MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t

26/1999

Algebraic Number Theory

27.06. - 03.07.1999

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The meeting has been organized by Christopher Deninger (Münster), Peter Schneider (Münster) and Anthony Scholl (Durham).

The subject of the conference was Algebraic Number Theory and Arithmetic Algebraic Geometry. In particular, the following aspects have been treated in the talks:

- Iwasawa Theory
- Galois group actions
- Modular Forms and Galois representations
- Langlands correspondences
- p-adic Hodge Theory
- rigid analytic methods in Arithmetic Geometry
- classical and p-adic polylogarithms
- Higher dimensional class field theory
- Cycles and Chow groups

Some speakers reported on their proofs of outstanding conjectures (Henniart, Lafforgue on local respectively global Langlands correspondences; Fontaine on the "weakly admiss. $\Rightarrow$ admiss."-conjecture (joint work with Colmez)).

VORTRAGSAUSZÜGE

Amnon Besser (Be'er-Sheva, Israel)

p-adic regulators for K-theory of fields and p-adic polylogarithms (joint work with Rob de Jeu)

Goncharov conjectured the existence of complexes, for a field F :

$$\begin{array}{lccccccccc} \Gamma(F,n): & B_n & \to & B_{n-1}\otimes F_{\mathbb{Q}}^{\times} & \to & B_{n-2}\otimes\wedge^2 F_{\mathbb{Q}}^{\times} & \to & \dots & \to & \wedge^n F_{\mathbb{Q}}^{\times}, \\ \text{in degrees} & 1 & & 2 & & 3 & & \dots & & n+1, \end{array}$$

where B_n is generated by symbols $[x]_n,\ x\in mathbbP^1(F)-\{0,1,\infty\}$, and an isomorphism $H^r(\Gamma(F,n))\xrightarrow{\sim} H^r_{\mathcal{M}}(F,\mathbb{Q}(n))$. Zagier further conjectured that when F is a number field the composition

$$H^1(\Gamma(F,n))\to H^1_{\mathcal{M}}(F,\mathbb{Q}(n))\to H^1_{\mathcal{D}}(F,\mathbb{R}(n)) = (\bigoplus_{\sigma:F\to\mathbb{C}}\mathbb{R})^{(-1)^{n-1}},$$

where the eigenspace with respect to the action of complex conjugation on the embeddings is given by $[x]_n\to(P_{Zag,n}(x^\sigma))_\sigma$, where $P_{Zag,n}$ is a version of the complex polylogarithm. de Jeu constructed a complex $\widetilde{\mathcal{M}}_{(n)}$ with similar properties and a map $H^1(\widetilde{\mathcal{M}}_{(n)})\to H^1_{\mathcal{M}}(F,\mathbb{Q}(n))$ and proved the analogue of Zagier's conjecture.

In this work we treat the p-adic analogue: when $[K:\mathbb{Q}_p]<\infty$ we have a syntomic regulator $H^1_{\mathcal{M}}(K,\mathbb{Q}(n))\to K$ (Gros, Niziol, Nekovář, Besser). We state the following conjecture:

Conjecture: If $F\hookrightarrow K$ where F is a number field, then the composed

$$reg_p : H^1(\widetilde{\mathcal{M}}_{(n)})\to H^1_{\mathcal{M}}(F,\mathbb{Q}(n))\to H^1_{\mathcal{M}}(K,\mathbb{Q}(n))\to K$$

is given by $[x]_n\mapsto L_{\mathrm{mod},n}(x)$, where $L_{\mathrm{mod},n}$ is a version of Coleman's p-adic polylogarithm.

We prove this conjecture when $|x|=|x-1|=1$ and derive consequences for syntomic regulators of cyclotomic elements in K-theory.

Don Blasius (UCLA)

Motives for Hilbert Modular Forms

According to a basic theorem of Shimura, there is attached to each holomorphic newform f of weight 2, with rational Hecke eigenvalues, an elliptic curve E_f, defined over $\mathbb{Q}$, such that

$L(E_f, s) = L(f, s)$. It is expected that this result will generalize to totally real fields F; the result is known if $[F : \mathbb{Q}]$ is odd or f belongs to the discrete series at some finite prime v. We show that if the Hodge Conjecture is true, then the sought elliptic curve always exists. More generally, if the Hecke field of f is $\mathbb{T}_f$, there exists (under the same hypothesis) an abelian variety A_f, defined over F, with $\dim A_f = [\mathbb{T}_f : \mathbb{Q}]$, having a natural action of $\mathbb{T}_f$, such that $L(A_f, s) = L(f, s)$. (Thus, Shimura's full result over $\mathbb{Q}$ will generalize to F.)
For methods, we employ base change and the Jacquet-Langlands correspondence to find a motive for $L(\mathrm{Sym}^2, f, s)$ in the cohomology of a Shimura surface. A formal argument produces over $\mathbb{C}$ an abelian variety B of dimension $2[\mathbb{T}_f : \mathbb{Q}]$ whose H^2 contains a submotive isomorphic to $Sym^2(f)$. The Hodge conjecture is invoked to descend B to a number field L and relate it's l-adic representations to those attached to f by Taylor. From this point, the conclusion follows easily that the sought A_f is a factor of $R_{L/F}B$.

