2$.

(2) *The maps* $\mathrm{L}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{gs}}) \to \mathrm{L}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{s}})$ *and* $\mathrm{GW}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{gs}}) \to \mathrm{GW}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{s}})$ *are isomorphisms on* π_k *for* $k \geq d-1$ *and injective for* $k = d-2$.

In particular, the map $\mathrm{GW}^{\mathrm{q}}_k(R) \to \mathrm{GW}^{\mathrm{s}}_k(R)$ *is an isomorphism for* $k \geq d+3$.

Let R be a ring of integers in a number field. This is an example of a Noetherian ring of global dimension 1. We can describe all of its quadratic and symmetric L-groups in terms of Witt groups of symmetric and quadratic forms, the number of primes dividing 2, and the Picard group of R and its 2-completion R_2. All these groups are finitely generated, so we obtain the following consequence of Theorems 1 and 8:

Theorem 9. *Let* R *be the ring of integers in a number field. Then the higher Grothendieck–Witt groups* $\mathrm{GW}^{\mathrm{x}}_k(R)$ *are finitely generated.*

The fact that this is a consequence of Theorems 1 and 8 relies on the finite generation of the algebraic K-groups of R, a result due to Quillen. The case $\mathrm{x} = \mathrm{q}$ in Theorem 9 can also be shown using a combination of a cofinality result together with homological stability for the groups of automorphisms of hyperbolic forms over R and finiteness results known for these groups of automorphisms. In the symmetric case, to the best of our knowledge, this method is not known to work in the generality presented above.

Finally, we show the following result, which for the case $R = \mathbb{Z}$ was conjectured by Berrick and Karoubi:

Theorem 10. *Let* R *the ring of integers in a number field. Then the map*

$$\mathrm{GW}(\mathcal{D}^{\mathrm{p}}(R); Q^{\mathrm{s}}) \longrightarrow \mathrm{GW}(\mathcal{D}^{\mathrm{p}}(R[\tfrac{1}{2}]; Q^{\mathrm{s}})$$

is a 2-local equivalence in degrees ≥ 1.

Together with results of Berrick and Karoubi on $\mathrm{GW}^{\mathrm{s}}_k(\mathbb{Z}[\frac{1}{2}])$, the results of this talk give an almost full calculation of the higher Grothendieck–Witt groups of the integers. Such a calculation was also recently announced by Schlichting [1].

Remark 11. *Throughout the talk, I assumed that* R *is a commutative ring, as the applications I discussed were about Dedekind rings. Many of the general results, however, do not rely on* R *being commutative and work more generally in the presence of what we call a module with involution* M. *This includes* $M = R$ *for a ring with involution* R, *but also* M *being a general line bundle over a commutative ring* R.

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Parametrised algebraic surgery

Fabian Hebestreit
(joint work with Wolfgang Steimle)

Building on the talk of Markus Land I explained how classical Grothendieck-Witt groups of rings fit into the framework of Poincaré categories introduced by Lurie and developed in joint work with B. Calmès, E. Dotto, Y. Harpaz, M. Land, D. Nardin, T. Nikolaus and W. Steimle, [1].

To state the main results recall from Land's talk that a Poincaré ∞-category consists of a small stable ∞-category $\mathcal{C}$ and a reduced, quadratic functor $\Phi\colon \mathcal{C}^{\mathrm{op}} \to \mathrm{Sp}$, subject to a non-degeneracy condition. For such object we define a Grothendieck-Witt spectrum $\mathrm{GW}(\mathcal{C}, \mathcal{Q})$ via a version of the hermitian $\mathcal{Q}$-construction. This spectrum has many favourable properties, which allows us to explicitly determine its homotopy type in many situations as was detailed by Land. It remains to show that spectra of interest arise via this construction.

To this end let R be a ring and (M, σ) an invertible module with involution over R, that is M is an $R \otimes R$-module, together with an involution $\sigma\colon M \to M$ such that

(1) σ is linear over the flip involution on $R \otimes R$,
(2) M is finitely generated projective when restricted to an R-module along either inclusion of R into $R \otimes R$ (σ provides an equivalence between these restrictions), and
(3) the natural map
$$R \to \mathrm{End}_R(M)$$
is an isomorphism (where again we regard M as an R-module by restriction along either inclusion).

Simple examples include

(1) a commutative ring R and any finitely generated projective, $\otimes_R$-invertible R-module M (such as $M = R$) regarded as an $R \otimes R$-module via the multiplication $R \otimes R \to R$, with involution $\pm\mathrm{id}_M$, or
(2) a ring R with anti-involution τ, and $M = R$ made into an $R \otimes R$-module using the involution on the right factor and then given the involution $\sigma = \pm\tau$.

To such data come associated three functors
$$\mathrm{Quad}_M, \mathrm{Sym}_M, \mathrm{Ev}_M, \colon \mathrm{Proj}(R)^{\mathrm{op}} \to \mathrm{Ab}$$
that take a finitely generated projective (left) R-module P to the abelian group of M-valued quadratic, symmetric or even, that is
$$\mathrm{Hom}_{R\otimes R}(P \otimes P, M)_{\mathrm{C}_2}, \quad \mathrm{Hom}_{R\otimes R}(P \otimes P, M)^{\mathrm{C}_2}$$
$$\text{or} \quad \mathrm{im}\left[\mathrm{nm}\colon \mathrm{Hom}_{R\otimes R}(P \otimes P, M)_{\mathrm{C}_2} \to \mathrm{Hom}_{R\otimes R}(P \otimes P, M)^{\mathrm{C}_2}\right],$$
respectively, where nm denotes the norm map and the (co)invariants are taken with respect to the conjugation action using the flip on $P \otimes P$ and the involution on M. Associated to these functors are on the one hand groupoids of unimodular

forms of the given type which we will call $\mathrm{Unimod}^r(R,M)$ with $r \in \{q,s,e\}$, where an object is a pair (P,q) with q is an element of $\mathrm{Quad}_M(P), \mathrm{Sym}_M(P)$ or $\mathrm{Ev}_M(P)$ as appropriate, whose associated map $q_\# \colon P \to \mathrm{Hom}_R(P,M)$ is an isomorphism. This groupoid is symmetric monoidal under orthogonal sum, and thus gives rise to an E_∞ space of the same name. One then classically defines the Grothendieck-Witt space $\mathcal{GW}^r(R,M)$ in analogy with the algebraic K-theory of R as the homotopical group completion of this E_∞-space, in formulae

$$\mathcal{GW}^r(R,M) = \mathrm{Unimod}^r(R,M)^{\mathrm{grp}}.$$

On the other hand, the functors $\mathrm{Quad}_M, \mathrm{Sym}_M$ and Ev_M admit animations (or in more classical terminology: non-abelian derived functors) $\mathcal{Q}_M^{gr} \colon \mathcal{D}^p(R)^{\mathrm{op}} \to \mathrm{Sp}$, again with $r \in \{q,s,e\}$; here $\mathcal{D}^p(R)$ denotes the perfect derived category of R.

