Tagungsbericht 03/1999

Orders in Arithmetic and Geometry

17.01. – 23.01.1999

This conference, led by Jürgen Ritter (Augsburg) und Martin J. Taylor (UMIST, Manchester), was the second on the above subject held in Oberwolfach. The main topics were

1. Galois Module Structure
2. Integral Group Rings
3. Quadratic Forms.

The order of the individual talks ensured a constant change between these topics, thereby preventing participants from getting lost in a series of lectures on subjects they do not work on themselves. This greatly increased interest and brought about many discussions. The abstracts of talks, which follow below, demonstrate how lively this part of mathematics has become, which new paths in research work have been opened, and which classical problems have been solved since the previous meeting in 1995.

Abstracts of talks

Agboola, Adebisi

Metrised Class Invariants

Let F be a number field with ring of integers $\mathfrak{o}_F$, and let $\mathcal{G}/\mathfrak{o}_F$ be a finite, flat, commutative $\mathfrak{o}_F$-group scheme which is killed by N. Write $\mathcal{G}^D/\mathfrak{o}_F$ for the Cartier dual of $\mathcal{G}$. Each $\mathcal{G}$-torsor may be viewed as a line bundle on $\mathcal{G}^D$, and in this way one obtains a "class invariant homomorphism" $\psi : H^1(\mathfrak{o}_F, \mathcal{G}) \to \mathrm{Pic}\,(\mathcal{G}^D)$. This homomorphism was first introduced by W. Waterhouse and was later used by M. Taylor to study the Galois structure of rings of integers in certain wildly ramified extensions. In my talk I described some results of G. Pappas and myself concerning a metrised version of the homomorphism ψ. We show that each $\mathcal{G}$-torsor may be endowed with canonical norms at the infinite places of F, and we use them to construct a homomorphism $\psi_M : H^1(\mathfrak{o}_F, \mathcal{G}) \to \widehat{\mathrm{Pic}}\,(\mathcal{G}^D)$, where $\widehat{\mathrm{Pic}}\,(\mathcal{G}^D)$ denotes the Arakelov Picard group of $\mathcal{G}^D$. By taking $\mathcal{G}$ to be the N-torsion subgroup scheme of an abelian scheme $\mathfrak{A}/\mathfrak{o}_F$, we are able to describe a relationship between ψ_M and the so-called circle pairing of Mazur and Tate. We also describe an "Iwasawa theoretic" version of ψ_M, and we show that this is injective.

Bayer-Fluckiger, Eva

Multiples of trace forms

Let k be a field, G a finite group, and let L be a G-Galois algebra over k. The trace form $q_L : L \times L \to k$, $q_L(x,y) = \mathrm{Tr}_{L/k}(xy)$, is a G-form. If φ is a quadratic form over k, then the tensor product $\varphi \otimes q_L$ is also a G-form. The G-Galois algebra L determines (up to conjugacy) a homomorphism $f_L : \Gamma_k \to G$ (where Γ_k is the absolute Galois group of k). f_L induces a homomorphism $f_L^* : H^1(G, \mathbb{Z}/2) \to H^1(k, \mathbb{Z}/2)$. If $x \in H^1(G, \mathbb{Z}/2)$, set $x_L = f_L^*(x) \in k/k^2 \simeq H^1(k, \mathbb{Z}/2)$. The aim of this talk was to discuss the following conjecture: *Set $d = \mathrm{cd}_2(\Gamma_k)$. Let L and L' be two G-Galois algebras, and let φ be a quadratic form. Suppose that $\varphi \in I^{d-1}$ (where I is the ideal of even dimensional forms of the Witt ring $W(k)$). Then we have* $q_L \cong q_{L'}$ *as G-forms* $\iff$ $e_{d-1}(\varphi)x_L = e_{d-1}(\varphi)x_{L'} \in H^d(k, \mathbb{Z}/2)$ *for all $x \in H^1(G, \mathbb{Z}/2)$ where e_{d-1} is the $(d-1)$th cohomological invariant,* $e_{d-1} : I^{d-1}/I^d \xrightarrow{\sim} H^{d-1}(k, \mathbb{Z}/2)$.

The conjecture holds for $d \leq 2$. This generalizes results obtained jointly with J.-P. Serre, J. Morales and M. Monsurro.

Bley, Werner

Euivariant Tamagawa numbers, Fitting ideals and Iwasawa theory

Let L/K be a finite Galois extension of number fields with group G. In a series of papers Burns and Flach defined Equivariant Tamagawa Numbers for certain motives over number fields. These are elements of $K_0(\mathbb{Z}G, \mathbb{R})$, which is the Grothendieck group of the fibre category of $- \otimes_{\mathbb{Z}} \mathbb{R}$. In the special case of the basic Tate motive $h^0(\mathrm{Spec}\ L)$ with coefficients in $\mathbb{Q}[G]$ we write $T\Omega(L/K)$ for this Tamagawa number. Throughout the talk we suppose that G is abelian. In this case one can give a natural and explicit description of $T\Omega(L/K)$ in terms of the (first) Fitting ideal of a finite, cohomologically trivial $\mathbb{Z}[G]$-module, which arises when computing a Tate sequence. This description is in particular amenable to investigation of $T\Omega(L/K)$ by using Iwasawa theory. Note that the Stark conjecture (which is proved for $L/\mathbb{Q}$) abelian) implies that $T\Omega(L/K) \in K_0(\mathbb{Z}G, \mathbb{Q}) \simeq \oplus_p K_0(\mathbb{Z}_p G, \mathbb{Q}_p)$.

