Mathematisches Forschungsinstitut Oberwolfach

Report No. 16/2023

DOI: 10.4171/OWR/2023/16

Random Graphs: Combinatorics, Complex Networks and Disordered Systems

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26 March – 31 March 2023

Abstract. Since the turn of the millennium the theory of random graphs has advanced by leaps and bounds. Random graphs have found very many applications, and many unexpected applications, in a remarkable variety of disciplines, and they are nowadays studied by mathematicians with various backgrounds (combinatorialists, probabilists, mathematical physicists), computer scientists and physicists. While this diversity has led to a proliferation of new models, questions and results, the community also has shattered, and by now different methods, terminologies and research agendas have come to coexist without much interaction. The workshop brought together distinct communities, who don't typically interact at their own intra-community events, but have each made significant contributions to the recent advancements in the theory. The workshop provided an effective platform for exchanging ideas, sharing insights, and building bridges across their respective domains.

Mathematics Subject Classification (2020): 05C80, 05C82, 05C85, 60B20, 60Cxx, 60K35, 68Q87, 82-xx.

Introduction by the Organizers

The workshop *Random Graphs: Combinatorics, Complex Networks and Disordered Systems* organized by Amin Coja-Oghlan (Technical University of Dortmund), Tobias Friedrich (Hasso-Plattner-Institute, Potsdam), Mihyun Kang (Graz University of Technology) and Konstantinos Panagiotou (University of Munich) took place in the week 26 March – 31 March 2023. Among the 43 on-site and 4 online participants, there were mathematicians, computer scientists and physicists from

several European countries, north America, Israel and Cyprus. The program of the workshop comprised of five 50-minute-long plenary lectures and twenty-three shorter, 25-minute talks. Moreover, in order to provide a stage for younger researchers, six 15-minute talks were presented by PhD students at the meeting. In addition, a very inspiring one-hour-long problem session was held on Tuesday night.

Originally scheduled for May 2020, the workshop faced an unfortunate cancellation due to the outbreak of the coronavirus pandemic, and it could only take place three years later. This year's event, however, was also met with unexpected hurdles, when a widespread strike of public transportation and trains occurred at the end of March, unfortunately preventing five registered participants from attending. Despite the various setbacks and obstacles, the workshop was ultimately a great success. The participants engaged in rich discussions, shared valuable insights and perspectives, and build connections across the different communities represented at the event. We (the organizers) are deeply grateful to MFO for providing them with the opportunity to host the workshop. We look forward to continuing the momentum and collaboration established at the event, with hopes of significantly advancing through cross-discipline interaction the theory in the future.

The theory of random graphs properly commenced in the beginning of the 1960's, when Erdős and Rényi published their famous 'giant component' paper. Their main discovery, stating that random graphs exhibit phase transitions, where macroscopic changes ensue from minuscule parameter alterations, has been the lodestar of the theory ever since. The study of phase transitions also ties the theory of random graphs to statistical physics, where the term originates. Additionally and foresightfully, Erdős and Rényi predicted that random graphs will in time serve as models of complex networks.

Since the early 2000s the theory of random graphs rapidly evolved into an interdisciplinary subject. Indeed, this evolution was sparked to a fair extent by the advent of 'network science', which can perhaps roughly be described as the notion that complex networks with similar characteristics arise in different contexts, and that the prevalence of certain special network properties entail that subjects that a priori appear to be remote should actually be treated by alike methods. This impetus was also received with great excitement in the computer science community, where specialised algorithms for complex network structures have been a growing theme, as well as in probability theory, where sophisticated methods have been seized upon to study increasingly complex network models mathematically.

A further boost to random graphs came from advances in statistical physics in the early 2000s. In that discipline random graphs are models of 'disordered systems' such as spin glasses, a subject of notorious difficulty and complexity. At first the line of work led to analytic but non-rigorous 'predictions' on several questions in the theory of random graphs that had come to be deemed extremely challenging. One example is the task of determining the chromatic number of the Erdős-Rényi random graph, a problem that was already posed in the 1960's and that still

awaits a solution. Additionally, physicists proposed novel so-called 'message passing algorithms' for actually solving optimisation problems on random structures. Experiments impressively demonstrated the superiority of the physics-inspired algorithms over classical approaches. Impressed by these successes, and inspired by the new perspective opened up by the physics ideas, mathematicians and computer scientists with diverse backgrounds took on the challenge of putting the physics 'prediction' on a rigorous mathematical basis. The ensuing body of rigorous work has not only transformed the theory of random graphs, but also highlights new, perhaps unexpected applications. An example of this is the construction of novel error-correcting codes, so-called 'low density parity check' codes, that achieve the Shannon capacity and that at the same time allow for efficient (message passing) algorithms for decoding. Additional celebrated results that emerged from this line of thinking include work on satisfiability thresholds, a question at the junction of computer science and random graphs, and inference problems such as the stochastic block model, an intriguing problem at the junction of random graphs, network science and statistics.

It is evident that over the past two decades the area of random graphs has been advanced by researchers with different backgrounds and objectives. What they have in common is an extraordinarily rich research topic, with an equally remarkable potential impact on a wide variety of subjects and disciplines. Yet the current state of affairs is that these communities are largely disjoint. They meet at different conferences and operate largely independently of each other. While the mathematical challenges arising in these disciplines have many similarities, and while there is a common thrust, different methods and different languages prevail. However, the communities can learn much and mutually benefit from each other.

In this workshop we managed to bring together researchers with different backgrounds, namely from the random graphs and the probabilistic combinatorics community, the disordered systems community at the junction of mathematics, computer science and statistical physics, and the complex networks community within computer science. While most of the participants were mathematicians, quite a few came from other disciplines, particularly computer science and physics. When preparing for the workshop, we highlighted the cross-discipline nature of the workshop, and asked the participants to prepare their presentations accordingly. During the workshop, we saw lively discussions, perhaps more lively than otherwise at mathematical conferences, about the utility of certain methods, the merit of certain conjectures and the validity of certain research objectives. The workshop facilitated the establishment of many important links among the participating communities, which we believe will have a significant and visible impact in the near future. The valuable connections forged at the event will inspire new collaborations and ideas that will advance the theory and contribute to its ongoing development.

