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Mathematical Aspects of String Theory

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Introduction

The conference was organised by David R. Morrison (Duke University, Durham) and by Werner Nahm (Physikalisches Institut der Universität Bonn).

String theory is the field of mathematical physics which has created by far the most intense interactions between mathematicians and theoretical physicists in the past years. It contains a perturbative domain which is described by two–dimensional superconformal quantum field theories and mathematically well defined. Non–perturbative string theory (M–theory) has made much progress in recent years and promises new insights, but it is not yet well understood from any point of view.

Perturbative string theories have a moduli space with many rational points (this may generalise to the non–perturbative domain). Their classification is very far from being complete, but makes steady progress. The understanding of theories with continuous parameters, most importantly of the sigma models on Calabi–Yau manifolds, is rapidly gaining in maturity, such that decisive break–throughs seem possible in the near future.

Non–perturbative string theory involves solitonic objects (branes) of higher dimension. Since strings can end on them, the study of boundary states in perturbative string theory has become important. Further interesting new results concern the breaking of supersymmetry.

A major tool in non–perturbative string theory is the study of its low mass states, which yield conventional quantum field theories. The conjectured dualities of some of them have been put into a firm mathematical context by Seiberg and Witten. One expects these dualities to lift to all states of the string theory. Moreover, they also involve physics in more than ten dimensions (membranes in eleven dimensions and F–theory).

Dualities between string theories yield deep relations between so far unconnected objects in algebraic field theory. In particular, the mirror symmetry between varieties should generalise to certain bundles on them. So far, this is best understood for varieties constructed by elliptic or K3 fibrations.

In many cases, the mathematical consequences of the physical conjectures can be checked independently. This also means that some standard objects of physics like quantum field theories gain in mathematical interest and credibility. We do not know yet if string theory is a true model of nature, but it certainly contributes a lot to bridging the gap between physics and mathematics.

Collection of Abstracts (in chronological order)

Monday:

Peter Goddard (DAMTP, Cambridge)

Axiomatic conformal field theory

A rigorous approach to conformal field theory which, as with the historical origins of string theory, states from a family of amplitudes; these amplitudes are meromorphic densities. They define a meromorphic conformal field theory (or chiral algebra), leading naturally to the definition of topological vector spaces between which the vertex operators act as continuous operators. The assumption of Möbius invariance enables the proof of the duality relation $V(\varphi,z)V(\psi,\zeta) = V(V(\varphi,z-\zeta)\psi,\zeta)$ for vertex operators. Up to this point the theory is extremely general but the key assumption of cluster decomposition is a substantial restriction which implies the uniqueness of the vacuum and the spectrum of L_0, the scaling generator, is bounded below.

Representations of the meromorphic theory (chiral algebra) can be introduced in a way analogous to the definition of the theory itself. The representation condition is equivalent to the existence of a suitable (family of) state(s) in the meromorphic theory itself. The conditions satisfied by these states lead naturally to the definition of Zhu's algebra. Zhu's theorem states that the representations of Zhu's algebra are in essence in correspondence with the representations of the meromorphic theory. The finite–dimensionality of Zhu's algebra is a key criterion in defining an amenable class of conformal field theories.

(This is joint work with Matthias Gaberdiel.)

Anne Taormina (Durham, England)

Representations of the affine Lie superalgebra $\widehat{SL(2|1)}$ and N=2 strings

The general theory of admissible representations of affine Lie algebras and superalgebras developed by Kac and Wakimoto shows the existence of a large class on noninteger level representations sharing many important features of the integrable representations (in particular, nice transformations under the modular group) when the level k is of the form $u(k+h^\vee) = k^\circ + h^\vee$, with $h^\vee$ the dual Coxeter number of the corresponding Lie (super)algebra, and k°, $u \in \mathbb{Z}\mathbb{Z}_+$.

In this talk, the affine Lie superalgebra $\widehat{sl(2|1)}$ at fractional level k of the form $k+1 = \frac{p}{u}$, $p,\ u \in \mathbb{Z}_+$, $\gcd(p,u) = 1$ is shown to play a rôle in the description of noncritical N=2 strings when the matter coupled to supergravity is minimal, i.e. is taken in an N=2 super Coulomb gas representation with central charge $c_{\text{matter}} = 3(1-2\frac{p}{u}$. Some aspects of the theory of its admissible representations are discussed in the more general context of the exceptional affine Lie superalgebra $\widehat{D(2|1;\alpha)}_k$. In particular, two field representations of $\widehat{sl(2|1)}$ at level $k = \frac{1}{u} - 1$ are discussed. The first is an analogue of the Wakimoto modules which are needed in the description of the physical spectrum of the noncritical N=2 strings. The second can be constructed out of the representations of two $\widehat{sl(2)}$ affine Lie algebras at dual levels $k = \frac{1}{u} - 1$ and $k' = u - 1$.

Ronen Plesser (Duke University, Durham)

T–duality can fail

(joint work with Paul Aspinwall)

T–duality is the statement that string theory in a spacetime of the form $M \times S^1$, where S^1 has radius R, is invariant under the exchange $R \to \frac{1}{R}$. This important, intrinsically stringy symmetry has been used to learn much about the theory. In this work we showed that in some circumstances nonperturbative effects destroy the symmetry.

To all orders in the perturbation expansion, string theory is determined by CFT. In CFT the duality follows in a direct way from the presence of an enhanced $SU(2)$ gauge symmetry when $R = 1$. In compactification on T^2 the $\mathbb{Z}_2^2$ symmetry is enhanced to $SL(2,\mathbb{Z})$. In compactification of the heterotic string on $K3 \times T^2$, we show that T–duality is "broken" in the following sense: the classical monodromies of "flat" coordinates are modified when the heterotic coupling is non–zero. The locus corresponding to $R = 1$ splits much as in Seiberg–Witten theory into two loci about which we have infinite monodromy. This makes it impossible to define a "size" variable in a consistent manner for $R \approx 1$ and smaller, so that the statement of T–duality loses its meaning.

Computations including nonperturbative effects are possible by string–string duality which relates the entire question to a problem in tree–level type IIA strings on a Calabi–Yau threefold.