The main result of the talk was then the following:

Theorem 1. *The natural map*

$$\mathcal{GW}^r(R,M) \longrightarrow \Omega^\infty \mathrm{GW}(\mathcal{D}^p(R), \mathcal{Q}_M^{gr})$$

is an equivalence for all R and M as above.

For applications of this result, I refer the reader to Land's talk, but let me just mention that previous work on the left hand side was largely restricted to the case where 2 in R is a unit [6, 7] (in which case in particular the three types of form under consideration turn out to agree), whereas no such assumption is required for our analysis of the right hand side in [1].

The result is akin to the (weight version of the) theorem of the heart in algebraic K-theory, see e.g. [2], a generalisation Waldhausen's sphere theorem and thus Quillen's "+=Q"-theorem. This is usually proven by employing the machinery of exact or Waldhausen categories, the requisite generalisation of which to the setting above is not sufficiently developed at present (unless 2 is appropriately invertible), but see [5, 8].

We will therefore proceed in a completely different manner: As I will explain momentarily, the hermitian $\mathcal{Q}$-construction defining the right hand side can be regarded as an algebraic type of cobordism category, and the proof of the theorem follows a strategy developed by Galatius and Randal-Williams to relate the homotopy theory of diffeomorphism groups to that of cobordism categories. Before explaining the analogy, let me therefore briefly recall that for a d-dimensional vector bundle γ, the ∞-category Cob_d^γ is defined to have objects closed $d-1$-dimensional manifolds M equipped with a bundle map $TM \oplus \mathbb{R} \to \gamma$ and morphisms are cobordisms between such pairs. The higher structure of this category is arranged so that there are equivalences

$$\mathrm{Hom}_{\mathrm{Cob}_d^\gamma}(M,N) \simeq \sum_W \mathrm{BDiff}_\partial^\gamma(W)$$

where the sum (i.e. disjoint union) runs over all diffeomorphism types of γ-cobordisms between M and N. Letting W range through the closed d-dimensional

γ-manifolds there arises a map

$$\sum_W \mathrm{BDiff}^\gamma(W, D^d) \longrightarrow \mathrm{Hom}_{\mathrm{Cob}_d^\gamma}(S^{d-1}, S^{d-1}) \longrightarrow \Omega|\mathrm{Cob}_d^\gamma|$$

by removing the fixed disc from W and replacing it with a pair of pants. Boundary connecting sum gives the left hand side the structure of an E_d-space Man_γ, often referred to as a moduli space of manifolds, and the simplest case of the results of Galatius and Randal-Williams [4] can be stated as:

Theorem 2. *For $n > 2$ the map just described induces an equivalence*

$$(\mathrm{Man}_{\gamma_n}^{\mathrm{high}})^{\mathrm{grp}} \longrightarrow \Omega|\mathrm{Cob}_{2n}^{\gamma_n}|,$$

where γ_n is the tautological bundle over $\tau_{>n}\mathrm{BO(2n)}$ and the source (before group completion) is the collection of path components of Man_{γ_n} spanned by $n-1$-connected manifolds.

This result is one key step in their computation of the homology of the spaces $\mathrm{BDiff}(W, D^{2n})$ for highly connected, and even dimensional W (though their results also cover non-highly connected manifolds and diffeomorphism groups not fixing discs, but intrestingly not odd dimensional manifolds). The proof proceeds by regarding the left hand side as $\Omega|\mathrm{Cob}_{\mathrm{high}}^{\gamma_n}|$, where $\mathrm{Cob}_{\mathrm{high}}^{\gamma_n}$ denotes the subcategory of $\mathrm{Cob}_{2n}^{\gamma_n}$ spanned by n-connected $2n-1$-manifolds (i.e. homotopy spheres) and their $n-1$-connected cobordisms, and then filtering the difference between these categories by incrementally relaxing the connectivity constraints, first on objects then on morphisms. They then show that none of the filtration steps changes the homotopy type under consideration, increasing the connectivity of manifolds by means of a process they termed "parametrised surgery".

There are two further principal ingredients into their work: The group completion theorem of McDuff and Segal which allows one to analyse the homology of the group completion in terms of the homology of the constituents, and the celebrated theorem of Galatius, Madsen, Tillmann and Weiss on the homotopy type of the cobordism category [3]:

Theorem 3. *For every d and γ there is a canonical equivalence*

$$|\mathrm{Cob}_d^\gamma| \simeq \Omega^{\infty-1}\mathrm{MT}\gamma,$$

the so-called scanning map, where the right hand side denotes the Thom spectrum of $-\gamma$.

Since this makes the cohomology of the cobordism category (and its loop space) fairly easily accessible, one can extract information about diffeomorphism groups.

Now, for a Poincaré category we define in [1] an ∞-category $\mathrm{Cob}(\mathcal{C}, \mathcal{Q})$ with objects the Poincaré objects in $(\mathcal{C}, \Sigma\mathcal{Q})$, i.e. pairs (X, q) with $q \in \Omega^{\infty-1}\mathcal{Q}(X)$, satisfying an appropriate unimodularity condition, and morphisms from (X, q) to (Y, q') diagrams of the form

$$X \xleftarrow{f} W \xrightarrow{g} Y$$

together with an identification $f^*(q) \simeq g^*(q')$ that satisfy a relative unimodularity condition. Think about unimodularity for objects in $\mathcal{D}^p(R)$ (in the absolute case) as Poincaré duality, this condition is an abstract version of Lefschetz duality for the "cobordism" W relative to its "boundary pieces" (X, q) and (Y, q') (though I will not spell it out here). A span as above is therefore an abstraction of Ranicki's notion of an algebraic cobordism between Poincaré chain complexes, which hopefully suffices to justify the name algebraic cobordism category. Since this ∞-category is rigorously build using the hermitian $\mathcal{Q}$-construction as is the Grothendieck-Witt spectrum, there is a straight forward relation between the two; as part of [1] we obtained the following formal analogue of the theorem of Galatius, Madsen, Tillmann and Weiss:

Theorem 4. *For every Poincaré ∞-category $(\mathcal{C}, \mathcal{Q})$ there is a canonical equivalence*

$$|\mathrm{Cob}(\mathcal{C}, \mathcal{Q})| \simeq \Omega^{\infty-1}\mathrm{GW}(\mathcal{C}, \mathcal{Q}).$$

Furthermore, there are canonical maps

$$\mathrm{Unimod}^r(R, M) \longrightarrow \mathrm{Hom}_{\mathrm{Cob}(\mathcal{D}^p(R), \mathcal{Q}^{gr}_M)}(0, 0) \longrightarrow \Omega|\mathrm{Cob}(\mathcal{D}^p(R), \mathcal{Q}^{gr}_M)|$$

which under the above equivalences group complete to the map from the first theorem. This translation allows us to treat the theorem of the heart as an algebraic analogue of the results of Galatius and Randal-Williams, and to transport their parametrised surgery techniques into the algebraic setting (whence the title of this talk).

