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Mathematics and its Ancient Classics Worldwide: Translations, Appropriations, Reconstructions, Roles (hybrid meeting)

Organized by
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Abstract. The workshop analyzes the constitution, recovery, and role of the classical texts in mathematical practice throughout history. It aims at problematizing the notion of "classic", to make it a historical category and to study the rhetorical, pedagogical, and institutional mechanisms that contribute to secure the status of classic to specific texts. So far, the focus of the historiography has dealt mostly with Greek classics and their impact on Western European societies. We aim to expand the focus of our enquiry culturally and chronologically in two ways. We want to address the reception and transformation of these "classics" outside Europe in different historical periods. We are particularly interested in the roles played by this classical tradition within Islamicate societies, South-East and East Asia. Secondly, we are interested in the ancient mathematical writings in Arabic, Chinese, Sanskrit and other languages that, at certain time periods in these other parts of the world and elsewhere, were perceived as classics. Widening the focus should allow us to inquire into questions such as: what did classical texts mean for various types of actors? How were they available to them? How did they read them? In the contexts of which institutions and with which expectations? The important role classical works have played in mathematical history pose deep methodological questions with far-reaching implications for the history and philosophy of mathematics. In mathematics conceptual and methodological innovations are thought to be legitimized only by appeal to mathematical arguments and consistency. Yet, legitimation has involved in many crucial episodes giving a prominent role to classical works. The mathematical classics have repeatedly been the source and grounds for new ideas and techniques. There is therefore a deep, complex tension between innovation and tradition. We are interested in how innovation has often been legitimized by re-reading old texts, concepts, and methods-old texts whose principles and methods were utterly different from the ones they contributed to sustain. What can this teach us about the nature of mathematical argument, and mathematical practice?

Mathematics Subject Classification (2010): 01-xx.

Introduction by the Organizers

The workshop *Mathematics and its Ancient Classics Worldwide: Translations, Appropriations, Reconstructions, Roles*, organised by Karine Chemla (Paris), Vincenzo De Risi (Berlin) and Antoni Malet (Barcelona) was well attended with over 52 participants with broad geographic representation from all continents. The aim of the workshop is described in the following lines.

Why does mathematics, that is, a discipline in which progress has regularly been valued, have classics, and in particular, why classics from Antiquity? Which role have these classics played in mathematical practice in different time periods, and how have actors read and used them? The suggested workshop aims to analyze with a multidisciplinary perspective the role of the classical texts in mathematical practice throughout history. To do so we would like to focus not only on different facets of the recovery and preservation of texts that have been perceived at different moments of history as great classics of mathematics, but also on studying how they have been handled and what role they have played in different ages and within different scholarly cultures. More to the point, the workshop would aim at studying the ways in which the great classics of mathematics (including not only Euclid's, Archimedes', Apollonius', Nicomachus' and Diophantus' works, but also *The Ten Canons of Mathematics* (*Shi bu suanjing*), the *Āryabhaṭīya*, and al-Khwārizmī's algebra) have been preserved, read, translated, commented, rewritten, and put to use in different historical periods, in diverse cultural and intellectual contexts, within different institutions, and for different purposes.

The relevance of ancient Greek mathematics for the European mathematical traditions has already been an active area of research. We possess a number of substantial studies on the so-called medieval Latin Euclid and Archimedes. Those studies have provided important insights on the Greek and Arabic sources of the earliest translations of Euclid's *Elements* and *Data*, and of Archimedes' writings available to Western European scholars. Those studies have clarified as well the main features of the early translations, particularly in matters concerning accuracy and faithfulness in translations as compared with the oldest and most reliable Greek sources. Our knowledge of the fortunes and status of Euclid and other Greek mathematicians whose writings were considered classical in Renaissance and early modern Europe is sketchier and less systematic. In particular, fewer historians have studied how concretely mathematical texts perceived as classics were actually edited, read, and for which purposes. Some of these issues are addressed in Benjamin Wardhaugh's project entitled "Reading the Classics" (Oxford). This workshop aims to widen its scope and embrace these questions more broadly. For what regards Western Europe, for instance, it is from the 16$^{\text{th}}$ century on that the appropriations of what is perceived as the classical tradition becomes more diversified, while the relation between the classical tradition, the practice of mathematics, and mathematical innovations becomes multi-faceted and gains complexity. What rhetorical, pedagogical, and institutional mechanisms contribute to secure the status of classic to specific texts? What do classics mean in these different traditions of scholarship, and how were they used differently? These are some of the questions that we intend to address.

So far, the focus of this introduction has been in Western European societies, but we aim to expand the focus of our enquiry culturally as well as chronologically, and this in two ways. First, writings that were considered classics were those that were taken to other areas of the planet and translated into different languages. Hence the status of classic has had a major impact on the circulation of mathematical knowledge and practices. We want to address this issue, by considering the reception and transformation of these "classics" outside Europe in different historical periods. For reasons that will fully appear below, we are particularly interested in the roles played by this classical tradition within Islamicate societies, South-East and East Asia. Second, Western Europe was not, by far, the only context in which practitioners of mathematics viewed specific mathematical writings as classics: the phenomenon exists (almost) wherever there is mathematical activity (and the counterexamples are also interesting for us to ponder.) In line with this remark, we are interested in the ancient mathematical writings in Arabic, Chinese, Sanskrit and other languages that, at certain time periods in these other parts of the world, were perceived as classics. Widening the focus in this way should allow us to inquire into questions such as: what did classical texts mean for various types of actors? How were they available to them? How did they read them? In the contexts of which institutions and with which expectations?

There is a long tradition of important studies about mathematics in Islamic societies and the ways in which they creatively fused at least Sanskrit and Greek mathematical bodies of knowledge and practices. Yet the sheer volume of the sources available and their complexity make evident that crucial facets of the ways in which Greek and Sanskrit mathematical sources perceived as classics in the Arabic world were read and contributed to shape new mathematical works are still awaiting close scrutiny. What did it mean for works to be deemed classics in the Arabic world? Which kind of attitude towards these texts are documented and which kinds of mathematical practices did they elicit? These are some of the questions we aim to address. Recent research on these topics will probably provide new insights on the appropriation of the classical tradition generally.

We are also particularly interested in the emerging field that focuses on the acculturation of (what in actors' view were) "classics" of mathematics to other parts of the world, particularly in South-East and East Asia. In addition to considering how the writings that for some were classical were perceived by the others, we are interested in the way in which this introduction of classical mathematical literature interacted with mathematical classicism in these other parts of the world. The methods, structure, and argumentative style of Greek geometry have been pointed up as characteristic features not only of European mathematics but of European culture generally - as opposed to, say, Indian science and civilization, where grammar and formal analysis play a prominent role (Bronkhorst, 2001). A similar belief in cultural differences that shaped the confrontation of mathematical cultures pervades P.M. Engelfriet's work, the first comprehensive account of the translation of Clavius's first six books of Euclid's *Elements* into Chinese in 1607. We would like instead to revisit this transfer from the viewpoint of a history of classical mathematical literature. Our interest in the reception of mathematical classics belongs to a global perspective and aims at answering global issues. In what follows we expose some of the questions and issues our workshop would address. Following some of our actors' views, we have been using the terms "classical mathematical tradition" to refer to a number of works written in the classical Greek world. These works have achieved the category of mathematical classics in some contexts. However, controversies and disagreements about what counts as a classic, and modifications in the classical status of given works are not unheard of in given historical periods. Euclid and Archimedes have enjoyed a secure position as authorities at least from the High Middle Ages on, more continuously than Apollonius, Diophantus, Nicomachus and Pappus have. In fact, the latter benefited from a much more dubious position. The same holds true for China: Classics from the collection *The Ten Canons of Mathematics* that was shaped in the 7^{th} century and that defined a classical tradition until at least the 13^{th} century were for the most part simply lost in the subsequent centuries, until the end of the 18^{th} century, when a major scholarly enterprise, commissioned by imperial institutions, reshaped the mathematical classics of the past for the future. It appears important to clarify what are the mechanisms at play in enhancing or weakening the authoritative status of mathematical works.

Classical works pose deep philological questions, some of which have been open through centuries. Establishing critical editions of canonical texts has often required a combined effort of historians, philologists, and mathematicians, unless these have worked separately and with wholly different conceptions of what it meant to produce an edition of a classical work for the present. We aim to gather state of the art contributions on methods for restoring the text of the Classics, according to different ideas of what "restoring" meant at different time periods. We aim as well to discuss the role of translations in both preserving classical works and making them more widely available, and how translations have contributed to shape the public understanding of given classics.

Along with textual matters and textual transmission our workshop would address the uses classical authoritative works have been put to. Classical works have played different roles, besides their being the depositaries of past knowledge. There is indirect evidence about the uses of some classics in education. We know that some have proved instrumental as icons in philosophical and mathematical debates, but also in political debates. At given times some of them have proved decisive in triggering mathematical innovations. We would like to discuss in detail case studies that provide insights in classical works' performance of social and cultural roles. This includes their use in the classroom and what they meant for mathematical education in different historical periods, but also their status as authoritative texts in learned, research and political institutions. Why were the classics appealed to as guarantors of certainty and correctness at some crucial stages of mathematical history, and as warranting cultural identity at others?

A related, important topic of investigation is the study of the commentaries added to classical mathematical texts. These are absolutely widespread in ancient, medieval and modern times as well, and a classical text is seldom published without some apparatus. In the past, commenting a classical text was itself an important aspect of mathematical research, and many important mathematicians (such as for instance Tartaglia, Pascal, Wallis, Barrow, Mercator, Leibniz, or Legendre) put forward their own views in the form of commentaries on Euclid's *Elements* or Archimedes' works. The commentaries on classical texts take many forms, and may aim at preserving and integrating the original text or at subverting it altogether. In many cases, the difference between commentary and original text is clearly marked, but at other times it fades away and the boundaries between text and paratext disappear. This has produced a few reflections, in the past, on the opportunity of drawing a distinction between primary sources and deuteronomic texts (Netz 1998, Chemla 1999). We would like to discuss on the role of commentaries in producing new mathematics and transforming classical texts. More generally, classics were available to readers not only with commentaries, but with notes, specific layouts and typographies. Indices, numbers, and other types of material devices were added to them in relation to the use that actors would have. We welcome studies of the varying textual realizations of the classics as a guide towards how actors intended them to be read and used.

We want to focus as well on the motivations for reading classics as they have been articulated in different contexts: what did readers think they would find in classics? In which ways philosophical discourses and methodological issues contributed to fashion old texts into classical texts? Once a text became a classic, were there specific modes of reading it, as opposed to the reading of other types of mathematical texts? The important role classical works have played in mathematical history pose deep methodological questions with far-reaching implications for the history and philosophy of mathematics. Mathematics receives inputs from the social and cultural context in which it is cultivated, and some of them prove crucial in stimulating and even directing conceptual innovations. In mathematics, however, conceptual and methodological innovations can only be legitimized by appeal to mathematical arguments and consistency. Yet, this has involved in many crucial episodes giving a prominent role to classical works. The mathematical classics have been more than once and twice the source and grounds for new ideas and techniques. There is therefore a deep, complex tension between innovation and tradition in mathematics. Innovation has often been legitimized by re-reading old texts, concepts, and methods - old texts whose principles and methods were utterly different from the ones they contributed to sustain. What can this teach us about the nature of mathematical argument, and more generally about mathematical practice?

These questions appear closely related to the global issues we would like to address in connection with the circulation of classics among (conceptually and geographically) distant cultures. Mathematics has been long regarded as the most abstract, culturally unbiased, and theoretically free science. And yet it has been recognized that classics do not circulate easily through different cultural geographies. The workshop would study how classics originating in one scientific culture were appropriated by other scientific cultures, thus problematizing the apprehension of mathematical ideas and the distortions caused by well rooted (but often tacit) cultural frames.

Workshop (hybrid meeting): Mathematics and its Ancient Classics Worldwide: Translations, Appropriations, Reconstructions, Roles

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The Classics through the Eyes of their Commentators

Yang Hui's (13$^{\text{th}}$ century) understanding and treatments of problems in chapter "right-angled triangle" of the mathematical classic *The Nine Chapters*

Célestin Xiaohan Zhou

The *Ten Mathematical Canons* edited in the 7$^{\text{th}}$ century provide good specimens for us to approach the questions pertaining to "the constitution, recovery, and role of the classical texts in mathematical practice throughout history" raised in the workshop. Among the ten classics, *The Nine Chapters on mathematical procedures* (hereafter, *The Nine Chapters*) is the most influential one, and the most important point for this workshop is that *The Nine Chapters* was handed down over centuries, and abundant commentaries have been composed on the book. These commentaries are crucial, since they give us evidence about how ancient readers interpret, understand, or misunderstand, treat the text of the classic.

In 1261 C.E. Yang Hui took *The Nine Chapters* with commentaries by Liu Hui (the third century), Li Chunfeng (the seventh century), and Jia Xian (the eleventh century) to compose his own commentaries on all the layers of texts. This book is known under the title *Mathematical Methods Explaining in Detail The Nine Chapters* (hereafter, *Mathematical Methods*). Yang is also the earliest known scholar who has rearranged the problems of the classic into his categories, which in turn form *Reclassifications* as the last section of the book. Chapter "base and height" dealt with problems relating to what today is known as right-angled triangle. Yang Hui's treatments of the problems in this chapter are fully representative in terms of using mathematical problems for presenting a mathematical procedure/method.

A comparison between the texts of the classic and different layers of commentaries from different sources shows that nine problems (4, 5, 6, 11, 13, 16, 21, 29, and 31) of *Mathematical Methods* are not from *The Nine Chapters*. Why did Yang/Jia add the nine problems in *The Nine Chapters*? Why did Yang Hui choose the added problems instead of the original ones from *The Nine Chapters* in his *Reclassification*? Why did Yang Hui change the order of the problems in *Reclassifications*?

The theoretical reflection in Chemla's paper [5] about "a mathematical problem" provided a methodological tool of my presentation to address Yang Hui's dealing with problems of the classic. By analyzing Liu Hui's commentary to *The Nine Chapters*, she points out that "problems did not boil down to being statements requiring a solution, but were also used as providing a situation in which the semantics of the operations used by an algorithm could be formulated in order to establish its correctness" and the reason of changing the numerical values consists in the fact that "the commentators introduce material visual tools to support their proofs [...] the numerical values given in the problems refer to these tools (blocks *qi*)".

With regard to Yang Hui's *Mathematical Methods*, a research on the added problems and the original ones appears to be meaningful for us to inquire into these questions: Is there a continuity in the use of problems in the sense Chemla has revealed? In chapter "base and height", are the ways of approaching the mathematical problem, such as adding a problem subsequent to the original one, which changes the values or the situation of the original one, similar to those reflected by the classic and its former commentators?