By applying the Main Conjecture of Iwasawa theory one can prove the following theorem: *Let l_1 and l_2 be distinct odd primes, $a, b \in \mathbb{Z} > 0$, and let L be any subfield of $\mathbb{Q}(\zeta_{l_1^a})^+\mathbb{Q}(\zeta_{l_2^b})^+$. Suppose that neither l_1 nor l_2 splits in $\mathbb{Q}(\zeta_{l_1^a})^+\mathbb{Q}(\zeta_{l_2^b})^+$. Then $T\Omega(L/\mathbb{Q})_p = 0$ for $p \neq 2$.*

Burns, David

Equivariant special values and epsilon constants

Let $\mathfrak{A}$ be an order in a finite dimensional semi-simple $\mathbb{Q}$-algebra A. For any field E with $\mathbb{Q} \subset E \subset \mathbb{C}$ there is a canonical homomorphism $\partial_{\mathfrak{A},E} : \zeta(A \otimes_{\mathbb{Q}} E)^\times \to K_0(\mathfrak{A}, \otimes E)$ such that $\ker(\partial_{\mathfrak{A},E}) = \zeta(\mathfrak{A})^\times$. Here, ζ indicates the respective centre. For any global field K and any motive M defined over K and with coefficients A we propose to study special values of the $\zeta(A \otimes_{\mathbb{Q}} \mathbb{C})$-valued L-function which is associated to M. We hope to describe the image of such special values under $\partial_{\mathfrak{A},\mathbb{R}}$ in terms of invariants of $\mathfrak{A}$-structures associated to M. We point out that several well known conjectures can be phrased in this way, and we prove two results in this direction. The first is concerned with equivariant epsilon factors associated to Artin L-functions, and the second with special values of zeta functions of varieties defined over finite fields and which admit an étale action of a finite group G. The latter result has connections with the function field analogue of the Chinburg conjecture $\Omega(3) = W(L/K)$.

Byott, Nigel

Comparing integral Hopf-Galois structures

Let H be a Hopf algebra, free of finite rank over R, and let A be an H-module algebra. We say A is an H-Galois extension of R (or A is H-Galois, or H gives A a Hopf-Galois structure) if $\mu : A \otimes_R H \to \mathrm{End}_R(A)$, $\mu(a \otimes h)(b) = a \times (h \cdot b)$, is bijective. Hopf-Galois structures as a separable field extension L/K are classified by a result of Greither and Pareigis. For L/K elementary abelian of degree p^2, there are p^2 Hopf Galois structures. Taking $R = \mathfrak{o}_K$ for a finite extension K of $\mathbb{Q}_p$, one can obtain Hopf-Galois extensions using Kummer theory for formal groups. Lubin-Tate groups give rise to many examples where $\mathfrak{o}_L$ is free over a Hopf-order in some H making L Hopf-Galois, but not free over its associated order in the group algebra. The smallest examples (for $p \neq 2$) are elementary abelian of degree p^2. We investigate this phenomenon more systematically by considering elementary abelian extensions L/K of degree p^2 ($p \neq 2$, $\mathbb{Q}_p(\zeta_p) \subseteq K$), totally ramified with distinct ramification numbers $t_1 = pj - 1$, $t_2 = p^2 i - 1$. We determine when $\mathfrak{o}_L$ is $\mathcal{E}$-Galois for a Hopf order $\mathcal{E}$ in some H such that L is H-Galois. We then consider the effect of changing H. This depends on i and j. The relevant part of the (i, j)-space falls into three regions where, respectively, $\mathfrak{o}_L$ is Hopf-Galois in p of the p^2 Hopf-Galois structures on L, $\mathfrak{o}_L$ is Hopf-Galois in only one structure (but this need not be the classical one), and $\mathfrak{o}_L$ can only be Hopf-Galois in the classical structure. The Lubin-Tate example lies on the boundary between the second and third region. Here, exceptionally, $j \equiv 1 \pmod{p}$. Away from this boundary, $\mathfrak{o}_L$ can only be Hopf-Galois if j is divisible by p.

Cassou-Noguès, Philippe

Galois module structure for wild extensions

Let $N/\mathbb{Q}$ be a finite Galois extension and G the Galois group $G(N/\mathbb{Q})$. Assume that the ring of integers $\mathfrak{o}_N$ of N is locally free over a well chosen order Λ in $\mathbb{Q}[G]$. Our aim is to compare, in the locally free class group $Cl(\Lambda)$ of Λ, the class U_N defined by $\mathfrak{o}_N$ with the so called "root number class",

W_N, associated with the Artin root numbers for symplectic characters of G. This has successfully been done by Leopoldt when G is abelian and Λ is the associated order, and by M. Taylor when $N/\mathbb{Q}$ is tame and $\Lambda = \mathbb{Z}[G]$ (Fröhlich's conjecture). In this joint work with M. Taylor we define, under certain restrictions on the first ramification group of the primes wildly ramified in $N/\mathbb{Q}$, a "good order" Λ in $\mathbb{Q}[G]$, an adjusted ring of integer $\check{\mathfrak{o}}_N$ which is locally free over Λ, and prove in $Cl(\Lambda)$ the equality $\check{U}_N = W_N$ with $\check{U}_N$ denoting the class associated with $\check{\mathfrak{o}}_N$. This result is an improvement of results of Holland and Wilson and of Queyrut.