To properly state the output, recall from Goodwillie calculus that every quadratic functor $\mathcal{Q}\colon \mathcal{C}^{\mathrm{op}} \to \mathrm{Sp}$ uniquely decomposes into a symmetric biexact part $\mathrm{B}_{\mathcal{Q}}\colon \mathcal{C}^{\mathrm{op}} \to \mathrm{Sp}$ (the cross effect) and an exact part $\mathrm{L}_{\mathcal{Q}}\colon \mathcal{C}^{\mathrm{op}} \to \mathrm{Sp}$ (the linear approximation) determined by the existence of a natural cartesian square

$$\begin{array}{ccc} \mathcal{Q}(X) & \longrightarrow & \mathrm{L}_{\mathcal{Q}}(X) \\ \downarrow & & \downarrow \alpha \\ \mathrm{B}_{\mathcal{Q}}(X, X)^{\mathrm{hC}_2} & \longrightarrow & \mathrm{B}_{\mathcal{Q}}(X, X)^{\mathrm{tC}_2} \end{array}$$

where the superscripts in the lower row denote the homotopy fixed point, and Tate construction, respectively. In the case of $\mathcal{C} = \mathcal{D}^p(R)$ these functors are automatically of the form

$$\mathrm{L}_{\mathcal{Q}}(X) \simeq \hom_R(X, N) \quad \text{and} \quad \mathrm{B}_{\mathcal{Q}}(X, Y) \simeq \hom_{R\otimes R}(X \otimes Y, M)$$

for some $N \in \mathcal{D}(R)$ and $M \in \mathcal{D}(R \otimes R)$ equipped with an involution satisfying a derived version of the properties listed on the first page. Under these identifications α is induced by an R-linear map $N \to M^{\mathrm{tC}_2}$. In these terms, the main result is:

Theorem 5. *For a triple (M, N, α) as above the natural map*

$$\mathrm{Pn}^{\mathrm{high}}(\mathcal{D}^p(R), \mathcal{Q}^{\alpha}_M)^{\mathrm{grp}} \longrightarrow \Omega|\mathrm{Cob}(\mathcal{D}^p(R), \mathcal{Q}^{\alpha}_M)|$$

is an equivalence, whenever M is concentrated in degree $2n$ and N is n-connective.

Here the left hand side denotes the space of Poincaré objects for $\mathcal{Q}$ that happen to be $n-1$-connected (and thus concentrated in degree n by duality), just as in the theorem of Galatius and Randal-Williams, and we think of M (or equivalently the bilinear part) as determining the dimension of the Poincaré objects and N as an analogue of the tangential structure encoded by γ in the geometric setting. For the functors $\mathcal{Q}^{gr}_M$ the bilinear part is easily seen to be given by M regarded as a chain complex in degree 0, and essentially by definition we thus have

$$\mathrm{Unimod}^r(R, M) \simeq \mathrm{Pn}^{\mathrm{high}}(\mathcal{D}^p(R), \mathcal{Q}^{gr}_M),$$

the Poincaré objects concentrated in degree 0.
To apply this result then we, finally, use the somewhat surprising result from [1], that the linear parts of $\mathcal{Q}^{gq}_M, \mathcal{Q}^{gs}_M$ and $\mathcal{Q}^{ge}_M$ are given by $\tau_{\geq 2} M^{\mathrm{tC}_2}$, $\tau_{\geq 0} M^{\mathrm{tC}_2}$ and $\tau_{\geq 1} M^{\mathrm{tC}_2}$, respectively, all of which are evidently 0-connective.

Remarks. (1) Another interesting example to which the result above applies is the quadratic functor $\mathcal{Q}^q_M(X) = \mathrm{Hom}_{R\otimes R}(X \otimes X, M)_{\mathrm{hC}_2}$, parametrising derived quadratic forms. Its linear part simply vanishes. The associated Grothendieck-Witt theory has not appeared in the literature so far, but the L-theory associated to this Poincaré ∞-category is the classical quadratic L-theory of Ranicki and Wall (the same is not correct for $\mathcal{Q}^{gq}_M$).

(2) In contrast, a non-example is $\mathcal{Q}^s_M(X) = \mathrm{Hom}_{R\otimes R}(X \otimes X, M)^{\mathrm{hC}_2}$ which gives rise to the usual symmetric L-theory spectrum of R; in this case the linear part is classified by M^{tC_2}, which is periodic.

(3) Just like the result of Galatius and Randal-Williams the restriction to even numbers is essential in the result. For odd numbers the final surgery in the middle dimension encounters an obstruction.

(4) Relaxing the connectivity assumptions on $\mathrm{L}_{\mathcal{Q}}$ to $n-k$-connectivity still allows one to concentrate objects in a band of degrees $[n-k, n+k]$ (and similarly for cobordisms) without affecting the homotopy type of $|\mathrm{Cob}(\mathcal{D}^p(R), \mathcal{Q})|$, and since one then stays away from the middle dimension this statement has an analogue for M concentrated in degree $2n+1$. Applying this to the quadratic functors $\Omega^{2n}\mathcal{Q}^{\mathrm{gs}}_M$ (where $k = n$), one obtains a proof that $\mathrm{L}_{2n}(\mathcal{D}^p(R), \mathcal{Q}^{gs}_M)$ is Ranicki's short symmetric L-group of R (and similarly in odd degrees), which forms the basis of our analysis of $\mathcal{GW}^s(R, M)$ in [1].

(5) The result is also not restricted to derived categories, but rather works whenever $\mathcal{C}$ admits a bounded weight structure and $\mathrm{B}_{\mathcal{Q}}$ and $\mathrm{L}_{\mathcal{Q}}$ satisfy appropriate (co)connectivity assumptions. In this form the result extends to cover for instance the quadratic Grothendieck-Witt spectra of E_1-rings, the visible Grothendieck-Witt spectra of categories of parametrised spectra (which recover the visible LA-theory of Weiss and Williams), and also so-called hyperbolic Poincaré ∞-categories, in which case our result recover the theorem of the heart in algebraic K-theory.