Through analyzing the nine added problems divided into three groups, we revealed that the change of values was related to using the numbers to check the relationships between items attached to the right-angled triangle. The change of units was probably related to Yang/Jia's intention of conveying the mathematical thought that the relations between the three lengths of the right-angled triangle do not change even though the units of these lengths changed.

The intention of adding some problems was closely related to the varied new subordinate methods in *Reclassifications*. In *Reclassifications*, each subordinate method was exemplified by a problem from *Mathematical Methods*. Moreover, the intention of presenting a kind of symmetry between these methods made Yang/Jia change the required item of problem. In some "procedures of calculation", the method "inscribed area" was used. Moreover, two added problems were solved by this method too. Through subtly changing the "question" part of a problem, the mathematical method used for solving it became a typical method of "inscribed area" Yang intended to demonstrate.

Yang Hui's use of problems, such as his change of the numerical values and the situation of a problem, are closely related to the third-century commentary Liu Hui's practice of problems for presenting mathematical procedures/methods. But even though Yang Hui also had the attempt to check the correctness of the method provided in the text by his added problem and his commentaries, in contrast with Liu Hui, who has given convincing proof for the correctness of the procedure, Yang has not fully achieved his goal through his accounting for his change of numerical values and situations of the added problem. Yang Hui's treatments and rearrangements of these problems were accepted by the 15$^{\text{th}}$ century Wu Jing, and they influenced Wu Jing's compilation of the "ancient problems" in Great *Compendium*.

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Interpreting the intentions of a mathematical classic. Technical readings, exemplifying and proofs in commentaries to the *Āryabhaṭīya* (499) and the *Brāhmasphuṭasiddhānta* (628)

Agathe Keller

The *Āryabhaṭīya* (499) and the *Brāhmasphuṭasiddhānta* (628) are two early astronomical classics containing mathematical chapters, written as it is usual for Sanskrit treatises, in more or less aphoristic verses. How do different commentaries read these texts? Do they do it in the same way? What kind of mathematical work goes into these readings? After a presentation of the historiography of the category of *śāstra* (treatise, maybe "classic") and the new research it has initiated, this presentation turns to different commentaries examining how they understand their role of providing the meaning (*artha*) of the treatise and the different uses of solved examples (*uddeśaka*) in relation to this effort. More largely what is at stake are the different ways in which commentators have tried to explain and prove the algorithms they were working with.

For the last thirty years in Indology there has been a renewed approach and debates on the genre of scholarly texts and the kind of knowledge it produced, the issue being how to contextualize and historicize it. Indeed, scholarly texts in Sanskrit have preserved a standard form during a very long time: they are formed of treatises (*śāstra* - also a name for systematic knowledge) of prose or versified more or less aphoristic rules (*sūtra*) on the one side and commentaries (*bhāṣya*, *vākhyā*, etc.) on the other. Treatises in astral science have further specialized technical names (*tantra*, *siddhānta* and *karaṇa* notably).

There is a historiography then that considers that Sanskrit knowledge has permanently this form: a historiography shaped maybe by a comparison with a historiography of European science emancipating itself from scholastic commentaries. S. Pollock suggests also to understand this apparent permanence by the values of what would have been "a Sanskrit knowledge system" which considers itself universal and thus outside of time and space [4, 5]. The idea of a knowledge system wouldn't be so much of an orientalist's artefact as an actor's category. The values attached to the emic category of *śāstra* would explain why contextual histories are so difficult to undertake. Specifying this initial statement Pollock himself, and many of his students also, have underlined how such a point of view has strong political (brahmanical) undertones, and should be nuanced in discipline, time and across other South Asian languages [1].

It is maybe one of the names listed above as emic categories of types of texts difficult to historicize, that could contain a candidate as a Sanskrit name for "classics" - treatises as well as commentaries - although none fits exactly: 99% of the corpus of Sanskrit texts are classics in as much as they exist because they have been transmitted to us by continuous copy in a place where the medium for preserving texts is very fragile.

In mathematics, the a-historical point of view is superposed with a certain perception of text: one that views the *sūtras* as formulas that mathematical commentaries illustrate and irregularly demonstrate [3]. It is also possible to show how this point of view on Sanskrit mathematical texts is the product of an orientalist historiography endorsed by nationalist scholars such as the important manual published by Datta and Singh (Datta and Singh 1935).

I would like to show that the shape of commentary and treatise can actually produce very different texts, with different relations between the treatise and the commentary. More, I would like to explore these differences by asking the question of what kind of meaning and purpose (*artha*) commentaries give to the treatises. What kind of mathematical work does this entail? And in particular what relation to mathematical proof does this work have?

The *Āryabhaṭīya* (499) and the *Brāhmasphuṭasiddhānta* (628) are two early astronomical classics containing mathematical chapters, written as it is usual for Sanskrit treatises, in more or less aphoristic verses. They both have given birth to two different astronomical schools and have been commented upon by commentaries who themselves have been emulated by others, and thus might also be considered as "classics". I first show that the oldest preserved commentaries to both texts are very different from one another. They do not focus on the truth of the assertions they comment, but they do contribute a mathematical work on the treatise.

I argue that the mathematical chapter of the *Āryabhaṭīya* is made of a compilation of very condensed verses which the 7$^{\text{th}}$ century *Bhāskara* reads in a technical way arguing for a non etymological/literal reading of technical words, a non-linear literal reading of certain verses (read by considering just part of the words of a verse to make sense of them), and by understanding the algorithms evoked by the author of the *Āryabhaṭīya* as statements about the algorithm, not descriptions of them. *Bhāskara* comments Āryabhaṭa's *assertions* : his reflections concern language first and its relation to the mathematical content Āryabhaṭa wants to convey.

His commentary is not concerned directly and only with showing that Āryabhaṭa's assertion are true, but this does not mean that the veracity of Āryabhaṭa's *sūtras* is not an issue, part of Bhāskara's commentary is intent in removing "doubts" (*sandeha*) about certain rules.

Many different kinds of reasonings and explanations around and about the rules are evoked and/or carried out, but proof is not the only systematic focus.

By contrast, Brahmagupta's treatise is much easier to read, much more bulkier and ambitious. It is critical of the *Āryabhaṭīya*. It has a mathematical chapter that is our focus here but also chapters on metrics and algebra, maybe seen as belonging to the general scope of astral science. Pṛthūdaka's 7$^{\text{th}}$ century commentary is called *Commentary with explanation* (*Vāsanābhāṣya*). My current investigation is about the meaning the term "explanation" can take for Pṛthūdaka. To approach this question, I suggest to look more widely at how Pṛthūdaka understands his role as a commentator. By looking at rules concerning arithmetic sequences provided by Brahmpagupta (BSS.12.17-18) and Pṛthūdaka's commentary on them I underline the importance of a feature unique to mathematical commentaries (vss astronomical commentaries for instance), that is, their list of solved examples (*uddeśaka*). I argue that these examples are here used by Pṛthūdaka to explore the scope of the rules he comments. First by exploring the different kinds of mathematical topics in which the rule can be applied, and then the numerical values (or the different type of operands) that enter the rules. He does this particular exercise by adding to the solved examples a textual part called "variations" (*udaharaṇīya*), in which givens and results in the examples are changed. Pṛthūdaka makes clear that these variations are modes of investigation. Examples then are not merely illustrations of a "formula", they explore its domains of application. More, examples appear as tools to explore the rule's reach. But there is something even more than that: it is within solved examples that "proofs" (*upapanna*) and "explanations" (*vāsanā*) are put forth. The reasonings that proofs and explanations provide show to us that the ways of discussing the validity of an algorithm might not be through a statement that discusses "its truth". In the commentary on the rules for arithmetical expansions, Pṛthūdaka's "proof" consists in showing that two independent geometrical representations of the sum of the terms of an arithmetical sequence exist and that these representations are equivalent while they both represent different arithmetical operations to compute the sum of terms. The "explanation" given in this part of the commentary, re-reads one rule in algebra to show that this reading enables the derivation of the other rule. The reasoning, general, uses nonetheless the specific numerical values of a solved example. In both cases, and contrary to a certain historiography of proof in Sanskrit texts that considers that all "proofs" are of the same kind either geometrical or algebraical [7, 6], Pṛthūdaka uses different mathematical tools for his reasonings: he doesn't systematically use algebra or systematically geometry to explain and prove mathematical operations on arithmetical sequences. In the continuity of what his commentary reveals of its intentions through his work on examples and their variations, his explanations seem to aim at showing how all topics of astral science are related and can be used to explain a rule for another domain. Pṛthūdaka in this part of his commentary notes that his explanations aim at proficiency (*vyutpatti*): the term might be understood precisely as defining the range of meaning of an algorithm, from its original topic of application to the wider contexts where it takes different and new interpretations, like a word used in different contexts. The term *vāsanā* itself first means "perfume": explanations then might be of the different scents an algorithm takes according to the context in which it is made [2]. Or, since *vāsanā* is also an "impression", it might be about the different impressions it provides as the algorithm is inserted in a new topic and its relations between a known scope and the new scope of investigations are explored [6].

In all cases, I hope to have shown the variety of textual workings in mathematical treatises and commentaries, the fact that they do not focus on the "truth" to comment about mathematical topics even when the question of their validity is raised. The solved example as a way of stating general mathematical facts and as exploring the realm of an algorithm appear then as a specific way of reading a classic, and of discussing its validity.

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Al-Fārisī's commentary: fitting practical Arabic calculation into a classical Greek mold

Jeffrey Oaks

As Sonja Brentjes remarked during our meeting, there was no word in Arabic that corresponds to our "classic". I am free, then, to posit two tentative criteria for what should qualify a work to have been considered a classic in Islamicate civilization. The first is that it should have remained a starting-point for a particular branch of learning and continued to be cited, criticized, and praised by authors over an extended period of time. Thus Euclid's *Elements* in geometry and theoretical arithmetic and Ptolemy's *Almagest* in astronomy were classic works from the time of their translation by the early ninth century CE down at least to the eighteenth century. Al-Khwārazmī's ca. 825 CE *Book of Algebra (Kitāb al-jabr wa l-muqābala)*, however, did not achieve this status. While it is true that the fundamentals of algebra as presented in his book quickly became standard, his book soon became outdated so that writers after Abū Kāmil (late ninth c.) by and large ceased to reference it directly. His ideas and mode of presentation were absorbed into other more complete and better organized treatises, causing his own seminal book to become marginalized. This particular way of characterizing a classic, then, favors the comprehensive treatments of Euclid and Ptolemy over brief introductions like al-Khwārazmī's.

Another criterion that seems appropriate for the Islamicate context is that a work should have a respected provenance. Even if someone, say in the thirteenth century, were to have put together an astronomy text more comprehensive, up-to-date, and complete than Ptolemy's, it still might not have become a classic (again, if that would have had any meaning) because it would not be the hallowed foundation of the science that Ptolemy's book was. For this reason, Greek works were more likely to be considered classics. (Keep in mind that I made up these criteria during the meeting, so I am very liable to abandon them in favor of something better.)

Calling Euclid's and Ptolemy's books classics is not entirely the imposition of a foreign notion of "classic" on Arabic culture. Franz Rosenthal noted in his book *The Classical Heritage in Islam* that [3, p. 13]

> Islamic rational scholarship [...] depends in its entirety on classical antiquity, down to such fundamental factors as the elementary principles of scholarly and scientific research.

In mathematics it was the Greek marriage of philosophical principles of organization (categories) and ontology with the mathematical structure of postulates, propositions, and proofs, that impressed Arabic writers as the proper way to go about their research.

Today historians recognize that people in different cultures might practice ways of doing mathematics that are grounded in different ideas of the natures of numbers and geometric magnitudes. But naturally, in Islamicate societies, as in other premodern cultures, there was an unspoken presumption that mathematics has one nature, so that, for example, it should be possible to situate the arithmetic practiced and transmitted orally among merchants, surveyors, and secretaries in the Middle East in the framework of Greek mathematics. This was the goal of the Persian mathematician Kamāl al-Dīn al-Fārisī (1266/67-1319) in his *Foundations of Rules on Elements of Benefits (Asās al-qawāʿid fī uṣūl al-fawāʾid)*, a commentary on a textbook of his teacher Ibn al-Khawwām (1245-1325). In this book al-Fārisī attempted to give finger-reckoning, an oral method of practical calculation, a place in Aristotle's division of the sciences and to give its rules proofs in the manner of Euclid's *Elements*.

Arabic authors were not of one mind when it came to what constitutes a proper proof. Many, beginning with al-Khwārazmī and Ibn Turk, gave geometrical proofs for rules in algebra, for instance. But al-Fārisī, remaining faithful to Aristotle, maintained that proofs in arithmetic, and consequently also those in algebra, should be based in arithmetic, and indeed every one of his proofs is presented in the style of those in Euclid's *Elements* Books VII-IX. But there remained a problem with this project that he could not fully resolve. Because Euclid worked with an indivisible, noetic unit, his numbers are restricted to positive integers, while the numbers of finger reckoners include also fractions and irrational roots. Al-Fārisī accounted for fractions by identifying them with ratios of integers, but he made no attempt to explain the irrational roots that he himself used in his book.

Al-Fārisī was not the only author to make a serious attempt to bring a practical calculation technique into the Greek fold. To name just two other authors who are not quite as famous, Ibn Mun'im and al-'Uqbānī wrote similar books in the western part of the Islamic world. The project of al-Fārisī exhibits what appears to have been the commonly held view that Greek mathematics together with its philosophical foundation is mathematics done properly, and that all mathematical knowledge can and should be incorporated into it.

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Roundtable: "Commentaries on the Classics"

Karine Chemla, Gregg De Young, Ivahn Smadja

Karine Chemla: "Commentaries on the Classics"

K. Chemla's contribution to the round table "Commentaries on the Classics" began with a clarification: the commentaries she considers should be distinguished from marginalia or notes taken on a book. By "commentaries", she means a specific genre of text, explicitly chosen qua genre by its author, and that can be characterized by its essential dependence on a base-text. It is typically in this type of work that we might find emic terms for "Classic" as well as resources to interpret what actors mean by this term. Indeed, commentaries contain evidence of how actors read classics and of the expectations they had with respect to such types of text. For instance, Sanskrit commentators refer to the *artha* of a classic, which Keller suggests translating as "meaning/intention", whereas Chinese commentaries regularly refer to two types of meaning (*yi* and *yi'*) of a classic. How they view these different types of meaning in the case of mathematical classics and which mathematical work must be carried out to bring these meanings to light are issues worth pursuing to research classics as an actor's category and to understand what the study of commentaries can bring to the history of mathematics. Commentaries also offer precious resources with respect to the techniques of reading classics, in cases where we might be at a loss to make sense of a text (as is often the case with the form of the *sutra* with which Sanskrit mathematical classics are formulated). They thus contain evidence that should help us avoid reading ancient texts in an anachronistic fashion. Commentaries are also of help when we might have difficulties understanding why a given text was considered a classic. An example of this is the Chinese classic *The Nine Chapters on Mathematical Procedures* which consists mainly of problems that look practical and procedures solving them. Commentaries provide resources to perceive abstraction in this book as an actor's category and to see that practitioners of mathematics established an analogy between how this mathematical book was formulated and how Confucius chose to express historical knowledge when he edited the canon *Spring and Automn Annals*. In other words, for these scholars, mathematical classics were not divorced from other canonical texts but were written in ways comparable to the latter.