It may be hoped that this result may be proved by direct construction, using for example Shimura varieties attached to unitary groups. However, the construction above leads to the following general question: Is the Mumford-Tate group of the category of motives defined by Shimura varieties (i.e. occuring in their cohomology) simply-connected?

Christophe Breuil (CNRS, Université d'Orsay)

Classification of group schemes and applications
("p-adic" Riemann conditions, Shimura-Taniyama-Weil conjecture)

Let p be a prime, $p \neq 2, k$ a perfect field of char. p, $W = W(k)$ the Witt vectors, $K_0 = \text{Frac }(W), K$ a finite totally ramified extension of $K_0, \mathcal{O}_K$ the ring of integers and π a fixed uniformizer in $\mathcal{O}_K$.

Let S be the p-adic completion of $W[u, \frac{u^{ie}}{i!}]_{i \in \mathbb{N}}$, $\mathrm{Fil}^1 S$ the kernel of the map $S \to \mathcal{O}_K,\ u \mapsto \pi$, and $\varphi : S \to S$ the lifting of Frobenius induced by $\varphi(u) = u^p$. Let $\varphi_1 = \frac{\varphi}{p}|_{\mathrm{Fil}^1 S}$, one has $\varphi_1 : \mathrm{Fil}^1 S \to S$.

Define $'(\mathrm{Mod}/S)$ to be the category whose objects are:

- an S-module M
- a sub-module $\mathrm{Fil}^1 M \supset \mathrm{Fil}^1 S \cdot M$
- a semi-linear application $\varphi_1 : \mathrm{Fil}^1 M \to M$ $s.t.$ $\varphi_1(sx) = \varphi_1(s)\frac{\varphi_1^{(E(u)x)}}{\varphi_1^{(E(u))}}$
$(s \in \mathrm{Fil}^1 S, x \in M)$

(where $E(u)$ in the minimal polynomial of π), and whose morphisms are the obvious compatible ones.
Finally, let (ModFI/S) be the full subcategory of $'$(Mod$/S$) such that:

- $M \simeq \underset{i\in I}{\oplus} S/p^{n_i} \cdot S, |I| < \infty$
- $\varphi_1(\mathrm{Fil}^1 M)$ spans M over S

and define a strongly divisible module to be any M in $'$(Mod$/S$) s.t.:

- $M \simeq S^d$
- $\varphi_1(\mathrm{Fil}^1 M)$ spans M over S
- $M/\mathrm{Fil}^1 M$ has no p-torsion

Theorem 1. There is an equivalence of categories between $(ModFI/S)$ and the category of finite flat p-groups over $\mathcal{O}_K$ such that $ker(p^n)$ is still flat over $\mathcal{O}_K$ for any n. This equivalence preserves short exact sequences.

Theorem 2. There is an equivalence of categories between strongly divisible modules and p-divisible groups over $\mathcal{O}_K$.

The proof uses syntomic techniques, introduced by Mazur, Fontane, Messing in the case where $K = K_0$. Using this, on can get:

Theorem 3 (p-adic Riemann conditions). Assume $k \subset \overline{\mathbb{F}}_p$. Then every crystalline representation V of $Gal(\overline{K}/K)$ with Hodge-Tate weights $\in \{0, 1\}$ is isomorphic to $T_pG \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$ for some p-divisible group $G/\mathcal{O}_K$.

This is already known for $[K : K_0] \leq p - 1$ (Fontaine, Messing, Laffaille).

Corollary 1. Let A be an abelian variety over K (and $k \subset \overline{\mathbb{F}}_p$).
(i) A has good reduction over $\mathcal{O}_K \Leftrightarrow T_pA \otimes \mathbb{Q}_p$ is crystalline.
(ii) A has semi-stable reduction over $\mathcal{O}_K \Leftrightarrow T_pA \otimes \mathbb{Q}_p$ is semi-stable.

Corollary 2. The period map of Rapoport-Zink for p-divisible groups is surjective.