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Arc matching complexes and finiteness properties

Kai-Uwe Bux

We describe a class of simplicial complexes that combine matching complexes in graphs with the arc complex associated to a surface. Connectivity properties of these complexes are related to higher finiteness properties of groups.

Let Σ be a surface, possibly with boundary; and let O denote a finite set of marked points, which we regards as *obstacles*, i.e., when we consider something up to isotopy, the isotopy shall be relative to O. We fix a subset $N \subseteq O$ of distinguished obstacles, which we call *nodes*.

Let Γ be a graph with node set N. A *drawing* of Γ onto Σ is an embedding of the geometric realization $|\Gamma|$ into Σ that is the identity on N and meets the boundary $\partial\,\Sigma$ only at nodes. We consider drawings up to isotopy relative to obstacles.

Let $\mathcal{G}$ be a collection of graphs with node set N that is closed with respect to taking subgraphs. Then $\mathcal{G}$ can be considered as a simplicial complex (a k-simplex is a graph in $\mathcal{G}$ with $k+1$ edges). A $\mathcal{G}$-*drawing* is the drawing of a graph $\Gamma \in \mathcal{G}$ onto Σ. The $\mathcal{G}$-drawings also form a complex, which we call the *arc complex* associated to $\mathcal{G}$ and Σ.

There are many interesting examples of complexes of graphs, i.e., the complex of forests in a given graph Γ or the complex of *matchings* in Γ. A matching in Γ is a collection of pairwise disjoint edges. Clearly, any subset of a matching is a matching. The connectivity properties of many graph complexes are well understood. We shall investigate the higher connectivity of associated arc complexes.

In this talk, we focus on one particular example: the arc complexes associated to the matching complex of a complete bipartite graph. Let $\mathcal{A}(m,n,k)$ be the arc complex associated to the complete bipartite graph with m hollow nodes and n solid nodes. The underlying surface Σ shall be the disk. We place the hollow nodes

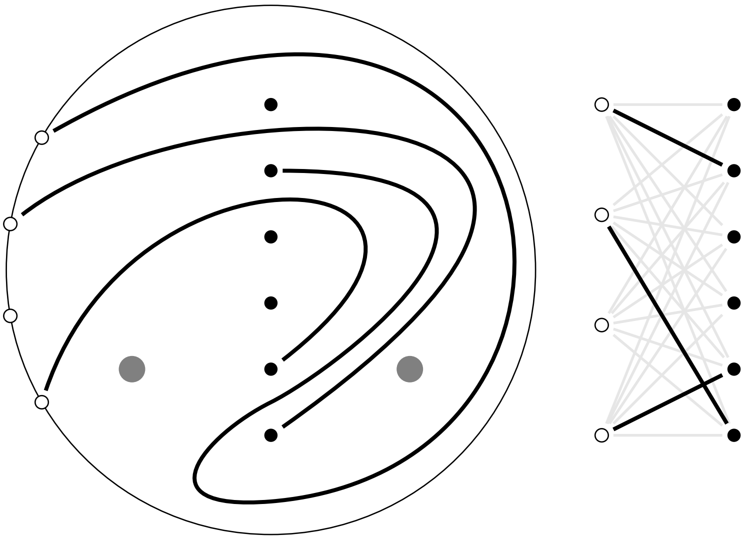

Figure 1. An arc matching in $\mathcal{A}(4,6,2)$ with its corresponding matching in $\mathcal{M}(4,6)$.

on the boundary and the solid nodes in the interior where k additional obstacles are marked. The connectivity of the underlying matching complex is known:

Theorem 1 ([1, Theorem 1.1]). *For the complete bipartite graph $K_{m,n}$ on m white and n black nodes, the matching complex $\mathcal{M}(m,n)$, also called the* chess board complex, *is ν-connected where $\nu(m,n) = \min\left(m, n, \left\lfloor \frac{m+n+1}{3} \right\rfloor\right) - 2$.*

In this talk, we sketch a proof of a corresponding connectivity result for the associated arc complex:

Theorem 2. *$\mathcal{A}(m,n,k)$ is ζ-connected, where $\zeta(m,n) := \min\left(m, \left\lfloor \frac{n+1}{2} \right\rfloor\right) - 2$.*

Remark 3. *The obvious projection $\mathcal{A}(m,n,k) \to \mathcal{M}(m,n)$ has poorly behaved fibers. Hence connectivity properties of the base do not readily transfer to the total space. This explains in part why the connectivity bounds for both complexes differ.*

Remark 4. *Ken Brown has devised a method (now standard) for deriving higher finiteness properties of groups. In his seminal paper* [2]*, he discusses the family of Houghton groups H_m and deduces that H_m is of type F_{m-1} but not of type F_m from the connectivity of the matching complexes $\mathcal{M}(m,n)$ for $n \to \infty$.*

In his PhD thesis [4]*, Franz Degenhardt has introduced braided versions H_m^{br} of the Houghton groups and studied their finiteness properties. He was able to show the analogue of Brown's result for the types F_1, F_2, and F_3. As he describes explicitly low-dimensional skeleta of classifying spaces, higher finiteness properties are not within reach of his methods.*

It turns out, that Brown's method can be adapted to deal with braided Houghton groups. The arc matching complexes $\mathcal{A}(m,n,0)$ arise as relative links and their connectivity (again for $n \to \infty$) determines the finiteness properties of H_m^{br}. Thus Theorem 2 implies that H_m^{br} is of type F_{m-1} but not of type F_m.

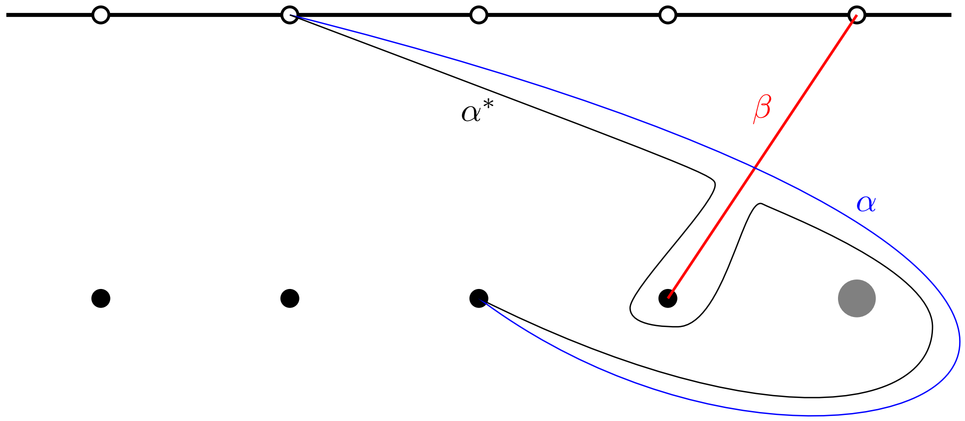

FIGURE 2. Surgery towards the end of β.