Dieter Lüst (Humboldt–Universität, Berlin)

Gauge theories from branes

The recent progress in the understanding of non–perturbative effects in string theory is largely linked to the discovery of several kinds of duality symmetries (S–duality, T–duality) and of D–branes as the solitons in type IIA, B superstrings. D–branes give rise to non-abelian gauge interactions where the nonabelian gauge bosons correspond to open strings which can move on the world volume at the D–branes. Non–trivial gauge models with matter fields and reduced number of supersymmetries (N=2 and N=1 supersymmetry in four dimensions) are obtained by placing the D3 branes on a transversal singularity of ADE type (non–compact Calabi–Yau space). In particular, we discussed hyper quotient singularities (generalised conifold singularities) in this talk. In a T–dual picture the ADE singularities can be equivalently described by a net of NS 5–branes with the D–branes suspended in between. This picture is quite convenient, since the configuration of NS and D–branes can be embedded into 11–dimensional M–theory. In this way one can obtain non–perturbative informations about the corresponding gauge theories, and the conjecture is that (part of) the non–perturbative moduli space of the gauge theories is described by the moduli space of the brane embedded in M–theory. In particular, for N=2 supersymmetric models the Seiberg–Witten curve emerges in a geometric way as the shape of the embedded M–theory branes. We show that for N=1 supersymmetric gauge theories a supersymmetric 3–cycle (special Lagrangian submanifold in $\mathbb{C}^3$) plays the analogous role of the Seiberg–Witten curve and encodes many non–perturbative properties of N=1 gauge theories.

Terry Gannon (University of Alberta, Edmonton)

Fusion rings and their symmetries

Arguably, there are few things the world needs less than another formal treatment of fusion rings. My main justification is the surprising lack of cross–fertilisation between the different areas in which this type of algebraic structure arises. In my talk I tried to sketch a general theory, give a few examples of where fusion rings arise, and point out a few directions for future study which seem natural.

I also alluded to the question of automorphisms of the fusion rings corresponding to affine algebras, or equivalently symmetries of their fusion coefficients. There are the permutation π of the level k highest weights $\mathrm{P}_+^{\mathrm{k}}(\mathrm{g})$ which obey $\mathrm{N}_{\pi\lambda,\pi\mu}^{\pi\nu} = \mathrm{N}_{\lambda,\mu}^{\nu}\ \forall \lambda,\mu,\nu \in \mathrm{P}_+^{\mathrm{k}}(\mathrm{g})$. This is equivalent to finding all pairs (π,π') of permutations which obey $\mathrm{S}_{\lambda,\mu} = \mathrm{S}_{\pi\lambda,\pi\mu}\ \forall \lambda,\mu$, where S is the (Kac–Peterson) matrix diagonalising the fusions. These symmetries have been classified for all affine algebras, and most of them correspond to symmetries of the extended Coxeter–Dynkin diagram of g.

TUESDAY:

Antonella Grassi (University of Pennsylvania, Philadelphia)

On the topological Euler characteristic of CY 3–folds and the anomaly formula

(based on work with D. Morrison)

We consider an elliptic Calabi–Yau 3fold X, with sections. There is a natural way to associate a group G to X (if dim X = 2, via Kodaira's classification, one can assign to each singular fiber the algebraic group corresponding to the Dynkin diagram of the configuration of the exceptional divisors). This correspondence has no intrinsic explanation (at least not yet) within algebraic geometry; it is instead natural from the point of view of physics (F–theory/heterotic duality and gauge theory). In particular, an "anomaly formula" must vanish.

We show that, under certain "general" assumptions, there is a simple mathematical formulation of the anomaly formula. The analysis of the anomaly formula (via the topological Euler characteristic of CY 3folds) implies a natural correspondence between representations of Lie groups, 3fold singularities and their resolutions.

Our results are in agreement with the predictions in the physics literature but we cover also other cases.

Bobby Samir Acharya (Queen Mary College, London)

M–theory, Joyce orbifolds and super Yang–Mills

M–theory compactified on a 7–manifold with G_2–holonomy J (henceforth referred to as a Joyce manifold) gives a model for physical theories in four dimensional Minkowski space-time which at low energies (and at a smooth point in the moduli space $\mathcal{M}(\mathrm{J})$ of G_2 holonomy metrics) can be described as N=1 supergravity theories. The matter content may be summarised as $\mathrm{b}_2(\mathrm{J})$ U(1) vector multiplets and $\mathrm{b}_3(\mathrm{J})$ chiral multiplets ($\mathrm{b}_i(\mathrm{J})$ are the Betty numbers of J). Such theories are relatively uninteresting physically due to the fact that the gauge symmetry is abelian: $\mathrm{U}(1)^{\mathrm{b}_2(\mathrm{J})}$.

By constructing a "local model" we proposed that interesting physics arises when J becomes singular as one moves around in the moduli space $\mathcal{M}(J)$.
The "local model" is topologically a non–compact Joyce orbifold $O :\equiv \frac{\mathbb{C}^2\times T^3}{G}$, with G a finite subgroup of G_2.
The singularities of O are of the form $\frac{\{\underline{0}\}\times T^3}{K}$, where $\{\underline{0}\} \in \mathbb{C}^2$ is the singularity in $\frac{\mathbb{C}^2}{\Gamma(ADE)}$ with $\Gamma(ADE)$ a finite subgroup of SU(2) and $K \cong \frac{G}{\Gamma(ADE)}$ acts freely on T^3.
This model gives rise at low energies to super Yang–Mills theory with ADE gauge group, N=1 supersymmetry and no matter multiplets.
We calculated the superpotential and argued that it is generated by fractional membrane instantons which "wrap" the singularities of O. This superpotential agrees precisely with that which may be calculated from the field theory. This is the main result.
It shows that

(i) M–theory "understands" quantum Yang–Mills theories which are strongly interacting in the infrared (low energy) regime. The real world is described by just such a Yang–Mills theory (QCD); we just do not understand everything we would like to about QCD.

(ii) The dynamically generated superpotential is generated by "instantonic" objects of fractional charge which are difficult to see in field theory.

The result we described is intrinsically quantum in nature since it is subtly related to anomalies (or lack of them) in M–theory. We hope it will have some bearing on understanding nature.