Workshop: Random Graphs: Combinatorics, Complex Networks and Disordered Systems

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Abstracts

Where the really hard problems really are, really?

Florent Krzakala

I wish to discuss the phase space of optimization problems such as the discrete perceptron, submatrix localization, and graph coloring, and discuss the structure of the phase of solution, revisiting the traditional picture of the clusters, highlighting in particular with the idea of the connection between structure and computational hardness.

Aligning sparse random graphs with a message-passing algorithm

Guilhem Semerjian

A pair (G, H) of correlated Erdős-Rényi random graphs on n vertices is generated by drawing independently for each of the $n(n-1)/2$ couples on vertices a pair of correlated Bernouilli random variables of mean λ/n and of correlation parameter s, and including the corresponding edge in G and H according to these random variables. Consider now G' obtained from H through a random permutation of its vertices; the graph alignment problem is to infer the permutation from the observation of the pair (G, G'). Depending on the values of the mean degree λ and the correlation s this task might be, in the large size limit where n diverges, either information-theoretically impossible, possible with a polynomial time algorithm, or possible but requiring a priori an exponential time. This talk will summarizes a series of recent results on the boundaries of these phases, emphasizing in particular the role played by Otter's constant (related to the growth rate of the cardinality of unlabelled trees) as a correlation threshold.

Is it easier to count communities than find them?

Fiona Skerman
(joint work with Cynthia Rush, Alex Wein and Dana Yang)

Random graph models with community structure have been extensively studied. For both the problems of detecting and recovering community structure, an interesting landscape of statistical and computational phase transitions has emerged. A natural unanswered question is: might it be possible to infer properties of the community structure (for instance, the number and sizes of communities) even in situations where actually finding those communities is believed to be computationally hard? We show the answer is no. In particular, we consider certain hypothesis testing problems between models with different community structures, and we show in the low-degree polynomial framework that testing between two options is as hard as finding the communities. Our methods give the first computational lower bounds for testing between two different "planted" distributions,

whereas previous results have considered testing between a planted distribution and an i.i.d. "null" distribution.

Algorithmic thresholds for optimization problems on sparse random graphs (some hints from simple models)

Federico Ricci-Tersenghi

Focusing on some fundamental constraint satisfaction problems defined on sparse random graphs (e.g. random k-SAT and random graph coloring) I show how tools and ideas from statistical physics can help in identifying the algorithmic threshold for some broad classes of algorithms.

The hitting time of clique factors

Matija Pasch
(joint work with Annika Heckel, Marc Kaufmann and Noela Müller)

In a recent paper, Kahn gave the strongest possible, affirmative, answer to Shamir's problem, which had been open since the late 1970s: Let $r \geq 3$ and let n be divisible by r. Then, in the random r-uniform hypergraph process on n vertices, as soon as the last isolated vertex disappears, a perfect matching emerges. In the present work, we transfer this hitting time result to the setting of clique factors in the random graph process: At the time that the last vertex joins a copy of the complete graph K_r, the random graph process contains a K_r-factor. Our proof draws on a novel sequence of couplings, extending techniques of Riordan and the first author. An analogous result is proved for clique factors in the s-uniform hypergraph process ($s \geq 3$).

Free energy of a diluted spin glass model with quadratic Hamiltonian

Wei-Kuo Chen

In an important work, Shcherbina-Tirozzi (ST) proposed a mean-field spin glass model with a concave Hamiltonian defined on the Gaussian spin configuration space. By using tools from the convex geometry, they computed the limiting free energy and used it to derive the famous Gardner formula in the spherical perceptron model. In this talk, I will focus on a diluted variant of the ST model with a quadratic Hamiltonian and I will discuss the existence and expression for the limiting free energy at any temperature and external field.

Percolation phase transition for the marked random connection model

Markus Heydenreich
(joint work with Gabriel Berzunza)

We investigate a spatial random graph model whose vertices are given as a marked Poisson process on $\mathbb{R}^d$. Edges are inserted between any pair of points independently with probability depending on the Euclidean distance of the two endpoints and their marks. Upon variation of the Poisson density, a percolation phase transition occurs under mild conditions: for low density there are finite connected components only, while for large density there is an infinite component almost surely. Our interest is on the transition between the low- and high-density phase, where the system is critical. We establish that if the dimension is high enough and the mark distribution satisfies certain conditions, then an infrared bound for the critical connection function is valid. As a consequence, we obtain that various critical exponents exist and take on their mean-field values. We achieve this result through combining the recently established lace expansion for Poisson processes with spectral estimates in Hilbert spaces. We finally present an asymptotic expansion of the critical density as a function of the dimension.

The asymptotic distribution of cluster sizes for supercritical percolation on random split trees

Cecilia Holmgren

We consider the model of random trees introduced by Devroye (1998), the so-called random split trees. The model encompasses many important randomized algorithms and data structures. We then perform supercritical Bernoulli bond-percolation on those trees and obtain the asymptotic distribution for the sizes of the largest clusters.

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Independent sets in random subgraphs of the hypercube

Gal Kronenberg
(joint work with Yinon Spinka)

Independent sets in bipartite regular graphs have been studied extensively in combinatorics, probability, computer science and more. The problem of counting independent sets is particularly interesting in the d-dimensional hypercube $\{0,1\}^d$, motivated by the lattice gas hardcore model from statistical physics. Independent sets also turn out to be very interesting in the context of random graphs.

