Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 26/1999

Partial Differential Equations

13.06.1999 – 19.06.1999

This meeting was organized by L. C. Evans (Berkeley), S. Müller (MPI Leipzig) and E. Kuwert (Freiburg). The 22 delivered talks addressed a broad spectrum of questions for nonlinear partial differential equations. Most of the considered problems arise in geometry or in physics, as for example submanifolds with prescribed curvature relations, harmonic maps, crystal shapes, phase boundaries, Hamilton-Jacobi equations or quantum particles. The used methods, which are more universal than the problems and can be useful even for different types of equations, include variational techniques, estimates for fully nonlinear equations, viscosity solutions, measure-theoretic tools, Strichartz estimates and others.

For the success of the conference, which had 40 participants from about ten different countries, it was important that the program allowed sufficient time for informal talks and discussions.

Ernst Kuwert

The following abstracts are in chronological order.

Regularity for parabolic equations in two space variables
(Ben Andrews)

It has been known for a long time that the regularity theory for elliptic equations in two space variables is somewhat special: Morrey and Nirenberg provided $C^{1,\alpha}$ estimates for solutions of such equations, depending only on the ellipticity constants and L^∞ bounds for coefficients. This also leads to a $C^{2,\alpha}$ estimate for solutions of fully nonlinear elliptic equations in two variables of the form

$$F(D^2u, Du, u, x) = 0,$$

depending only on ellipticity and bounds on the first derivatives of F. In contrast, in dimensions three and higher, the known $C^{2,\alpha}$ estimates for fully nonlinear equations (due to Evans and Krylov) require that F be concave (or convex) in the first argument, and the estimates depend on second derivatives of F in the other arguments.

In the parabolic case, a correspondingly simple result is known for the case of one space variable (due to Kruzhkov), but there has not been any analogous result for the case of two space variables. I fill this gap by proving $C^{2,\alpha}$ estimates for solutions of fully nonlinear uniformly parabolic equations in two space variables,

$$du/dt = F(D^2u, Du, u, x, t)$$

with estimates depending only on the ellipticity constants and first derivatives of F.

Repulsion between holomorphic and antiholomorphic bubbles in almost-harmonic maps, and asymptotics of the harmonic map flow
(Peter Topping)

We present an analysis of bounded-energy low-tension maps between 2-spheres. By deriving sharp estimates for the ratio of length scales on which bubbles of opposite orientation develop, we show that we can establish a 'quantization estimate' which constrains the energy of the map to lie near to a discrete energy spectrum. One application is to the asymptotics of the harmonic map flow; we find uniform exponential convergence in time, in the case under consideration.

Crystals in a Cone: Equilibrium Shapes in the Plane
(Robert McCann)

A crystal in equilibrium with its melt will have minimum free energy among all shapes of its size. In our case, this energy consists of a surface term measuring the crystal-fluid interface along each direction, plus a potential energy reflecting the effect of a background field such as gravity (or hysteresis in some dynamical growth models). When the gravitational potential is convex, as for crystals on a table or in a cone, these energies are minimized separately by convex sets (given by Wulff's construction in the case of surface energy). However, the total energy may not be. In this talk we discuss recent work with Felix Otto showing why equilibrium shapes in the plane consist of disjoint unions of at most two convex components. The possibility of disconnected equilibria in the physically relevant case of three-dimensions remains an open question...

Regularity of Lipschitz free boundaries for two-phase problems for fully nonlinear elliptic equations

(Mikhail Feldman)

We consider a 2-phase free boundary problem, in which the positive and negative parts of a solution satisfy elliptic equations, and a condition, involving normal derivatives from positive and negative sides holds on the free boundary in a weak sense. The equations are assumed to be fully nonlinear, uniformly elliptic, not necessarily convex. We prove that if the free boundary is locally a graph of Lipschitz function, then it is C^1_α smooth. This is an extension of a part of the regularity theory developed by L. Caffarelli for two-phase problems for the Laplacian equation, and of the result by P. Wang for fully nonlinear convex equations.

On the Discontinuity Set of Minimizers of Mumford–Shah Functional

(Nicola Fusco)

Let us consider the following functional

$$G(K,u) = \int_{\Omega\setminus K} \left[|\nabla u|^2 + \alpha(u-g)^2\right] dx + \beta\mathcal{H}^{N-1}(\Omega\cap K)$$

where $\alpha, \beta > 0$, Ω is a bounded open set in $\mathbb{R}^N$, K is a closed set, $g \in L^\infty(\Omega)$, $u \in C^1(\Omega\setminus K)$ and $\mathcal{H}^{N-1}$ denotes the $(N-1)$-dimensional Hausdorff measure.