Victor V. Batyrev (Universität Tübingen)

Stringy Hodge numbers

Let X be a quasi–projective normal algebraic variety with at worst log–terminal singularities and $\varrho : Y \to X$ a log–resolution of singularities such that the support of the exceptional locus of ϱ is a normal crossing divisor $D = D_1 \cup \ldots \cup D_r$ and

$$K_Y = \varrho^* K_X + \sum_{i=1}^{r} a_i D_i \quad (\text{all } a_i > -1).$$

We introduce a rational function $E_{st}(X; u, v) \in \mathbb{Q}[u, v]$ by the following formula:

$$E_{st}(X; u, v) = \sum_{J\subset\{1,\ldots,r\}} E(D_J^0; u, v) \prod_{j\in J} \frac{1 - uv}{1 - (uv)^{a_j+1}}$$

where

$$\begin{aligned} E(Z; u, v) &= \sum_{p,q} e^{p,q}(Z) u^p v^q \\ e^{p,q}(Z) &= \sum_{k\geq 0} (-1)^k h^{p,q}(H_c^k(Z, \mathbb{C})) \quad \text{and} \\ D_J^0 &= \{y \in Y : y \in D_j \Leftrightarrow j \in J\}. \end{aligned}$$

The important fact about $E_{st}(X;u,v)$ is the following:

Theorem: $E_{st}(X;u,v)$ does not depend on the choice of the resolution of singularities $\varrho : Y \to X$.

1. The function $E_{st}(X;u,v)$ allows to define stringy Hodge numbers $h^{p,q}_{st}(X)$ of singular projective algebraic varieties X.

Def.: Assume that X is a projective algebraic variety with at worst Gorenstein canonical singularities and $E_{st}(X;u,v)$ is a polynomial. Then we define $h^{p,q}_{st}(X)$ by the formula

$$E_{st}(X;u,v) = \sum_{p,q} (-1)^{p+q} h^{p,q}_{st}(X) u^p v^q.$$

2. The function $E_{st}(X;u,v)$ allows to formulate the topological mirror duality test even in the case, when $E_{st}(X;u,v)$ is not polynomial. For a mirror pair $(X,\hat{X})$ one expects:

$$E_{st}(X;u,v) = (-u)^d E_{st}(\hat{X};u^{-1},v) \quad d = \dim X = \dim \hat{X}.$$

3. The specialisation $u = v = 1$ gives the following formula for the stringy Euler number of X:

$$e_{st} = \sum_{J \subset \{1,\dots,r\}} e(D^0_y) \prod_{j \in J} \frac{1}{a_j + 1},$$

where $e(D^0_J)$ is the usual Euler number of D^0_J. In the case, when $X = V/G$ where G is a finite group acting on V and the covering $V \to X$ is umramified in codimension 1 one has:

Theorem:

$$e_{st} := \frac{1}{|G|} \sum_{\substack{g,h \\ [g,h]=1}} e(V^g \cap V^h), \quad \text{where } V^g = \{x \in V : gx = x\}.$$

This formula for $e_{st}(X)$ has appeared in the paper "Strings on orbifolds" by Dixon, Harvey, Vafa and Witten in 1989.

Viacheslav V. Nikulin (Steklov Mathematical Institute)

Algebraic surfaces with finite polyhedral Kähler cone

We consider non–singular projective algebraic surfaces X over $\mathbb{C}$ with finite polyhedral Kähler cone ($X \in$ fpkc).

Classification of 3–folds (e.g. Calabi–Yau) with fpkc would be of great interest, and one can consider the 2–dimensional case as a model. We expect that surfaces $X \in$ fpkc have very interesting quantum cohomology related with automorphic forms (e.g. Borcherds type automorphic products).

A surface $X \in$ fpkc has natural invariants $\varrho = \operatorname{rk} NS(X)$, $\delta_E = \max_{D \in Exc(X)}\{-D^2\}$, $p_E = \max_{D \in Exc(X)} p_a(D)$. Here Exc(X) is the set of all exceptional curves on X. We prove

Theorem:

$X \in$ fpkc has an ample effective divisor h such that $h^2 \leq N(\varrho, \delta_R)$, and has a very ample divisor h′ such that $h'^2 \leq N(\varrho, \delta_E, \varrho_E)$. Here we suppose that $\varrho \leq 3$. In this sense the set $X \in$ fpkc is bounded for fixed $\varrho \leq 3$, δ_E, p_E.
We give examples which show that the theorem is not valid if one does not fix one of the invariants ϱ, δ_E, p_E.
Because of the theorem, it is interesting to classify $X \in$ fpkc for small ϱ, δ_E, p_E.
For $\delta_E = 1$, $p_E = 0$ we get non–singular Del Pezzo surfaces. We give a classification of $X \in$ fpkc with $\delta_E = 2$, $p_E = 0$. It contains K3, Enriques, minimal resolutions of Del Pezzo with Du Vál singularities, rational surfaces and nef(–K).

Ralph Blumenhagen (Humboldt–Universität, Berlin)
Non–tachyonic orientifolds of type 0B in various dimensions
Type 0B string theory is a non–supersymmetric string theory in ten space–time dimensions. It is plagued with an inconsistency namely the appearance of a tachyon in the spectrum. It was shown, that one can define an orbifold of this model, in which the tachyonic mode is projected out. One can cancel all RR–tadpoles by introducing two kinds of D9–branes in the background. However, a dilaton tadpole survives which can be cured by the Fischler–Susskind mechanism. Equipped with this ten–dimensional string theory, compactifications to six and four dimensions were investigated. After cancelling all RR–tadpoles by introducing suitable D–branes, we arrived at non–supersymmetric anomaly free low energy effective field theories. Phenomenologically, these models have some interesting features, as a purely bosonic gauge group and fermionic matter in non–singlet representations of the gauge group.