Corollary 1 (i) was known (Fontaine, Coleman, Iovita); the implication "$\Rightarrow$" of (ii) was known before too (Fontaine). The objects killed by p of (ModFI/S) classify *all* finite flat group schemes over $\mathcal{O}_K$ killed by p. By developing a decent data formalism on such objects and using it to compute the dimension of tangent spaces of specific deformation problems of potential Barsotti-Tate representations (joint work with B. Conrad, F. Diamond and R. Taylor), one can settle the last cases of the Shimura-Taniyama-Weil conjecture following of course Wiles' (+ Taylor's) strategy. Thus:

Theorem (Wiles,...): Every ellipic curve over $\mathbb{Q}$ is modular.

Kevin Buzzard (Imp. College, London)

Galois representations and weight 1 modular forms

My talk was a report on the current status of Taylor's program for attacking Artin's conjecture for representations $Gal(\overline{\mathbb{Q}}/\mathbb{Q}) \to GL_2(\mathbb{C})$ and also contained some remarks on some of the technicalities behind one of the results used in the program. The talk centered around a theorem of myself and Taylor which, broadly speaking, was a "weight 1 analogue" of Wiles' lifting theorem (if $\overline{\rho}$ is modular, and lots of technical conditions hold, then ρ is modular, $\rho : G_{\mathbb{Q}} \to GL_2(\mathbb{Z}_p)$ etc.).
I explained how this lifting theorem already gave new examples of Artin's conjecture, and also how, modulo some technical details, would give many more new examples. The key problem is that to get these many new examples one has to prove many 'standard' results about mod p modular forms in the case when $p = 2$. The main standard result which is still unproved is a "companion forms" result, proved by Gross (modulo unchecked compatibilities) for all primes and by Coleman and Voloch (not assuming these compatibilities) for $p > 2$. It now seems that if this result were to be unconditionally established for $p = 2$ then one would be able to prove that many icosahedral Galois representations $Gal(\overline{\mathbb{Q}}/\mathbb{Q}) \to GL_2(\mathbb{C})$ which were odd, would come from weight 1 modular forms and would hence have the property that their L-functions had analytic continuation and functional equation.

Henri Darmon (McGill University, Montreal)

The p-adic Birch and Swinnerton-Dyer Conjecture

This talk summarizes the results of different works, all joint with Massimo Bertolini, and Adrian Iovita and Michael Spieß.

Let $E/\mathbb{Q}$ be an elliptic curve, and $L(E,s)$ its Hasse-Weil zeta-function. Thanks to Breuil's lecture, we know E is modular, so there exists

$$\varphi : \mathcal{H}/\Gamma_0(N) \to E(\mathbb{C}),$$

and $(2\pi)^s\Gamma(s)L(E,s) = \int_0^\infty y^{s-1}f(iy)dy$, where $2\pi i f(z)dz = \varphi^*(\omega)$, so that f is a cusp form of weight 2 on $\Gamma_0(n)$. The Birch and Swinnerton-Dyer conjecture predicts

$$\mathrm{ord}_{s=1}L(E;s) = \text{ rank } (E(\mathbb{Q})).$$

But both conjectured inequalities are very mysterious.

If $E/\mathbb{Q}_p$ has multiplicative reduction, there is a rigid analytic uniformization $\varphi : \mathcal{H}_p/\Gamma \to E(\mathbb{C}_p)$, with Γ a p-adic arithmetic subgroup of $SL_2(\mathbb{Q}_p)$. Following an idea of Schneider and Iovita-Spieß, we define $L_p(E, s)$ by taking a kind of p-adic Mellin transform of the rigid analytic modular form of weight 2 on Γ attached to E. We are then able to

- give formulae for $L'_p(E, 1)$ relating these values to p-adic periods, and Heegner points on E;
- extend the definition of $L_p(f, 1)$ to forms f of higher even weight $k \geq 2$, and express $L'_p(f, \frac{k}{2})$ in terms of an L-invariant of Teitelbaum, and in terms of certain Heegner cycles in the Chow groups of Kuga-Sato varieties (joint work with Iovita and Spieß);
- prove $\mathrm{ord}_{s=1} L_p(E; s) \geq$ rank $(E(\mathbb{Q}))$; in fact we supply a more precise lower bound for $\mathrm{ord}_{s=1} L_p(E; s)$ which is conjectually sharp.

Jonathan Dee (Cambridge, UK)

Chow Groups for self-products of CM Elliptic Curves

I discuss certain properties of the image of the cycle class map for cycles of arbitrary codimension on the d-fold self-product of an elliptic curve over $\mathbb{Q}$ with complex multiplication.