The existence of minimizers for this functional is now well understood, but there are still open problems concerning the regularity of the discontinuity set.

Recently Ambrosio, Fusco and Pallara proved that if (K,u) is a minimizing pair, then there exists a closed set $\Sigma \subset K$ such that $\mathcal{H}^{N-1}(\Sigma) = 0$ and $K \setminus \Sigma$ is locally a $C^{1,\alpha}$ hypersurface for some $\alpha > 0$. Other regularity results have been obtained in two dimensions by Bonnet, David and Semmes. However, more precise conjectures have been made by Mumford and Shah and by De Giorgi concerning the dimension of the singular set Σ.

In the talk we have presented a result by Ambrosio, Fusco and Hutchinson concerning the reduction of the dimension of the set Σ. More precisely we prove that if (K,u) is a minimizing pair and $|\nabla u| \in L^p_{\text{loc}}(\Omega)$, then the Hausdorff dimension of Σ is less than or equal to $\max\{N - p/2, N-2\}$. In the talk we have discussed in particular the two dimensional case, where this result is related to a well known conjecture on the summability of the gradient of conformal maps.

Uniqueness of Equilibrium Configurations in Solid Crystals

(Wilfrid Gangbo)

Despite that $M \to h(\det M)$ is not coercive, we prove that the functional $E[u] := \int_\Omega (\det D\mathbf{u}) - F \cdot \mathbf{u})\, dx$ admits a unique minimizer over the set $\mathcal{U}_\lambda$ of all orientation-preserving deformations $\mathbf{u} \in C^1(\Omega)^d$ that are homeomorphisms from $\overline{\Omega}$ onto $\overline{\Lambda}$ provided that $\det DF$ is positive, h is strictly convex, smooth and satisfies suitable growth conditions at 0^+ and $+\infty$ and $F(\Omega)$, Λ are convex. This is done by introducing a relaxation of $\inf_{\mathcal{U}_\Lambda} E$ and identifying a problem dual to the relaxed problem. Next, given $\mathbf{u}_0 \in \mathcal{U}_\lambda$ we study the pure displacement boundary value problem that consists of minimizing E over $\mathcal{U}_0$ the set of maps $\mathbf{u} \in \mathcal{U}_\Lambda$ with prescribed boundary values $\mathbf{u}_0$. We show that the infimum of E over $\mathcal{U}_\Lambda$ and $\mathcal{U}_0$ coincide and conclude that in general the pure displacement boundary value problem does not admit a minimizer.

Semilinear wave equations with critical data

(Patrick Gérard)

The purpose of this talk is the qualitative study of solutions to critical semilinear equations of the type $\Box u + |u|^{m-1} u = 0$, $(u, u_t)|_{t=0} \in \dot{H}^s \times \dot{H}^{s-1}$, assuming global existence for the Cauchy problem. We use a special decomposition of waves describing defects of compactness for Strichartz inequalities. In a first part, some emphasis is made on the applications of this decomposition to the 3D quintic wave equation*. Then we sketch more recent generalizations: equations outside a convex obstacle* or with lower regularity $s < 1$ – including the critical case $s = \frac{1}{2}$, where new phenomena occur.

On the Bernstein Problem for maximal affine Hypersurfaces

(Neil Trudinger)

The Bernstein problem for maximal affine hypersurfaces, as posed by Chern and Calabi, asks whether a locally uniformly convex function in Euclidean n-space, whose graph locally maximizes affine area in $(n+1)$-space, must be a quadratic polynomial. The Euler equation is a nonlinear elliptic equation of fourth order. In this talk, I report on research with X. J. Wang,which provides the affirmative answer in the case of two dimensions, thereby confirming the original conjecture of Chern. For higher dimensions, the result is established under a "uniform strict convexity" condition.