WEDNESDAY:

Michael Flohr (King's College, London)
Logarithmic conformal field theory on Riemann surfaces and applications to strings, branes and Seiberg–Witten models
Logarithmic conformal field theory (LCFT) is a generalisation of conformal field theory where correlation functions may exhibit logarithmic divergencies.
A prominent example is the simple b–c ghost system of two anticommuting fields of spin 1 and 0, which has central charge $c = -2$. Following old ideas of Knizhnik, b–c systems on Riemann surfaces Σ can be considered as b–c systems on $\mathbb{P}^1$, where Σ is represented as a branched covering of $\mathbb{P}^1$ and the effect of branch points is simulated by appropriate vertex operators $V_q(z) = :\exp(iq\varphi(z)):$ (φ being a free field) which are added to the spectrum of the CFT. We concentrate on the simplest case, Σ being hyperelliptic. It is shown that including the branch point vertex operator $V_{\frac{1}{2}}$ necessarily leads to a LCFT. This can already be seen in the case $\Sigma =$ torus, represented as a double covering of $\mathbb{P}^1$ which 4 branch points. The b–c system represents 1– and 0–differentials. Periods of such differentials are expressed as correlation functions, e.g. $\pi_\gamma = \oint_\gamma \omega = \langle\langle V_{\frac{1}{2}}(e_1) V_{\frac{1}{2}}(e_2) V_{\frac{1}{2}}(e_3) V_{\frac{1}{2}}(e_4)\rangle\rangle_{(\gamma)}$,

where $e_1, \ldots, e_4$ are the branch points of the torus and $\langle\langle \ldots \rangle\rangle$ means that the correlation function is divided by its free part. The curve γ defines the contour of the screening charge integration, yielding a basis of conformal blocks. In the above case, two linearly independent blocks exist, one of them exhibiting a logarithmic singularity. Such logarithms occur precisely, when screening charge integration contours get pinched due to insertions of operator product expansions. As a consequence, the theory must be enlarged by logarithmic partners of vertex operators, $\Lambda_q(z) = \frac{\partial}{\partial z} V_q(z)$. In the case of the $c = -2$ b–c system, one still can construct a consistent theory, but there exist many more consistent LCFTs.
One particular application is Seiberg–Witten theory of N=2 supersymmetric Yang–Mills theory. The scalar modes are given by the periods of a certain meromorphic 1–form λ_{SW} associated to a Riemann surface Σ which encodes the moduli space of the Seiberg–Witten theory. Representing Σ and this 1–form as above with vertex operators allows to compute these periods as correlation functions, which can be expressed in terms of Lauricella F_D–functions. Asymptotic regions of the moduli space, where certain BPS states become light, are particularly simple, since they correspond to branch points flowing together (shrinking cycles), i.e. insertions of OPEs.
Other applications include the computation of periods of Calabi–Yau string compactifications.
A physical interpretation is yet incomplete, but might be related to a description of Seiberg–Witten theory in terms of intersecting branes, the latter forming an interacting "gas" described by our LCFT.

Phillipe Ruelle (Université Louvain–la–Neuve)
Symmetries in boundary conformal field theories
Boundary conditions in a boundary conformal field theory can be examined in the light of an internal symmetry, of any. This talk has focussed more specifically on the Virasoro minimal models on a cylinder (or an annulus). A minimal model is specified, from its modular invariant partition function, by a pair of simple Lie algebras (A,G) where G is of ADE type, and has a symmetry group equal to the group of automorphisms of the Dynkin diagram of G (with one exception). That symmetry can be thought of as a generalisation of the spin flips of the Ising model.
The torus twisted partition functions can be explicitly computed and give information about the fields that arise in the different monodromy sectors. Similarly on the cylinder, the partition functions can be determined once the boundary conditions are known. For the minimal models, all this can be made very explicit. It turns out that the most essential features are encoded in the product Dynkin graph $A \times G$.
The boundary conditions are indeed labelled by the nodes of $A \times G$, and using the torus data, one can see that the symmetry group acts on them by automorphisms of the Dynkin graph of G. Restricting to the invariant boundary conditions, one can then compute the twisted cylinder partition function and determine the charges of the various fields. In turn, these provide non–trivial selection rules on the reflection coefficients and on the boundary operator product coefficients.

Christoph Schweigert (ETH Hönggerberg, Zürich)

D–brane conformal field theory and traces on bundles of conformal blocks

It is explained that conformally invariant boundary conditions can be associated to the irreducible representations of a finite–dimensional semi simple algebra, the classifying algebra.

The classifying algebra generalises the fusion algebra: its structure constants are the traces of the action of certain automorphisms on spaces of conformal blocks (Such traces also appear in the Verlinde formula for non–simply connected groups.). Various conjectures for such formulae have been discussed.

THURSDAY:

Katrin Wendland (Physikalisches Institut der Universität Bonn)

Aspects of conformal field theory on K3

(joint work with Werner Nahm)

Any supersymmetric conformal field theory with central charge c = 6 and N = (4, 4) supersymmetry corresponds to string propagation on a K3 surface or a four dimensional torus. Given that the moduli space of Einstein metrics on a K3 surface is understood to quite some extent, a precise comprehension of the correspondence between conformal field theory and geometrical data might therefore provide a key to the classification problem of N = (4, 4), c = 6 superconformal field theories on K3.

Here – as a first step – we concentrate on "point by point" matches. One example of this is the K3 obtained as blown up $\mathbb{Z}_4$–orbifold of a four–torus, allowing a very detailed matching of conformal field theory and geometrical data. This model was conjectured to coincide with Gepner's $(2)^4$ and with the $\mathbb{Z}_2$–orbifold of a torus with SO(8)–lattice but vanishing B–field by Eguchi/Ooguri/Taormina/Yang, who showed that their partition functions agree. By studying the (1,0) current algebra and deformations of the theory, we give further evidence for the first correspondence but rule out the second. Adding simple currents to the $(2)^4$ and devoting the appropriate B–field for the $\mathbb{Z}_2$–orbifold we construct models with enhanced symmetry which we conjecture to agree. Partition function, current algebra and state by state matching support this strongly, as well as an agreement with the $SU(2)_1^4$ torus model. Here we consider the pure conformal field theory picture (leaving aside extended degrees of freedom), in which we seem to have found a crossing point of torus and K3 moduli spaces.

A next step to be taken would be the precise localisation of our examples within the "moduli space of K3 surfaces with B–field".