To prove Theorem 2, we consider the chain

$$\mathcal{A}(m-1, n-1, k+1) \subset \mathcal{A}(m, n-1, k+1) \subset \mathcal{A}(m, n, k)$$

of inclusions. Relative links are isomorphic to $\mathcal{A}(m-1, n-2, k+1)$ for the first inclusion and to $\mathcal{A}(m-1, n-1, k)$ for the second. Hence, induction applies to these links, and we deduce that the induced map in homotopy

$$\pi_d(\mathcal{A}(m-1, n-1, k+1)) \to \pi_d(\mathcal{A}(m, n, k))$$

is an epimorphism for $d \leq \zeta(m, n)$. It remains to show that it has trivial image, i.e., any d-sphere S in $\mathcal{A}(m-1, n-1, k+1)$ can be contracted within $\mathcal{A}(m, n, k)$.

We contract S inside $\mathcal{A}(m, n, k)$ using a method pioneered by Alan Hatcher [5]. Fix an arc β connecting the hollow node with label m to the solid node with label n. We homotope S into the star of β, where it dies. To this end, we draw all arcs (i.e. vertices) of S simultaneously onto Σ. If none of them intersects β, we have nothing to do. Otherwise, let α be the arc whose point of intersection is closest (along β) to the solid endpoint of β. We do surgery along β to obtain α^* (see Figure 2). Since no arc from S intersects the final segment of β cut off by α, any arc from S in the link of α also lies in the link of α^*.

The final step is to homotope S so that is uses α^* instead of α. That reduces the number of intersections with β and we can proceed until S has moved into the star of β. The main difficulty stems from the fact that α and α^* are not connected by an edge in $\mathcal{A}(m, n, k)$: they share end points. However, we shall not treat this problem here.

Remark 5. *In* [3], *the same program has been carried out for the braided Thompson's groups F^{br} and V^{br}. Both groups are of type F_∞ because the connectivity of arc matching complexes tends to infinity. In the case of F^{br}, the underlying graph complexes are the matching complexes associated to linear graphs, whereas in the case of V^{br} one considers the matching complexes over complete graphs.*

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Outer space for RAAGs

Corey Bregman

(joint work with Ruth Charney, Karen Vogtmann)

The classical symmetric space $Q_n = \mathrm{SL}(n,\mathbb{R})/\,\mathrm{SO}(n,\mathbb{R})$ parametrizes marked lattices in $\mathbb{R}^n$, i.e., discrete embeddings $\alpha\colon \mathbb{Z}^n \hookrightarrow \mathbb{R}^n$, up to rotation. Alternatively, the quotient $T^n = \mathbb{R}^n/\alpha(\mathbb{Z}^n)$ is a torus equipped with a flat metric, and the marking α gives a particular choice of basis for $\pi_1(T^n)$. Thus Q_n can also be regarded as the space of marked, flat n-tori. There is a natural left action of $\mathrm{GL}(n,\mathbb{Z})$ on Q_n which changes the marking, leaving the metric fixed. Since Q_n is connected, these changes in marking can be achieved by continuously varying the flat metric on the same underlying topological space, the n-torus. The action of $\mathrm{GL}(n,\mathbb{Z})$ on Q_n and the topology of the quotient is intimately connected with the algebraic structure of $\mathrm{GL}(n,\mathbb{Z})$, being used, for example, to compute its group cohomology and to prove it is a virtual duality group in the sense of Bieri and Eckmann [1].

By analogy with Q_n, Culler–Vogtmann introduced a finite-dimensional, contractible "Outer space" CV_n on which the outer automorphism group $\mathrm{Out}(F_n)$ of the free group F_n acts properly [2]. Points in CV_n are pairs (G,ρ) where G is a rank n metric graph and ρ is an identification of $\pi_1(G)$ with F_n. The metric assigns a positive length to each edge and the marking ρ specifies a homotopy equivalence $\rho\colon G \to R_n$, where R_n is the n-petaled rose (a wedge of n circles). In contrast to Q_n, the basic objects in CV_n have varying homeomorphism types, but each graph admits an obvious homotopy equivalence to R_n by collapsing a maximal tree. In this case, however, changes in marking are achieved by successively expanding one maximal tree (thereby changing the homeomorphism type) and then collapsing another. The action of $\mathrm{Out}(F_n)$ on CV_n has been an indispensable tool in the study of algebraic and dynamical properties of $\mathrm{Out}(F_n)$.

In this talk we study outer automorphism groups of right-angled Artin groups, a class which includes both $\mathrm{Out}(F_n)$ and $\mathrm{GL}(n,\mathbb{Z}) = \mathrm{Out}(\mathbb{Z}^n)$, with a view towards putting the above analogy on more formal footing. Recall that if $\Gamma = (V,E)$ is a finite simplicial graph, the *right-angled Artin group* (RAAG) A_Γ has a presentation

$$A_\Gamma = \langle v \in V \mid [v,w] = 1 \text{ if } (v,w) \in E\rangle.$$

That is, there is one generator for each vertex, and two generators commute if their corresponding vertices share an edge in Γ. If Γ has no edges, then A_Γ is free, while if Γ is a complete graph then A_Γ is free abelian. In this way, RAAGs form a natural class of groups interpolating between F_n and $\mathbb{Z}^n$.

The automorphism group $\mathrm{Out}(A_\Gamma)$ is generated by two main types of automorphisms. Elements in the "untwisted" automorphism group $U(A_\Gamma)$ involve generators which do not commute and therefore resemble automorphisms of F_n. In contrast, elements in the "twist" subgroup $T(A_\Gamma)$ involve generators which do commute, and thus are most similar to elementary matrices in $\mathrm{GL}(n, \mathbb{Z})$.