On the p-Poisson Equation and an Unconventional Moving Boundary Problem

(Gunnar Aronsson)

Flows of non-newtonian fluids give rise to a variety of interesting mathematical problems. The talk will focus on so-called Hele-Shaw flows of power-law fluids. This roughly means a

*jw. w/ H. Bahouri

*jw. w/ I-Gallagher

laminar flow in a narrow gap between parallel plates, and the fluid without any elasticity, but having variable viscosity. The fluid is here supposed to be strongly shear-thinning.It then makes sense to consider a limit case, where the mathematical description turns out to have interesting properties. The speaker will breafly describe two cases:

1. An injection problem, where a formal solution can be given to the corresponding moving boundary problem. This, in turn leads to geometric considerations. A nice application exists: injection molding of plastic objects.
2. A compression problem. Here, the fluid is contained between two plates, slowly approaching each other. Strictly speaking, this flow is called a generalized Hele-Shaw flow. Before passing to the limit, the flow is supposed to be governed by the p-Poisson equation. This is the problem referred to in the title. After passage to the limit, one finds a moving boundary problem in two dimensions, where the expansion of the crucial domain is governed by the curvature of the boundary, combined with a certain global variable. The ridge (skeleton) of the domain turns out to be an important concept here.

A common feature for the two problems is that by passing to the limit in a nonlinear PDE problem, one arrives at a purely geometric evolution problem. Thus, by sacrificing some accuracy in passing to the limit, one avoids solving any field equations. This leads to an enormous simplification of computations, at least in the first problems.

Singular integrals and the porous medium equation

(Herbert Koch)

The theory of singular integrals, which was developed by Calderón and Zygmund around 1950 in $\mathbb{R}^n$, had a profound impact on various areas of analysis. That theory relies on few properties of the Euclidean geometry and can be adapted to different geometric structures. Examples are the Kolmogorov equation of a particle in a random field, subelliptic operators on the boundary of strongly pseudo-convex domains in $\mathbb{C}^n$ and left invariant subelliptic operators on nilpotent Lie groups.

Other examples are operators which occur in homogenization, in the study of elliptic equations with strong drift, as well as those which come from linearizing the porous medium equation

$$\rho_t - \Delta\rho^m = 0 \text{ in } U \subset \mathbb{R}^n \times \mathbb{R}, \qquad m > 1.$$

The main result, analyticity of the free boundary for large times under weak assumptions on initial data, follows from modified Gaussian estimates of the fundamental solution of degenerate parabolic equations, which imply Harnack inequalities and fit into the theory of singular integrals.

Line energies for gradient vector fields in the plane

(Luigi Ambrosio)

We study the singular perturbation of $\int(1-|\nabla u|^2)^2$ by $\varepsilon^2|\nabla^2 u|^2$. This problem, first raised by Aviles and Giga in connection with the theory of smectic liquid crystals, has received in the last years considerable attention, due to potential applications to the theory of thin films and micromagnetics. The problem can also be thought as the natural second order version of the classical singular perturbation of the potential energy $\int(1-u^2)^2$ by $\varepsilon^2|\nabla u|^2$ and leads, as in the first order case, to energy concentration effects on hypersurfaces. In the two dimensional case we study the natural domain for the limiting energy, showing that it is strictly larger than the space of BV gradients satisfying the eikonal equation. Moreover we prove a compactness theorem in the natural domain for the limit energy.

Semilinear elliptic equations with critical exponential growth

(Michael Struwe)

In joint work with Adimurthi we characterize the blow-up profile and blow-up energy of solutions $u_k \overset{\omega}{\rightharpoonup} 0$ in$H_0^1(\Omega)$ of

$$-\Delta u_k = f_k(u_k) \text{ in } \Omega, \quad u_k > 0 \text{ in } \Omega, \quad u_k = 0 \text{ on } \partial\Omega$$

for nonlinerities $f_k(s) = s\, e^{\varphi_k(s)} \longrightarrow f(s) = s\, e^{\varphi(s)}$ locally uniformly of critical exponential growth (e.g. $\varphi(s) = 4\pi s^2$) on a domain $\Omega \subset\subset \mathbb{R}^2$.

The behavior of (u_k) is similar to the behavior of Palais-Smale sequences for critical equations in dimensions $n \geq 3$, like

$$-\Delta u = u^{\frac{n+2}{n-2}} \text{ in } \Omega, \quad u > 0 \text{ in } \Omega, u = 0 \text{ on } \partial\Omega.$$

In contrast to the latter case, however, in $n = 2$ dimensions the blow-up profile emerges after the nonlinear transformation

$$u_k \mapsto \eta_k(x) = \varphi_k(u_k(x_k + r_k x)) + \nu_k$$

and suitable change of scale with $r_k \to o, \nu \to -\infty$.