Gregg De Young: "Multiple roles of commentaries on the classics in late medieval studies of Euclid's *Elements*"

What is the function of a commentary vis-á-vis a classic? If we consider Euclid's *Elements* as the quintessential mathematical classic, we find that in the Islamicate world, commentaries appear almost at once after the initial translations into Arabic were made. These were typically discussion of the entire classic text (commentaries of al-Nayrīzī and al-Karābīsī). Such commentaries give us a window into how the mathematicians of the time were interacting with the classic text.

By the medieval period, many more commentaries on the *Elements* had been composed. Not a few of these were short treatises focused on specific sections of Euclid's *Elements* (book V and book X, for example) and even specific problems (such as proofs of Euclid's parallel lines postulate). I show the contents of three codices that span nearly three centuries. Each contains a collection of mathematical commentaries on the *Elements*. Each codex was copied by a single scribe and was organized in the same way: first a "long" Euclidean treatise (often the *Taḥrīr* of the *Elements* by Naṣīr al-Dīn al-Ṭūsī) followed by a number of shorter and more focused commentaries, most of them written prior to the 14$^{\text{th}}$ century AD. The central core of these collections remains surprisingly constant, although some treatises are added and others subtracted over time.

These commentary collections (presumably produced with a pedagogical purpose) reveal several interesting features that have sometimes been associated with classic texts:

(1) The actual classic (the *Elements*) tends to be replaced by an edition of the classic - al-Ṭusi's *Taḥrīr* of the *Elements*, which rapidly took on the status of a canonical text itself.
(2) The central core of these commentary collections, because of their historical longevity, also seem to take on something of the characteristics of classic texts.
(3) Only one of these collections (Munich, Bayerische Staatsbibliothek, cod. arab. 2697) contains extensive marginalia revealing additional layers of

interactions between the texts and later readers during the early Ottoman period. A nearly identical set of marginalia has been copied into another copy of al-Ṭūsī's *Taḥrīr* (Yale University Library, Yahuda 4848), showing that even collections of marginalia can sometimes take on characteristics of classics.

(4) Another of these collections (Istanbul, Feyzullah 1359) displays characteristics, such as copious use of gold ink and a dedication to Sultan Mehmet II, pointing to another use of the classics – as a tool to curry political and social patronage. The copyist was probably under no illusion that the potential patron would personally read or study his work. Nevertheless, his gift of a widely recognized "classic" collection of mathematical commentaries may have helped to affirm the legitimacy of the patron's rule, implying a wish that the patron would himself take on the characteristics of a classic – a long personal rule and an eternal influence on the life of the state.

Ivahn Smadja: " 'Restoration', 'restitution', 'divination': ways of recovering a lost past through reading commentaries on ancient Classics in the nineteenth-century"

In a variety of contexts, rereading mathematical classics proved to be a privileged way to legitimize innovations by anchoring them in tradition. However different they may be from the sources from which they were supposed to spring, these new methods, ideas, and techniques could thereby be partly imbued with the authority of the classics themselves – an aura only gained through complex social and historical processes, and whose transference in return often contributed to shape a sense of historical continuity. Historians of mathematics have shown that the relationship between the mathematical commentaries and the base text upon which they elaborated often played a significant role in this dialectic between innovation and tradition. Mathematical classics, however, were not only read from within a self-defining tradition, but they also offered themselves to what ethnologists call the "view from afar". What does it mean to read the "classics" whether of an alien tradition or of an irretrievably lost one? What kind of distance or otherness is then implied? And what are the means to overcome these? Extant commentaries in this case happened to be read with a different lens as promising access to otherwise unapproachable mathematics, whether the presumed "classics" be unintelligible or utterly lost.

In this connection, the focus will be on the ways some nineteenth-century European mathematicians devoted considerable efforts to make sense of the commentaries so as to recover the meaning of either famously lost texts of the Western tradition, or enigmatic non-Western ones recently made available in translation. In so doing, they happened to use the commentaries as a key to interpreting the base text, or even reconstituting it, when it was lost or corrupt. Two main features may suffice in a first stage to roughly characterize the specific context in which these nineteenth-century readings of ancient texts were produced: namely, on the one hand, the creative tension between philology and mathematics, both fields of knowledge being then fully engaged in concurrent, and at times conflicting, processes of differentiation, disciplinarization and professionalization, and, on the other hand, comparativism derived from the dominant paradigm in language studies.

Reviel Netz (1998) noted that it was only when textual criticism was established, apart from mathematics, as an independent activity, that is with its own goals and criteria – a turning point which he located in the 17^{th} or the 18^{th} century – that Western mathematics "cut its umbilical cord to the Greek canon", so that reference to the latter was no more simply mathematics, but definitely something else. More generally, it would be interesting to study the important transformations occurring from the 17^{th} to the 19^{th} century in the complex relationship between textual criticism (hence philology) and mathematics, and how these transformations affected the way ancient texts were read. In [1], Karine Chemla stressed in particular that the valuing and the quest of a purported original text, restituted as such without blemishes, has a history of its own which should be investigated more at length. This is a case in which debates in philology shaped similar ones in history of mathematic, especially in the German context [3]. The polarizing tension between *Sachphilologie* and *Wortphilologie* partly hinged on the status of ancient texts. Whereas the Berlin classicist August Boeckh for instance considered textual sources as being *realia* from the past, as were monuments, inscriptions and coins, his Leipzig opponent Gottfried Hermann focused exclusively on language, textual criticism and editing techniques.

The case study presented in this roundtable is part of a broader picture, namely the European reception of the work of the British Indologists on ancient Sanskrit mathematical sources. Among these, Henry Thomas Colebrooke (1817) translated two mathematical texts by the twelfth-century Indian astronomer Bhāskara, the *Lālāvatī* and the *Bīja-gaṇita*, and the mathematical chapters from the *Brāhma-sphuṭa-siddhānta*, an earlier astronomical treatise by the seventh-century mathematician Brahmagupta. In these translations, Colebrooke added notes to the base text in which he provided explanations, "proofs" and procedures, presumably excerpted from selected passages of the commentaries to the base text he had access to during his time in India - that is essentially Gaṇesa's *Buddhivilāsinī* (A.D. 1545), a commentary on the *Līlāvatī*, Kṛṣṇa's *Navānkura* (ca. A.D. 1600), a commentary on the *Bīja-gaṇita*, and Pṛthūdakasvāmin's *Vāsanābhaṣya* (ca. 860), a commentary on Brahmagupta's *Brāhma-sphuṭa-siddhānta*. By including the insights he gathered from the commentaries, he aroused an interest in these sources on the part of both European philologists and mathematicians, and also largely shaped the way they were later to address those. Colebrooke's European reception depended largely on national contexts. Whereas such German scholars as for instance A. W. Schlegel celebrated him as the only British scholar responding to their wish to raise a budding Sanskrit philology to the standards of classical philology, in the French context, Colebrooke first captured the attention of mathematicians around the École Polytechnique (Olry Terquem, Michel Chasles).

The case of the French mathematician Michel Chasles deserves special attention. His papers at the Académie des sciences de Paris bear witness to a particular type of mathematical endeavor, which he referred to with the labels "restoration", "restitution", "divination". He for instance attempted to make sense of a series of highly terse statements from Colebrooke's translation of Brahmagupta, in which he discerned a complete theory of rational cyclic quadrilaterals, that is, in his own terms, to "divine" or "restitute" Brahmagupta's theory. Archival research reveals hundreds of drafts devoted to tinkering with the information Colebrooke had extracted from the commentaries so as to organize those statements in a mathematically consistent way [4, 5]. Chasles's work on Brahmagupta would later prompt further work by the German mathematicians Ernst Kummer and Hermann Hankel. But at the same time, Chasles also set himself the task to "restore" Euclid's lost porisms, by drawing on the textual criticism and interpretation of his seventeenth-century predecessors Halley and Simson, as well as on ideas on geometry derived from Monge's teachings (see [5]). Typical of Chasles's approach is his forging new mathematical tools such as the "anharmonic ratio" out of his distinctive reading of some of Pappus's lemmas in his commentary on Euclid's lost porisms. On this score, Chasles left an even greater number of drafts and manuscripts that help documenting the gradual shaping of his so-called "restoration". How did he envision a common framework for both attempts?

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The Persistence of the Classics and their Long-Term Transformations

Notes on Euclid's canonization

Reviel Netz

The evidence on the production and early reception of Euclid's *Elements* is meagre and the most basic parameters must remain conjectural (including, famously, Euclid's very date). I shall survey some of the key facts and try to suggest an outline of the work's reception through antiquity.

The *Elements*, or A Classical Work in Translation: The representations of the Pythagorean theorem in China in the 17th century

Shuyuan, Pan

The recovery of Greek classics and their role in mathematical practices in Renaissance Europe has been the object of much discussion. However, the impact of these classics spread outside Europe. This talk focused on the first Chinese translation of Euclid's *Elements* in early modern times, taking the so-called Pythagorean theorem as a case in point.

European Jesuit missionaries arrived in China at the end of the 16th century, and from then on, they spared no effort to use knowledge from Europe to attract elites' attention and patronage. Thanks to the Chinese translations of various European books that Jesuits published in collaboration with Chinese literati in the 17th and 18th centuries, many mathematical classics and other works were introduced into China. Among this mathematical corpus, which in the eyes of the Chinese scholars represented what they called "Western learning" (*xixue*), the translation of Euclid's *Elements* was undoubtedly the most representative.

Mainly based on the first six books of Christoph Clavius's (1538 - 1612) *Euclidis Elementorum liber XV* (1574), the Chinese translation in six chapters (juan) was carried out by Matteo Ricci (1552 - 1610) and Xu Guangqi (1562 - 1633) and it was published in 1607, under the title *The Origin and Basis of Quantities* (*Jihe Yuanben*). Ricci and Xu not only translated definitions, postulates, axioms, and propositions, but they also frequently translated passages of Clavius' commentary on them. Therefore, a considerable quantity of explanations and criticisms of Clavius as well as notes that he collected from previous commentators were introduced along with the Greek text of the classic. Further material, including quotations from an ancient Chinese classic, and Ricci and Xu's reflections were also inserted. The Chinese translation thus became a mixture of diverse elements. On the other hand, the translation embodied strong uniformity with respect to the form. The definitions of each book (as well as, for Book I, the postulates and the axioms) were set as a separated part: the Beginning (*shou*) of the corresponding chapter in the Chinese text. On the basis of the formal division of a proposition described by Proclus, which Clavius followed, Ricci and Xu made their reformulation for the structure of propositions. The *enunciation* was naturally arranged at the very beginning of a proposition. In the following paragraph, for theorems, *setting-out* and *definition of goal* were combined into a division designated as "exposition" (*jie*); for problems, the two parts were incorporated into the division "method" (*fa*) which mainly corresponded to *construction*. Another division "argument" (*lun*) contains the *proof* with the related *construction*. From a formal viewpoint, the indentation of the paragraphs starting with the terms "exposition", "method", "argument" also differs from that of the enunciation paragraph. In addition, Ricci and Xu also classified the material that they selected and appended to the main part into divisions to which they attached various designations and for which they adopted distinctive indentations. As a result, the translation was further systematized and standardized, thereby being involved in a process of classic-formation.

The mathematical procedure named "the Base and the Height Procedure" (*Gou-Gu shu*) in ancient China and the equivalent methods in medieval and early modern times represented general statements about the relationships between the sides of a right-angled triangle, thus partly corresponding to the Pythagorean theorem. In the Chinese text of Euclid, the Pythagorean theorem and its proof were translated as the main part of the 47^{th} proposition of Chapter 1. Interestingly enough, in the sentence about the construction of the line drawn from the right angle, a blank occurs in the copies printed in the Late Ming dynasty (before 1644), suggesting that Ricci and Xu revised their previous translation. Meanwhile, two diagrams that derived from Clavius's text were drawn, corresponding respectively to the case when the right-angled triangle is isosceles and not. About these two cases, in his expression of the definition of goal, Clavius added the specification that the two sides containing the right angle "are equal or unequal", to emphasize that the proof from the *Elements* is general. Ricci and Xu probably wrote the translation about the construction in a particular way corresponding to the isosceles case and made the deletion after being aware of the generality of the proof. Besides the Pythagorean theorem itself, four of the propositions that Clavius collected in his scholion were translated as added propositions (*zengti*) or additions (*zeng*) in the Chinese text. Aiming at finding out a side of a right-angled triangle when knowing its two other sides, the fourth proposition, which was presented as a problem, was in some sense similar to "the Base and the Height Procedure" or equivalent methods. However, the translated text had more arithmetical features than the original proposition in Clavius's scholion and borrowed some traditional Chinese mathematical terms. This reveals the implicit and subtle influence of the local knowledge on Xu's treatment of the translation. Through this case study, we intend to show how a mathematical classic was read and studied in the process of translation, and how some transformations of the classic met the local knowledge and further brought about new transformations in this cross-cultural context.

The representations of the Pythagorean theorem in China were not confined to the Greek classic, and did not come to an end in the 17^{th} century. A method to transform two squares into one square was presented as a problem in *Complete Reasonings of the Measurement* (*Celiang quanyi*, 1631), another compilation of translated material that comprised comprehensive discussions on practical geometry. The Chinese translation of Ignace-Gaston Pardies's (1636 - 1673) *Elémens de géométrie* (1671), which was originally prepared as Emperor Kangxi's textbook around 1691 and was also entitled *Jihe yuanben*, introduced the Pythagorean theorem again with a proof that relied on a use of proportions. The two texts about the Pythagorean theorem and the comparison between them and the Chinese Euclid are worthy of further study.

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The paradigm of science: axiomatic methods in the Euclidean tradition

Vincenzo De Risi

My talk develops my research on the history of axiomatics in the tradition of Euclid's *Elements*. In particular, I argue that the axiomatic method, which has been considered for centuries as the model of science, and which would be exemplified in a paradigmatic way by Euclid's *Elements*, has undergone in the course of history some important transformations.