Theorem. Let $E|\mathbb{Q}$ be an elliptic curve, with complex multiplication by $\mathcal{O}_K$, K quadratic imaginary. Put $X = E^d$, and assume E has good reduction at p, $p > max\{2i + 1, d\}$, $d \geq i \geq 1$.
Then

$$\mathrm{Im}\ (cl_X) \cap\ \mathrm{Fil}^2 H^{2i}_{ét}(X, \mathbb{Q}_p(i)) = 0,$$

where $cl_X : CH^i(X)_{\mathbb{Q}} \to H^{2i}_{ét}(X, \mathbb{Q}_p(i))$ is the cycle class map.
Furthermore, if p does not divide c_i then

$$cl_{X,\mathbb{Z}_p}(CH^i(X))_{tors} = 0,$$

where c_i is a certain explicit product of L-values.

To prove this theorem we use a local-global principle due to Nekovář in this setting which reduces us to bounding a certain Selmer group. This is achieved by using Rubin's proof of the Main Conjecture and an explicit reciprocity law.

I. Fesenko (Nottingham)

Bizarre pro-p-groups and Coates-Greenberg's problem

Let $T = \{\sum a_i t^{1+iq} : a_0 = 1, a_i \in \mathbb{F}_p\}$, $q = p^r, r \geq 1$. With respect to substitution, T is a torsion free finitely generated pro-p-group in which every non-trivial closed subgroup of an open subgroup is open.

Theorem. Let $r > 1$, $p > 2$ and $F/\mathbb{Q}_p$ be an unramified extension of degree $\geq q+1$. Then there is a Galois extension L/F with Gal $(L/F) \simeq T$; every such L/F is an arithmetically profinite extension; and L/F doesn't have infinite p-adic Lie subextensions.

This answers affirmatively Coates-Greenberg's problem set in Oberwolfach in 1994.

Jean-Marc Fontaine (Université d'Orsay)

Semi-stable p-adic Galois representations
(joint work with Pierre Colmez)

Let K be a complete field of characteristic 0 with perfect residue field of char. $p > 0$. Let $\overline{K}$ be an alg. closure of K, $G_K = Gal(\overline{K}/K)$. The category of semi-stable representations of G_K is a subtannakien category of the category of all p-adic representations and we have a functor

$$V_{st} : \text{ semi-stable representations } \rightarrow \text{ filtered } (\phi, N)\text{-modules}$$

which is a fully faithfuld $\otimes$-functor. We prove that the essential image is, as conjectured, the category of so-called weakly admissible filtered (ϕ, N)-modules.

Alexander Goncharov (Brown University/MPI)

Galois groups and geometry of modular varieties

We study the map on Lie algebras induced by the map

$$Gal(\overline{\mathbb{Q}}/\mathbb{Q}(\sqrt[l^\infty]{1})) \longrightarrow \mathrm{Aut}(\mathbb{L}_N^{(l)})$$

where $\mathbb{L}_N^{(l)}$ is the pro-nilpotent Lie algebra over $\mathbb{Q}_l$ corresponding to

$$\hat{\pi}_1^{(l)}(\mathbb{P}^1 \setminus \{0, \mu_n, \infty\}, \mathcal{V}_\infty)$$

let us denote it by $\mathfrak{g}_N^{(l)}$. We formulate a precise conjecture on the structure of the bigraded Lie algebra

$$\oplus \, \mathrm{Gr}^{D,W}_{\cdot,\cdot} \mathfrak{g}_N^{(l)}$$

where D and W are the so-called depth and weight filtration, respectively.

Guy Henniart (Université d'Orsay)

The Langlands conjecture for GL_n over p-adic fields

Let p be a prime number and F a finite extension of $\mathbb{Q}_p$. Let $\overline{F}$ be an algebraic closure of F and W_F the Weil group of $\overline{F}$ over F. The Langlands conjecture predicts the existence of a system of canonical bijections, for any integer $n \geq 1$, between the set $\mathcal{G}_F^0(n)$ of irreducible degree n (complex) representations of W_F and the set $\mathcal{A}_F^0(n)$ of supercuspidal smooth irreducible (complex) representations of the locally compact totally disconnected group $GL_n(F)$. Such maps $\sigma \mapsto \pi(\sigma)$ should be compatible with local class field theory, twisting by characters of $F^\times$, taking contragredients, and above all preserve L and ε-factors for pairs: for $\sigma \in \mathcal{G}_F^0(n)$, $\sigma' \in \mathcal{G}_F^0(n')$, we should have

$$L(\sigma \otimes \sigma', s) = L(\pi(\sigma) \times \pi(\sigma'), s),$$

$$\varepsilon(\sigma \otimes \sigma', s, \psi) = \varepsilon(\pi(\sigma) \times \pi(\sigma'), s, \psi),$$

where ψ is a nontrivial additive character of F and the factors on the right have been defined by Jacquet, Piatetskii-Shapito and Shalika.
Last summer M. Harris and R. Taylor proved the existence of the required bijections, providing in fact a geometric local model for the correspondence. Their proof was global and relied on a detailed analysis of the bad reduction of some Shimura varieties attached to unitary groups over number fields.
I reported on a shorter proof, found in december 1998, which is still global but more straightforward, and uses Shimura varieties only at primes of good reduction, where the situation was analyzed by Kottwitz in the early 90's. This shorter proof, however, yields no geometric information. Both global proofs back explicit information; an explicitation of the correspondence is under investigation (work with C. Bushnell).