Hypersurfaces with mean curvature given by a trace

(Reiner Schätzle)

We consider smooth, oriented n-hypersurfaces $\Sigma_j = \partial E_j$ with interior E_j whose mean curvature is given by the trace of a function in the ambient space $u_j \in W^{1,p}(\mathbb{R}^{n+1})$

$$\vec{\mathbf{H}}_{\Sigma_\mathbf{j}} = \mathbf{u_j}\, \nu \mathbf{E_j} \quad \text{on } \Sigma_\mathbf{j}, \tag{1}$$

where νE_j denotes the inner normal of Σ_j. We investigate (**??**) when $\Sigma_j \to \Sigma$ weakly as varifolds and prove that Σ is an integral n-varifold with bounded first variation which still satisfies (**??**) for $u_j \to u, E_j \to E$. p has to satisfy

$$p > \frac{1}{2}(n+1)$$

and $p \geq \frac{4}{3}$ if $n = 1$. The difficulty is that in the limit several layers can meet at Σ which creates cancellations of the mean curvature.

Keywords: surfaces, varifolds, fully non-linear elliptic equations.

AMS Subject Classification: 49 Q 15, 53 A 10, 35 J 60.

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The geometry of dissipative evolution equations: the porous medium equation

(Felix Otto)

We show that the porous medium equation

$$\partial_t \rho - \triangle \rho^m \;=\; 0 \quad \text{in } (0,\infty) \times R^N,$$

which describes the flow of a gas of variable density $\rho(t,x) \geq 0$ through a porous medium, is a gradient flow of

$$E(\rho) \;=\; \left\{ \begin{array}{ll} \int \frac{1}{m-1}\rho^m & \text{for } m \neq 1 \\ \int \rho \log \rho & \text{for } m = 1 \end{array} \right\}$$

on the manifold $\mathcal{M}$ of density functions of given mass, endowed with a specific metric tensor g, making $(\mathcal{M}, g)$ a Riemannian manifold (actually non–flat for $N > 1$).

We argue that this gradient flow interpretation (there are others) is both physically and mathematically natural. It is physically natural: The functional E has the meaning of the free energy and the metric tensor g encodes the dissipation mechanism. It is mathematically natural: The time asymptotics, that is, the convergence of a rescaled solution $\hat\rho$ to the Barenblatt profile $\hat\rho^*$, can be easily understood and quantified in this framework:

- $\hat\rho$ evolves according to the gradient flow of $F = E + \alpha\, M$, where

$$M(\rho) \;=\; \int_{R^N} \frac{1}{2}\,|y|^2\,\rho(y)\,dy \quad \text{and} \quad \alpha \;=\; \frac{1}{(m-1)\,N+2}.$$

- $\hat\rho^*$ is the unique minimizer of F on $\mathcal{M}$.
- F is strictly convex with respect to the geometry of $(\mathcal{M}, g)$ in the sense of

$$\text{Hess}F_{|}\rho \;\geq\; \alpha\,\text{id} \quad \text{for all } \rho \in \mathcal{M}.$$

These three observations imply by basic Riemannian calculus that $\hat\rho$ converges to $\hat\rho^*$ with rate α, for instance in the induced metric of $(\mathcal{M}, g)$, which we indentify with the Wasserstein metric. Please consult http://www.math. ucsb.edu / otto/publications.html for details.

Aubry-Mather Theory and Periodic Solutions of the Forced Burgers Equation

(Weinan E)

Consider a Hamiltonian system with Hamiltonian of the form $H(x,t,p)$ where H is convex in p, and periodic in x and $t, x \in \mathbb{R}^1$. It is well-known that its smooth invariant curves correspond to smooth $\mathbb{Z}^2$-periodic solutions of the PDE

$$u_t + H(x,t,u)_x = 0.$$

In this paper, we establish a connection between the Aubry-Mather theory of invariant sets of the Hamiltonain system and $\mathbb{Z}^2$-periodic weak solutions of this PDE by realizing the Aubry-Mather sets as closed subsets of the graphs of these weak solutions. We show that the complement of the Aubry-Mather set on the graph can be viewed as a subset of the *generalized* unstable manifold of the Aubry-Mather set. The graph itself is a backward invariant set of the Hamiltonian system.

The basic idea is to embed the globally minimizing orbits used in the Aubry-Mather theory into the characteristic fields of the above PDE. This is done by making use of one and two-sides minimizers, a notion introduced in [**?**] inspired by the work of Morse on geodesics of type A [**?**]. The asymptotic slope of the minimizers, also known as the rotation number, is given by the derivative of the homogenizes Hamiltonian, defined in [**?**]. As an application, we prove that $\mathbb{Z}^2$-periodic weak solution of the above PDE with given irrational asymptotic slope is unique. A similar connection also exists in multi-dimensional problems with convex Hamiltonian, except that in higher dimensions, two-sided minimizers with specified asymptotic slope may not exist.