It is legitimate to argue, in fact, that Euclid had a conception of the principles of mathematics so different from that which was later developed in the modern world, that the *Elements* cannot be considered to have been developed according to the axiomatic method. Euclid's *Elements* are introduced by two sets of principles: the properly geometric "postulates", and the "common notions" dealing with certain properties of equality. I argue that Euclid's postulates were not semantic principles expressing the properties of space, as they are normally regarded. Rather, they licensed the possibility of constructions of figures that are useful in proving geometrical theorems. In this respect, the *denial* of a postulate, or the claim that a postulate could be false (something that actually happened in Antiquity), does not produce a different geometry, but limits the deductive power of geometry. If it is not allowed to draw a line from any point to any point (Postulate 1), the consequence is not, according to Euclid, that the space modeled by the axioms of geometry is geodesically disconnected. Rather, the geometer is not allowed to draw straight lines and therefore would to be able to prove some theorems in elementary geometry in the way in which Euclid does. These theorems, however, would nonetheless be *true*: the postulate is not a semantic claim about the nature and properties of figures, but rather a proof-theoretic tool. The same can be said of the Parallel Postulate, which is indeed liable to such a constructive and "syntactic" interpretation: should the Parallel Postulate be denied, Euclid would not be able to prove *Elements* I, 32 on the interior angle sum of a triangle. This does not mean, however, that the interior angle sum of a triangle would be less, or more, than 180°. It only means that the interior angle sum, which still is 180°, cannot be proven with the proof offered by Euclid in the *Elements*. This fact explains why alternative geometries and axiomatic systems were not conceived in Antiquity.

Something similar can be said for the "common notions" that introduce, as a second set of principles, Euclid's *Elements*: according to Aristotle's testimony, they were regarded as inferential rather than semantic principles. Common notions, in short, did not express (in the epistemology of Euclid) certain properties of mathematical objects, but rather certain properties of mathematical argumentation.

In general, then, it is possible to offer a homogeneous "inferentialist" interpretation of postulates and common notions as logical principles rather than semantic principles. In this sense, they were not considered as "axioms" in the modern sense of the term, and the Greek geometers did not consider mathematics to be grounded on a number of indemonstrable assumptions. It should also be noted that the axiomatic system of the *Elements* was not followed in any other work of Greek mathematics, because the later Hellenistic authors simply had no need to respond to the dialectical challenges that had produced the postulates and common notions found in Euclid's *Elements*.

I further discuss the development of the conceptions of axioms from the times of Euclid to the birth of modern axiomatics. In Late Antiquity, a few of Euclid's postulates began to be regarded as semantic statements expressing some truths about geometrical figures. At the same time, these principles were considered to be provable, and we witness the invention of the first proofs of the Fourth Postulate and the Parallel Postulate. In the same centuries, the Aristotelian commentators offered some very creative interpretations of the *Posterior Analytics*, and attempted to merge Aristotle's theory of principles with Euclid's axiomatics. They provided a first philosophical interpretation of Euclid's postulates and common notions, and insisted on their dependence on Euclid's definitions, thus setting the stage for the medieval developments in epistemology. This point is of extreme importance for the conception of the classics of mathematics: a text now considered classic, Euclid's *Elements*, began to be accompanied by extensive commentaries that found their origin in part in the commentaries of another classic text, Aristotle's *Analytics*. We are thus witnessing a hybridization between two commentary traditions.

I deal with Scholastic conceptions of axioms, and expound the epistemological view that was to become standard from the twelfth to the eighteenth century. According to the first medieval commentators of the *Posterior Analytics*, indeed, all axioms can be proven starting from the definitions of the terms employed. As a consequence, mathematics was seen as an axiomless science: all principles are provable. This offers a further historical reason for the fact that no alternative mathematical theory was developed in the Middle Ages or in the Early Modern Age. I show that this Scholastic theory of the derivability of axioms from definitions became extremely widespread and was endorsed by mathematicians and philosophers throughout the centuries. The theory passed into Late Scholasticism, and from there it became standard in the Renaissance and the Early Modern Age. It had several adherent also outside Scholastic thought, and was one of the few Scholastic doctrines not to be challenged in the Modern Age. Hobbes, Spinoza, Leibniz, Borelli, Euler, just to mention a few philosophers and mathematicians agreed on the demonstrability of axioms from the definitions. A few other authors (such as Ramus or Pascal) had (partially) different opinions, but I show that their disagreement was very localized and did not invest the whole extension of the scholastic theory.

Finally, I show how the downfall of Scholasticism in the eighteenth century had as a consequence the relinquishing of this view. I take into consideration, in particular, the development of the epistemology of principles in German late-Scholastic thought (especially Wolff), and the opposition to it by philosophers and scientists such as Lambert and Kant. I find that Lambert played indeed the most important role in this connection, and that he was possibly the first author to develop a rich epistemology according to which the definitions were grounded on the axioms rather than the other way around. Lambert's epistemology may be seen, in fact, as the most important milestone in the constitution of the modern conceptions of axioms further developed in the nineteenth century. I argue that Lambert's influence was indirect in the further developments of epistemology, and it mainly passed through Bolzano's works, which were deeply imbued with Lambert's ideas. The important point is that just as the tradition of scholastic commentaries on the *Elements*, which mixed Euclid and Aristotle (and much else) together, had produced a new conception of axiomatics, so too the demise of that tradition produced a new epistemology of principles.

Thus, one can conclude that the axiomatic model of science was largely the product of the transformation of Euclid's *Elements* into a classical text. Once the *Elements* began to be used widely in different eras, geographical areas, and milieus in general, the original meaning of the principles of demonstration was radically altered. Thus it was the tradition of commentaries on the Elements that produced the idea of an "axiomatic method". This latter was not the peculiarity of Greek mathematics, and it was not a uniform model throughout history, but it was instead a changing historical construction that lived its most important moments and turning points in the hands of many generations of commentators.

Roundtable: "Readers of Classics"

Angela Axworthy, Veronica Gavagna, Orna Harari

Angela Axworthy: "Readers of Classics. Sixteenth-century commentators of the *Elements* of Euclid"

The aim of my presentation is to offer examples of how the objects, finality and methods of Euclid's *Elements*, and of ancient geometry in general, were interpreted by sixteenth-century commentators of the *Elements*, and more particularly by Oronce Fine (1494–1555), Jacques Peletier (1517–1582), François de Foix-Candale (1512–1594), John Dee (1527–1608/1609), Henry Billingsley (d. 1606) and Christoph Clavius (1538–1612).

I show that, for these authors, the fact of commenting on a canonical text such as Euclid's *Elements*, which was extensively circulated and commented on over the sixteenth century thanks to the development of printing techniques and to changes in the status of mathematics, not only enabled authors to prove their mathematical skills, but also to adapt its content to their intended readership and to their pedagogical or epistemological agenda, as well as to display their conception of mathematics and of mathematical practice.

By looking at the prefaces or epistles of these commentaries, I show that commentators of Euclid greatly drew from the Neoplatonic commentary on the first book of the *Elements* by Proclus for their interpretation of ancient mathematics and of Euclidean geometry in general. In this context, Proclus's commentary was read in the light of other sources of premodern philosophy of mathematics, such as Averroes or Thomas Aquinas, leading at times to original conceptions.

This is illustrated by the case of Oronce Fine. When commenting on Euclid's *Elements*, he interpreted the averroist notion of *demonstratio simpliciter* (i.e. the most scientifically powerful demonstration), which was canonically represented by Euclidean demonstrations, according to Proclus's description of the double movement (analytic and synthetic) of geometrical knowledge. Another example is that of John Dee. In his *Mathematicall preface* to Billingsley's 1570 English Euclid, Dee used Proclus's representation of the intermediary place of mathematics between physics and metaphysics to assert the ontological transcendence of mathematical objects while justifying the double orientation of mathematics toward contemplation and concrete applications, legitimating thereby a more practical treatment of Euclid's demonstrations. To this conception, was related Billingsley's own reading of Euclid, who asserted the intelligible status of mathematical objects while promoting a hands-on approach to geometry. Billingsley related indeed geometrical objects to instrumental procedures and explained Euclid's abstract concepts through concrete objects, such as paper polyhedra to cut out and fold, which he invited the reader to construct and paste within his book. The case of Clavius (in his 1574 commentary on Euclid and later editions) followed a similar pattern, insofar as he adapted Proclus's Neoplatonic assertion of the ontologically intermediate status of mathematical objects to the abstractionist stances of Jesuit Aristotelianism, using this, as did John Dee, to emphasize the wide applicability of geometry to the material world.

Considering the pedagogical, philological, scientific or philosophical comments that were appended to Euclid's text, I show that, in the cases of Fine, Billingsley and Clavius, two of whom had to devise a mathematical education program for their respective institutions, the reading of Proclus's commentary on Euclid allowed them to promote and develop a constructive approach to the geometrical books of the *Elements*. Hence, the use of genetic definitions, which displayed the mode of generation of lines and figures, was more systematically applied to the interpretation of the definitions by property contained in Euclid's plane geometry. In some cases (Billingsley and Clavius), these genetic definitions were explicitly related to instrumental procedures, relating the abstract discourse proposed in the *Elements* to the teachings of practical geometry. According to the same approach, these authors did not hesitate to relate arithmetic and geometry in their commentary on Book II, overthrowing the clear separation Euclid had established between numbers and magnitudes. A different attitude was proposed by Foix-Candale, who challenged Euclid's genetic definition of the sphere in his 1566 commentary, considering that this definition displayed the accidental rather than the essential mode of being of geometrical objects. Foix-Candale, as Jacques Peletier before him, also criticized the use of superposition as a means to demonstrate the congruence of figures in Prop. I.4, I.8 and III.24 of the *Elements*, regarding it as a mechanical rather than as a geometrical procedure. By contrast, Billingsley took up Foix-Candale's distinction between genetic definitions and definitions by property, but asserted the epistemic importance of the former by designating them as *causal definitions*. He anticipated thereby Isaac Barrow's later assertion of the foundational status of genetic definitions in geometry on account of their causal nature.

Jacques Peletier, in his 1557 Euclid, did not only openly reject Euclid's use of geometrical superposition, but also made significant objections to other parts of Euclid's discourse, notably to his concept of angle, given the incomparability of the angle of contact to any rectilinear angle (in Prop. III.16). His critical attitude was legitimated by a conception of mathematics as a non-historical and eternal body of knowledge open to discovery and revision by any skilled mathematician.

Orna Harari: "Readings of Classics"

My contribution focuses on the question for what purpose Greco-Roman thinkers read classical texts. Surveying various sources such as Strabo's Geography, introductions to late-antique commentaries on Aristotle's *Categories*, and Galen's commentary on Hippocrates' *On Fractures*, I show that reading classical texts was regarded as a means of inquiry. Reading these texts could play this role because Greco-Roman thinkers did not regard classical texts as authoritative and therefore they aimed to distinguish the matters on which these authors of these texts are right from those on which they are wrong. Further, they held that the authority of classical texts and the truth found therein admits degrees, and therefore could regard several texts and writers as authorities, and rank them in different ways. In the second part of my contribution I exemplify these points through an examination of one lemma from Proclus' commentary on the first book of Euclid's

Elements (*Elements* I.7) showing how Proclus reads two classics in a complex way. On the one hand, he reads Euclid in light of Aristotle and exclusively discusses the methodological, rather than the mathematical, aspects of the theorem being commented on. On the other hand, he does read not Aristotle's methodological considerations into the Elements but adapts them to Euclid's form of reasoning.

Veronica Gavagna: "Two Renaissance readers of Euclid: Tartaglia and Maurolico"

The two Renaissance mathematicians Niccolò Tartaglia (1499-1557) and Francesco Maurolico (1494-1575) were both readers and editors of Euclid's *Elements*. The thrust that pushed them to plan a new edition was their common dissatisfaction with the available printed editions of the *Elements*: nevertheless, their purposes and their approach to the text were completely different. On one side, Niccolò Tartaglia was an abacus teacher who lived in a humanist background: the Classics, in particular Euclid and Archimedes, represented a bridge between the world of practitioners and the world of the scholars. Treatises on practical geometry usually strongly depended on the *Elements*, albeit mostly intended as a toolbox. The practitioners were interested in results, constructions and procedures, but not really in Euclidean proofs: the truth of theorems and problems was tested through applications in concrete numerical cases. Tartaglia's 1543 vernacular edition was aimed at offering a theoretical justification of results in an accessible language, together with comments clarifying the more abstract parts through concrete examples. On the other side, Francesco Maurolico was alien to the abacus world: he was a mathematician whose research program was mainly focused on the recovery of the Classics. In his opinion, the extant texts were irreparably corrupted by centuries of transcriptions, therefore mathematicians were authorized to rework the texts in order to transform them into correct, solid bases useful to develop new ideas. To set up a reliable text of the *Elements* Maurolico was guided by three criteria: "emaculare", or to correct the numerous mistakes of the printed editions, "e duobus unum opus coaptare", or to merge the available different traditions (even replacing proofs and adding its own ones when he felt it necessary), and "reddere Euclidem faciliorem", or to simplify and shorten, where possible, the Euclidean proofs. The Classics were therefore read with different eyes according to the reader's purposes: continuing the exploration of the various types of readers, and consequently of the various uses, of Classics such as the *Elements* during the Renaissance, is bound to shed light on the nature and the role of Classics in the course of history.

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Roundtable: "Diagrams and The Changing Materiality of the Classics"

Mordechai Feingold, Eunsoo Lee, Yelda Nasifoglu

Yelda Nasifoglu: "The reception of mathematical diagrams in early modern England"

In this brief presentation, I consider some key issues with the reception of mathematical diagrams in England in the late seventeenth and early eighteenth centuries, particularly in the context of education, and highlight some contradictory views. Throughout the early modern period, the iconography of mathematics, especially geometry, involved the depiction of a diagram or the drawing instruments, such as compass or ruler, used to generate the shapes. Indeed, material evidence from the Savilian Collection at the University of Oxford show that drawing diagrams was an integral part of teaching geometry in the seventeenth century. Yet this same period saw the increasing use of analytical geometry, which favoured the mathematical text over the diagrams, which in turn came to be seen as superfluous or as mere illustrations of the text. At the turn of the eighteenth century, such algebraic approaches would be criticised for their incompatibility with the classical texts, calling for a return to the graphical approach.

Political Dimensions of Classics

Between Restoration and Reform. The reception of ancient mathematics in later seventeenth-century England

Philip Beeley

The paper considers two contrasting trends which defined the reception of ancient mathematical texts in England during the second half of the Seventeenth Century. On the one side, emanating from vice-chancellor John Fell's desire to establish a scholarly press at Oxford, similar to what he had observed on the continent, there was the ambitious but only partly realized plan to edit "the ancient Mathematicians Greek & Latin in one and twenty volumes" devised by the mathematician and Arabist Edward Bernard. The key to this plan was the meticulous editing of classical texts to the highest philological standards and its most noted products, albeit some thirty years later, were David Gregory's edition of Euclid's works and Edmond Halley's edition of Apollonius's *Conics*. On the other side, there was the recognition among scholars at England's two universities that existing curricula and texts for the teaching of mathematics needed to be revised and reformed because they were no longer suited to the intellectual and economic needs of the country. For this purpose, classical texts were also required, but preferably translated into the vernacular and employing modern algebraic notation.