The number of independent sets in the hypercube $\{0,1\}^d$ was estimated precisely by Korshunov and Sapozhenko in the 1980s and recently refined by Jenssen and Perkins.
In this talk we will discuss new results on the number of independent sets in a random subgraph of the hypercube. The results extend to the hardcore model and rely on an analysis of the antiferromagnetic Ising model on the hypercube.

The rank of random graphs

Matthew Kwan
(joint work with Margalit Glasgow, Ashwin Sah, Mehtaab Sawhney)

Two landmark results in combinatorial random matrix theory, due to Komlós and Costello–Tao–Vu, show that discrete random matrices, and symmetric discrete random matrices, are typically nonsingular. In particular, in the language of graph theory, when p is a fixed constant, the biadjacency matrix of a random Erdős-Rényi bipartite graph $G(n,n,p)$ and the adjacency matrix of an Erdős-Rényi random graph $G(n,p)$ are both nonsingular with high probability. However, very sparse random graphs (i.e., where p is allowed to decay rapidly with n) are typically singular, due to the presence of "local" dependencies such as isolated vertices. In this work we give an essentially complete characterisation of such local dependencies, answering a question due to Vu.

The degree-restricted random graph process is far from uniform

Lutz Warnke
(joint work with Mike Molloy, Erlang Surya)

The switching method has been successfully used to analyze many uniform random graph models, in particular random graphs with a given degree sequence. In this talk we discuss how we adapt the switching method to the degree-restricted random graph process, demonstrating that this combinatorial enumeration technique can also be used to analyze stochastic processes (rather than just uniform random models, as before).

More concretely, the degree-restricted random process is a simple algorithmic model for generating graphs with degree sequence $D_n = (d_1, ..., d_n)$: starting with an empty n-vertex graph, it sequentially adds new random edges so that the degree of each vertex v_i remains at most d_i. It is natural to ask whether the final graph of this process is similar to a uniform random graph with degree sequence D_n.

We show that, for degree sequences D_n that are not nearly regular, the final graph of the degree-restricted random process differs substantially from a uniform random graph with degree sequence D_n. Here the switching method allows us to sidestep some technical difficulties that arise when one tries to use the differential equation method.

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Karp's patching algorithm on random perturbations of dense digraphs

Alan Frieze

We consider the following question. We are given a dense digraph D_0 with minimum in- and out-degree at least αn, where $\alpha > 0$ is a constant. We then add random edges R to D_0 to create a digraph D. Here an edge e is placed independently into R with probability $n^{-\epsilon}$ where $\epsilon > 0$ is a small positive constant. The edges of D are given edge costs $C(e), e \in E(D)$, where $C(e)$ is an independent copy of the exponential mean one random variable $EXP(1)$ i.e. $\Pr(EXP(1) \geq x) = e^{-x}$. Let $C(i,j), i,j \in [n]$ be the associated $n \times n$ cost matrix where $C(i,j) = \infty$ if $(i,j) \notin E(D)$. We show that w.h.p. the patching algorithm of Karp finds a tour for the asymmetric traveling salesperson problem that is asymptotically equal to that of the associated assignment problem. Karp's algorithm runs in polynomial time.

Partitioning problems via random processes

Oliver Cooley

(joint work with Michael Anastos, Mihyun Kang and Matthew Kwan)

The *majority colouring conjecture* of Kreutzer, Oum, Seymour, van der Zypen and Wood states that any directed graph has a *majority* 3*-colouring*, i.e. a colouring of the vertices with 3 colours such that for every vertex v, at most half of the out-neighbours of v have the same colour as v. We prove that this conjecture holds for almost all digraphs in a very strong sense: For any probability function $p = p(n)$, the binomial random digraph $D(n,p)$ has such a majority 3-colouring with high probability. (The most interesting range is when $p = \Theta(1/n)$.) The proof uses a carefully designed randomised algorithm which iteratively converges on a majority 3-colouring.

Another famous problem of a similar flavour is that of finding an *unfriendly bisection* in a graph, namely a balanced bipartition in which each vertex has at most half of its neighbours in its own class, or the the corresponding *friendly* version (at most half of its neighbours in the other class). In the latter direction, Ban and Linial conjectured that for any integer d, only finitely many d-regular graphs fail to have a friendly non-trivial (but not necessarily balanced) bipartition; on the other hand Bollobás and Scott conjectured that any graph admits a bisection in which every vertex is almost unfriendly, in the sense that it has at most one at most one more neighbour in its own class than the other. We use randomised iterative algorithms once again to prove that any graph G of not too large maximum degree has an *almost* friendly bisection and an *almost* unfriendly bisection, in which almost all of the vertices satisfy the appropriate (exact) condition.

Maximally stable local optima in random graphs and spin glasses: phase transitions and universality

Yatin Dandi

(joint work with David Gamarnik and Lenka Zdeborová)

In this work, we provide a unified analysis of stable local optima of Ising spins in Hamiltonians having pair-wise interactions and partitions in random weighted graphs where a large number of vertices possess sufficient single-spin-flip stability. We consider partitions on random graphs where a large number of vertices possess sufficient friendliness/unfriendliness. Equivalently, we characterize approximate local-optima in Ising models having local magnetic fields of sufficiently large magnitude, where the edge-weights equal the interaction coefficients in the Hamiltonian. For n nodes, as $n \to \infty$, we prove that the maximum number of vertices possessing such a stability undergoes a phase transition from $n - o(n)$ to $n - \Theta(n)$ around a certain value of the stability, proving a conjecture from [1]. Through a universality argument, we further prove that such a phase transition occurs around the same value of the stability for different choices of interactions (ferromagnetic, anti-ferromagnetic) for sparse graphs as $n \to \infty$ in the large degree limit. Furthermore, we show that after appropriate re-scaling, the same value of the threshold characterizes such a phase transition for the case of fully connected spin-glass models. Our results also allow the characterization of possible energy values of maximally stable approximate local optima. Our work extends and proves seminal results in statistical physics related to metastable states, in particular, the work of [2].