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Closed hypersurfaces of prescribed Gaussian curvature in Lorentzian manifolds

(Claus Gerhardt)

Let N be a globally hyperbolic Lorentzian manifold with compact Cauchy hypersurface, $\Omega \subset N$ open and $0 < f \in C^{l,\alpha}(\overline{\Omega})$. Then, we prove the existence of a closed, space-like hypersurface $M \subset \Omega$ whose Gaussian curvature equals f, provided $\partial\Omega$ consists of two advanced hypersurfaces acting as barriers and provided these is a strictly convex function in Ω.

Dynamics of a quantum particle in a potential

(Robert L. Jerrard)

We consider solutions of the equation

$$i\,u^2_t + \frac{\varepsilon}{2}\Delta u^2 + \frac{1}{\varepsilon}(|u^\varepsilon|^{p-1}u^\varepsilon - V(x)\,u^\varepsilon) = 0, x \in \mathbb{R}^n, t > 0$$

$$u^\varepsilon(x,0) = \phi^\varepsilon(x) \sim \left(\frac{x - x_0}{\varepsilon}\right) e^{\frac{iv_0 \cdot x}{\varepsilon}}$$

in the limit $\varepsilon \to 0$, where s is a grand state solution of an associated elliptic equation. The data may be thought of as corresponding to a quantum particle at position x_0 with velocity v_0 (although this is arguably not completely correct, physically). We show that at later times there exist y^ε such that

$$\frac{1}{\varepsilon^n}\||u^\varepsilon(\,\cdot\,,t)| - s\left(\frac{\cdot\, - y^\varepsilon(t)}{\varepsilon}\right)\|^2_{L^2} \to 0 \quad \forall t > 0.$$

and $y^\varepsilon(t) \to x(t)$, where x solves $x'' = -DV(x)$, $x(0) = x_0$, $x'(0) = v_0$.

Moser-Trudinger Inequalities and Liouville Systems

(Guofang Wang)

Let Σ be a closed surface and $A = (a_{ij})$ an $n \times n$ matrix with non-negative entries. We consider the following system.

$$
-\Delta u_i = M_i \frac{e^{\Sigma_j a_{ij} u_j}}{\int e^{\Sigma_j a_{ij} u_j}} \quad \text{for any } i \in I = \{1, 2, \dots, n\}. \tag{1}
$$

To find solutions of (1), we consider the corresponding variational problem. For simplicity, we assume that A is positive definite. In this case, we consider

$$
F_M = \frac{1}{2} \sum_{i=1}^{n} \int a_{ij} \nabla u_i \nabla u_j - \sum_{i=1}^{n} M_i \log \int_{\Omega} \exp(\sum_{i=1}^{n} a_{ij} u_j)
$$

for $u = (u_i, u_2, \dots, u_n)$ with $u \in H_0^{1,2}(\Omega)$ and $M = (M_1, M_2, \dots, M_n) \in (\mathbb{R}_+)^n$.
For any subset $J \subseteq I$, define

$$
\Lambda_J = 8\pi \sum_{j \in J} M_j - \sum_{i,j \in J} a_{ij} M_i M_j.
$$

Generalizing a result of Chipot, Shafrir and Wolansky, we obtain

Theorem 1. *If $\Lambda_J(M) > 0$ for any subset $J \subseteq I$, then F_M has a lower bound.*

We propose a conjecture.

Conjecture 2. *Φ_M has a lower bound, if*

$$
\Lambda_J \geq 0 \quad \textit{for any } J \subseteq I.
$$

Theorem 3. *Let A be a symmetric nonnegative row-stochastic matrix, i. e.*

$$
a_{ij} \geq 0 \textit{ for all } i, j \quad \textit{and} \sum_{j \in I} a_{ij} = 1 \textit{ for all } i \in I.
$$

Assume, in addition, that A is invertible. Then Conjecture 2 holds.

Lagrangian immersions with prescribed Maslov form

(Knut Smoczyk)

We investigate the following problem: Assume L is an initial Lagrangian immersion with Maslov form H. Let m be a closed one-form on L representing the same cohomology class as H. Can one deform L into a new Lagrangian immersion such that the new maslov form coincides with m?