Bruce Hunt (MPI für Mathematik in den Naturwissenschaften, Leipzig)

CY–fibered Calabi–Yau manifolds

A specially interesting class of Calabi–Yau varieties are given by those which possess a fibration, X → B. In this case the fiber must also be Calabi–Yau and there are strong restrictions on the base B. In general the complex modulus of the fiber does change, but

there are cases where one gets fibrations of constant modulus. One construction of these was discussed in this talk.
Let V_1, V_2 be the weighted hypersurfaces defined by

$$\begin{aligned} V_1 &= \{x_0^2 + p(x_1, \dots, x_n) = 0\} \subset \mathbb{P}_{(w_0,\dots,w_n)} =: \mathbb{P}_{\underline{w}} \\ V_2 &= \{y_0^2 + q(y_1, \dots, y_m) = 0\} \subset \mathbb{P}_{(v_0,\dots,v_m)} =: \mathbb{P}_{\underline{v}} \end{aligned}$$

$X = \{p(t_1, \dots, t_n) - q(z_1, \dots, z_m) = 0\} \subset \mathbb{P}_{(v_0w_1,\dots,v_0w_n,w_0v_1,\dots,w_0v_m)} = \mathbb{P}_{\underline{v},\underline{w}}$

Theorem:

The map

$$\begin{aligned} \Phi : \mathbb{P}_{\underline{w}} \times \mathbb{P}_{\underline{v}} &\to \mathbb{P}_{\underline{v},\underline{w}} \\ ((x_0, \dots, x_n), (y_1, \dots, y_m)) &\mapsto (y_0^{\frac{w_1}{w_0}} x_1, \dots, y_0^{\frac{w_1}{w_0}} x_n, x_0^{\frac{v_1}{v_0}} y_1, \dots, x_0^{\frac{v_1}{v_0}} y_m) \end{aligned}$$

restricts to $V_1 \times V_2$ to a generically finite to one map. If $\gcd(v_0, w_0, \ell) = 1$, then $X \simeq V_1 \times V_2/\mu_\ell$ ($\mu = \ell^{\underline{\text{th}}}$ roots of unity) is the quotient of $V_1 \times V_2$ by a group of order ℓ. If $w_0 > 1$, then if X has a Calabi–Yau resolution $\tilde{X}$, $\tilde{X}$ has a fibration onto a desingularisation B of $V_1/\mu_\ell \Leftrightarrow V_2$ has a Calabi–Yau resolution.
An interesting consequence of this is in the case of usual projective space, i,e., unit weights.

Corollary:

Let $X = \{x_1^d + \ldots + x_n^d = 0\} \subset \mathbb{P}^{n-1}$ be a Fermat hypersurface of degree d, $\{n_1, \dots, n_\lambda\}$ a partition of n with $n_i \geq 2$ $\forall$i, x_{ik}, $i = 1, \dots, \lambda$, $k = 1, \dots, n_i$ corresponding coordinates. Then X is birational to the quotient of $\Pi(\lambda$ Fermat hypersurfaces)

$$X_1 \times \ldots \times X_\lambda, \quad X_i = \{x_{i0}^d + \ldots + x_{in_\lambda}^d = 0\}$$

by a group $\mathbb{Z}/d\mathbb{Z}$ acting on the x_{i0}, $i = 1, \dots, \lambda$.

Claudio Bartocci (Università di Genova)
Mirror symmetry for K3 surfaces In his talk delivered at the '94 International Congress of Mathematicians M. Kontsevich conjectured that mirror symmetry could be interpreted as an equivalence of triangulated categories over mirror pairs of Calabi–Yau manifolds X and $\hat{X}$. This conjecture fits convincingly into the setting of Strominger–Yau–Zastow interpretation of mirror symmetry. According to their approach – very roughly speaking, both X and $\hat{X}$ are Calabi–Yau manifolds admitting a fibration whose fibers are special Lagrangian tori; the mirror dual should be regarded as the moduli space of deformations of a special Lagrangian torus equipped with a flat U(1)–bundle. In order to state Kontsevich's conjecture, we have to define a suitable modification of the so–called Fukarya category on X, that we will denote by SF(X). Objects of SF(X) are pairs $U = (M, \mathcal{E})$, where M is a special Lagrangian submanifold of X and $\mathcal{E}$ is a flat vector bundle on M. Morphisms turn out to be rather complicated. Since two special Lagrangian cycles intersect (generically) in a finite number of points, we set $\mathrm{Hom}(U_1, U_2) = \bigoplus_{p \in M_1 \cap M_2} \mathrm{Hom}_{\mathbb{C}}(\mathcal{E}_1|_p, \mathcal{E}_2|_p)$.

The vector space $\mathrm{Hom}(\mathrm{U}_1, \mathrm{U}_2)$ has a $\mathbb{Z}$–graduation induced by the Maslov index defined at each point p. Moreover an A_{00}–category structure is defined, by introducing suitably defined linear maps:

$$\mathrm{m_k} : \mathrm{Hom}(\mathrm{U}_1, \mathrm{U}_2) \otimes \ldots \otimes \mathrm{Hom}(\mathrm{U_k}, \mathrm{U_{k+1}}) \longrightarrow \mathrm{Hom}(\mathrm{U}_1, \mathrm{U_{k+1}}).$$