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Supercritical site percolation on the hypercube: small components are small

Sahar Diskin

(joint work with Michael Krivelevich)

In the site percolation model, a random induced subgraph $G[R]$ of a given graph G is formed by putting every vertex v of G into a random subset R with probability p and independently. One then researches typical properties of $G[R]$, such as the sizes of its connected components. In this talk, we consider site percolation on the d-dimensional binary hypercube Q^d in the supercritical regime, that is, with probability $p = \frac{1+\epsilon}{d}$. In 1994, Bollobás, Kohayakawa, and Łuczak showed that the largest component of $Q^d[R]$ in this regime is typically of order $\Theta(2^d/d)$ (proportional to the typical size of R), and that with high probability, all the other

components are of order $O(d^{10})$. They conjectured that, with high probability, all the components besides the giant are in fact of order $O(d)$ (note that $O(d) = O(\ln |V(Q^d)|)$ is optimal, and at large is analogous to the case of supercritical $G(n,p)$). We resolve their conjecture, showing that in the supercritical regime, typically all the components of $Q^d[R]$, besides the giant, are of order $O(d)$.

Properties of recursive trees with independent fitnesses

Tejas Iyer
(joint work with Bas Lodewijks)

We study a general model of recursive trees, where nodes are equipped with random weights, arrive one at a time and connect to existing nodes with probability proportional to a general function of their degree and their weight. We study the limiting infinite tree associated with this model, and show that, under a certain 'explosive' regime, the limiting tree has only a single node of infinite degree, and finite height, or a single infinite path with every node having finite degree. We provide sufficient criteria to determine which occurs.

A large-deviations principle for all the components in a sparse inhomogeneous random graph

Heide Langhammer

We study a sparse inhomogeneous random graph with N vertices and edge probabilities that depend on the types of the vertices. We count the connected components of such a graph by defining empirical measures with different rescalings and study their limiting behaviour as $N \to \infty$. As a main result we derive a large deviations principle for those empirical measures, that provides detailed information about the exponential decay of probabilities of rare events via its rate function. Analyzing the latter we recover the phase transition regarding the emergence of a giant component, which is already well understood, and beyond that can describe other effects that have not been studied in the literature before.

Computational aspects of Gibbs point processes

Marcus Pappik
(joint work with Tobias Friedrich, Andreas Göbel, Maximilian Katzmann and Martin Krejca)

Gibbs point processes are a popular model for the distribution of interacting particles in a region of Euclidean space. Among the most important computational tasks related to such a point process are sampling from its Gibbs distribution and computing its partition function. However, until recently, very few rigorous algorithmic results for these problems existed. In this talk, I focus on a recent approach that is based on reducing these computational problems to a discrete hard-core model on a carefully constructed family of geometric random graphs. This allows

us to make use of the rich algorithmic literature on the hard-core model. As a result, we obtain efficient approximation and approximate sampling algorithms for arbitrary repulsive pair potentials ϕ up to a fugacity of $\lambda < \mathrm{e}/C_\phi$, where C_ϕ is the temperedness constant of ϕ.

The expected degree distribution in transient duplication divergence models

Yin Yuan Lo
(joint work with Andrew D. Barbour)

We study the degree distribution of a randomly chosen vertex in a duplication-divergence graph, paying particular attention to what happens when a non-trivial proportion of the vertices have large degrees, establishing a central limit theorem for the logarithm of the degree distribution. Our approach, as in [3] and [2], relies heavily on the analysis of related birth-catastrophe processes.

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Random colorings from the Potts model on 'regular' trees

Guus Regts
(joint work with Ferenc Bencs, David de Boer and Pjotr Buys)

Consider a random q-coloring of the vertices of an infinite d-regular tree conditioned on a fixed coloring of the vertices at distance at least k from the root. In this talk I consider the question whether or not the marginal distribution of the root vertex tends to the uniform distribution as k tends to infinity in the setting when the random coloring is drawn according to the Potts model.

Limits of random cubic planar graphs

Benedikt Stufler

Planar graphs are graphs that admit a crossing-free embedding into the 2-sphere. We call such a graph cubic, if each vertex is adjacent to precisely three edges. Cubic planar graphs and related classes have been enumerated by [11, 28] via analytic and combinatorial methods. The average number of perfect matchings in a random cubic planar graph was determined in [31]. The work [32] established a Uniform Infinite Cubic Planar Graph as their quenched local limit. The paper [29] determined the typical number of triangles in 3-connected cubic planar graphs. Related research directions concern 4-regular planar graphs [27], cubic graphs on general orientable surfaces [16], and cubic planar maps [15]. In particular, [9] determined the geodesic two- and three-point functions of random cubic planar maps, after assigning independent random lengths with an exponential distribution to each edge.

For any even number $n \geq 4$ we let C_n denote the uniform random simple connected cubic planar graph with n labelled vertices and hence $3n/2$ edges. The graph distance on C_n is denoted by d_{C_n}. We let μ_{C_n} denote the uniform measure on the vertex set of the graph C_n. We let $(\mathbf{M}, d_{\mathbf{M}}, \mu_{\mathbf{M}})$ denote the Brownian sphere established independently in [19] and [22]. See Figure 1 for an illustration. Our main result shows that the Brownian sphere describes the asymptotic global geometric shape of C_n.

Theorem 1. *There exists a constant $\gamma > 0$ such that*

$$\left(\mathsf{C}_n, \gamma n^{-1/4} d_{\mathsf{C}_n}, \mu_{\mathsf{C}_n}\right) \xrightarrow{d} (\mathbf{M}, d_{\mathbf{M}}, \mu_{\mathbf{M}})$$

in the Gromov–Hausdorff–Prokhorov sense as $n \in 2\mathbb{N}$ tends to infinity.