We use a modified mean curvature flow to attack this problem and derive longtime existence results for Lagrangian graphs over the Clifford torus and for Lagrangian immersions in the cotangent bundle of flat manifolds. The proof is based on a Harnack principle and an energy estimate.

Uniqueness properties for Ginzburg-Landau vortices

(Tristan Rivière)

We are considering the solutions of the complex Ginzburg-Landau Equation

$$\Delta u + \lambda u(1 - |u|^2) = 0 \tag{1}$$

on a bounded 2-dimensional domain for large coupling constants λ (the strongly repulsive case) and for a given Dirichlet boundary condition into the circle. In collaboration with Frank Pacard, we prove that, in the generic case, when the limiting configuration has vorticity of order + or - 1 (e.g. [**?**]), the following property holds: if two solutions are sufficiently close to this limit and if they cancel at the same points with the same multiplicities then the two solutions coincide. By the mean of this property

$$\text{zero of } u = \text{zero of } v \quad \Rightarrow \quad u = v$$

and a perturbation argument, we describe how to deduce a complete picture of the set of solutions of (**??**) in the generic case for limiting vortices of degree ± 1.

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Minimizing Volume among Lagrangian Cycles

(Jon G. Wolfson)

Let (N, ω) be a symplectic $2n$-manifold equipped with symplectic form ω. Let g be the metric $g(-,-) = \omega(J-,-)$, where J is a compatible almost complex structure. Define the Lagrangian homology $H_n^L(N;\mathbb{Z})$ to consist of the classes $\alpha \in H_n(N;\mathbb{Z})$ that can be represented by Lagrangian immersions. We consider the problem of finding a canonical representative of $\alpha \in H_n^L(N;\mathbb{Z})$ by minimizing volume (with respect to g) over Lagrangian cycles representing α. The most geometrically interesting result is obtained when the metric g is Kähler-Einstein.

If H denotes the mean curvature vector along an immersed Lagrangian set, then $\sigma_H = H \lrcorner\, \omega$.

Theorem 1. *Let (N, g, ω) be a compact symplectic 4-manifold. Then $H_n^L(N;\mathbb{Z})$ is generated by (branched) Lagrangian immersions Σ whose mean curvature 1-form σ_H satisfies a Hodge-type system of the form*

$$\begin{aligned} d\sigma_H &= Ric_{|\Sigma} + \rho_{|\Sigma}, \\ \delta\sigma_H &= 0. \end{aligned}$$

where Ric denotes the Ricci curvature and ρ is a 2-form that vanishes if N is Kähler.

In the case that (N, g, ω) is a compact Kähler-Einstein surface, $H_n^L(N;\mathbb{Z})$ is generated by (branched) Lagrangian immersions that are classical minimal surfaces.

This talk reported on joint work with R. Schoen.

A Harnack Inequality for the Inverse Mean Curvature Flow

(Gerhard Huisken)

In this joint work with T. Ilmanen (ETH Zürich) we study families $F : M^n \times [0,T) \to (N^{n+1}, \overline{g})$ of closed embedded hypersurfaces in a Riemannian manifold, which move in direction of their inverse mean curvature, i. e.

$$\frac{d}{dt}F(p,t) = \frac{1}{H}\nu(p,t), \quad p \in M^n, \quad t \in [0,T).$$

Here $H > 0$ is the mean curvature and ν the exterior unit normal of the evolving surfaces. In previous work we established the existence of weak solutions $M_t^n = \partial\{x \in N^{n+1} | u(x) < t\}$ arising as level sets of a scalar function $u : N^{n+1} \longrightarrow \mathbb{R}$ satisfying

$$\operatorname{div}\left(\frac{Du}{|Du|}\right) = |Du|$$

together with a minimization principle, if N^{n+1} is asymptotically Euclidean in an approproate sense. The new result provides a lower bound for the mean curvature in the class of starshaped surfaces, showing that in this case the weak solution coincides with a smooth classical solution. One particular result is the following:

Theorem 1. *If $M_t^n \subset \mathbb{R}^{n+1}$ is a weak solution of mean curvature flow such that for some $t_0 > 0$ M_t^n is starshaped, i. e. $\langle F(p,t_0), \nu(p,t_0)\rangle > 0$ $\forall p \in M^n$. Then M_t^n is smooth for all $t > t_0$ and the mean curvature satisfies a lower bound of the form*

$$H(p,t) \geq c(t-t_0)^{\frac{1}{2}} \cdot \exp(-ct).$$

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