Uwe Jannsen (Regensburg)

Class field theory for varieties over local field

Let X be a smooth, projective variety over a finite extension K of $\mathbb{Q}_p$, and consider the reciprocity map

$$\varrho : H^d(X, \mathcal{K}_{d+1}) \to \pi_1(X)^{ab},$$

$d = \dim X$, into the abelianized fundamental group. Here

$$H^d(X, \mathcal{K}_{d+1}) = coker(\underset{y\in X_1}{\oplus} K_2(k(y)) \xrightarrow{tame} \underset{x\in X_0}{\oplus} k(x)^\times)$$

is the cokernel of the tame symbol, where X_i is the set of points of dimension i on X, and the map ϱ is induced by the classical maps $k(x)^\times \to Gal(\overline{k(x)}|k(x))^{ab}$ for the local fields $k(x)$ and the canonical maps $Gal(\overline{k(x)}|k(x))^{ab} \to \pi_1(X)^{ab}$. The following results were obtained in joint work with Shuji Saito:

Theorem 1. Assume that the Milnor-Kato conjecture holds for $K_3^M(F)/l$, for all function fields F of smooth surfaces on X (this holds for the prime $l = 2$).

(a) If X has good reduction, then

$$\varrho \otimes \mathbb{Z}/l^w : H^d(X, \mathcal{K}_{d+1})/l^w \overset{\sim}{\to} \pi_1(X)^{ab}/l^w \text{ for all } w \geq 1.$$

(b) If X has semi-stable reduction, with special fibre Y, there is an exact sequence

$$H_2(\Gamma_Y, \mathbb{Z}/l^w) \to H^d(X, \mathcal{K}_{d+1})/l^w \overset{\varrho\otimes\mathbb{Z}/l^w}{\longrightarrow} \pi_1(X)^{ab}/l^w \to H_1(\Gamma_Y, \mathbb{Z}/l^w) \to 0,$$

where Γ_Y is the simplical complex describing the configuration of the irreducible components $Y_1, \ldots, Y_r$ of Y, provided
(i) $l \neq p$ or (ii) $l = p$ and Y is ordinary or (iii) $l = p$ and $d \leq p - 2$.

Theorem 2. Under the above assumptions, the groups $ker\ (\varrho \otimes \mathbb{Z}/l^w)$ are finite of order bounded independent of w. The same holds for the groups $ker\ (\varrho \otimes \mathbb{Z}/w)$, if the Milnor-Kato conjecture is true modulo all primes l.

Guido Kings (Münster)

Geometry of elliptic polylog and computation of l-adic Eisenstein classes

In this talk we contruct a new "geometric" approach to the elliptic polylogarithm sheaf which allows to compute its l-adic realizations. This gives a description of the l-adic Eisenstein classes which is needed for the Tamagawa number conjecture for modular forms, CM

elliptic curves or Dirichlet characters. The idea is as follows: Let E/K be an elliptic curve and $[l^t] : E \to E$ the multiplication by l^t. Over E we construct a ramified covering exactly ramified at $E[l^t]$ in a norm compatible way. To do this we use a covering of the generalized Jacobian for E, with modulus $E[l^t]$, which we pull back to $E - E[l^t]$. The covering is a $T_{E[l^s]}[l^r]$-torsor where $T_{E[l^s]}$ is the torus of the generalized Jacobian. The resulting class in $H^1_{\acute{e}t}(E - E[l^t],\ T_{E[l^s]}[l^r])$ comes via the boundary map from a function $E - E[l^t] \to T_{E[l^s]}$. Taking the $\varinjlim_s$ and $\varprojlim_t \varprojlim_r$ we get the class of the elliptic polylogarithm.

Mark Kisin (Sydney)

Unit L-Functions and a Conjecture of Katz

Let X be a finite type $\mathbb{F}_p$-scheme, Λ a complete local $\mathbb{Z}_p$-algebra, with finite residue field. For a Λ-adic lisse sheaf $\mathcal{L}$ on X one defines

$$L(X, \mathcal{L}) = \prod_{x \in |X|} \det{}_\Lambda(1 - \phi^{d(x)} T^{d(x)} | \mathcal{L}_{\overline{x}})^{-1}$$

where $d(x) = [k(x) : \mathbb{F}_p]$; ϕ is the geometric Frobenius; and $\overline{x}$ is a geometric point over x.