Kontsevich's conjecture claims the existence of an equivalence of triangulated categories between the derived category of SF(X) and D Coh($\hat{\mathrm{X}}$) (a, more precisely, as recently suggested by Polishchuk, a refined version of this category endowed of an A_{00}–category structure). The conjecture was proven by Polishchuk and Zastow in the case of elliptic curves.
In my talk I show that Kontsevich's conjecture holds for K3 surfaces. We consider an elliptic, M–polarised K3, X, equipped with a fibration whose fibers are special Lagrangian tori. It is then possible – generically on the moduli space of M–polarised K3 surfaces – to fix a hyper–Kähler metric on X such that the K3 surfaced $\hat{\mathrm{X}}$ obtained by a $\frac{\pi}{2}$ rotation of the complex structure of X is $\check{\mathrm{M}}$–polarised (we are using Nikulin–Dolgachev's terminology). Thus, X and $\hat{\mathrm{X}}$ are a mirror pair. The category SF(X) is essentially trivial due to the following facts: 1. special Lagrangian cycles on X always intersect transversally, since they are algebraic cycles on $\hat{\mathrm{X}}$; 2. the Maslov index is trivial; 3. generically, $\mathrm{m_k} = 0$ for all $k \neq 2$, and m_2 is then an associative composition. It should be noticed that the triviality of SF(X) is related to the triviality of the quantum cohomology of K3 surfaces. Thus, it is possible to identify SF(X) with an additive subcategory C($\hat{\mathrm{X}}$) of the category of coherent $\mathcal{O}_{\hat{\mathrm{X}}}$–modules: C($\hat{\mathrm{X}}$) is the category of coherent sheaves supported on a divisor of $\hat{\mathrm{X}}$.
In order to obtain a mirror map satisfying physical requirements, we have to compose the equivalence $\mathrm{D}(\mathrm{SF}(\mathrm{X})) \xrightarrow{\sim} \mathrm{D}(\mathrm{C}(\hat{\mathrm{X}}))$ with the equivalence of the derived category given by the relative Fourier–Mukai transform $\mathrm{S} : \mathrm{D}(\hat{\mathrm{X}}) \to \mathrm{D}(\overline{\mathrm{Jac}(\hat{\mathrm{X}})})$, where $\overline{\mathrm{Jac}(\hat{\mathrm{X}})}$ is the compactified Jacobian of the elliptically fibered K3 $\hat{\mathrm{X}}$.

Matthias R. Gaberdiel (DAMTP, Cambridge)
Non–BPS Dirichlet branes
It is explained how Dirichlet branes can be constructed and analysed without reference to their space–time supersymmetry properties. In this approach they are described by coherent boundary states of the closed string theory that satisfy a number of conditions (i) the boundary states are physical states, i.e. are GSO–invariant and invariant under the appropriate orbifold (or orientifold) projections, and (ii) the open strings that are induced by the boundary state have consistent interactions with the original closed string theory. For the case of the familiar Type II theories these conditions reproduce the known D–brane spectrum, but these techniques can also be used for theories that break supersymmetry completely (such as Type 0B or its orientifold) or for Dirichlet branes that break supersymmetry completely in an otherwise supersymmetric theory. Non–BPS Dirichlet branes of the latter kind play an important role in understanding string–string duality beyond the

BPS spectrum, in particular for the string theories

$$\begin{array}{ccc} \text{IIB } \mathrm{T}^4/\mathrm{I}_4(-1)^{\mathrm{F_L}} & \xrightarrow{\quad\mathrm{T}\quad} & \text{IIA } \mathrm{T}^4/\mathrm{I}_4 \ \{= \text{K3 at orbifold point}\} \\ \Big| \mathrm{S} & & \Big| \mathrm{S} \\ \mathrm{IIBT}^4/\Omega\mathrm{I}_4 & & \mathrm{HeteroticT}^4 \end{array}$$

where perturbative stable non–BPS states of the theories in the lower line correspond to non–BPS D–branes in the theories in the top line.
(joint work with Oren Bergman)

Tony Pantev (University of Pennsylvania, Philadelphia)
Mirror symmetry and vector bundles
We consider several features of the quantum mirror symmetry duality as manifested in the moduli spaces of Euclidean D–branes of type II string compactification.
For the type I sector of the compactification the moduli space of the Euclidean D–branes is identified with a component of the moduli space of semistable sheaves on a Calabi–Yau 3–fold X. We use secondary Abel–Jacobi maps and Fourier–Mukai transforms along the fibers of elliptic fibrations to construct special coordinates in this moduli space.
Concretely if $\mathrm{S} \stackrel{\pi}{\to} \mathrm{B}$ is an elliptic K3 surface with an involution $\sigma : \mathrm{S} \to \mathrm{S}$ acting along the fibers of π and such that σ acts as -1 on the holomorphic (2,0) form on S consider the Borcea–Voisin CY 3–fold X obtained from resolving $(\mathrm{S} \times \mathrm{E})/(\sigma, \pm 1)$ for an elliptic curve E.
If $\mathrm{p} : \mathrm{X} \to \mathrm{Q}$ is the elliptic fibration on X induced by $\pi : \mathrm{S} \to \mathrm{B}$ and if $\mathcal{N}$ is a component of the moduli space of rank n sheaves on X that are of degree 0 along the fibers of p, then one builds natural coordinates on $\mathcal{N}$ in two steps. First identify $\mathcal{N}$ with a moduli space of spectral data $(\mathrm{C}, \mathcal{L})$ where $\mathrm{C} \subset \mathrm{X}$ is a surface covering Q n:1 and $\mathcal{L} \to \mathrm{C}$ is a line bundle. This is achieved via a standard relative Fourier–Mukai transform along the fibers of $\mathrm{p} : \mathrm{X} \to \mathrm{Q}$. The next step is to consider Green's secondary AJ map for $\mathrm{C} \subset \mathrm{X}$: The extension class $\mathrm{e}_{\mathrm{C}\subset\mathrm{X}}$ of the extension of MHS

$$0 \longrightarrow \frac{\mathrm{H}^2(\mathrm{C})}{\mathrm{H}^2(\mathrm{X})} \longrightarrow \mathrm{H}^3(\mathrm{X}, \mathrm{C}) \longrightarrow \mathrm{Ker}(\mathrm{H}^3(\mathrm{X}) \to \mathrm{H}^3(\mathrm{C})) \longrightarrow 0$$

can be interpreted as a homomorphism

$$\mathrm{e}_{\mathrm{C}\subset\mathrm{X}} : \mathrm{Ker}\left(\mathrm{H}^3(\mathrm{X}) \to \mathrm{H}^3(\mathrm{C})\right)_{\mathbb{Z}} \longrightarrow \frac{\mathrm{H}^2(\mathrm{C})_{\mathbb{Z}}}{\mathrm{H}^2(\mathrm{X})_{\mathbb{Z}}} \otimes \mathrm{S}^1.$$

Evaluating on the holomorphic 3–form of X and projecting on $\mathrm{H}^{2,0}(\mathrm{C})$ we obtain a map

$$(\text{moduli of C's}) \longrightarrow \mathrm{H}^0(\Omega^2_{\mathrm{C}}) \Big/ \left(\frac{\mathrm{H}^2(\mathrm{C})_{\mathbb{Z}}}{\mathrm{H}^2(\mathrm{X})_{\mathbb{Z}}}\right)$$

which can serve as coordinates.
This map matches Vafa's coordinates in the A–model and provides a basis for comparing the appropriate correlators.