Childs, Lindsay

Counting Hopf Galois structures on Galois extensions of fields

If L/K is a Hopf Galois extension of number fields with (cocommutative) K-Hopf algebra H, and $\mathfrak{A}_{L/K}$, the associated order of $\mathfrak{o}_L$ in H, is a Hopf order in H, then $\mathfrak{o}_L$ is a locally free $\mathfrak{A}_{L/K}$-module. My talk discussed problems arising from trying to understand when $\mathfrak{A}_{L/K}$ is Hopf. The three approaches were: (1) Find conditions on L/K, K local, so that $\mathfrak{A}_{L/K}$ is Hopf, when L/K is Galois with group G and $H = KG$. It turns out that congruence conditions on break numbers (when L/K is totally ramified) are necessary but not sufficient for $\mathfrak{A}_{L/K}$ to be Hopf. (2) Construct families of Hopf orders in KG which are candidates to be $\mathfrak{A}_{L/K}$ for some L. (3) Look at L/K as an H-Hopf Galois extension for various K-Hopf algebras H. In this direction we let L/K be Galois with group G, then the number $s(G)$ of Hopf Galois structures on L/K depends only on G. By a translation of Byott, $s(G) = \sum e(G, N)$ where N runs through isomorphism types of groups N of the same cardinality as G and $e(G, N)$ is the number of equivalence classes of regular embeddings of G into the holomorph of N. We described some joint work with S. Carnoban which computed $e(G, N)$ for some N and G: for G simple, $e(G, G) = 2$; $s(S_n) \geq \sqrt{n!}$ for S_n = symmetric group, $n \geq 5$. We conjecture that $s(G) = 2$ if G is simple.

Chinburg, Ted

Duality and Hermitian Galois Structure

My talk was about some joint work with G. Pappas and M. Taylor on generalizing to tame G-covers of schemes the theory of the discriminant $d(\mathfrak{o}_N, \mathrm{Tr}_{N/\mathbb{Q}})$ in the Hermitian class group $HCl(\mathbb{Z}G)$ of the ring of integers $\mathfrak{o}_N$ in a tame G-extension N/K of number fields. The algebraic problems one has to resolve are how to define a discriminant $d(P^\bullet, \langle\ ,\ \rangle)$ for a perfect complex $P^\bullet$ of $\mathbb{Z}G$-modules having symmetric cohomology pairings $\langle\ ,\ \rangle_t : H^t(P^\bullet) \otimes \mathbb{Q} \times H^{-t}(P^\bullet) \otimes \mathbb{Q} \to \mathbb{Q}$, as well as how to define complexes up to homotopy which have Euler characteristics equal to a de Rham invariant $\chi(X/Y)$ in $Cl(\mathbb{Z}G)$ studied earlier with B. Erez when $X \to Y$ is a tame G-cover of projective regular flat schemes over $\mathbb{Z}$. Using work of Dold and Puppe we define a Hermitian class $\chi_H(X/Y)$ in $HCl(\mathbb{Z}G)$ lifting $\chi(X/Y)$. The construction suggests it may be useful to consider the Galois structure of simplicial G-sheaves.

Cougnard, Jean

Stably free ring of integers

Let $N/\mathbb{Q}$ be a tame extension, Galois with Galois group G. By a theorem of M. Taylor one can decide when the ring of integers $\mathfrak{o}_N$ is stably free, that is, has class 0 in the projective class group $Cl(\mathbb{Z}[G])$.

That means that $\mathfrak{o}_N \oplus \mathbb{Z}[G] \simeq \mathbb{Z}[G]^2$. One wants to decide if one can cancel and get a normal integral basis. Cancellation may fail if G has as quotient a binary polyhedral group. An extensive study of this algebraic problem has been done by Swan (1982). The smallest group for which cancellation fails for modules is a group of order 16 : $H_8 \times C_2$ (there are four distinct isomorphism classes of rank one modules which are stably free). It is shown that there are infintely many rings of integers in each isomorphism class. One tool of the proof is to produce an explicit construction of normal intergral bases (when they exist); it uses Martinet's normal integral basis criterion and Witt's construction of quaternion extensions of order 8.

Fesenko, Ivan

Nonabelian reciprocity maps

For a local field F and a Galois extension L of F, which is totally ramified and either finite or infinite arithmetically profinite, several reciprocity maps $\mathfrak{N}_F$ from $G(L/F)$ to specific subquotients A of $\mathbb{F}_p{}^{sep}[[X]]^*$, which are G_F-modules, are defined. They are bijections and 1-cocycles with natural functorial properties; ramification filtration being mapped onto some natural filtration on A induced by the principal unit group filtration of $\mathbb{F}_p{}^{sep}((X))$.

Gruenberg, Karl

Capitulation

If K/k is a finite Galois extension of number fields with group G, then the natural homomorphism of ideal class groups $\alpha : cl_k \to cl_K$ is a G-equivariant map, called the capitulation homomorphism. If K/k is unramified, one knows that $H^1(G, E_K) \hookrightarrow cl_k \xrightarrow{\alpha} cl_K^G \twoheadrightarrow H^2(G, E_K)$ is exact. Hence, if G is cyclic, by the Herbrand quotient being $1/|G|$, we have $|G|$ divides $|\ker \alpha|$. This is Hilbert's Theorem 94. Again, if $K = \tilde{k}$, the Hilbert class field of k, then $G \simeq cl_k$ and $|G| \Big| |\ker \alpha|$ (i.e., $\operatorname{im} \alpha = 1$): this is the Principal Ideal Theorem. Miyake raised the question whether always $|G| \Big| |\ker \alpha|$ when K/k is abelian (and unramified). This was answered affirmatively by H. Suzuti (Nagoya Math. J. 121 (1991)). By Artin's work, the problem translates to one in group theory: given a finite extension $A \hookrightarrow H \twoheadrightarrow G$ with G and A abelian, does $|G|$ divide $|\ker t|$, where t is the homomorphism $H/[H, H] \to A$ induced by the transfer $v_{H\to A}$. We call $\ker t$ a transfer kernel (for G). I presented a new proof of this result found by A. Weiss and myself. The first step is to translate the problem to module theory: The given group extension determines a $\mathbb{Z}G$-module extension $A \hookrightarrow B \overset{\pi}{\twoheadrightarrow} \Delta G$ (ΔG the augmentation ideal), where H is hidden in B as $H = \{x \in B \mid \pi x = g - 1 \text{ for some } g \in G\}$. The norm $\hat{G} = \sum_{g\in G} g$ on B restricts to $v_{H\to A}$ on H and the transfer kernel is now recognised as $H^{-1}(G, B)$. Theorem: *G is a finite abelian group, X is a finite abelian group. (i) X is a transfer kernel (for G) $\Longleftrightarrow$ (ii) $X \simeq M_G$ (co-invariants) for some Λ-module M ($\Lambda = \mathbb{Z}G/(\hat{G})$) with $\mathbb{Q}M \supseteq \mathbb{Q}\Lambda \Longleftrightarrow$ (iii) $|G|X = 0$ and $|G| \Big| |X|$.* A question: given $A \hookrightarrow H \twoheadrightarrow G$ with H finite and G, A abelian; does there exist an unramified extension K/k such that $H = G(\tilde{K}/k)$, $G = G(K/k)$? If so, then the class of groups described in (iii) is exactly that of capitulation kernels.