Bernard composed a synopsis listing not only the twenty-one volumes to be included in Fell's publication programme, but also existing editions, manuscripts in public and college libraries, commentaries, annotated volumes, relevant modern authors, and so on, to be consulted. In some cases, such as Apollonius of Perga, reference was additionally made to codices known to exist in major European libraries, of which copies could be procured or had been so already. Euclid was the first named author, followed by Archimedes, Apollonius, Pappus, and Diophantus. Volume six was to be devoted to a small collection of Alexandrian authors, while all of volumes seven to sixteen were to be devoted to the writings of Ptolemy. The authors and themes foreseen for the remaining five volumes were Hero of Alexandria, the writings of anonymous Greek authors, Vitruvius, practical geometry, and Victorius of Aquitaine. The most comprehensively outlined volume of all is the first on the list, comprising the complete works of Euclid of Alexandria, and Bernard immediately set to work at producing this. His aim for the Greek text of the *Elements* was to start out from the 1533 Basel edition, while the Latin translation was to be based on that published by Commandino in 1572. Both texts were to be collated against codices found amongst the holdings of the Bodleian Library. Demonstrations were to be expressed succinctly at the foot of the corresponding pages using algebraic notation William Oughtred and Richard Rawlinson that Isaac Barrow had employed int his 1655 edition. By adopting such an approach, Bernard evidently intended to signify that the aim of the critical edition he was producing was not be seen exclusively in antiquarian terms. The edition eventually produced by Gregory dropped this idea.

Bernard encountered two major problems. The first was of his own making: his all-encompassing plan for Euclid was over-ambitious and he clearly became overwhelmed by the sheer quantity of material he amassed. His efforts did not go entirely to waste, however, for Gregory later made use of Bernard's papers in producing his Euclid edition. The second problem was due to the subscription model that Fell had adopted for his publication programme. Before the printing process could begin in earnest, a sufficient number of members of the learned public needed to commit to purchasing one or more copies. As was as common practice with this publication model, a specimen page was printed to provide prospective buyers with an indication of what they could expect. In the case of Bernard's proposed Euclid edition three such specimens have survived with significant differences in textual presentation. They along with Bernard's extant scholarly correspondence allow us to identify at least three attempts to re-launch the edition during two decades after the initial undertaking collapsed in the early 1670s. Bernard did his best to garner the support of influential scientific figures, but his friend Thomas Smith came to the damning conclusion that there was simply no market for what was intended. The practically-orientated mathematicians in London got by perfectly well with the geometry contained in their inexpensive manuals, while young members of the gentry were averse to studying Greek or Latin, and certainly to a high level.

Bernard was also involved in the second edition of an ancient mathematical text that was realized at Oxford around the same time. Already in December 1668 he had travelled to Leiden to make a transcription of the three books of Apollonius's

Conics which only survived in Arabic, namely Books V-VII, from the so-called Banū Mūsā codex owned by Golius. Unfortunately, nothing came of his editorial plans at that time, but in late 1696, Bernard succeeded in purchasing the codex at auction in Leiden and brought it back to Oxford. While the planned Euclid edition he worked on had failed to generate any scholarly interest, things were quite different with the *Conics*, because of the perceived importance of this work for understanding ancient methods of analysis. In fact, there had been various competing initiatives to produce a new and reliable edition of the *Conics* since the 1630s involving amongst others Christian Rau (Ravius), John Pell, and Golius, not to mention those by Commandino and Borelli.

The decision to revive the planned Oxford editions of Euclid's works and Apollonius's *Conics* was made as part of wider plans at the turn of the century to reform the teaching of mathematics at that university, coinciding with two new appointments to the Savilian chairs: David Gregory as Bernard's successor as astronomy professor and Edmond Halley as John Wallis's successor as geometry professor. The key figures in these reforms were Henry Aldrich, Dean of Christ Church, and himself a mathematician, and Arthur Charlett, Master of University College. It was Aldrich who brought together Wallis, Gregory and the classicist John Hudson to finalize plans to publish "all of Euclid's works in Greek and Latin in one volume", from which the magnificent 1703 edition resulted. Aldrich then reached a similar agreement with Gregory and Halley that the Savilian professor of geometry complete the edition of Apollonius's *Conics* that Gregory had begun. In both cases it was recognized that important preliminary work had been carried out by Bernard, although this tended to be underplayed or even suppressed. In both cases it was also clear that different hands contributed decisively to the finished publications: Wallis and Hudson in the case of Euclid, Gregory in the case of Apollonius. But it was crucial to the strategic plan of renewal that the editions should stand as monuments to the Savilian chairs moving into the next century.

Like Gregory's Euclid, Halley's edition of the *Conics* was conceived as an auspicious contribution to the Republic of Letters, an embodiment of the highest standards of philological precision and painstaking scholarship. The reader was to be sure at all times that with very few minor exceptions the authentic text – albeit partly in translation – was being presented. Both editions primarily served to set standards of scholarship that would somehow reflect back on the institution that produced them, but they served little or no role within the primary function of that institution, the instruction of youth. It is noticeable that in them appeal is made to their value for the wider Republic of Letters. The contrast to Halley's Latin translations of Apollonius's *Cutting-off of a Ratio* (*De sectione rationis*) and reconstruction of Apollonius's *Cutting-off of an Area* (*De sectione spatii*), published four years earlier, could not be greater. There in the preface he allows himself the freedom to set out the often-expressed trope that the achievements of moderns such as Viète, Wallis or Newton "should in now way lessen the glory of the ancients who brought geometry to perfection". But even more significantly, Halley at the same time uses Apollonius, and indeed Pappus, in order to attack Descartes, notably for falsely accusing the ancients of ignorance. Pappus, Halley suggests, was probably the first to recognize that the four-line locus was initially composed by Euclid and that Apollonius had indicated "not obscurely" how it could be carried out. As one of Newton's main promoters, Halley proceeds to cite Newton's anomalous geometrical construction of the four-line locus in Book I of the *Principia*. This problem had been central to Descartes's *Géométrie* and Newton was convinced that the solution in purely geometrical terms which he presented was more elegant than the analytic one. Thus, Apollonius is ultimately mobilized in later seventeenth-century England in the battle between Cartesian and Newtonian mathematics: another side of the multifaceted reception of ancient mathematics in the early modern era.

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Rethinking the ancient mathematical text: Ming-Qing scholars' critical reflections on *The Gnomon of the Zhou [Dynasty]*

Qi Han

During the late Ming and mid-Qing periods, European science and religion were introduced into China. Chinese scholars got interested in European mathematics and used it as a tool to study traditional Chinese mathematics, so they focused on *The Gnomon of the Zhou [Dynasty]*, the oldest Chinese mathematical book.

Xu Guangqi (1562–1633), one of the first Chinese scholars to contact with Western science, stressed the importance of *The Gnomon of the Zhou [Dynasty]* in Chinese mathematics. In his book *Explanations of Gougu*, he found out the similarity between the *Gougu* method and the Pythagoras theorem in Western mathematics.

In the 1660s, the anti-Christianity movement greatly impressed the younger the Kangxi Emperor (1654–1722). In 1678, he summoned scholars to test their knowledge in ancient Chinese astronomy, which stimulated Mei Wending (1633–1721), and led to his deep concentration on *The Gnomon of the Zhou [Dynasty].*

After the calendar case, the Kangxi Emperor proposed his theory of the Chinese origin of Western learning. To propagate the Emperor's idea, Mei Wending deliberately expanded his research on *The Gnomon of the Zhou [Dynasty]* and wrote *The Supplement to the Doubts on the Calendar*.

Kangxi's interest in *The Gnomon of the Zhou [Dynasty]* exercised a great influence on the imperial compilers of *The Basic Mathematical Principles*. As one of the chief editors, Mei Juecheng (1681–1763) was an advocate of Kangxi's theory. He played a crucial role in the compilation of the *Ming History*, making "Loss of rites rescued by barbarians" become an official theory, so the idea of the Chinese origin of Western learning had spread to all the fields of mathematical sciences.

From the late sixteenth century onward, Jesuits wrote many letters home about China. French Jesuit Dominique Parrenin (1665–1741) mentioned Chinese ancient mathematics several times in his letter, in which Voltaire showed great interest. Other French Jesuits, like J.-F. Foucquet (1665–1741) and Antoine Gaubil (1689–1759), also did some research on *The Gnomon of the Zhou [Dynasty]*. They tried to prove that China had its tradition of astronomy and mathematics in order to refute some European scholars' negative views of ancient Chinese science.

In the mid-eighteenth century, Chinese scholars were aware of the possible contributions that European methods could make to traditional Chinese mathematical astronomy. With support from the Qianlong Emperor (1711–1799), Dai Zhen (1724–1777) tried to study traditional science based on Western science and systematically collated the text of *The Gnomon of the Zhou [Dynasty]*. He helped the official opinions on Western learning transmit to the Jiangnan region.

In the late eighteenth and early nineteenth century, An Qingqiao (1756–1829), opposed the Kangxi's theory of the Chinese origin of Western learning. He held the view that East and West Ocean may share the same basic principle, but doubted that the Western method originated from *The Gnomon of the Zhou [Dynasty]*.

The study of *The Gnomon of the Zhou [Dynasty]* represents an interesting episode in the historiography of Chinese science. The emperor Kangxi and Mei Wending continued to propagate the theory of the Chinese origin of Western learning. It became as a tool for comparing and establishing relationships between Chinese and European mathematics. The new approach to traditional mathematics and new Western methods, helped the Chinese to achieve a better understanding of their own traditional science.

The seventeenth-century Jesuit Euclid

Antoni Malet

Two successful 17th-century Jesuit editions of Euclid's *Elements*. Early modern Jesuit editions of Euclid's *Elements* are heavily indebted to Clavius'. Two editions inspired by Clavius' and written by Jesuit authors André Tacquet and Claude-François Milliet Dechales, are remarkable not only by the way they depart from Clavius but above all for their success and popularity. Probably they count among the mathematical books most times reprinted and translated in Western Europe in the 17^{th} and 18^{th} centuries. If there is any truth in the saying that the reiterated repetion of an idea makes it true or at least respectable, regardless of the intrinsic value of the idea, there is no question that the mathematical ideas conveyed by Tacquet's and Dechales' editions of the *Elements* – be they orthodox or hetherodox vis-á-vis the classical Euclid – these ideas gained a good amount of respectability.

(i) André Tacquet's (1612–1660) *Elementa geometriae planae ac solidae* (Anvers, 1654) was a highly successful edition many times reprinted both in the Latin original (at least twelve editions from 1654 to 1783) and in the English version prepared by the Newtonian William Whiston (1667–1752), *The Elements of Euclid* (William Whiston, ed. and trans., London, 1714, with at least nine further editions up to 1791). Under the mentorship of Gregoire de Saint Vincent, Tacquet was originally interested in, and wrote about, sophisticated methods of higher geometry applied to the quadrature of new surfaces and bodies. His original results were published in a book on pure geometry, *Cylindricorum et annularium libri*, printed in 1651. Thereafter, in 1652 the Jesuit General required Tacquet to write a full *cursus* of mathematics adapted for use in Jesuit colleges. Thus, Tacquet's heavily edited version of Euclid's *Elements*, first printed in 1654, was meant to be the opening treatise of one large pedagogical work of encyclopedic nature. Tacquet's arithmetic, *Arithmeticae theoria et praxis* (Leuven, 1656), the second tract of his pedagogical commission, although not so popular as his geometry also knew multiple editions. Tacquet's unfinished treatises on astronomy, optics, catoptrics, practical geometry, and military architecture were posthumously published together in 1669 as *Opera mathematica*. Tacquet's Euclid's influence extended to the Spanish speaking world by way of Jakub Kresa's translation, *Elementos geométricos de Euclides* (Brussels, 1689). A Moravian Jesuit who taught for some years at Madrid, Kresa acknowledged his edition to be but a simplified Spanish translation of Tacquet's.

(ii) Claude-François Milliet Dechales' (1621-1678) edition of Euclid's *Elements*, even more successful than Tacquet's, was many times times reprinted in Latin (*Euclidis Elementorum libri octo: Ad faciliorem captum accommodati*, Lyon, 1660, with at least 7 further edition up to 1700); French (*Les Elemens d'Euclide expliquez d'une maniere nouvelle et tres-facile*, trans. Jacques Ozanam, no less than twelve editions from 1672 to 1778); English (*The Elements of Euclid explained in a new but most easie method*, no less than ten editions from 1685 to 1748); and Italian (*Gli Elementi d'Euclide*, no less than 4 editions from 1749 to 1797). Dechales wrote an impressive mathematical encyclopedia, *Cursus seu mundus mathematicus* (Lyon, 1674) filling (in its second, 1690 edition) four large folio volumes and almost 3000 pages in total. It contains 30 tracts that cover all the

> mathematical sciences, starting with pure geometry – Dechales' edition of the *Elements* – followed by mechanics, astronomy, optics, geography, architecture, and so on. The 1690 edition also included a long historical tract, *Tractatus proemialis de progressu matheseos, et illustribus mathematicis*, which opens the whole *Cursus*.

The similarity in structure, order, and pedagogical motivation in Tacquet's and Dechales' encyclopedias needs be stressed, and also the similar location of the *Elements* in them. Notice finally that the similarity extends to the relevance and location both authors grant to history as prefatorial narrative to their works, as Tacquet opened his edition of the *Elements* in 1654 with a short historical preface titled "Historica narratio de ortu et progressu Matheseos". It was included in all the many subsequent Latin and English editions. It is likely that the editions of the *Elements* here considered, besides being read in Jesuit colleges, ended up in the private libraries of the nobility and the learned professions and rich burghers who sent their children to Jesuit colleges – civic officers, noblesse de robe, lawyers and physicians, rich merchants, and so on. According to the scanty evidence at hand, the children of families such as these would make up near 90% of all students matriculated in Jesuit colleges.

The Jesuit *Elements*, a summary. A summary of the major technical novelties in Tacquet's and Dechales' editions must stress that while they are obviously inspired by Clavius', they are more innovative than Clavius was in respect of Commandino's canonical version. There are minor differences between Tacquet's and Dechales's, but they share an important number of similarities when compared to Clavius'. Among them we find the major role motion plays in both conceptualizing and defining lines and geometrical figures, by making prominent use of Clavius' notion of *ductus*, which Saint Vincent developed. Distances (which are absent in Euclid's original text) are taken for granted and applied to define lengths, areas, and so on. Moreover, distance is given a major role in defining parallelism. Axioms and postulates are reorganized. Furthermore, the editions of Dechales and Tacquet leave out the three so-called arithmetical books (VII to IX) and Book X, on irrationals (more about this below).

A highly distinctive feature of their editions of the *Elements* is the substantial modifications introduced in Book V and to Euclid's difficult and cumbersome conceptualization of proportionality, a matter that raised doubts and discussions among leading mathematicians throughout the 16^{th} and 17^{th} centuries. Dechales and Tacquet (finding inspiration in Clavius and even more so in Gregoire de Saint-Vincent) assumed an arithmetical understanding of magnitudes and their ratios, at odds with the letter and the spirit of Euclid's text. Dechales' and Tacquet's historical texts enlighten us on the reason they had for rewritting the *Elements* as they did.