The cohomological L-function $L(\mathbb{F}_p, H^*_c(\mathcal{L}))$ is defined by

$$L(\mathbb{F}_p, H^*_c(\mathcal{L})) = \prod_{i=0}^{\infty} \det{}_\Lambda(1 - \phi T | H^i_c(X \otimes \overline{\mathbb{F}}_p, \mathcal{L}))^{(-1)^{i+1}}.$$

We show a conjecture of Katz which says that the quotient $L(X, \mathcal{L})/L(\mathbb{F}_p, H^*_c(\mathcal{L}))$ is a p-adic analytic, invertible function on the closed p-adic unit disc $|T| \leq 1$.

Laurent Lafforgue (CNRS, Université d'Orsay)

La correspondance de Langlands sur les corps de fonctions

Soit X une courbe projective lisse géométriquement connexe sur un corps fini $\mathbb{F}_q$ et F le corps des fonctions de X. Soit $\mathbb{A}$ l'anneau des adèles de F et $W_F = G_F \times_{\hat{\mathbb{Z}}} \mathbb{Z}$ son groupe de Weil. On démontre qu'il existe une correspondance bijective, préservant les fonctions L, entre l'ensemble des représentations automorphes cuspidales de $GL_r(\mathbb{A}), r \geq 1$, et l'ensemble des représentations l-adiques continues de W_F irréductibles de dimension r, comme conjecturé par Langlands.

La méthode généralise celle inaugurée par Drinfeld dans sa preuve de la conjecture pour $r = 2$. Elle consiste à étudier la cohomologie l-adique des champs de Chtoucas de Drinfeld de rang r au-dessus du point générique de $X \times X$, avec l'action des correspondances de Hecke.

J. Nekovář (Cambridge, UK)

Duality theorems in Iwasawa theory

Duality theorems for "Selmer complexes" imply duality theorems for various complexes in Iwasawa theory. For example, let $p \neq 2$ be a prime, K a number field, K_∞/K an extension with $Gal(K_\infty/K) = \Gamma \simeq \mathbb{Z}_p^r$, S any finite set of primes. Write $K_\infty = \cup K_\alpha$, K_α/K finite; let S_α be the set of primes of K_α above S,

$$A_{\alpha,S} = Pic(\mathcal{O}_{K_\alpha,S_\alpha})_{p^\infty}.$$

Put $\Lambda = \mathbb{Z}_p[[\Gamma]]$.

Theorem. There exists a canonical epimorphism in the category
(Λ-modules)/(pseudo-null Λ-modules)

$$\varprojlim_\alpha A_{\alpha,S} \longrightarrow Ext^1_\Lambda((\varprojlim_\alpha A_{\alpha,S})^\wedge, \Lambda)$$

The same formalism allows one to describe a variant of a p-adic height for Selmer complexes in terms of a "Bockstein map".

Alexander Schmidt (Heidelberg)

Singular Homology and Class Field Theory of Varieties over Finite Fields

The talk reports on joint work with M. Spieß. We give a generalization of Kato's and Saito's unramified classfield theory for smooth projective varieties over finite fields to the case of smooth, quasiprojective varieties and tame coverings. For this we use the singular homology groups $h_*(X)$ defined by Suslin.

Let k be a finite field, let $\overline{X}$ be a smooth, projective and geometrically connected variety over k. Let $X \subset \overline{X}$ be open. Let $\pi_1^t(X)^{ab}$ be the abelianized tame fundamental group of X. Let $(h_0(X))^0$ and $(\pi_1^t(X)^{ab})^0$ be the kernel of the surjections $h_0(X) \to h_0(\mathrm{Spec}(k)) \simeq \mathbb{Z}$ and

$\pi_1^t(X)^{ab} \to G_k^{ab} \simeq \hat{\mathbb{Z}}$, respectively.

Theorem. There exists a natural homomorphism (reciprocity map)

$$rec : h_0(X) \to \pi_1^t(X)^{ab}$$

which induces an isomorphism $rec_0 : (h_0(X))^0 \xrightarrow{\sim} (\pi_1^t(X)^{ab})^0$ of finite abelian groups which fits into the commutative diagram

$$\begin{array}{ccccccccc} 0 & \to & (h_0(X))^0 & \to & h_0(X) & \to & \mathbb{Z} & \to & 0 \\ & & \downarrow rec_0 & & \downarrow rec & & \downarrow & & \\ 0 & \to & (\pi_1^t(X)^{ab})^0 & \to & \pi_1^t(X)^{ab} & \to & \hat{\mathbb{Z}} & \to & 0 \end{array}$$

In particular, rec induces an isomorphism between the profinite completions of $h_0(X)$ and $\pi_1^t(X)^{ab}$.