Past approaches to the study of $\mathrm{Out}(A_\Gamma)$ have been largely algebraic (see, for example [3, 4, 5]). Here we focus on constructing an analogue of outer space in order to apply geometric methods. As the examples of Q_n and CV_n demonstrate, in order to obtain a space on which $\mathrm{Out}(A_\Gamma)$ acts, one should parametrize metric spaces whose fundamental group is A_Γ. In [6], Charney–Stambaugh–Vogtmann constructed an outer space K_Γ for $U(A_\Gamma)$. Points in K_Γ correspond to certain marked, locally CAT(0) cube complexes with fundamental group A_Γ called *Γ-complexes*. There is a canonical locally CAT(0) cube complex with fundamental group A_Γ, known as the *Salvetti complex* $\mathbb{S}_\Gamma$, which has a k-torus for each k-clique in Γ. This plays the analogous role of the rose R_n in CV_n, in the sense that every Γ-complex X, though not necessarily homeomorphic to $\mathbb{S}_\Gamma$, admits homotopy equivalence $c\colon X \to \mathbb{S}_\Gamma$ which collapses a subcomplex to a point.

In order to realize the automorphisms in the twist subgroup $T(A_\Gamma)$, we vary the flat metric along tori in Γ-complexes. We define a space $\mathcal{O}_\Gamma$ consisting of metric spaces (X, d) marked with a homotopy equivalence $h\colon X \to \mathbb{S}_\Gamma$. Each X is homeomorphic to a Γ-complex and d is a locally CAT(0) metric, obtained by replacing cubes with arbitrary parallelotopes. As in the special cases of $\mathrm{GL}(n, \mathbb{Z})$ acting on Q_n and $\mathrm{Out}(F_n)$ acting on CV_n, $\mathrm{Out}(A_\Gamma)$ acts on $\mathcal{O}_\Gamma$ by changing the marking. The main theorem states,

Theorem 1. *For any right-angled Artin group A_Γ, the space $\mathcal{O}_\Gamma$ is finite-dimensional, contractible and the group $\mathrm{Out}(A_\Gamma)$ acts properly.*

In particular, $\mathcal{O}_\Gamma$ is a rational classifying space for $\mathrm{Out}(A_\Gamma)$. The main theorem gives a unified construction of an outer space for all RAAGs, and paves the way for further investigation of $\mathrm{Out}(A_\Gamma)$ by geometric means. For example, both Q_n and CV_n have natural compactifications by certain degenerate actions and it is natural to wonder if $\mathcal{O}_\Gamma$ can be compactified in this way as well. Another direction concerns the fixed sets of finite subgroups, and in particular whether they are nonempty and contractible. Proving the latter would imply that $\mathcal{O}_\Gamma$ is a finite-dimensional $\underline{E}G$, or classifying space for proper actions.

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Expansions, Completions and Automorphisms of welded tangled foams

Marcy Robertson
(joint work with Zsuzsanna Dancso)

Welded tangles are knotted surfaces in $\mathbb{R}^4$. We say that welded tangles admit "foamed vertices" if we allow surfaces to merge and split. In [BD16] Bar-Natan and Dancso show that the resulting welded tangled foams carry an algebraic structure, similar to the planar algebras of Jones, called a circuit algebra. In forthcoming work with Dancso and Halacheva ([DHR20]), we provide a one-to-one correspondence between circuit algebras and a form of rigid tensor category called wheeled props. This is a higher dimensional version of the well-known algebraic classification of planar algebras as certain pivotal categories. Using this, we show that homomorphic expansions of welded tangled foams are isomorphisms of certain (completions of) wheeled props which are, in turn, in one-to-one correspondence with the solutions to the Kashiwara-Vergne conjecture in Lie theory.

In joint work in progress with Dancso, we this categorical description of welded tangled foams, wF to show that the homotopy automorphisms of the rational completion of wF are isomorphic to the group of symmetries KV, which act on the solutions to the Kashiwara-Vergne conjecture. Moreover, we explain how this approach illuminates the close relationship between the group KV and the prounipotent Grothendieck-Teichmüller group conjectured in [AT12].

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Vanishing results for chromatic localizations of algebraic K-theory

Georg Tamme
(joint work with Markus Land, Lennart Meier)

In this talk, I summarized some results of the paper [2]. It is a classical result of Waldhausen [4] that a 1-connective map of connective ring spectra which is a rational equivalence induces a rational equivalence on algebraic K-theory. From the viewpoint of chromatic homotopy theory, rationalization is just the zeroth step of a series of localizations on the category of spectra. We prove the following analog of Waldhausen's result at higher chromatic heights.

Theorem 1. *Let $n \geq 1$ be an integer. Let $A \to B$ be an n-connective map of connective ring spectra which is a $T(0) \oplus \cdots \oplus T(n)$-equivalence. Then the induced map of K-theory spectra*

$$K(A) \to K(B)$$

is again a $T(0) \oplus \cdots \oplus T(n)$-equivalence.

Here we have fixed an implicit prime p, and $T(n)$ denotes the telescope $V_n[v_n^{-1}]$ of a v_n-self map v_n on a p-local finite spectrum V_n of type-n. By convention $T(0) = H\mathbb{Q}$. From the theorem, we deduce several vanishing results for chromatic localizations of algebraic K-theory of ring spectra, for example:

Corollary 2. *Let $K(m)$ denote Morava K-theory of height m at p. Then $K(K(m))$ vanishes $T(n)$-locally and hence also $K(n)$-locally for $0 < n < m$.*

The following result can be seen as a version of the first theorem with a weaker assumption on the connectivity for a restricted class of ring spectra.

Theorem 3. *Let $n \geq 1$ be an integer. Then $T(n)$-local K-theory is truncating on $T(1) \oplus \cdots \oplus T(n)$-acyclic ring spectra in the following sense: For every $T(1) \oplus \cdots \oplus T(n)$-acyclic ring spectrum A, the natural map*

$$K(A) \to K(\pi_0 A)$$

is a $T(n)$-local equivalence.

Note that for $n \geq 2$ the theorem is equivalent to the statement that $K(A)$ vanishes $T(n)$-locally because Mitchell [3] has proven that the K-theory of discrete rings vanishes $T(n)$-locally for every $n \geq 2$.

Combining Theorem 3 with work of Hahn [1] we deduce the following redshift result.

Corollary 4. *Let A be a $K(1)$-acyclic $\mathbb{E}_\infty$-ring. Then $K(A)$ vanishes $T(n)$-locally for all $n \geq 2$.*

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Embedding calculus for knot spaces

Danica Kosanović

This talk expands on the one we gave earlier this year at the Oberwolfach Workshop *2008* "Low-dimensional Topology". Back then we presented our main results from [5], while this time, as the audience was more familiar with the Goodwillie–Weiss embedding calculus, we discussed the more technical results from that work.