An independent proof of the scaling limit is given in parallel work [7]. Originating from the study of combinatorial models [20, 13], the Brownian sphere has attracted the interest from researchers of a broad range of fields, including discrete mathematics, probability theory and mathematical physics. It was shown to be equivalent to the $\sqrt{8/3}$-Liouville quantum gravity sphere by [24, 23, 25, 26]. The Brownian sphere is also known to be universal in the sense that it arises as scaling limit of random elements of various classes of planar maps [1, 3, 6, 10, 2, 21, 18]. However, the methods for establishing such limits break down when considering graphs that are not embedded into the plane.

Previously known scaling limits for classes of graphs that are not embedded into the plane, such as the continuum limits of the Erdös–Rényi model [5, 4], differ from the Brownian sphere. The diameter of random graphs from subcritical classes was studied in [14], and these models have a more tree-like shape as shown in the work [30], which established Aldous' Brownian continuum random tree [8] as their scaling limit. Graph classes that are critical in a specific sense [17] were believed to exhibit a shape more similar to the Brownian sphere, because large deviation bounds for random unrestricted planar graphs [12] show that their diameter grows at the order $n^{1/4+o_p(1)}$. The exponent 1/4 hints at the Brownian

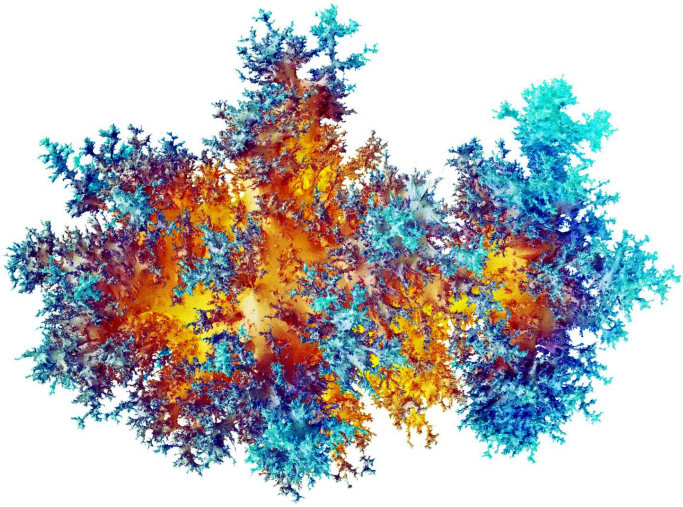

FIGURE 1. The Brownian map, approximated by a random simple triangulation of the two-dimensional sphere with 4 million triangles.[1].

sphere as scaling limit. However, describing the asymptotic shape at precisely the conjectured critical scale $n^{1/4}$ poses a serious challenge.

The present work establishes $n^{1/4}$ as the correct scale of the critical class of cubic planar graphs and establishes for the first time the Brownian sphere as scaling limit of a model of random graphs that are not embedded into the plane. Our approach features a new method that allows us to relate graph distances on random planar structures to first-passage percolation distances on their 3-connected cores. We believe this method to be helpful in understanding the geometry of further classes of non-embedded planar graphs.

Our main result successfully establishes $n^{1/4}$ as the accurate scale of the critical class of cubic planar graphs and, for the first time, confirms the Brownian sphere as the scaling limit of a random graph model that is not embedded in the plane. Our novel approach relates graph distances on random planar structures to first-passage percolation distances on their 3-connected cores. We anticipate that this

[1]In the depicted drawing, the colours indicate the closeness centrality in the dual map, while the vertex coordinates were determined through a spring-electrical embedding algorithm. The triangulation process used to generate the map was performed with the author's freely available open source software called `simtria`, which can be accessed on github: `https://github.com/BenediktStufler/simtria`. Additional visualizations of randomly generated surfaces can be found on the author's personal homepage at `https://www.dmg.tuwien.ac.at/stufler/`.

method will prove useful in comprehending the geometry of additional categories of non-embedded planar graphs.

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Successive shortest paths and MSTs

Gregory Sorkin

Let G be a complete graph on n vertices, with i.i.d. edge weights drawn from the uniform distribution $U(0,1)$ or the exponential distribution with mean 1. Given two random vertices s and t, it is known that the shortest path P_1 from s to t has cost asymptotically $\ln n/n$. It is also known that the MST T_1 of G has cost asymptotically $\zeta(3)$. We introduce "successive" versions of both problems.

If the edges of P_1 are deleted from G, what is the cost of the cheapest remaining path P_2? In general, what is the cost of the cheapest path P_k edge-disjoint from all previous paths? Where $w(P_k)$ is the cost of P_k, for the uniform-distribution model we show that, uniformly for all k from 1 to $n-1$, $w(P_k)/(2k/n+\ln n/n)$ converges to 1 in probability. We show an analogous result for the exponential model, and, for both models, the corresponding result for the expectation conditioned on P_k's existence (joint work with Stefanie Gerke and Balázs F. Mezei).

Likewise, what is the weight $w(T_k)$ of the cheapest spanning tree edge-disjoint from all previous ones? In the uniform model, the exponential model, and a Poisson multigraph model, we show that each $w(T_k)$ converges in probability to some γ_k, with $2k - 2\sqrt{k} < \gamma_k < 2k + 2\sqrt{k}$; in the Poisson multigraph model, $\mathbb{E}[w(T_k)] \to \gamma_k$. Kruskal's algorithm defines forests $F_k(t)$, each initially empty and eventually equal to $T(k)$, with each arriving edge added to the first $F_k(t)$ where it does not create a cycle. The size of the largest component of each forest is predictable, with $C_1(F_k(t))/n$ converging to some $\rho_k(t)$. We conjecture that for

large k, the functions ρ_k tend to time translations of a single function, $\rho_k(2k+t) \to \rho_\infty(t)$, and that $\gamma_k \to 2k-1$ (joint work with Svante Janson).