Leila Schneps

Galois Actions on Fundamental Groups

Whenever V is a variety defined over $\mathbb{Q}$, there is a canonical outer action of $Gal(\overline{\mathbb{Q}}|\mathbb{Q}) =: G_{\mathbb{Q}}$ on its geometric π_1 (denoted $\pi_1(\overline{V})$). Following ideas of Grothendieck, we would like to identify the image of $G_{\mathbb{Q}}$ inside Out $(\pi_1(\overline{V}))$. We first consider $V = \mathbb{P}^1 - \{0, 1, \infty\}$ so that $\pi_1(\overline{V}) \cong \hat{F}_2$ (the profinite free group on 2 generators). Because the Galois action preserves inertia we obtain an injection

$$G_{\mathbb{Q}} \hookrightarrow \hat{\mathbb{Z}}^\times \times \hat{F}_2'$$

$$\sigma \longmapsto (\lambda, f)$$

via σ acting on $\hat{F}_2 =< x, y >$ by $x \longmapsto x^\lambda, y \to f^{-1}y^\lambda f$. In order to study the image of $G_{\mathbb{Q}}$ in $\hat{\mathbb{Z}}^\times \times \hat{F}_2'$, we introduce the automorphism group of a tower of fundamental groups: if we have a collection of $\mathbb{Q}$-varieties linked by $\mathbb{Q}$-morphisms f, the geometric fundamental group functor transforms it to a collection of profinite fundamental groups linked by induced homomorphisms f_*, and $G_{\mathbb{Q}}$ maps to the automorphism group of the whole collection (called a tower).
Two different towers give 2 different theorems:

Theorem 1 (F. Pop). If T is the tower corresponding to the collection of $\mathbb{P}^1 - \{n\ pts/\mathbb{Q}\}, n \geq 1$, and if $Out^*(T)$ denotes the automorphism group *preserving inertia*, then

$$G_{\mathbb{Q}} = \mathrm{Out}^*(T).$$

Theorem 2. (Drinfeld, Schneps, Lochak, Nakamura, Hatcher ...) If T is the tower of $\pi_1's$ of moduli spaces of Riemann surfaces, then $G_{\mathbb{Q}} \subset Out^*(T)$ and $Out^*(T)$ can be explicitly computed as a subset of $\hat{\mathbb{Z}}^\times \times \hat{F}_2'$ of elements satisfying a few simple properties.

Future goal: describe $G_{\mathbb{Q}}$ itself inside $\hat{\mathbb{Z}}^\times \times \hat{F}_2'$!

Ehud de Shalit (Jerusalem)

Residues on the Bruhat-Tits building of $GL_{d+1}(\mathbb{Q}_p)$ and de Rham cohomology of p-adic symmetric spaces

Let $[K : \mathbb{Q}_p] < \infty$, V_K a $(d+1)$-dimensional vector space over K and

$$\mathcal{X} = \mathbb{P}(V^*) - \bigcup_{a \in \mathbb{P}(V_K)} H_a$$

($H_a :=$ the hyperplane "$a = 0$") Drinfeld's p-adic symmetric space of dimension d. $G = PGL(V_K)$ acts on $\mathcal{X}$. The de Rham cohomology $H^k_{dR}(\mathcal{X})$ (closed rigid analytic forms modulo exact) $(0 \leq k \leq d)$ was computed by Schneider and Stuhler (Inv. Math 105,47-122 (1991)).

We propose a somewhat different approach, which also solves some questions left open by Schneider and Stuhler.

Let $\mathcal{T}$ be the Bruhat-Tits building of G, and $\hat{\mathcal{T}}_k$ the oriented k-simplices. We define the notion of a harmonic k-cochain as a certain map $\hat{\mathcal{T}}_k \to K$ satisfying "harmonicity conditions". If ω is a closed k-form we define its "residue" $\mathrm{res}_\sigma \omega$ along σ using a power-series expansion in terms of appropriate coordinates on X_σ, the open set in $\mathcal{X}$ mapping onto $|\sigma|$ under the canonical reduction map $\mathcal{X} \to |\mathcal{T}|$.
The cochain $c_\omega(\sigma) := \mathrm{res}_\sigma \omega$ is harmonic, it vanishes iff ω is exact, and the resulting map $\omega \mapsto c_\omega$ is an isomorphism

$$H^k_{dR}(\mathcal{X}) \xrightarrow{\sim} C^k_{\mathrm{har}}$$

onto the space of all harmonic k-cochains.

The proof relies on a detailed combinatorial study of the sub-algebra of logarithmic classes (those obtained from $d\log(\frac{a}{b})$, $a,\ b \in V_K - \{0\}$), using ideas borrowed from the theory of hyperplane arrangements.