Hertweck, Martin

Embedding of group bases in integral group rings

This talk is on finite subgroups of the unit group of an integral group ring $\mathfrak{o}X$, where X is a finite group and $\mathfrak{o}$ a ring of algebraic integers. 1. *To the conjectures of Zassenhaus.* Let $B_1, \dots B_h$ be the blocks of $\mathbb{Q}X$, U a subgroup of finite index in the augmentation 1 unit group $V(\mathbb{Z}X)$ and U_i, V_i the respective projections of U, V in B_i. Is U_i conjugate to a subgroup of X_i in $B_i^\times$? The corresponding question for p-adic blocks has an affirmative answer for the principal block if X is p-constrained (F^*-Theorem). If the above question itself has a positive answer and $\mathbb{Z}X = \mathbb{Z}Y$, then Y is a subdirect product of the groups X_i. 2. *The normalizer problem and the isomorphism problem.* In 1997 I presented two non-isomorphic groups X and Y of *even* order with $\mathbb{Z}X = \mathbb{Z}Y$. An important role in my construction played $\mathrm{Out}_{\mathbb{Z}}(G) = \frac{\mathrm{Aut}\,(G) \cap \mathrm{Inn}\,(\mathbb{Z}G)}{\mathrm{Inn}\,(G)}$ for some $G < X$, which was not trivial. Since $\mathrm{Out}_{\mathbb{Z}}(G)$ is always an (elementary abelian) 2-group, there is no direct generalization to groups of odd order. Proposition: *There are* $\mathfrak{o}$ *and* G *such that* $\mathrm{Out}_{\mathfrak{o}}(G)$ *is any given finite abelian group.* Conversely, $\mathrm{Out}_{\mathfrak{o}}(G)$ is always abelian. 3. *Construction of non-isomorphic* X, Y *of order* $p^a q^b$ *such that* $\mathfrak{o}X \simeq \mathfrak{o}Y$ *for some* $\mathfrak{o}$. Given X, try to construct some group basis Y of $\mathfrak{o}X$ as a subgroup of $\prod X_i$. For that it suffices to work over the semi-localization $\mathbb{Z}_\pi X$ ($\pi = \{p \mid |X|\}$).The group X is formed as $X = Q \rtimes P$, Q a q-group, P a p-group, p, q given primes. The talk concentrates on discussing this case with an explicitly given P of order p^{21}.

Jehanne, Armand

Two dimensional Galois representations

The goal of my talk is to give a method for constructing Galois representations $\rho : G_{\mathbb{Q}} \to Gl_2(\mathbb{C})$, with quadratic odd determinant, such that if $\overline{\rho}$ is the induced projective representation, the image of $\overline{\rho}$ is isomorphic to the alternating group A_5. My plan is the following. If we assume Serre's conjecture on modular forms, then there is bound with respect to the restrictions of ρ to the decomposition groups of the unramified prime numbers l. We use it to get (conjecturally) the number field $\overline{\mathbb{Q}}^{\ker \rho}$ as the splitting field of a polynomial of degree 24. Finally, I give a test by means of which one checks that the polynomial we found has indeed a Galois group isomorphic to A_5.

Kimmerle, Wolfgang

On Hecke orders, integral group rings and finite reflection groups

Let (W, S) be an irreducible Coxeter system, W finite. Denote by $H(W)$ its associated generic Iwahori-Hecke order over $A = \mathbb{Z}[q, q^{-1}]$. The concern of the first part of the talk is the classification of the A-order automorphisms of $H(W)$. This reports on joint work with F. Bleher and M. Geck.
Theorem: *Let* $\mathrm{Autcent}\, H(W)$ *be the central automorphisms of* $H(W)$, $\mathrm{Aut_n}\,(H(W))$ *the normalized automorphisms, and denote by* τ *the Alvis-Curtis duality. Then (a)* $\mathrm{Aut}\; H(W) = \mathrm{Aut_n}\; H(W) \rtimes \langle \tau \rangle$, *(b)* $\mathrm{Aut_n} H(W) = \mathrm{Autcent}\, H(W) \rtimes \mathrm{Out}\ \Delta$, *where* $\mathrm{Out}\ \Delta$ *is the group of "outer" graph automorphisms.*
In most cases is $\mathrm{Out}\ \Delta = 1$, i.e., $H(W)$ is rigid. The main ingredients in the proof are results on the integral group ring RW, R denoting a G-adapted integral domain of characteristic 0. In particular, the isomorphism problem and the normalizer problem are valid for RW. Moreover the "$Z2$-conjecture" of Zassenhaus holds for the automorphisms.
The second part considers for $\mathbb{Z}G$, G a general finite group, the normalizer problem. Theorem: *Let*

$\mathrm{Aut}_p\,(G) = \{\sigma \in \mathrm{Aut}\ G$*; σ is modulo an inner automorphism the identity on a Sylow p-subgroup*$\}$. *Put* $\mathrm{Aut}_{\mathrm{l}}\,(G) = \cap_{p\in\pi(G)}\mathrm{Aut}_p\,(G)$. *Assume a) G has no composition factor of order 2, b) for each non-abelian composition factor X we have* $\mathrm{Aut}_{\mathrm{l}}\,(X) = \mathrm{Im}\,X$. *Then the normalizer problem has a positive answer for* $\mathbb{Z}G$.