We refer the reader to that earlier report for precise statements of the main results. In brief, we show that *the evaluation maps* $ev_n\colon \mathrm{Emb}_\partial(I,M) \to T_n(M)$ *from the space of (long) knots in a* 3*-manifold* M *to its corresponding Taylor tower stages are* 0*-connected*; this completes some of the missing cases of the connectivity estimates of Goodwillie and Klein [4], and is also closely related to the theory of finite type knot invariants of Vassiliev. Namely, for classical knots $M = I^3$ we get that $\pi_0 ev_n$ are *universal rational* additive Vassiliev invariants, the first such not using integration over configuration spaces, and having potential of being universal over the integers. Combining the work of Boavida de Brito and Horel [1] with ours gives further evidence for that conjecture: $\pi_0 ev_n$ is a universal type $\leq n-1$ additive invariant p-locally for $p \geq n-2$.

We believe that our intermediate results may be of independent interest. In the current work in progress we use them to study the space of properly embedded arcs $\mathrm{Emb}_\partial(I,M)$ in an oriented connected smooth manifold M of any dimension $d \geq 3$ with non-empty boundary, which agree near boundary with a fixed proper embedding $\mathrm{U}\colon I \hookrightarrow M$ (an arbitrarily chosen basepoint). Let us give some details.

Degree one. It turns out that the first Taylor stage $T_1(M) = \mathrm{Imm}_\partial(I,M)$ is the space of immersed arcs. Note that for $d > 3$ we simply have $\pi_0\mathrm{Emb}_\partial(I,M) \cong \pi_1 M \cong \pi_0\mathrm{Imm}_\partial(I,M)$, so it is natural to try to determine the lowest homotopy group distinguishing embeddings from immersions. In other words, we are asking about the connectivity of the natural inclusion $ev_1\colon \mathrm{Emb}_\partial(I,M) \hookrightarrow \mathrm{Imm}_\partial(I,M)$.

It follows by general position arguments[1] that it is at least $(d-3)$-connected:

$$\pi_k ev_1\colon \pi_k\mathrm{Emb}_\partial(I,M) \to \pi_k\mathrm{Imm}_\partial(I,M)$$

is an *isomorphism* for $0 \leq k \leq d-4$ and a *surjection* for $k = d-3$. To determine the kernel in the last case, one can first consider the homotopy fibre

$$\overline{\mathrm{Emb}}_\partial(I,M) := \mathrm{hofib}_{\mathrm{U}}(ev_1),$$

[1] Namely, the double point set of a k-parameter family of immersions generically has dimension $k + d - 2(d-1)$. This is negative if $k \leq d-3$.

the *space of embeddings modulo immersions.* By the argument above, $\overline{\mathrm{Emb}}_\partial(I,M)$ is $(d-4)$-connected, and we can find $\pi_{d-3}\overline{\mathrm{Emb}}_\partial(I,M)$ using the *second layer in the Taylor tower*

$$\mathsf{F}_2(M) := \mathrm{fib}_{\mathrm{U}}\big(p_2 \colon T_2(M) \to T_1(M) = \mathrm{Imm}_\partial(I,M)\big)$$

and the evaluation map $ev_2 \colon \overline{\mathrm{Emb}}_\partial(I,M) \to \mathsf{F}_2(M)$.

Theorem 1. *For any $d \geq 3$ there is an explicit homotopy equivalence*

$$\chi \colon \mathsf{F}_2(M) \to \Omega^2\big(\mathbb{S}^{d-1} \vee \Sigma^{d-1}\Omega M\big).$$

In particular, $\mathsf{F}_2(M)$ is $(d-4)$-connected and $\chi_* \colon \pi_{d-3}\mathsf{F}_2(M) \xrightarrow{\cong} \mathbb{Z}[\pi_1 M]$.
Moreover, there is a map of sets

$$\rho \colon \mathbb{Z}[\pi_1 M] \to \pi_{d-3}\overline{\mathrm{Emb}}_\partial(I,M)$$

which is a group homomorphism if $d \geq 4$, and which satisfies

$$\chi_* \circ \pi_{d-3} ev_2 \circ \rho = \mathrm{Id}_{\mathbb{Z}[\pi_1 M]}.$$

The realisation map ρ is defined using ‘band-summing into a meridinal sphere’ as in Figure 1. As a consequence, we see that $\pi_{d-3}ev_2$ is surjective, but by results of Dax [2], and Goodwillie and Klein [4], it is *also injective*, so ρ is an isomorphism.

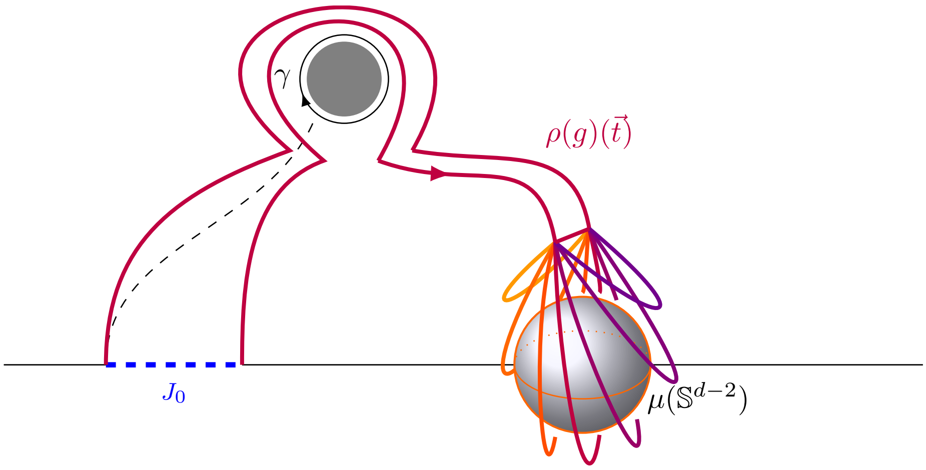

FIGURE 1. Here $g = [\gamma] \in \pi_1 M$ and for various values of $\vec{t} \in \mathbb{S}^{d-3}$ we depicted ‘time samples’ $\rho(g)(\vec{t}) \in \mathrm{Emb}_\partial(I,M)$ in the family.

General degree. We analogously have higher Taylor layers

$$\mathsf{F}_{n+1}(M) := \mathrm{fib}_{\mathrm{U}}\big(T_{n+1}(M) \xrightarrow{p_{n+1}} T_n(M)\big)$$

and the evaluation maps

$$ev_{n+1} \colon \ \mathrm{hofib}_{\mathrm{U}}\big(\mathrm{Emb}_\partial(I,M) \xrightarrow{ev_n} T_n(M)\big) \to \mathsf{F}_{n+1}(M).$$

Let $\mathsf{Tree}_{\pi_1 M}(n)$ be the set of rooted binary planar trees with n leaves each decorated by an element of $\pi_1 M$, and let

$$\mathsf{Lie}_{\pi_1 M}(n) := \frac{\mathbb{Z}[\mathsf{Tree}_{\pi_1 M}(n)]}{\text{Jacobi, antisymmetry}}$$

be the quotient by the usual relations satisfied by Lie brackets.