Progress and the *Elements*. Dechales' historical treatise makes an apology for the progress of mathematics, which he makes a characteristic of the discipline in opposition to natural philosophy. He set forth a description of mathematics' lineal progress, from simple techniques in Egypt gradually advancing up to today's knowledge. In Dechales's history there is no discontinuity between the past and the present, on the contrary, continuity in time, in subject matter, and in method is strongly suggested, and the contributions of the past are presented as building blocks of our present mathematical knowledge, thus cumulatively built.

Dechales does not make explicit the metaphor of the building that grows cumulatively, piece by piece, by successively adding blocks that make it larger, higher, stronger. Yet that image is forcefully suggested in different ways. Already for Thales, the founding father, the results he contributed are identified by referring to their location in Euclid's *Elements* – and this is true for most results mentioned from Thales on. Thus Dechales credits Thales with [Book] I [Proposition] 5, and I 15, and III 31 of the *Elements* (p. 7). Pythagoras contributed I 32 and I 47. Oenopides of Chios, I 12 and I 24 (p. 7). Eudoxus is attributed the invention of Book V (p. 8), and so on and so forth. Continuity in mathematics Dechales assumes in theoretical as well as in practical terms. He claims land mensuration in Egypt must have been close to what 17$^{\text{th}}$ century surveyors practice. As for theoretical stuff, Dechales mentions results that first appear in ancient times to be completed by later (often early modern) authors, as if no major changes conceptual or methodological intervene. Commandinus, Clavius, Saint Vincent, and others, are mentioned as "perfecting", "expurgating" or generally improving works of the past as if they were contemporaries of the works' original authors. A presentist piecemeal understanding of mathematical knowledge pervades Dechales' history, a uniform and continuous thread linking all results from ancient times up to the 17$^{\text{th}}$ century.

In Dechales' historical picture, mathematics grows little by little, by accumulating one particular truth over another. The results in the *Elements*, identified by their numbers somehow serve to embody or materialize the piecemeal growth of the discipline. The *Elements* thus becomes both a template for growth and a monument parading the fruits of progress.

Mathematics and natural philosophy. As Dechales put it, the special nexus that bounds together the indubitable principles of mathematics to the most concealed things, rejecting probabilities and accepting only demonstrations, is what makes mathematics uniquely certain. Mathematics teaches that all things must be examined not by the senses but under the standard (*norma*) of right reason. This is why Euclid's *Elements* reflect the most perfect, ideal organization of knowledge. It is a golden book, says Dechales, among the most important ones ever written because it sets out the "true idea of science" (p. 8).

Dechales strongly vindicates by way of examples the value of mathematics for physics. He highlights mathematics' 17$^{\text{th}}$-century role in increasing *physica* with many "non-contemptible, noble and pleasing" results that were unknown to the Ancients (p. 2). He underlines the decisive role of mathematics in settling debates about the shape of the world and in astronomy. Among other things, he mentions the motions, distances and order of planets and stars, their influences, the nature of sun and moon, the new observations with telescopes, the new satellites around planets, the nature of the rainbow, the laws of dioptrics and catoptrics.

Then he goes on to stress that mathematics is unavoidable in studying the "more hidden mysteries of nature", the ocean's ebb and flow, magnetism, the forces of gravitation and percussion, and also in studying the *fabrica* of the human body, the working of sense organs, and so on (p. 2).

Usefulness in mathematics. For the technical modifications introduced in the Jesuit editions of the Elements under consideration, they appear related to Dechales' understanding of usefulness as set forth in his historical tract. Usefulness is certainly applied to works that solve practical problems, like Kepler's *Stereometria doliorum*, but it is also applied to G.A. Borelli's (1608-1679) *Euclidis restitutus* (1658), a substantial attempt to reorganize the results in books I to VI in Euclid's *Elements* in order to avoid Euclid's theory of proportionality in Book V. On the other side, theoretical difficult *per se* does not make a work useful, on the contrary. Wallis's two treatises on the cycloid and cissoid, *Tractatus Duo, Prior de Cycloide ... et de Cissoide...* (1659), Dechales dismisses by a phrase he uses often, "these matters are more curious than useful." Dechales also qualifies as "ingenious but useless matter" results that other mathematicians considered theoretically relevant.

In particular, says Dechales, books VII to IX of the Elements, the so-called arithmetical books, set forth properties of numbers but not all properties, just those necessary for the theory of incommensurability. Thus they serve as introduction to Book X, where incommensurable quantities are dealt with. Properly speaking, Dechales goes on, books VII to X should not be counted as part of the *Elements*, since the theory of incommensurable quantities contains "more curiosity than utility". The theory is not needed for understanding other parts of mathematics and even less so for the common applications of mathematics or for "explaining some physical effect" (*ad effectum aliquem physicum explicandum*) – the only plausible positive argument, adds Dechales, would be the theory's potential use in eliciting some "slight arguments" on the infinite divisibility of the continuum. Theoretical sophistication and originality per se have no claim to superiority in Dechales' *Tractatus*. He gives the same status to practical and theoretical works. Practical geometries, mathematical tables, and treatises on instruments appear generously, and sometimes are given pride of place (p. 18). Tables and treatises on how to make tables also appear prominently. Simon Stevin's (1548–1620) practical geometry is singled out for praise within his *Hypomnemata mathematica* (1605–08). Clavius's *Geometria practica* is put above everything else that Clavius wrote about geometry, including his edition of the *Elements*.

For Dechales, the logarithm is the paradigm of "useful" invention, and presents it as the tool that has deeply renovated all the applications of mathematics. He twice claims that its invention justifies the thought that we are indeed more learned than our ancestors (p. 19 and p. 34). The logarithm plus advances in trigonometry plus the invention of new methods and instruments in geodesy plus the method of indivisibles count among the strongest evidence of progress in mathematics. The three classical problems – the quadrature of the circle, the duplication of the cube, and the trisection of angles by constructions with straight edge and compass – prompt Dechales to contrast the progress made in the last two centuries with the inability to solve them, and hence to question their status. From a modern perspective, Dechales stresses, numerical solutions to them – solutions endowed with as much accuracy as one wishes – have been achieved. Efforts to find solutions by straight edge and compass constructions are, accordingly, niceties "more curious than useful." Dechales declares the solutions to the classical problems in the terms in which the Ancients set them forth, to be useless.

In agreement with the importance Dechales gives to practical applications and the knowledge of nature, he understands geometrical magnitude as fully arithmetized, in the sense that geometrical objects are assumed measurable or numerically expressible. It is on the basis of this non-Euclidean understanding of geometrical magnitude that Dechales not only elevates the status of practical mathematics but considers logarithms the jewel of the crown of 17$^{\text{th}}$ century mathematics. It is also on this basis that he would dismiss the relevance of the three classical construction problems. This is not the place to take the argument further, but it is worthwhile stressing that the arithmetization of geometrical magnitude had no theoretical foundation in the 17$^{\text{th}}$ century. The measure of magnitudes makes possible to handle magnitudes through their numerical measures, but requires an arithmetic understanding of the notions of ratio and proportionality in Book V of the *Elements* – notorious notions through the 16$^{\text{th}}$ and 17$^{\text{th}}$ centuries which Dechales and Tacquet and many others attempted to reform. The tacit arithmetization of geometrical magnitudes, so conspicuous in Dechales' history, was a common feature in 17$^{\text{th}}$ century mathematics that logically impinged on the notions of ratio and proportionality, and which forcefully contributed to make the nature of these notions a hotly debated early modern issue. Dechales' history allows us to understand why the Jesuit editions were militant against Euclid's Book V.[1]

Refashioning the Classics in Different Ages

Diophantus redivivus: is Diophantus an early-modern classic?

Catherine Goldstein

Many early-modern mathematical books incorporated at least a part of Diophantus' *Arithmetica*, from Jacques de Billy's *Diophanti Redivi Pars prior et posterior* to John Kersey's *Third and Fourth Books of the Elements of algebra* or Jacques Ozanam's *Recréations mathématiques.* Diophantine questions regularly circulated among mathematicians of the time in the context of exchanges of problems or challenges [12]. It is thus tempting to consider Diophantus's opus magnum as a classic. However, I argued in my talk that, while Diophantus was indeed a classical

[1]Deschales' historical tract is fully analyzed in A. Malet, *Milliet Dechales as Historian of Mathematics*, forthcoming in Perspectives on Science.

author for early-modern mathematicians, his main work did not become a classical book.

The first point is easy to establish. According to the definition of the *Dictionnaire de l'Académie* [1, vol. I, p. 197],

> "classical" [is] only in use in this sentence: *a classical author*. That is: an ancient and much approved Author who is an authority on the subject he deals with. *Aristotle, Plato, Livy, &c. are classical Authors.*

Statements establishing such a status for Diophantus abound. To give just one example among many, in his 1660 *Diophantus geometra*, Billy claims [4, Lectori Benevolo…]:

> Who does not know Diophantus does not deserve the name of mathematician ; indeed, what Cicero is among the Orators, Virgil among the Poets, Aristotle among the Philosophers, Saint Thomas among the Theologists, Hippocrates among the Physicians, Justinian among the Lawmakers, Ptolemy among the Astronomers, Euclid among the Geometers, is the very same as what Diophantus is among the Arithmeticians ; he who surpasses all the others by a long interval is their coryphaeus, and easily their prince.

Still, several historiographical issues are at stake. First of all, at least two authoritative versions of Diophantus' *Arithmetica* are referred to by most mathematicians, Franciscus Vieta's *Zetetica* [20] and the heavily commented edition, with a Latin translation, published by Claude-Gaspard Bachet de Meziriac [3]. These texts belong to different genres and provide their readers with different organizations and selections of Diophantine material, different symbolisms and textual marks, and even different ideas of what constitutes an adequate solution [13] (see also Abram Kaplan's contribution to this workshop).

On the other hand, both, in various degrees, present Diophantus as the father of algebra, a clue followed by most early-modern mathematicians who followed them. Billy, mentioned above, for instance, goes on [4, Lectori Benevolo…]:

> Diophantus remained within the limits of Arithmetic, not the vulgar kind that is taught to children and merchants, but another, more subtle one, that one calls Algebra and that is the science of unknown numbers starting from hypotheses.

Even this agreement poses problems: one of its consequences is that Diophantine questions are usually treated in textbooks on algebra, another that algebraic formulas are often presented as a natural and valid generalization of Diophantus's original, unique, solution in fractions. Some, like Fermat, advocated at the time against this trend, but in favor of a search for integer solutions only. However, none of these developments correspond to the current idea about Diophantine questions, with its emphasis on the description and computation of rational solutions [11, 18].

A last, intriguing, issue concerns the French scene particularly. In French literature, "classical" has been more and more associated with the new standards emphasized in the framework of, or parallel to, the courtly culture of Louis XIV's times; opposed to the heavy and contrived volumes of the schools [9], classical texts, in this sense, were supposed to share a set of characteristics, such as elegance, simplicity, naturalness and correctness [19, 17]. A question is thus whether Diophantine problems found a home in mathematical books that can be qualified as classical, and how.

To study these issues, I analyzed in the talk several works linked to Diophantus's *Arithmetica*, focussing for the sake of time to French authors of the second half of the seventeenth century: Billy's *Diophantus geometra* [4] and *Diophantus redivivus* [5, 7], Ozanam's manuscript *Diophante reduit à la specieuse* and *Traité des simples, des doubles and des triples égalités pour la solution des problèmes en nombres* [6], as well as his *Nouveaux elemens d'algebre* [14], Jean Prestet's *Elemens de mathematiques* (both editions) [15, 16, 2], and finally Bernard Frenicle de Bessy's posthumous *Traité des triangles rectangles en nombres* [11]. For each of them, I surveyed the selection and organization of the material, as well as the individual treatment of some of the problems and their solutions. This shows that if Diophantus' *Arithmetica* remained an inspiration for new isolated mathematical problems, in algebra, number theory and even in Euclidean geometry, they entered into a large variety of partial reconstructions, within different genres. There was no agreement at the time on what was a satisfactory solution to Diophantine problems: even with algebraic formulas, for instance, and the claims of their authors to go further than Diophantus in providing infinitely many solutions instead of a single one, no proof of the infinity of the solutions was ever given, nor that they were all obtained. Attempts to restructure Diophantine problems and eventually integrate them in a text adapted to the new fashion (such as Ozanam's *Traité des simples, des doubles and des triples égalités* or Frenicle's *Traité des triangles rectangles en nombres*) either were not published in their original form or relied on a severe selection of a few problems and topics.

Contrary to other authors, thus, there was no Diophantus for the *honnête homme*. And early-modern Diophantus appears as classical author without a classical text.

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Vernacular mathematics in medieval Kerala

Roy Wagner

In this talk (based on joint work with Arun Ashokan and Vrinda PM), we will focus on Malayali *Kaṇakkatikāram* treatises. *Kaṇakkatikāram* is the title of elementary mathematical treatises that focused on measurements, calculation techniques and practical-recreational word problems. They enjoyed substantial distribution in medieval and colonial Tamil Nadu and Kerala. We will discuss their content, linguistic and stylistic form, context of use, relation to actual practice, the cultural values that they express and the political-economic reality that they reflect.

The title *Kaṇakkatikāram* is a compound of two words: *kaṇakku*, meaning "number", "calculation", "accounting" or (in a compound) "calculator/accountant", and *atikāram*, a Dravidian version of the Sanskrit *adhikāra*. In a literary context, the latter term means "topic" or the subject of a section of a treatise, but in a broader sense it refers to "rule" and "official authority". Thus, the title *Kaṇakkatikāram* means something like "topics (or rules) of calculation (or of accounting)", but in an oblique reading could also signify "the authority of calculation (or of the accountant)".

The subject of the *Kaṇakkatikāram* can be loosely described as "practical mathematics" – but this category will be problematized in the talk. It contains invocations, a "table of content" (a list enumerating the verses and subjects covered in the treatise, which includes land, gold, rice, capacity, weight...), lists of fractions, decimal powers and measurement units, explanations of some mathematical operations (like multiplication of fractions and summation of some finite series) and mathematical word problems organized according to topic.

Kaṇakkatikāram treatises exist in both Tamil and Malayalam. Based on linguistic and internal evidence, they can be tentatively dated to around the 15$^{\text{th}}$ century. Their geographical origin is most likely mixed. Different *Kaṇakkatikāram* manuscripts display substantial variety in content and ordering. While there are some verses that have variants in all Malayali versions, most verses can be found only in some versions, and some manuscripts leave entire sections out. The most consistently repeated common verses are a couple of introductory verses and some lists of units of measurements. The least consistent are the word problems, which vary substantially across the various versions, and are simply absent in some manuscripts. This variety means that we shouldn't consider the *Kaṇakkatikāram* as an authorial text, but as genre of mathematical treatises held together by a common theme and a few common verses.