FRIDAY:

Antoine Coste (Université Paris–Sud)
Questions about $SL_2(\mathbb{Z}/N\mathbb{Z})$ representations occurring in RCFT's
(In Memoriam C. Itzykson) $SL_2(\mathbb{Z})$ representations carried by characters of rational conformal field theories have many interesting properties:
We have checked W. Nahm's "working hypothesis" formulated at the birth of this field: For $sl(N)_k$, $\Gamma(m)$ is in the kernel if m is the order of T. A proof uses classical $\mathcal{O}$ series results.
Another point of view is that there are additional relations satisfied by S, T together with $T^m = 1$ in order to make the quotient a finite quotient of $SL_2(\mathbb{Z}/m\mathbb{Z})$.
One can explicitly enumerate the $q^3(1 - \frac{1}{p})$ elements $SL_2(\mathbb{Z}/m\mathbb{Z})$ when q is p primary. However, presentation of $SL_2(\mathbb{Z}/m\mathbb{Z})$ by generators including Cartan–torus elements and relations still fascinates me.
Galois properties of these representations are also striking: $\sigma(S_{ab}) = \varepsilon_\sigma(a)S_{\sigma(a)\,b)} = \cdots$.
Let me point to attention of reader that Altschuler, Ruelle and Thiran pointing out its cocycle nature, recently simplified the study of $\hat{\mathcal{G}}$ sign in terms of sl(2) factors.
What properties characterise these representations?
Another such property is that Pasquier Verlinde formula gives structure constants of a based $\mathbb{Z}_{sing}$.
Factoring it by $\mathbb{Q}$ we get an algebra and it has been explained that it contains divisions of 0 because the absence of nilpotents since Kawai makes it $\mathbb{Q}$ isomorphic to a product of number fields.
For $\widehat{sl(2)}_{n-2}$ this corresponds to factorisation of Tchebicheff polynomials into irreducibles.
We know in general how to build up idempotents.
It is possible to formulate various axiomatics for these mathematical objects in order to include various situations such as centers of group algebras, representation rings, algebraic integer rings.
So many and so various relationships with smart mathematics.

Andreas Wißkirchen (Physikalisches Institut der Universität Bonn)
Landau–Ginzburg vacua of string, M– and F–theory at c=12
(joint work with Monika Lynker and Rolf Schimmrigk)
Theories in more than ten dimensions play an important role in understanding nonperturbative aspects of string theory. Four dimensional compactifications of such theories can be constructed via Calabi–Yau (CY) fourfolds. These models will be analysed particularly efficiently in the Landau–Ginzburg (LG) phase of the linear $\sigma-$model, when available. We focus on those $\sigma-$models which have both a LG phase and a geometric CY phase described by a hypersurface in weighted projective five–space. Assuming the hypersurface to

be transverse, one can construct an algorithm to find all possible weight systems. Some of the pertinent properties of these 1,100,055 models, such as cohomology, mirror symmetry and fibration structure, are presented. Using this data we plan to construct many dual pairs

$$\mathrm{F}(\mathrm{CY}_4) \longleftrightarrow \mathrm{Het}(\mathrm{V} \to \mathrm{CY}_3)$$

where V is a vector bundle of rank ≥ 3 (or a sheaf) over the CY threefold.

Dražen Adamović (University of Zagreb)
Representation theory of some irrational vertex algebras In this talk we consider vertex algebras associated to highest weight representations of affine Lie algebras and superconformal algebras. Vertex algebras associated to affine Lie algebras at an integer level are rational and their irreducible representations are WZWN–models. On a rational admissible level the representation is much complicated. We present the decomposition result for $\widehat{\mathrm{sl}}_2$ vertex algebras (obtained in joint work with A. Miles). In the second part of the talk we apply the representation theory of $\widehat{\mathrm{sl}}_2$ vertex algebras to vertex algebras associated to the N=2 superconformal algebra and describe its irreducible modules.

Markus Rosellen (MPI für Mathematik, Bonn)
Mirror symmetry of Frobenius manifolds In the first part of my talk I gave an introduction to Frobenius manifolds (FM). I motivated this structure as "topological special geometry" on the deformation space of a topological CFT, gave the definition and the A– and B–model of a CY–manifold (Quantum cohomology, Barannikov–Kontsevich construction, respectively) as two very important examples of it. In the second part I discussed in detail the FM structure of Dubrovin on Hurwitz spaces H which is the 1–dimensional (global) case of Landau–Ginzburg models/Saits frameworks which constitute the third large class of FMs. I introduced the hierarchy of primitive forms which pull back the Grothendieck residue pairing to flat metrics on H having all the same rotation coefficients. In the third part I stated the theorem that on any semisimple FM the space of flat metrics with fixed rotation coefficients (i.e. a solution of the Darboux–Egoroff equations) form such a hierarchy. I compared this hierarchy of metrics (where the multiplication of the FM is fixed) with the deformations of the multiplication at tree–level by gravitational descendant (where the metric on the FM is fixed).

Berichterstatter: *Andreas Wißkirchen, Bonn*

E-mail addresses of the participants:

Bobby Samir Acharya	r.acharya@qmw.ac.uk
Dražen Adamović	adamovic@math.hr
Claudio Bartocci	bartocci@dima.unige.it
Victor V. Batyrev	batyrev@bastau.mathematik.uni-tuebingen.de
Tilman Bayer	uzs210@uni-bonn.de
Ralph Blumenhagen	blumenha@physik.hu–berlin.de
Peter Bowcock	peter.bowcock@durham.ac.uk
Antoine Coste	coste@qcd.th.u–psud.fr
Micℏael Flohr	flohr@mth.kcl.ac.uk
Jürgen Fuchs	jfuchs@itp.phys.ethz.ch
Matthias R. Gaberdiel	m.r.gaberdiel@damtp.cam.ac.uk
Terry Gannon	tgannon@math.yorku.ca
Peter Goddard	p.goddard@damtp.cam.ac.uk
Antonella Grassi	grassi@math.upenn.edu
Bruce Hunt	bhunt@mis.mpg.de
Jürgen Jost	jjost@mis.mpg.de
Dieter Lüst	luest@physik.hu–berlin.de
David R. Morrison	drm@math.duke.edu
Werner Nahm	werner@avzw02.physik.uni–bonn.de
Viacheslav V. Nikulin	slava@nikulin.mian.su
Tony Pantev	tpantev@math.upenn.edu
Ulf Persson	ulfp@math.chalmers.se
Ronen Plesser	plesser@cgtp.duke.edu
Andreas Recknagel	anderl@aei–potsdam.mpg.de
Markus Rosellen	rosellen@mpim–bonn.mpg.d
Philippe Ruelle	ruelle@fyma.ucl.ac.be
Emanuel Scheidegger	esche@ls–wess.physik.uni–muenchen.de
Martin Schlichenmaier	schlichenmaier@math.uni–mannheim.de
Christoph Schweigert	schweige@itp.phys.ethz.ch
Markus Spitzweck	msp@math.uni–bonn.de
Jakob Stix	stix@math.uni–bonn.de
Anne Taormina	anne.taormina@durham.ac.uk
Gerard Watts	gmtw@mth.kcl.ac.uk
Katrin Wendland	wendland@avzw02.physik.uni–bonn.de
Andreas Wißkirchen	wisskirc@avzw02.physik.uni–bonn.de
Noriko Yui	yui@ny.mast.queensu.ca
Eberhard Zeidler	ezeidler@mis.mpg.de

Tagungsteilnehmer

Prof.Dr. Bobby Samir Acharya
School of Mathematical Sciences
Queen Mary and Westfield College
University of London
Mile End Road

GB-London , E1 4NS

Prof.Dr. Drazen Adamovic
Department of Mathematics
University of Zagreb
Bijenicka 30

10000 Zagreb
CROATIA

Prof.Dr. Claudio Bartocci
Dipartimento di Matematica
Universita di Genova
Via Dodecaneso 35

I-16146 Genova

Dr. Victor V. Batyrev
Mathematisches Institut
Universität Tübingen

72074 Tübingen

Tilman Bayer
Mathematisches Institut
Universität Bonn
Beringstr. 1

53115 Bonn

Dr. Ralph Blumenhagen
Fachbereich Physik
Humboldt Universität Berlin
Invalidenstr. 42

10115 Berlin

Dr. Peter Bowcock
Dept. of Mathematical Sciences
The University of Durham
Science Laboratories
South Road

GB-Durham , DH1 3LE

Prof.Dr. Antoine Coste
53 Rue des Meuniers

F-92150 Suresnes

Dr. Michael Flohr
Department of Mathematics
King's College London
University of London
Strand

GB-London WC2R 2LS

Dr. Jürgen Fuchs
Division TH
CERN

CH-1211 Geneve 23

Dr. Matthias R. Gaberdiel
Fitzwilliam College
University of Cambridge
Department of Mathematics

GB-Cambridge CB3 ODG

Prof.Dr. Terry Gannon
Dept. of Mathematical Sciences
University of Alberta

Edmonton , AB T6G 2G1
CANADA

Prof.Dr. Peter Goddard
St. John's College
Department of Mathematics

GB-Cambridge CB2 1TP

Prof.Dr. Antonella Grassi
Department of Mathematics
University of Pennsylvania
209 South 33rd Street

Philadelphia , PA 19104-6395
USA

Prof.Dr. Bruce Hunt
Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26

04103 Leipzig

Prof.Dr. Jürgen Jost
Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26

04103 Leipzig

Prof.Dr. Dieter Lüst
Institut für Physik
Humboldt Universität Berlin
Unter den Linden 6

10099 Berlin

Prof.Dr. David R. Morrison
Dept. of Mathematics
Duke University
P.O.Box 90320

Durham , NC 27708
USA

Prof.Dr. Werner Nahm
Physikalisches Institut
Universität Bonn
Nußallee 12

53115 Bonn

Prof.Dr. Viacheslav V. Nikulin
Max-Planck-Institut
Gottfried-Claren-Str. 26

53225 Bonn

Dr. Tony Pantev
Department of Mathematics
University of Pennsylvania

Philadelphia , PS 19104-6395
USA

Prof.Dr. Ulf Persson
Department of Mathematics
University of Michigan

Ann Arbor , MI 48103
USA

Prof.Dr. Ronen Plesser
Department of Physics
Duke University
P.O. Box 90305

Durham , NC 27708-0305
USA

Dr. Andreas Recknagel
MPI für Gravitationsphysik
Schlaatzweg 1

14473 Potsdam

Markus Rosellen
Max-Planck-Institut für Mathematik
Vivatsgasse 7

53111 Bonn

Prof.Dr. Philippe Ruelle
Institut de Physique Theorique
Universite Catholique de Louvain
Chemin du Cyclotron, 2

B-1348 Louvain-la-Neuve

Emanuel Scheidegger
Fachbereich Physik
Universität München
Theresienstraße 37

80333 München

Dr. Martin Schlichenmaier
Fakultät für Mathematik und
Informatik
Universität Mannheim

68131 Mannheim

Christoph Schweigert
Institut für Theoretische Physik
ETH Zürich
Hönggerberg

CH-8093 Zürich

Markus Spitzweck
c/o Prof.Dr. W. Nahm
Physikalisches Institut
Universität Bonn
Nussallee 12

53115 Bonn

Jakob Stix
Venusbergweg 3

53115 Bonn

Dr. Anne Taormina
Dept. of Mathematical Sciences
The University of Durham
Science Laboratories
South Road

GB-Durham , DH1 3LE

Dr. Gerard Watts
Department of Mathematics
King's College London
University of London
Strand

GB-London WC2R 2LS

Prof.Dr. Noriko Yui
Department of Mathematics and
Statistics
Queen's University

Kingston, Ontario K7L 3N6
CANADA

Katrin Wendland
Physikalisches Institut
Universität Bonn
Endenicher Allee 11 - 13

53115 Bonn

Prof.Dr. Eberhard Zeidler
Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26

04103 Leipzig

Dr. Andreas Wisskirchen
Physikalisches Institut
Universität Bonn
Nußallee 12

53115 Bonn