On recent advances in group testing

Max Hahn-Klimroth
(joint work with Noela Müller)

Consider n items, each of which is characterised by one of two possible features in $\{0,1\}$. We study the inference task of learning these types by queries on subsets, or pools, of the items that only reveal a form of coarsened information on the features - in our case, the sum of all the features in the pool. This is a realistic scenario in situations where one has memory or technical constraints in the data collection process, or where the data is subject to anonymisation. Sometimes, this problem is called quantitative group testing problem.

We are interested in the minimum number of queries needed to efficiently infer the features, in the setting where the feature vector is chosen uniformly while fixing the frequencies, and one of the features, say 0, is dominant in the sense that the number $k = n^\theta, \theta \in (0,1)$, of non-zero features among the items is much smaller than n. It is known that in this case, all features can be recovered in exponential time using no more than $O(k)$ queries. However, so far, all *efficient* inference algorithms required at least $\Omega(k \ln n)$ queries, and it was unknown whether this gap is artificial or of a fundamental nature. Here we show that indeed, the previous gap between the information-theoretic and computational bounds is not inherent to the problem by providing an efficient algorithm based on Approximate Message Passing (AMP) that succeeds with high probability and employs no more than $O(k)$ measurements. This solves a prominent open question for the quantitative group testing problem.

Entropy of sparse unlabeled random graphs

Dmitri Krioukov

Even though the structure of a network is an unlabeled graph, a vast majority of network models in network science and random graphs are models of labeled graphs. The difference between labeled and unlabeled random graph models is typically not a serious issue if graphs are dense. In sparse graphs, however, this difference may be huge, and with important consequences.

We first review what's known about unlabeled Erdős-Rényi (ER) graphs. Even though the leading term of their entropy is the same as in labeled ER graphs (a necessary but not sufficient condition for model equivalence), their degree distributions are very different. In the configuration model (random graphs with a given degree sequence), the leading entropy terms may have different prefactors in labeled and unlabeled graphs. Our main results are tight lower and upper bounds for the entropy of labeled and unlabeled sparse one-dimensional random geometric graphs. We prove that their entropies scale differently, indicating that the leading

contribution to the "randomness" of labeled graphs is from their random labeling, versus their random structure. These results suggest a need for "unlabeled network science", reexamining the adequacy of certain models of random labeled graphs in applications to the statistical analysis of the structure of real-world networks.

This project was motivated by earlier results, briefly reviewed as well, on the convergence of the Ollivier curvature of random geometric graphs to the Ricci curvature of their Riemannian manifolds, which recently led to the first-ever exact derivation of the diameter of a compact hyperbolic manifold.

Supercritical percolation on the hypercube – likely properties of the giant component

Michael Krivelevich

(joint work with Sahar Diskin, Joshua Erde and Mihyun Kang)

A random subgraph of the binary d-dimensional hypercube Q^d is one of the most classical and researched models of bond (edge) percolation. In this model, the base graph is the binary hypercube Q^d (vertices are 0/1-vectors with d coordinates, two are adjacent if they differ in exactly one coordinate), and each edge of Q^d is retained independently with probability $p = p(d)$.

It is known since the seminal work of Ajtai, Komlós and Szemerédi in 1982 that the model undergoes phase transition at $p = 1/d$, and in the supercritical regime $p = (1 + \epsilon)/d$, $\epsilon > 0$ a small constant, there is typically a unique component of size linear in $|V(Q^d)| = 2^d$, the so-called giant component.

We investigate typical combinatorial properties of the giant component, with an emphasis on, and a key being, its typical expansion. Among the properties we address are: edge- and vertex-expansion, diameter, length of a longest cycle, mixing time of a lazy random walk.

Our methods extend smoothly to the general setup of supercritical percolation on a product of many bounded size connected regular graphs.

Best response dynamics on random graphs

Nikolaos Fountoulakis

(joint work with Jordan Chellig and Calina Durbac)

We consider population games on a binomial random graph $G(n, p)$. These games are determined through 2-player symmetric game with 2 strategies played between the incident members of the vertex set. Players/vertices update their strategies synchronously: at each round, each player selects the strategy that is the best response to the current profile of strategies its neighbours play. We show that such a system reduces to generalised majority and minority dynamics. We show rapid convergence to unanimity for p in a range that is determined by a certain quantity of the payoff matrix. In the presence of a bias among the pure Nash equilibria of the game, we determine a sharp threshold for p, above which the

largest connected component reaches unanimity with high probability, and below which this does not happen. We also discuss the case where the game has more than 2 strategies. In particular, we consider payoff matrices with 3 strategies. We show convergence to unanimity after a bounded number of steps under certain conditions of the payoff matrix.

Cycles in the Mallows model

Tobias Müller
(joint work with Jimmy He and Teun Verstraaten)

We study random permutations of $1, \dots, n$ drawn at random according to the Mallows distribution. For $n \in \mathbb{N}$ and $q > 0$, the distribution Mallows(n, q) samples a random permutation Π_n of $1, \dots, n$ in such a way that each has probability proportional to $q^{\mathrm{inv}(\pi)}$, where $\mathrm{inv}(\pi)$ is the number of *inversions*. That is, pairs $1 \le i < j \le n$ for which $\pi(i) > \pi(j)$. In a formula:

$$\mathbb{P}(\Pi_n = \pi) = \frac{q^{\mathrm{inv}(\pi)}}{\sum_{\sigma \in S_n} q^{\mathrm{inv}(\sigma)}},$$

for all $\pi \in S_n$ where S_n denotes the set of permutations of $1, \dots, n$.

This distribution was introduced in the late fifties by C.L. Mallows in the context of "statistical ranking models" and has since been studied in connection with a diverse range of topics.