In particular, in degree one we have $\mathsf{Lie}_{\pi_1 M}(1) = \mathbb{Z}[\pi_1 M]$, and Theorem 1 can be generalised to an arbitrary degree as follows.
Let us denote by $N'B(\underline{n})$ the set of those words w in a Hall basis for the free Lie algebra on $\{x^i, x^{i'} : i \in \{1, \dots, n\}\}$, in which for each $1 \leq i \leq n$ at least one of the letters x^i or $x^{i'}$ appears. Let l_w denote the word length of w and $l'_w \leq l_w$ the number of letters in w with a prime.

Theorem 2 (See [5] for the proof of the statement in the first paragraph, and for the second in the case $d = 3$; the general case is work in progress)**.** *For any $d \geq 3$ there is an explicit homotopy equivalence*

$$\chi\colon \mathsf{F}_{n+1}(M) \to \Omega^n \prod_{w \in N'B(\underline{n})}^{\text{weak}} \Omega\Sigma^{1+(d-2)l_w}(\Omega M)^{\wedge l'_w}$$

Hence, $\mathsf{F}_{n+1}(M)$ is $(n(d-3)-1)$-connected and $\chi_*\colon \pi_{n(d-3)}\mathsf{F}_{n+1}(M) \xrightarrow{\cong} \mathsf{Lie}_{\pi_1 M}(n)$.

Moreover, there is a map of sets

$$\rho_n\colon \mathsf{Lie}_{\pi_1 M}(n) \to \pi_{n(d-3)}\mathrm{hofib}_{\mathrm{U}}(ev_n)$$

which is a group homomorphism if $d \geq 4$, and which satisfies

$$\chi_* \circ \pi_{n(d-3)} ev_{n+1} \circ \rho_n = \mathrm{Id}_{\mathsf{Lie}_{\pi_1 M}(n)}.$$

Similarly as before, this shows that ρ_n is surjective for $d \geq 3$, while for $d \geq 4$ the theorem of Goodwillie and Klein shows it is also injective.

At the end of the talk we briefly outlined how these computations can give insight into some open problems in 4-dimensional topology: in a joint project with Peter Teichner [6] we classify up to isotopy properly embedded disks in a 4-manifold that have a dual sphere in the boundary, using the corresponding 1-parameter families of arcs they induce (in a different 4-manifold!) – a problem posed in [3].

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Diffeomorphisms of even-dimensional discs outside the pseudoisotopy stable range

Alexander Kupers
(joint work with Oscar Randal-Williams)

The topological group $\mathrm{Diff}_\partial(D^d)$, of diffeomorphisms of a d-dimensional disc which fix pointwise a neighbourhood of the boundary, is one of the fundamental objects of differential topology: for $d \neq 4$, Morlet's theorem says there is a weak equivalence

$$B\mathrm{Diff}_\partial(D^d) \xrightarrow{\simeq} \Omega_0^{2n} \tfrac{\mathrm{Top}(d)}{\mathrm{O}(d)},$$

and smoothing theory gives a description of $\frac{\mathrm{Homeo}_\partial(M)}{\mathrm{Diff}_\partial(M)}$ in terms of $\frac{\mathrm{Top}(d)}{\mathrm{O}(d)}$ for any d-dimensional smooth manifold M [1, Essay IV & V].

In [2], we give a complete description of the rational homotopy groups of $B\mathrm{Diff}_\partial(D^{2n})$ for $2n \geq 6$ in the range $* \leq 4n-10$.

Theorem 1. *Let $2n \geq 6$. Then in degrees $d \leq 4n-10$ we have*

$$\pi_d(B\mathrm{Diff}_\partial(D^{2n})) \otimes \mathbb{Q} = \begin{cases} \mathbb{Q} & \text{if } d \geq 2n-1 \text{ and } d \equiv 2n-1 \bmod 4, \\ 0 & \text{otherwise.} \end{cases}$$

In higher degrees, we obtain information outside certain *bands*:

Theorem 2. *Let $2n \geq 6$. Then in degrees $d \geq 4n-9$ we have*

$$\pi_d(B\mathrm{Diff}_\partial(D^{2n})) \otimes \mathbb{Q} = \begin{cases} \mathbb{Q} & \text{if } d \equiv 2n-1 \bmod 4 \text{ and } d \notin \bigcup\limits_{r\geq 2} [2r(n-2)-1, 2rn-1], \\ 0 & \text{if } d \not\equiv 2n-1 \bmod 4 \text{ and } d \notin \bigcup\limits_{r\geq 2} [2r(n-2)-1, 2rn-1], \\ ? & \text{otherwise.} \end{cases}$$

As we explain now, the non-trivial classes in these theorems are detected by *topological Pontrjagin classes*. Since $B\mathrm{O} \to B\mathrm{Top}$ is a rational equivalence, there are cohomology classes $p_i \in H^{4i}(B\mathrm{Top}; \mathbb{Q})$. We can pull these back to $B\mathrm{Top}(2n)$. Weiss discovered that, in contrast with $B\mathrm{O}(2n)$, the relations $p_{n+i} = 0$ or $e^2 - p_n = 0$ need not hold in $H^*(B\mathrm{Top}(2n); \mathbb{Q})$ [3]. Morlet's theorem provides an isomorphism $\pi_*(B\mathrm{Diff}_\partial(D^{2n})) \otimes \mathbb{Q} \cong \pi_{*+2n}(\frac{\mathrm{Top}(2n)}{\mathrm{O}(2n)}) \otimes \mathbb{Q}$ and in the diagram

$$\pi_{*+2n}(\tfrac{\mathrm{Top}(2n)}{\mathrm{O}(2n)}) \otimes \mathbb{Q} \longleftarrow \pi_{*+2n+1}(B\mathrm{Top}(2n)) \longrightarrow H_{*+2n+1}(B\mathrm{Top}(2n); \mathbb{Q})$$

the left map is an isomorphism for $* \geq 2n-1$. Thus the non-trivial classes in Theorems 1 and 2 give rise to homology classes in the right term, and we show that these pair non-trivially against the appropriate cohomology class p_{n+i} or $e^2 - p_n$.

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