McCulloh, Leon

Stickelberger modules in relative Galois module structure and class number formulas

Let G be a finite group and R_G its the character ring. The $\mathbb{Q}$-bilinear form $\langle\ ,\ \rangle : \mathbb{Q}R_G \times \mathbb{Q}G \to \mathbb{Q}$ defined by $\chi(s) = e^{2\pi i\langle\chi,s\rangle}$, $0 \leq \langle\chi, s\rangle < 1$, if $\deg\chi = 1$, and by $\langle\chi, s\rangle = \langle \mathrm{res}\,^G_{\langle s\rangle}\chi, s\rangle$ otherwise, leads to a Stickelberger map $\theta : \mathbb{Q}R_G \to \mathbb{Q}G$, $\theta(\chi) = \sum_{s\in G}\langle\chi, s\rangle s$ and a Stickelberger module $S_G = \theta(R_G) \cap \mathbb{Z}G$, which have applications to relative Galois module structure (GMS) and class number formulas (CNF), respectively. It is useful to note that $\theta^{-1}(\mathbb{Z}G) \cap R_G = A_G := \ker(\det : R_G \to \hat{G}^{ab})$ giving $\theta : A_G \to \mathbb{Z}G$. Regarding GMS: Let K be a number field, $\mathfrak{o} = \mathfrak{o}_K$, and $\Omega = \mathrm{Gal}(K^c/K)$. In the Fröhlich Hom-description of $Cl(\mathfrak{o}G) = \mathrm{Hom}_\Omega(R_G, J_{K^c})/\mathrm{Hom}_\Omega(R_G, K^{c^\times})\cdots$, replacing R_G throughout by A_G gives a map Rag: $Cl(\mathfrak{o}G) \to \mathrm{Hom}_\Omega(A_G, J_{K^c})/\mathrm{Hom}_\Omega(A_G, K^{c^\times})\cdots$. With an appropriate Ω-action on G, $\theta : A_G \to \mathbb{Z}G$ is an Ω-homomorphism, so we get $\theta^t : T \stackrel{\mathrm{def}}{=} \mathrm{Hom}_\Omega(\mathbb{Z}G, J_{K^c}) \to \mathrm{Hom}_\Omega(A_G, J_{K^c})$ and $\mathrm{Rag}' : Cl(\mathfrak{o}G) \to \mathrm{Hom}_\Omega(A_G, J_{K^c})/\cdots\theta^t(T)$. Then $\ker \mathrm{Rag}' \supseteq R(\mathfrak{o}G)$, the set of classes realized by rings of integers in tame Galois G-extensions of K. If G is abelian, equality holds, $R(\mathfrak{o}G)$ is a group and every class is realized by a field extension. Regarding CNF: If G is a p-group and p is odd,we conjecture: $|(\mathbb{Z}G/S_G)_{\mathrm{tor}}| = |Cl(\mathbb{Z}G)^-|\ |D(\mathbb{Z}G)^0|$ *(in the sense of R. Oliver).* This holds for G non-abelian of order p^3 and G abelian of homogeneous type $(p^n, \ldots, p^n)$. Calculations using MAGMA show it holds for all the groups of order $\leq 3^5$ in MAGMA's group library. Further calculation seems to show the same would hold for metabelian groups of order pq, p prime, $q \mid p-1$ (not necessarily prime).

Morales, Jorge

The Hasse-Witt invariant of the Killing form

We show that the Hasse-Witt invariant w_2 of the Killing form of a semisimple Lie algebra L can be expressed using the Tits invariant $\beta_L(\rho)$ associated to the weight $\rho = \frac{1}{2}\sum_{\alpha>0}\alpha$ (sum over the positive roots) and invariants associated to the symmetry group of the Dynkin diagram of L. By specifying L suitably, we recover Saltman's formula for w_2 of the trace form of a central simple algebra and Serre's formula for tale algebras.

Nebe, Gabriele

The group ring of $SL_2(p^f)$ over p-adic integers

In the first part of my talk I present methods which describe certain group rings of finite groups over p-adic integers up to Morita equivalence. The methods, in fact, apply to symmetric orders over such rings. The second part deals with the example $R[SL_2(p^f)]$, where R is the ring of integers in the unramified extension of degree f of $\mathbb{Q}_p$.

The basic algebra of the group algebra $\mathbb{F}_{p^f}SL_2(p^f)$ has been determined by Koshita using the explicit description of the projective indecomposable $\mathbb{F}_{p^f}SL_2(p^f)$-modules as tensor products of simple ones. I could lift this information to characteristic 0 and describe the endomorphism rings of the projective indecomposable $R[SL_2(p^f)]$-lattices, the homomorphism bimodules up to isomorphy, and the graduated hull of $R[SL_2(p^f)]$. This implies a purely combinatorial description of the irreducible $R[SL_2(p^f)]$-lattices, which can be formulated for $p = 2$ using the subsets of $\{1, \ldots, f\}$.