One strange feature of the *Kaṇakkatikāram* is lists of tiny numbers and measurements, which follow unintelligible scales with seemingly arbitrary ratios, which are meaningless in practical terms, and which are not in use in word problems. We will discuss possible explanations for these lists in terms of there possible cultural meanings or uses.

As we will show in our talk, the *Kaṇakkatikāram* clearly has to do with actual practical calculations, such as those related to the work of the goldsmith and carpenter, but it also has to do with esoteric knowledge, echoing divine rule and classical Sanskrit treatises. It is a mathematics textbook of sorts for non-elite communities, covering some of the skills required of merchants and accountants, but it also bestows on the teacher, student and performer the symbolic capital of knowledge that is not reducible to practice. It serves to display virtuosity in memorization, calculation and poetic skill, but also asserts the authority of the accountant.

Finally, we will suggest that perhaps the *Kaṇakkatikāram* is not simply an attempt to establish the authority of numeracy, but, in its most perplexing verses, also a satirical reflection challenging this authority. Perhaps the seemingly arbitrary and unintelligible lists of units pose a subtle critique against authoritative presumptions to represent law and order in the realm of numbers, while in fact, under the feigned channeling of rational and divine numeracy by accountants, there is nothing but arbitrary extraction of wealth.

Uses of Classics through History

Algebra and Historia: François Viète's reading of Diophantus

Abram Kaplan

François Viète's interpretation of Diophantus' *Arithmetica* played an important role in the specious logistic – the new algebra – that he developed at the end of the sixteenth century. Viète's reception of Diophantus differed notably from that of contemporary readers including Simon Stevin and Wilhelm Xylander. Recent efforts to locate Viète's new algebra in the context of currents of sixteenth-century humanism emphasize continuities between Viète's mathematics and that of earlier sixteenth-century humanists who wrote about algebra. Building on these accounts, I add another strand: Viète's investment in Renaissance practices of *historia*. I argue that epistemological and practical elements of *historia* explain some ways in which Viète's reception of Diophantus differed from that of his contemporaries. In particular, Viète's interpretation of Diophantus reflects a productive collision between the generalizing ideals of Renaissance method and the focus on particulars characteristic of *historia*.

Following Renaissance writers including Regiomontanus, Viète saw Diophantus as a practitioner of algebra. But this meant something different for Viète than it did for, say, Simon Stevin. Stevin's and Wilhelm Xylander's readings of Diophantus were shaped by the efflorescence of algebra in sixteenth-century Europe. As Giovanna Cifoletti and François Loget (among others) have stressed, sixteenth-century French algebra was shaped by epistemological ideals that derived from humanism. In particular, humanist rhetoric (with its ideals of brevity and clarity) and the humanist investment in pedagogy led mathematical writers to conceive of algebra as a well-ordered and, consequently, easily teachable art. Explicit rules or algorithms were an important part of making algebra teachable. Diophantus' *Arithmetica* lacked such rules even though it broached difficult problems: thus Xylander, working from a Greek manuscript, found it challenging and difficult, and Stevin, working from Xylander's Latin translation, found it subtle but lacking order. Each produced an edition/translation of part of the *Arithmetica* exhibiting Diophantus' algebraic practice.

Viète developed a new algebra using species rather than numbers. Surprisingly, he also claimed that Diophantus used that same specious logistic, but that the Alexandrian mathematician had dissimulated its use so that his solutions appear more subtle and marvelous. In his *Five Books of Zetetics*, Viète worked through some of the problems Diophantus broached in his Arithmetica. Elements of Renaissance *historia* explain both Viète's claim that Diophantus dissimulated his use of specious logistic and the use that Viète made, in the *Zetetics*, of Diophantus' problems.

Renaissance *historia* was a practice of knowledge-making concerned with the collection, identification, and organization of particulars. These particulars could be sourced from a wide range of places, including ancient texts, artisanal practice, nature, and personal experience. Viète looked to Diophantus' *Arithmetica* as one source for particular problems that could be used to stimulate the development of his specious logistic. Rather than treating the *Arithmetica* as a whole work (as Stevin and Xylander had), Viète focused on particular problems: he studied Diophantus' solutions to these problems and, in what he called "zetetics," discussed and generalized these solutions by means of his new algebra.

These zetetics could be invoked, in turn, as resources for the easy solution of further problems. Skilled practice of Viète's new algebra depended on understanding the composition and structure of equations that the zetetics studied. One route to this understanding was an internalization of these zetetics and, consequently, the ability to recognize *in situ* when they might be applied to the resolution of a problem (or when the equations to which they gave rise might aid in a manipulation). Characteristic of Renaissance scholarship, practitioners of *historia* understood their practice to give rise to a kind of erudition: a knowledge whose central act of mind was recognition. Mastering the analytic art entailed becoming erudite in zetetics and/or equations by internalizing the zetetics or their corresponding algebraic solutions as a new algebraic "toolbox".

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Roundtable: "Classics and Vernacular"

Michael Friedman, Jens Høyrup, Thomas Morel

Jens Høyrup: "Classics and abbacus vernacular"

Abbacus mathematics draws (though rarely) on three works that from its perspective can be characterized as "classics": Boethius's *Introduction to arithmetic*; Fibonacci's *Liber abbaci*; and al-Khwārizmī's *Algebra* (in Guglielmo de Lunis's translation).

A few 14th-century abbacus books claim to build on Boethius, though having absolutely nothing to do with him; this is pure namesdropping, an attempt to borrow authority though no substance. In two 15th-century "abbacus encyclopedias", gifts to high-status Florentine patrons, Boethius's names for ratios are presented, but in separate chapters with no application elsewhere. This, and their copying of al-Khwārizmī "because he seems the older" of algebra authors, can also be interpreted as an attempt to establish legitimacy within the Humanist cultural environment.

Finally, one abbacus book from around 1300 presents itself as made "according to the opinion of Fibonacci". While the basic matters taught in school, roughly half of the book, has nothing to do with the *Liber abbaci*, the other, sophisticated half is indeed copied from an otherwise unknown vernacular translation of this model – but demonstrably often without understanding, and without influence on later books. Even this, we can say, is an instance of "advanced namesdropping", making appeal to the authority of a recognized "culture hero", already known by name and fame but not for his too advanced substance.

The absence of influence reflects that these "classics" were unnecessary. Initially, the relatively simple matters to be taught in school were better borrowed from anonymous sources than from Fibonacci, who attempted to present the material "magisterially"; in the mid-15th century, advanced abbacus mathematics, in particular the algebra, had left the classics far behind.

Thomas Morel: "Practitioners, craftsmen and their Classics"

When classics were first translated into vernacular languages, this reflected not only an intellectual development but also the idea that such texts could be of a broader use beyond universities. It is obviously extremely difficult to get an overview, let alone a real sense of how Euclid's *Elements*, for instance, were read and used by craftsmen, experts and artisans. Still, it is worth trying to establish some categories, highlight specific aspects, and to ask several questions.

First, it seems that in most cases, the access to classics happened through intermediary figures such as Jean Errard's *La géométrie et practique généralle d'icelle* (1594), even as the original themselves were available, in French or other vernacular languages. Such texts can be copied verbatim or in part by practitioners. In the latter case, mentions of Greek authors might be erased, which does not mean that the influence was negligible. What exactly was transferred? Shorthand for demonstrations, methods and practical results, or a certain idea of what makes for 'good' mathematics?

Subterranean geometry makes for an interesting example, for this mostly handwritten tradition came to include portions of Christoph Puehler's *Kurtze vnd grundliche Anlaytung zü dem rechten Verstand Geometriae* (1563), whose practical geometry builds heavily on Euclid's *Elements*. Another phenomenon of patrimonialization can be observed among craftsmen. Texts are copied, commented upon, improved and used for teaching. Practical arithmetic, for instance, were among the most popular vernacular texts, from Adam Ries *Rechenbücher* to Willem Bartjens *Vernieuwde Cijfferinge*. François Barrême's *Compte faits*, initially published in 1667, were still in use two centuries later, as the author last name became a common noun. Can such works qualify as classics?

Michael Friedman: "Hebrew mathematical manuscripts, Latin translations and references to the classics in the 12th century"

Looking into one of the first mathematical manuscripts in Hebrew written in the 12th century by Abraham bar Hiyya: *A treatise on measurement of areas and volumes* (*Ḥibbur ha-Meshiḥa ve-ha-Tishboret*), which was later translated into Latin in 1145 as *Liber embadorum*, one may ask about the status of classics in this work (for example, Euclid – who is cited few times during the work). Indeed, *Ḥibbur* was composed more than a hundred years *before* the translation into Hebrew of Euclid's *Elements*, when the latter was made during the second half of the 13th century.

The references to the classics – either a name or a book – are seen clearly when comparing the various copies of *Ḥibbur* in Hebrew to the Latin translation. Already at the beginning of manuscript (at its first part, called Book I), the Latin translation mentions the name "Euclid", which the Hebrew manuscripts do not. Moreover, the Latin translation adds the five postulates and a list of eleven general assumptions – which do not appear in the Hebrew manuscripts; this may indicate that Plato of Tivoli, the translator into Latin, attempted to emphasize a more rigorous tradition for presenting geometry, at least in the sense, as Sonja Brentjes termed during the conference, that the name of Euclid functions as status symbol, or a house hold name. Nevertheless, the name of Euclid is mentioned twice in later sections of the Hebrew manuscripts, when Bar Hiyya (or the scribe) notes "as Euclid interprets [explains] in his book", though not saying which book is meant. These two references appear also in the Latin translation.

Euclid's name, as a status symbol, appears as an almost absent figure in the Hebrew text. This absence was perhaps deliberate: if *Ḥibbur* was a study text, one may suggest that his name (or the name of his book) was omitted in order not to "scare" students. This raises the question – what was posited explicitly as a classic in *Ḥibbur*? While newly coined mathematical terms are sometimes taken from Arabic (and Bar Hiyya in some cases states that explicitly), the sources which are stated and cited explicitly are the Jewish writings: the Bible and the Talmud. However, as can be expected, these classics are completely absent from the Latin translation. Hence, when speaking about the introduction of the classics in the vernacular, one should take into account the embedding of mathematics into the broader cultural context: in this case, the introduction of practical mathematics into the Jewish communities was done not necessarily or in the first place via a reference to someone named "Euclid" – but rather via the embedding of mathematical statements and proposition in a world of Rabbinic and Jewish sources.

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Francesco Maurolico and Archimedes

Riccardo Bellé, Pier Daniele Napolitani

Francesco Maurolico was one of the most original and productive mathematicians of the 16$^{\text{th}}$ century. His production amounts to about 5000 pages; the National Edition of his scientific works is going to print his Archimedean studies (volume 7) and has already published two volumes on optics (volume 10, [1]) and music (volume 9, [2]). For a complete transcription of Maurolico's scientific texts see www.maurolico.it.

Unfortunately, many of Maurolico's works were not published when Maurolico composed them but many years after his death. Their influence seems to be limited, even if some of his studies probably circulated thanks to the Jesuits background and in particular through Cristoph Clavius. Maurolico's version of Archimedean oeuvre, for example, was printed only in 1685.

Maurolico started his Archimedean studies from a partial and unreliable tradition: some incomplete translations and some medieval tracts. Maurolico knew the "real" Archimedes only after 1544, the year of the Basel *editio princeps*.

Maurolico's studies on Archimedes could be divided in two periods: from 1525 to 1540 and after 1544. Some works date to the first phase while others, like *Spiral Lines* and *Conoids and Spheroids*, were written only after 1544. In 1550, Maurolico also composed a new original text: the *Praeparatio ad Archimedis opera*, an introduction to the Archimedean corpus, connected with a revision of the *Sphere and Cylinder*. Marshall Clagett proved that the source of Maurolico in preparing his version of the *Sphere and Cylinder* (dated 1534) was the *Liber de curvis superficiebus* by Ioannes de Tinemue (see [3], vol. 3, pp. 798-99). In this work Ioannes used two suppositions: (1) it exists a straight line or a plane surface equal to a given line or a given surface; (2) the surface of a figure included by another one is smaller than the surface of the including one. Maurolico used these suppositions in his proofs by double contradiction, for example when he proved that the surface of the sphere is equal to the rectangle contained by the diameter and the circumference of its greatest circle.

In the Basel edition (1544) Maurolico discovered that Archimedes' *Sphere and Cylinder* opens with definitions, postulates and lemmas: a solid deductive structure. Maurolico could so compare his efforts with the original work by Archimedes, and he realized that the *Liber de curvis* was not fully satisfactory. The rethinking of the *Sphere and cylinder* led Maurolico to compose the *Praeparatio ad Archimedis opera*, as explained in its *Prooemium*:

> In the book *De sphaera et cylindro* I used an easier way. And so that no one thinks that I have assumed inadmissible principles when I suppose that there is a spherical surface . . . equal to any given surface I will demonstrate these same principles here.

So the role of the *Preparatio* is clearly expressed by Maurolico itself: giving mathematical basis to the techniques used in *Sphere and Cylinder*. Maurolico needed to justify the existence of a sphere of given surface and the relation between the surfaces of the figures included one in another. The last case is the simplest: Maurolico found in the *Sphere and cylinder* published in 1544 a suitable postulate (post. 4) that he simply added at the beginning of the *Praeparatio*.

More critical is dealing with the first unjustified passage in which existence is needed. Maurolico didn't find anything similar in Archimedes. In this case, Maurolico needed to prove the existence of a sphere of given surface, which he did in *Praeparatio*, prop. 20, based on postulate 1 of the *Praeparatio* itself: "It is possible to find two lines proportional to any two magnitudes". Maurolico, with this postulate, tried to express the proportionality between any two quantities through the proportionality between two lines.

We argue, in our critical edition to be published as volume 7, that the *Sphere and cylinder* was revised by Maurolico after composing the *Praeparatio*, but not completely. In fact, in the *Sphere and cylinder* there are: (1) wrong internal references, probably due to an incomplete revision; (2) many added corollaries which express in a way closer to the Archimedean form results Maurolico already proved with different statements.

The *Praeparatio* is more than a justification of the "easier way". The postulate 1 and the proposition 20 are very different from the lemmas of Archimedes' *Sphere and cylinder* (I.2 and I.3) which are not on the *existence* of lines, but on the possibility to find two lines whose ratio approaches a given ratio. Moreover, the lemmas are not general but designed for a specific proof. Another example of the different approach can be traced in the *Spiral Lines*. In his version of this text, Maurolico cuts out proposition 4, another Archimedean lemma, since it is no longer necessary.

The *Praeparatio* represents the first outcome of a process of meditation on the works of Archimedes that moved Maurolico to rethink critically his previous studies. This process had led Maurolico to conceive an overall revision of Archimedes' writings, basing his approach on the "theorems of existence", and on the possibility of reducing any relationship between quantities to a relationship between straight lines. However, the *Admirandi Archimedis Monumenta* of 1685 testify that this revision was not completed and texts such as the *Libellus de sphaera et cylindro* or the *De lineis spiralibus* were never revised in deep.