In the present work we will consider the cycle counts. That is, for ℓ fixed we study the vector $(C_1(\Pi_n), \dots, C_\ell(\Pi_n))$ where $C_i(\pi)$ denotes the number of cycles of length i in π and Π_n is sampled according to the Mallows distribution.

When $q = 1$ then the Mallows distribution is simply the uniform distribution on S_n. A classical result going back to Kolchin and Goncharoff states that in this case, the vector of cycle counts tends in distribution to a vector of independent Poisson random variables, with means $1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{\ell}$.

Surprisingly, the problem of finding analogues of this result for $q \neq 1$ has largely escaped attention until now. In the talk, I plan to discuss our proof of the fact that if $0 < q < 1$ is fixed and $n \to \infty$ then the cycle counts have linear means and the vector of cycle counts can be suitably rescaled to tend to a joint Gaussian distribution. Our results also show that when $q > 1$ there is a striking difference between the behaviour of the even and the odd cycles. The even cycle counts still have linear means and when properly rescaled tend to a multivariate Gaussian distribution, while for the odd cycle counts on the other hand, the limiting behaviour depends on the parity of n when $q > 1$.

On the chromatic number of graphons

Mikhail Isaev

We give an extension of Bollobás' classical result on the chromatic number of the binomial random graph to exchangeable random graphs associated with graphons. The asymptotic value was conjectured by Martinsson, Panagiotou, Su, and Trujić. We confirm this conjecture for block graphons and a special class of graphons that can be approximated by block graphons in L-infty norm. For general graphons, we verify the upper bound of the conjecture. The talk is based on [1] and [2].

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The Erdős-Gyárfás function $f(n, 4, 5) = \frac{5}{6}n + o(n)$ — so Gyárfás was right

Pawel Pralat
(joint work with Bennett, Cushman, and Dudek)

A $(4, 5)$-coloring of K_n is an edge-coloring of K_n where every 4-clique spans at least five colors. We show that there exist $(4, 5)$-colorings of K_n using $\frac{5}{6}n + o(n)$ colors. This settles a disagreement between Erdős and Gyárfás reported in their 1997 paper. Our construction uses a randomized process which we analyze using the so-called differential equation method to establish dynamic concentration. In particular, our coloring process uses random triangle removal, a process first introduced by Bollobás and Erdős, and analyzed by Bohman, Frieze and Lubetzky.

Constructing the scaling limit of the MST

Omer Angel
(joint work with Delphin Seniergues)

We give a new construction of the scaling limit of minimal spanning trees. The minimal spanning tree (MST) on the complete graph was previously shown to have a scaling limit, which is a random continuous tree of Hausdorff dimension 3 [1]. Towards a better understanding of the limit object, we consider the scaling limit of the MST on the Poisson weighted infinite tree (PWIT), which has the local structure of the complete graph with indeendent edge weights. We give a construvtion of the scaling limit in this setting as a chain of random trees, each of which is a result of an aggregation of CRTs of random sizes. The construction has parallels with both Ito's construction of Brownian motion in terms of excursions, and the stick breaking construction of the CRT as an aggregation of segments.

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Degree-biased spreading processes on random spatial graphs

Johannes Lengler

I present degree-biased epidemic spreading on Geometric Inhomogeneous Random Graphs (GIRGs). In this model, each edge receives a transmission delay that is the product of an iid random variable and a penalty factor that is some power of the product of the degrees or weights of the endpoints. Then we ask for the fastest path between two vertices in distance x. It turns out that this model is extremely rich: depending on the parameters, the number of infected vertices can jump to infinity in constant time; or grow at most poly-logarithmically; or grow polynomially, but strictly slower than linear; or grow linearly.

Kim-Vu's sandwich conjecture for random regular graphs

Jane Gao

The sandwich conjecture, proposed by Kim and Vu in 2004, is a well-known problem in random graph theory, which seeks to approximate a random regular graph $G(n, d)$ by sandwiching it between two Erdős-Rényi random graphs with approximately the same edge density. Kim and Vu conjectured that such a sandwiching exists for all $d \gg \log n$. I will discuss recent progress on this conjecture, as well as some related sandwiching problems and open questions.

Dynamics of random hyperbolic graphs

Fragksiskos Papadopoulos

Random hyperbolic graphs (RHGs) have been shown to be adequate for modelling real-world complex networks, as they naturally and simultaneously possess many of their common structural properties. However, existing work on RHGs has been mainly focused on structural properties of network snapshots, i.e., of static graphs, while little is known about the dynamical properties of RHGs. In this talk, we will consider the simplest possible model of dynamic RHGs in the cold regime (network temperature $T < 1$) and derive its most basic dynamical properties, namely the distributions of contact and intercontact durations. These distributions decay as power laws in the model with exponents that depend only on the network temperature T and are consistent with (inter)contact distributions observed in some real systems. Interestingly, these results hold irrespective of the nodes' expected degrees, suggesting that broad (inter)contact distributions in real systems are due to node similarities, instead of popularities. We will also see that several other properties, such as weight and strength distributions, group size distributions, abundance of recurrent components, etc., are also consistent with

real systems, justifying why epidemic and rumour spreading processes perform remarkably similar in real and modelled networks [1]. Furthermore, we will discuss a recent generalization of the model that incorporates link persistence [2], as well as results from dynamic RHGs in the hot regime (network temperature $T > 1$) [3]. In the hot regime, the intercontact distribution is nonnormalizable, which means that hot RHGs (including the configuration model that emerges for $T \to \infty$) cannot be used as null models for real temporal networks, in stark contrast to cold RHGs. We will conclude with future research directions.