Nguyen Quang Do, Thong

Galois module structure of class formations and free pro-p-extensions

Our data: $p \neq 2$, k a number field or a p-adic local field, K/k a finite Galois extension, $G = \mathrm{Gal}(K/k)$. We want to study the $\mathbb{Z}_p[G]$-module structure of the Iwasawa module X_K = Galois group of the maximal abelian pro-p-extension of K in the local case; = Galois group of the maximal abelian pro-p-extension of K which is S-ramified in the global case (S = a finite set of primes $\supset S_p \cup S_\infty \cup \mathrm{Ram}\,(K/k)$). This module has the cohomological properties of a class formation (assuming Leopoldt's conjecture in the global case). Using Gruenberg's and Weiss' theory of envelopes, we get a unified presentation of results of Jannsen, Wingberg and myself, namely Theorem A: *The homotopy class of X_K is determined by the isomorphism class of the $\mathbb{Z}_p$-torsion tX_K and by the character $\chi_K : H^2(G, tX) \xrightarrow{\text{nat}} H^2(G, X_K) \xrightarrow{\text{inv}} \mathbb{Q}_p/\mathbb{Z}_p$.* The torsion tX_K is a deep arithmetical invariant: for example, if K is totally real, it is related, by the Main Conjecture, to $\zeta_p(s, K)$. For $S = S_p \cup S_\infty$, a field K is called *p-regular* if $tX_K = 0$. A typical example is $\mathbb{Q}(\zeta_p)$, p regular. The character χ_K seems difficult to compute. Killing it, we get: $\chi_K = 0 \Longleftrightarrow$ *X_K is homotopic to (envelope of X_K) $\oplus$ (relation module of G) $\Longleftrightarrow$ every embedding problem ($G = \mathrm{Gal}(K/k), A$), A an abelian p-group, admits an S-ramified solution.* In view of this, we want to study the problem of embedding K/k in a free pro-p-extension L/k, i.e., $\mathrm{Gal}(L/k)$ is a free pro-p-group. Let ρ_k be the maximal rank of all such free pro-p-extensions. Leopoldt's conjecture obviously implies $\rho_k \leq 1 + r_2(k)$, but examples exist for which $\rho_k < 1 + r_2$ ($r_2 \neq 0$). Theorem B: *Let L/k be a free pro-p-extension of rank ρ. Then $1 + r_2 - \rho \geq \dim_{\mathbb{F}_p} H^1(G(K/k), tX_K)_p$, for any subextension K of L.* Theorem C: *Assume $r_2 \neq 0$. Then: $\rho_k = 1 + r_2 \Longleftrightarrow k$ is p-regular.*

Pappas, George

Galois module structure of unramified covers

Let G be a finite group which acts linearly on the lattice $\mathbb{Z}^n = \bigoplus_{i=1}^n \mathbb{Z}x_i$. Let I be a homogeneous ideal in the polynomial ring $\mathbb{Z}[x_1, \ldots, x_n]$; consider the graded ring $A = \mathbb{Z}[x_1, \ldots, x_n]/I = \bigoplus_{m \geq 0} A_m$ in which the graded pieces are finitely generated $\mathbb{Z}G$-modules. Let $X = \mathrm{Proj}\,(A)$ be the associated projective scheme and $X \xrightarrow{\pi} X/G = Y$ the quotient morphism. The main question addressed in my talk was: *Assuming π is unramified, is A_m, for $m >> 0$, a free $\mathbb{Z}G$-module?* We show that, when $G = \mathbb{Z}/p$, p a prime number $> \dim(X)$, a positive answer is a consequence of Vandiver's conjecture. Conversely, we show that a positive answer to this question (for $G = \mathbb{Z}/p$, $p > \dim X$) for all X would imply Vandiver's conjecture for p, provided one can realize, using unramified covers, Galois representations of a fixed shape.

del Rio, Angel

Structure Theorems of the Unit Group of Integral Group Rings

The structure of the unit group of the group ring $U(\mathbb{Z}A)$ of a finite abelian group A is well known by the work of Higman and Bass. This can be understood as a group ring version of Dirichlet's Unit Theorem. Today's talk reports on joint work with E. Jespers which concerns the structure of $U(\mathbb{Z}G)$

for an arbitrary finite group G. Hartley and Pickel proved that if G is a non-abelian finite group then $U(\mathbb{Z}G)$ is either finite (and, in this case, G is a Hamiltonian 2-group) or contains a non-abelian free subgroup. This can be considered as a group ring version of the Tits Alternative. If $\mathfrak{G}$ is a class of groups, then a group U is said to be virtually $\mathfrak{G}$ if and only if $\mathfrak{G}$ contains a subgroup of finite index of U. In previous joint work of the authors with G. Leal, the finite groups G so that $U(\mathbb{Z}G)$ is virtually a direct product of non-abelian free groups are classified and also characterized in terms of the Wedderburn decomposition of $\mathbb{Q}G$. It is easy to generalize the Hartley and Pickel result in order to obtain, in some cases, free products of free abelian groups of rank 2 as subgroups of $U(\mathbb{Z}G)$. The main result of our work lists the finite groups G so that $U(\mathbb{Z}G)$ is virtually a direct product of free products of abelian groups; it also characterizes them in terms of the Wedderburn decomposition of the rational group algebra. One of the main tools in the proof is the use of the theory of Bass-Serre on groups acting on trees (more concretely, the Dicks-Dunwoody Almost Stability Theorem).