Besides a biographical motivation, probably Maurolico's endeavor was too ambitious: to conceive a *mathematica prima* – the science of *quantitas generalis* – in which the theory of proportions, arithmetic and measurement of magnitudes had to be harmonized. Maurolico treated this theme explicitly in some theoretical works written in the '50s: *Sermo de quantitate*, *Sermo de proportione* and the preface of *Arithmeticorum libri duo*.

At the end of the '60s Maurolico tried in vain to make his project come true, motivated by the collaboration with the Jesuits of the local college. In this period, Maurolico came in contact with another Jesuit: Cristoph Clavius. Dates to this phase a compendium of Euclid's fifth book of the *Elements*, in which Maurolico presents his idea of "named" ratio (that of a number to a number) and compares ratios and named ratios.[1]

Clavius certainly possessed copies of Maurolico's writings on optics, gnomonics, conic sections, music theory, scientific instruments. It is possible to advance the hypothesis that his ideas spreaded in the Collegio Romano. What matters most is that Luca Valerio – one of the pupils of Clavius – takes inspiration from Archimedes. Starting from demonstrative techniques of Archimedes, he creates something profoundly new: at the center of the mathematical research are no longer objects-individuals (parabolas, circles, paraboloids, hyperboloids ...) but classes of figures defined by an abstract property. And yet Valerio – like Maurolico, again – confronted with his most daring innovations, when the form of geometric magnitude dissolves to leave space only for purely quantitative considerations, withdraws: as if frightened.

It is possible to find other examples of similar "failures" in the mathematics between the end of the 16^{th} and the beginning of the 17^{th} century: Cavalieri and the theory of indivisibles; Galileo and the law of falling bodies; Guidobaldo Dal Monte and its mechanics; Benedetti and the theory of proportions.

This proves that it is impossible to get out of the paradigm of Greek mathematics and to stay inside it.

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Didactic elements in pre-modern Chinese mathematical treatises

Alexei Volkov

In China, the "School of Computations" (*Suan xue*) was established during the Northern Zhou dynasty (557–581) in Chang'an city (modern Xi'an). Some authors suggest that the School actually had been established even earlier, under the Northern Wei dynasty (386-534). It was re-established by the late sixth and early seventh centuries under the rule of the Sui (581–618) and Tang (618–907) dynasties. For instruction in the School of the Northern Zhou dynasty Zhen Luan (fl. ca. 570) compiled a collection of a dozen mathematical treatises; it included older texts that he edited and commented as well as the texts that he compiled himself. In 656 Li Chunfeng (602–670) and his collaborators used these texts to produce a collection of 12 treatises to be used in the School of Computations as textbooks. In the second millennium, the School was re-established under the reign of the Song dynasty (960–1279), and several extant Tang dynasty texts were edited, printed

[1] See [4]

and used as textbooks. There exist descriptions of the mathematics instruction conducted in the School during the Sui, Tang, and Song dynasties; they specify the number of students, the lists of the textbooks, the periods of time allotted to the study of each book, as well as other details.

Recent works on the history of mathematics when describing the Chinese mathematical texts designed for, and used in, mathematics instruction, frequently portrayed them as "mathematical treatises *per se*", without taking into consideration their rather explicit didactical function. Their authors did not pay close attention to the numerical data and wording used in the problems, the involved measuring units, and other elements of the Chinese mathematical texts that may have had certain didactical value. This approach did not lead the researchers to a sensible explanation of certain peculiar features of the texts, for instance, of the presence of series of "mathematically identical" problems.

It appears reasonable to assume that a considerable number of pre-modern Chinese, as well as of Korean, Japanese, and Vietnamese mathematical texts, especially those styled as collections of mathematical problems, were originally compiled or adapted at subsequent stages to be used in educational context and thus are much closer, as far as their contents and use are concerned, to modern mathematical textbooks than to scholarly monographs. Placing the received mathematical treatises into the context where they originally belonged, namely, the context of mathematics instruction, may provide new insights into the ways in which they were read and interpreted.

J. L. Lagrange's practice of the mathematical conceptions by the ancients in his teaching of analysis

Xiaofei Wang

From 1795 to 1799, J. L. Lagrange taught analysis at the École Polytechnique. As a result, he published two important works, namely, the *Théorie des fonctions analytiques* (1797) and the *Leçons sur le calcul des fonctions* (1801). In these two works, Lagrange was motivated to rigorize the so-called infinitesimal analysis as he had been since his Berlin years. He proposed to establish the differential calculus on the method of series and claimed to give the rigor of the ancient demonstrations to the solutions of the principal problems of Analysis, Geometry, and Mechanics.

This talk first shows Lagrange's interest and cooperation with some scholars in a project of translating the Ancients' mathematical works and will clarify his motivation to work on these texts. Lagrange had been very interested in history and working on the ancient texts of mathematics. In this talk, I document and show that Lagrange paid much attention to both Peyrard's and Halma's projects of translating the works of Euclid, Archimedes and Ptolemy. Although it is not evident that Lagrange took part in the translating work in person, he cooperated with the two translators through giving support and advice to them. I thus assume that Lagrange often read the ancient Greek works and called for a good translation and edition of these works for the use of his own mathematical work. And he believed these works would be also useful to his contemporary mathematicians.

Further, this talk focuses on how Lagrange practiced the ancient conceptions and principles in his teaching of analysis for the purpose to meet the standard of rigor as one epistemological value in his pursuit. It interprets Lagrange's attitude toward the ancient works and his use of what he read from these works in his modern mathematical works. Through this part, I intend to formulate the view of Lagrange's efforts on reviving the ancient works as an integral part of his mathematical practice.

The talk draws the following conclusions. During his mathematical career, Lagrange had been reading, "translating" and using ancient Greek texts of mathematics. This constitutes a strong motivation for him to support the projects of translating important works by Ancient Greeks at the turn of the 19$^{\text{th}}$ century. Since his teaching at the École Polytechnique, Lagrange, together with J. B. Delambre, had supported his two young colleagues at this school, Peyrard and Halma, to implement the projects of translating and diffusing the important works by Euclid, Archimedes and Ptolemy. With all these activities, Lagrange intended to learn and get useful knowledge from the historical texts, and to use them in his mathematics. This reveals that all such activities were an integral part of his mathematical practice.

This case study reflects the uses of classics in the rigorization of the calculus, that had been an attempt for long in the 18$^{\text{th}}$ century.

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Roundtable: "Teaching and the Classics"

Sonja Brentjes, Michael N. Fried, Yiwen Zhu

Sonja Brentjes: "Classics in teaching in Islamicate societies"

In a general manner, teaching classics in Islamicate societies can be divided into two main periods:

(1) late 8$^{\text{th}}$ to 13$^{\text{th}}$ centuries;
(2) 14$^{\text{th}}$ to 19$^{\text{th}}$ centuries.

For the first period we have very few direct sources and depend on efforts to analyze extant texts of the mathematical sciences (number theory, geometry, astronomy, music with their sub-disciplines) with regard to their rhetoric, didactic tools, formal organization and possibly colophons and paratexts.

For the second period, we possess many texts explicitly marked as school texts plus a variety of sources about schools, teachers and famous texts.

For the first period we usually assume that Ptolemy's *Almagest*, Euclid's *Elements* and the so-called *Middle Books* were the taught classics. But there is no evidence of a continued teaching of either of those texts, although there is local evidence for teaching them in some cities and times. During the thirteenth century, these texts were newly edited by several scholars. Naṣīr al-Dīn Ṭūsī's (1201–1274) versions became the standard teaching books in many regions of the Islamicate world. The question whether new classics arose that were taught can be answered in the affirmative for those disciplines like algebra or calculation systems for which no classics from Antiquity existed. They became classics because they were taught. But their status as a classic was limited to certain regions and periods of time. Much more systematic research is needed to determine when, where and why certain texts of the mathematical sciences were considered a classic and when and why such a status dissolved. This applies, for instance, to Ptolemy's *Almagest* that despite its unchanged symbolic reputation did not become a school classic but was replaced by simpler and shorter texts.

Yiwen Zhu: "Zhen Luan, Li Chunfeng, and the ten mathematical Classics"

According to the historical records of the Tang dynasty (608-917), twelve mathematical treatises including *Ten classics* were used as textbooks in the School of Mathematics of the Imperial University. However, we don't know what the structure and the relationship between these treatises were. During this roundtable, I analyze this issue from three perspectives.

Firstly, by comparing Zhen Luan's (fl. 6^{th} century) works and Li Chunfeng et al.'s (602-670) works on mathematical treatises, I show Li Chunfeng's treatments on Zhen Luan's mathematical works.

Secondly, by analyzing the different roles of the 12 mathematical treatises in the curriculum, I show the structure of these treatises, and try to answer the question why only 10 books were viewed as classics.

Finally, I raise many open questions which are not only helpful for the issue, but also useful for the study of history of mathematical education in China.

Michael N. Fried: "Halley's posture towards Apollonius's works and its relevance for teaching Classics in modern mathematics classrooms"

The subject of this panel is the teaching of classics. As many of the talks in the conference have made clear, whatever classics may be, their nature and genesis are shaped by how they have been taught and studied. The books themselves were often geared towards a learner, books meant to teach something. In this panel contribution, rather than considering how a specific classic was taught in the past, I reflect on the teaching of historic mathematical texts in *modern* mathematics classrooms. The effort to bring historical texts into mathematics education today is different from what it was when classics, either the works themselves or versions of the texts, were read simply as the best source for mathematical knowledge (Euclid in particular comes to mind). In modern mathematics educational circles the effort to go back to historical texts is a way of introducing history *as such* into mathematics classrooms.

That effort, of course, presupposes that classics are indeed works of the past, works that are in some sense old. Being old means that teachers are fully aware there are modern alternatives for the mathematics they are teaching, sometimes more powerful alternatives. The basic educational question, then, is what does one learn from reading these texts? In a broader way this comes down to a question of what our relationship is not only to the texts themselves but to the past itself. It is not an obvious question, for as Oakeshott pointed out years ago (in [3], for example) there are different kinds of pasts and, accordingly, different kinds of relationships to the past.

The case of Edmond Halley's reading of Apollonius as reflected in his editions and especially his reconstructions of Apollonius's works – Book VIII of the *Conics* (1710) and *Cutting Off of an Area* (1706) – gives an example of a relationship to mathematical works of the past relevant for how one conceives bringing historical texts into the modern classroom. The reason is that, like us, Halley was both well aware of the power of his own modern mathematics, as is clear from, among other things, his preface to *Cutting Off of a Ratio and Reconstruction of Cutting Off of an Area* and, at the same time, had a genuine appreciation of the past, as can be seen not only in the same preface from 1706 but in his many purely historical/archeological papers throughout his career [2]. As a guide for modern education, he shows us how one can be a kind of *moderator* between past and present, respecting and learning from the "subtlety and inventiveness" of ancient thinkers on their own terms while not devaluing modern ideas and methods – an answer, in some ways, to modern educational efforts in which historical texts are used, in a Whiggist spirit, to promote standard mathematical curricula.

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Final Roundtable: "Mathematics and its ancient classics worldwide"
Courtney Roby, Ksenia Tatarchenko

Courtney Ann Roby: "Materiality and peculiarity in mathematical Classics"

On day 1, Sonja pointed out the need also to read marginalia and interlineal comments to really understand the history of these MSS, and today she has shown us some examples – opens up a very important set of questions about how to deal with two qualities – materiality and peculiarity – that may make us especially uncomfortable when considered in light of the "classics".

We might wish to consider the "classics" as "classical" purely because of the efficiency with which they can be appropriated by other authors, without thinking too much about the mechanics of that appropriation: Reviel's "toolbox", but as a black box.

But in fact, as Sonja points out, ideally we should always consider what the material matrix of that appropriation does to the process: mise en page, presence or absence (or abridgement) of the originating text, etc. What are the relationships between these components? How do various writing materials shape the process? Compare Reviel's ostraka to Thomas's beautifully hand-drawn bespoke manuscript versions of printed texts that allow one to incorporate at will both discourse and diagrams, plus Yelda on Aubrey on the power of creating diagrams not given to cement them in the reader's head.

Eunsoo's proposal to enact a new DH approach to coping with the proliferation of individual MS diagrams is a most welcome intervention – and in fact perhaps the affordances of digital media are such that we might find a way around the tendency to normalize a single version of the diagram and efface the proliferation of peculiar individualized versions of texts, because those peculiar and individualized encounters are themselves, I think , at the heart of what gives the "classic" its lasting importance.

Riccardo and Daniele's talk on Maurolico highlights an important issue: we may treat the "classic" as we have it as invariant, but Maurolico's gradual discovery of additional texts of Archimedes and different versions of the ones he started with highlights that the classic itself is something that must be discovered and rediscovered, and can itself change as an object of study (indeed, perhaps must change – even though I start with access to all of Archimedes' known works, I must consume them slowly, and my perception of the whole will change as I go like the blind men discovering the elephant). So Maurolico's personal history with the Archimedean corpus dramatizes a process that is going on with every encounter with a "classic". Relevant here as well is perhaps what Abram told us about Xylander's reconsideration of himself as a mathematician after confronting Diophantus: discovering the classic involves changing one's view not only of the classic but also of oneself.

Finally, a few slides from some definitely non-classical works: the Heronian *Stereometrica* and the *new P. Math.* Of course I'd never argue that Hero's works should be considered "classics," but in fact his own metrological work was a well-spring for a flood of later texts associated with his name, including the varied problem collections known as the *Stereometrica*. So we might wish to think of cases like this not as "classics," but maybe as "substrates" for the productive growth of commentaries, new problem-collections, and other complementary texts. And indeed, a subset of the problems in the *Stereometrica* seem to be drawn from an authentically Heronian work on vaults (Kamarika), for which Isidore wrote a commentary: here we see a few of those problems, which are arranged so as to walk the reader through a variety of problems on vaults, building from simpler to more complex structures so that the reader is eventually empowered to do calculations about volumes and areas of entire buildings.

Contrast this leisurely study with the short codex P. Math. recently edited by Bagnall and Jones. This codex is indeed one of the most robust known collections of mathematical problems on papyrus, but it covers a lot of territory in a relatively compact space. So the material matrix simply doesn't allow the reader or problem-solver to practice on simpler problems before moving to more complex ones. Hence the solver makes various errors when doing the kinds of calculation on a vault whose solutions are found in the *Stereometrica*.

Bagnall and Jones refer to the "miseducation" of the P. Math. solver, but in fact I would like to propose that the solver's peculiar errors are as interesting in their own way as a correct solution. They highlight both the peculiarity of individual encounters with canonical mathematical problems, as well as the key importance of the affordances of the material matrix Sonja pointed out for the potential variance in the ways those encounters are structured.

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