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Longest cycles in sparse random graphs

Michael Anastos
(joint work with Alan Frieze)

Let $L(c, n)$ be the length of the longest cycle of a sparse binomial random graph $G(n, p), p = c/n, c > 1$. Erdős conjectured that if $c > 1$ then w.h.p. $L(c, n) \geq \ell(c)n$ for some strictly positive function on $(1, \infty)$ that is independent of n. His conjecture was proved by Ajtai, Komlós and Szemerédi and in a slightly weaker form by Fernandez de la Vega. Henceforward there has been a line of research in trying to bound $L(c, n)$ for $c > 1$. In this talk we will discuss how one can identify a set of vertices that spans a longest cycle in $G(n, c/n)$ w.h.p provided that $c \geq 20$.

Open Problems

Label propagation on random graphs

Pawel Pralat

Consider the following variant of the widely popular, fast and often used "family" of community detection procedures referred to as label propagation algorithms. Initially, given a network, each vertex starts with a random label in the interval $[0, 1]$. Then, in each round of the algorithm, every vertex switches its label to the majority label in its neighbourhood (including its own label). Ties are broken towards smaller labels.

Consider the performance of this algorithm on the binomial random graph $G(n, p)$. It is known that for $np \geq n^{5/8+\varepsilon}$, the algorithm terminates with a single label a.a.s. [1]. (Note: For some technical reason, it was easier to analyze the following

variant of this algorithm: at the first round, ties are broken towards smaller labels, while at each of the next rounds, ties are broken uniformly at random; This property was previously known only for $np \geq n^{3/4+\varepsilon}$ [2].) Moreover, if $np \gg n^{2/3}$, a.a.s. this label is the smallest one, whereas if $n^{5/8+\varepsilon} \leq np \ll n^{2/3}$, the surviving label is a.a.s. not the smallest one. On the other hand, there is $\varepsilon > 0$ such that for any $np \leq n^{\varepsilon}$, the procedure a.a.s. terminates on $G(n,p)$ in a configuration where more than one label is present [3]. Simulations suggest that the behaviour of the process changes around $np = n^{1/5}$.

I am interested in investigating $G(n,p)$ model as well as random d-regular graphs.

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Coloring inhomogeneous random graphs

Mikhail Isaev

Given a graphon $W : [0,1]^2 \to (\varepsilon, 1-\varepsilon)$, where $0 < \varepsilon < 1/2$, consider the exchangeable random graph $G(n,W)$ on vertex set $\{1,2,\dots,n\}$ defined as follows: 1) sample n points $X_1, X_2, \dots, X_n$ uniformly at random from $[0,1]$; 2) add the edge $\{i,j\}$, where $1 \leq i,j \leq n$, independently with probability $W(X_i, X_j)$.

Question: Let $\chi(G(n,W))$ denote the chromatic number of $G(n,W)$. Is it true that $\frac{\log n}{n}\mathbb{E}[\chi(G(n,W))]$ converges as $n \to \infty$?

The answer is positive for block graphons, continuous graphons, and monotone graphons.

Resilience of perfect matchings in random hypergraphs

Matthew Kwan

Let $G_k(n,p)$ be the Erdős–Rényi k-uniform random hypergraph with n vertices and edge probability p. Johansson, Kahn and Vu famously answered *Shamir's problem*, showing that for any fixed constant k, if C is sufficiently large then whp $G \sim G_k(n, C\log n/n^{k-1})$ has a perfect matching. Asaf Ferber and I conjectured a *resilience* version of this theorem. A special case is the following:

Conjecture: For any fixed constant k, if C is sufficiently large and $\varepsilon > 0$ is sufficiently small then whp $G \sim G_k(n, C\log n/n^{k-1})$ has the following property. Even if we adversarially delete (at most) an ε-fraction of the hyperedges at each vertex, G still has a perfect matching.

Existing nonconstructive techniques for Shamir's problem do not seem to be effective in this "resilience" setting. The only constructive way I know how to find hypergraph perfect matchings is by absorption, and that type of method seems to encounter serious difficulties when p gets smaller than about $n^{k/2}$; I believe it would be interesting to prove the above conjecture even for $p = n^{k/2-0.01}$.

Random spanning trees of regular graphs

Alan Frieze

Let G be an r-regular, connected n-vertex graph, where r is assumed to be large. Suppose that each edge e of G is given an independent uniform $[0,1]$ cost X_e. Let L_n be the length of the minimum spanning tree. It was shown in [1], [2] under mild assumptions about expansion/connectivity that

$$\mathbb{E}[L_n] = \frac{n}{r}(\zeta(3) + \varepsilon_r) \text{ where } \varepsilon_r \to 0 \text{ as } r \to \infty.$$

It was further shown in [2] that without any assumptions, we have that

$$\mathbb{E}[L_n] \le \frac{n}{r}(\zeta(3) + 1 + \varepsilon_r).$$

Conjecture: For any r-regular, connected n-vertex graph

$$\mathbb{E}[L_n] \le \frac{n}{r}(\zeta(3) + 1/2 + \varepsilon_r), \tag{1}$$

This would be best possible. Start with $m = n/r$ copies $G_1, G_2, \dots, G_m$ of K_r. Choose $x_i, y_i \in V(G_i), i = 1, 2, \dots, m$ and (i) for $i = 1, 2, \dots, m$, delete the edge $\{x_i, y_i\}$ from G_i and then (ii) replace these edges by $\{x_i, y_{i+1}\}$ for $i = 1, 2, \dots, m$. This graph satisfies (1).

There is a related question: let G be an r-regular, connected n-vertex bipartite graph and now let L_n denote the weight of a minimum weight perfect matching.

Conjecture: There is ε_r such that $\varepsilon_r \to 0$ as $r \to \infty$ and such that

$$\mathbb{E}[L_n] \le \frac{n}{r}(\zeta(2) + 1/2 + \varepsilon_r),$$

and further, under mild assumptions about expansion/connectivity

$$\mathbb{E}[L_n] = \frac{n}{r}(\zeta(2) + \varepsilon_r)\ . \tag{2}$$

We note that [3] verifies (2) when $G = K_{n,n,p}, np \gg \log^2 n$.

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