Schoof, René

Finite flat group schemes over local Artin rings

Grothendieck asked whether a finite locally free group scheme G is always annihilated by its rank. For commutative G this was proved by Deligne in 1969. When the basis is the spectrum of a field or, more generally, a reduced scheme, G is annihilated by its rank by a result in SGA 3. One easily reduces the problem to a local group scheme G over a local Artin ring R with algebraically closed residue field of characteristic p. We show the theorem: *Let G be a finite flat group scheme over a local Artin ring whose maximal ideal $\mathfrak{m}$ satisfies $\mathfrak{m}^p = p \cdot \mathfrak{m} = 0$. Then G is annihilated by its rank.* Here, p denotes the characteristic of the residue field $R/\mathfrak{m}$.

de Smit, Bart

Linearly equivalent actions of solvable groups

Theorem (with H.W. Lenstra): *Let $n \in \mathbb{Z}_{\geq 1}$. The following are equivalent: (1) There exist solvable number fields of degree n which are not isomorphic, but which do have the same Zeta-function. (2) There exists a solvable group G with two non-conjugate subgroups of index n which induce the same permutation character of G. (3) There exist prime numbers p, q, r with $pqr \mid n$ and $p \mid q(q-1)$.* Using results of Feit, Guralnick and Wales we can determine for all $n \leq 2000$ whether condition (2) holds without the condition that G be solvable. For $n = 2001$ this is not known.

Snaith, Victor

Does the motivic complex of weight two have an Euler Characteristic?

Let G be a finite group and M, N be $\mathbb{Z}[G]$-modules. Each $\alpha \in \mathrm{Ext}^2_{\mathbb{Z}[G]}(M, N)$ represents a 2-extension, $N \rightarrowtail A \to B \twoheadrightarrow M$. If A, B may be chosen to be finitely generated and cohomologically trivial, then an Euler characteristic, $\chi_\alpha = [A] - [B] \in K_0(\mathbb{Z}[G])$, is defined. There are many naturally occurring examples: e.g. take L/K a Galois extension of local fields with group G and the canonical classes, $\alpha_r \in \mathrm{Ext}^2_{\mathbb{Z}[G]}(K_{2r}(L), K_{2r+1}(L))$ where $K_j(L)$ is the Quillen algebraic K-group. These exist for all r if L is a 2-adic local field. The case $r = 0$ is the classical case from local class field theory. All these extensions are natural with respect to passage to subgroups and quotients. Most important are quotients: for $H \triangleleft G$ we glue $0 \to N^H \to A^H \to B^H$ to $A_H \to B_H \to M_H$ to get $N^H \rightarrowtail A^H \to B^H \twoheadrightarrow M_H$. This gives the class for L^H from that for L.

Now Lichtenbaum constructed a 2-extension called $\Gamma(2)$, $K_3^{\mathrm{ind}}(L) \to C_{1,2}(L) \to C_{2,2}(L) \to K_2(L)$ for any field. When L is local, $K_3(L) = K_3^{\mathrm{ind}}(L)$ up to $\mathbb{Q}$-vector spaces, so we may ask whether for local $L/K, \Gamma(2)$ has an Euler characteristic. In the tamely ramified, characteristic p case I calculated the Euler characteristic of my α_1 and from that the Euler characteristics of all elements of $\mathrm{Ext}^2_{\mathbb{Z}[G]}(K_2(L), K_3(L))$. Then I showed that $K_3(L) \to C_{1,2}(L)$ and $C_{2,2}(L) \to K_2(L)$ induce zero maps in cohomology, which is necessary and almost sufficient to ensure that $\Gamma(2)$ has an Euler characteristic. Finally, if $\Gamma(2)$ satisfied the *Galois descent* naturality above, then it would be *unique*, not merely a complex in the derived category.

Symonds, Peter

Group Actions on Polynomial Rings

Let $S = \oplus S^i = k[x_1, \ldots, x_d]$, $k = \mathbb{F}_q = \mathbb{F}_{p^n}$ a finite field, and G a group of k-linear ring automorphisms preserving the grading. Equivalently, $M = S^1$ is a faithful kG-module and $S = S^*(M)$, the symmetric powers. We want to understand S as a kG-module (joint work with D. Haraguenzian). There are various motivational reasons from topology.

S is a type of Galois module. It is almost free in an asymptotic sense, and in fact, if $L = \prod_{\substack{v \in S^1 \\ v \neq 0}} v$ then $S[1/L]B$ is a free $S^G[1/L]G$ module or rank 1. (Simular results are true for any graded, finitely generated k-algebra which is an integral domain.) For many questions it is the Sylow-p subgroup of G that matters, so the universal example is the group $U_d(\mathbb{F}_q)$ of upper triangular matrices. This has invariants $d_x = x$, $d_y = \Pi(y + \lambda x)$, $d_z = \Pi(\tau + \mu y + \lambda x)$ etc., and $S^G = k[d_x, d_y, \ldots]$. We state the following conjecture: *For d variables, so $G = U_d(\mathbb{F}_q)$, we have, as kG-modules, $S \simeq \oplus_{\Lambda \subset \{1,\ldots,d\},\, d \in \Lambda} M_\Lambda \otimes k[x_\lambda, \lambda \in \Lambda]$, where each M_Λ is a finite sum of homogeneous pieces in degree $\leq \frac{q^d-1}{q-1} - d$ and M_Λ is projective relative to* $\mathrm{Stab_G}\,(x_\lambda, \lambda \in \Lambda)$. Corollary: *Only a finite number of isomorphism classes of indecomposable summands occur.* We can prove this for $d \leq 4$.

Taylor, Martin

Stiefel-Whitney classes associated to certain arithmetic varieties

This was a report on joint work (in progress) with Ph. Cassou-Noguès and B. Erez. Let G be a finite group and $\tilde{\pi} : \tilde{X} \to Y$ be a G-cover of arithmetic varieties, which are projective and flat over $\mathbb{Z}[\frac{1}{2}]$, and suppose that the cover is tame in the sense of Grothendieck and Murre; thus the branch locus b is a union of smooth divisors with strictly normal crossings. Let G_2 denote a 2-Sylow subgroup of G and suppose that G_2 acts freely on $\tilde{X}$. Fix a further subgroup H of G; set $X = \tilde{X}/H$ and let $\pi : X \to Y$ be the induced map. Suppose that $\tilde{X}, X$ and Y are all regular, and let $\mathfrak{D}_{X/Y}$ denote the ramification divisor associated to X/Y. Since the ramification is odd we can form $\mathfrak{D}_{X/Y}^{1/2}$, a divisor on X, and consider the orthogonal bundle $E = (\pi_* \mathfrak{D}^{-1/2}, \mathrm{Tr}_{X/Y})$. The principal aim of the talk was to describe formulae for the Stiefel-Whitney classes $w_n(E) \in H^n_{et}(Y, \mathbb{F}_2)$. The formulae described extend the original formulae of Serre and the more recent formulae of Esnault-Kahn-Viehweg for Dedekind schemes with tame odd ramification.

Wilson, Stephen

Results in exactness defect Grothendieck group and relative groups

We recall the theorem of Holland and Wilson: $\mathfrak{o}_N = W(N/K) = \Omega(N/K,2)$ *in* $K_0(\mathcal{C}^S)_{\mathrm{fd}}$. Here N/K is a Galois extension of algebraic number fields with group G; $\mathfrak{o}_N = \mathrm{int}\,(N)$; $W(N/K)$ is the Cassou-Noguès-Fröhlich class; $\Omega(N/K,2)$ is the second Chinburg invariant; S is the set of rational primes over which N/K has wild ramification; $\mathcal{C}^S$ is the category of finitely generated $\mathbb{Z}G$-modules which are cohomologically trivial outside S; $K_0(\mathcal{C}^S)_{\mathrm{fd}}$ is the defect Grothendieck group of $\mathcal{C}^S$ with respect to the factorisability defect. This theorem generalized or strenghtened results of some other authors, principally those of Fröhlich, Taylor and Chinburg. It implies the corollary: $\mathfrak{o}_N$ *is strongly factor equivalent to* $\mathfrak{o}_K G$. It is clear that, in applications, the factorisability defect is very closely related to factor equivalence. In fact, in a sense made precise in the talk, Theorem 2: *factorisability theories and exactness defects are* essentially *equivalent.* The connection is provided by a *universal exactness defect* $\phi_u : K_0^{\oplus}(\mathcal{E}) \to K_0^{\oplus}(\mathcal{C}, \otimes\mathbb{Q})$. Here $\mathcal{C}$ is a category of modules and $\mathcal{E}$ one of short exact sequences in $\mathcal{C}$; $\phi_u(E : M' \xrightarrow{i} M \xrightarrow{\pi} M'') = [M, (r, \pi_{\mathbb{Q}}), M' \oplus M'')$, where r is a splitting of $i_{\mathbb{Q}} : M'_{\mathbb{Q}} \to M_{\mathbb{Q}}$. The class group of $K_0(\mathcal{C}^S)_{\mathrm{fd}}$ is as good as that of $K_0^{\oplus}(\mathcal{C}^S)$. So, if we want a better classgroup we have to restrict the modules in $\mathcal{C}^S$. In fact, if we take $\mathcal{C} = \{M \in \mathcal{C}^S \mid M_p^{H(p)}$ is $G/H(p)$-cohomologically trivial, $p \in S\}$, where $H(p)$ is the group generated by the wild ramification groups over p, we have Theorem 1A: $(\mathfrak{o}_N) = \Omega(N/K,2)$ *in* $K_0(\mathcal{C})_{\mathrm{fd}}$. The first theorem and Theorem 1A can, no doubt, be *lifted* to theorems in the relative groups. Possibilities for future progress were also discussed.

Zimmermann, Alexander

A Picard group for derived categories

Based on J. Rickard's Morita theory for equivalences between derived module categories $D^b(\Lambda) \simeq D^b(\Gamma)$ is joint work with R. Rouquier where the idea of introducing Picard groups for the derived categories of bounded complexes of modules over an algebra Λ has been sketched. In fact, if Λ is an R-projective R-algebra over a commutative ring R, then Keller proved the existence of a two-sided tilting complex inducing an equivalence by tensor product provided that, as triangulated categories, $D^b(\Lambda)$ is equivalent to $D^b(\Gamma)$. Analogous to the Picard group of an R-algebra Λ we define $\mathrm{TrPic}_R\,\Lambda$ to be the group of isomorphism classes of autoequivalences of $D^b(\Lambda)$ induced by such a tensor product. We prove that the mapping $\mathrm{Pic}_{\mathrm{R}}\,\Lambda$ to $\mathrm{Aut}_R\,Z(\Lambda)$ of Fröhlich lifts to $\mathrm{TrPic}_R\,\Lambda$ and we define $\mathrm{TrPicent}\,\Lambda$ to be the kernel. Many properties of $\mathrm{Pic}_R\,\Lambda$ carry over to $\mathrm{TrPic}_R\,\Lambda$. Especially Fröhlich's localization sequence can be generalized, except that for an R-order Λ the right hand side fails to be surjective. For local R-algebras Λ, for hereditary R-orders Λ, for simple artinian R-algebras $\mathrm{TrPic}_R\,\Lambda$ is isomorphic to $\mathrm{Pic}_R\,\Lambda \times C_\infty$ as long as Λ is indecomposable. Nevertheless, for Brauer tree algebras new phenomenons arise. One gets a morphism of the braid group on p strings to $\mathrm{TrPic}_{\mathbb{Z}_p} B_0(\mathbb{Z}_p[S_p])$ where $\mathbb{Z}_p$ is the ring of p-adic integers, S_p the symmetric groups on p letters, B_0 the principal block and p a prime. For $p = 3$ the map is injective, the image being of order 4. The image of the standard braid generators in $\mathrm{TrPic}_{\mathbb{Z}_p} \mathbb{Z}_p[S_p]$ is infinite cyclic.

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