Atsushi Shiho (Université d'Orsay)

Crystalline Fundamental Groups and p-adic Hodge theory

In cohomology theory, there is a comparison theorem which connects p-adic étale cohomology and de Rham cohomology or crystalline cohomology, and it is called p-adic Hodge theory. In view of Deligne's philosophy that the theory of (pro-unipotent quotient of) rational π_1 should be also motivic there should be a similar comparison theorem also for rational π_1 (and rational π_n). We gave a definition of crystalline fundamental groups π_1^{crys} by the Tannaka dual of the category of unipotent isocrystals, and proved a rational π_1-version of Berthelot-Ogus theorem (comparison between de Rham and crystalline π_1) and crystalline conjecture (a part of p-adic Hodge theory). We can also prove the rational π_n-version of the above theorems, and it seems that our method can be used for the proof of rational $\pi_1(\pi_n)$-version of stable conjecture.

M. Strauch (Münster)

On the Jacquet-Langlands correspondence in the cohomology of the Lubin-Tate deformation tower

Let $F|\mathbb{Q}_p$ be a finite extension, $\mathbb{X}$ a formal one-dimensional o_F-module of F-height h over $\overline{\mathbb{F}}_p$, ϖ a uniformizer of $F, K_n = 1 + \varpi^n M_h(o_F) \subset K_0 = GL_h(o_F)$. Define the deformation functor $\mathcal{M}_{K_n}$ of deformations of $\mathbb{X}$ with Drinfeld level-n-structure over complete, local o_F^{nr}-algebras R with residue field $\overline{\mathbb{F}}_p$ by: $\mathcal{M}_{K_n}(R) = \{(X, \delta, \phi)\}/\simeq$, X being a formal o_F-module over R, $\delta : \mathbb{X} \to X_{\overline{\mathbb{F}}_p}$ a quasi-isogeny, and $\phi : (\varpi^{-n} o_F/o_F)^h \to X[\varpi^n]$ a Drinfeld basis. Drinfeld has shown that $\mathcal{M}_{K_n}$ is representable by a finite flat $o_F^{nr}[[u_1, \ldots, u_{h-1}]]$-algebra which itself represents $\mathcal{M}_{K_0}$ (this is due to Lubin and Tate). Let M_{K_n} be the rigid analytic space assoc. to $\mathcal{M}_{K_n}$. There is a natural action of $K_0 \times B^*$ on M_{K_n}, where B^* is the group of quasi-isogenies of $\mathbb{X}$. Then define for any open compact $K \subset G = GL_h(F)$ the space M_K as a quotient. On $H_c^i = \lim\limits_{\to} H_c^i(M_K \otimes \mathbb{C}_p, \overline{\mathbb{Q}}_l)$, there is a smooth action of $G \times B^* \times W_F$. The construction of such representations is due to H. Carayol, generalizing work of Deligne ($F = \mathbb{Q}_p, h = 2$). Put $H_c^* = \sum(-1)^i H_c^i$. Let π be a supercuspidal representation of G. Then the following theorem holds by work of M. Harris and R. Taylor.

Theorem. $\mathrm{Hom}_G(H_c^*, \pi) \simeq (-1)^{h-1}\mathfrak{JL}(\pi) \otimes (h\text{-dim'l } W_F\text{-representation})$, where $\mathfrak{JL}(\pi)$ is the representation of B^* corresponding to π via the Jacquet-Langlands correspondence.

Using a Lefschetz trace formula for adic curves proved by R. Huber we show how to obtain this theorem by purely local means, if $h = 2$.

For arbitrary h, we show how to reduce this theorem to the existence of a suitable trace formula for higher-dimensional adic spaces (work in progress).

Takeshi Tsuji (RIMS, Kyoto Univ.)

Geometric Galois cohomology of B_{crys}

For a smooth analytic space X, the constant sheaf $\mathbb{C}_X$ has a canonical resolution $\Omega^\bullet_{X/\mathbb{C}}$, which allows us to compare the singular cohomology and the de Rham cohomology of X. We want an analogue of this argument in the p-adic case. Let V be a complete discrete valuation ring of mixed characteristic $(0, p)$ with a perfect residue field and assume that V is absolutely unramified. Let K be the field of fractions of V. We consider a (sufficiently small) p-adically complete and separated ring A formally smooth over V. Then the rings $\overline{A}$, B_{dR} $(\overline{A})$ and B_{crys} $(\overline{A})$ with their action of $Gal(\overline{A}/A)(= \pi_1(A_K))$ play the role of $\mathbb{C}_X$ in the p-adic case, where $\overline{A}$ denotes the maximal extension of A unramified in characteristic 0. Around ten years ago, O. Hyodo constructed a canonical $\pi_1(A_{\overline{K}})$-acyclic resolution of $\hat{\overline{A}}_K$ of the form $? \otimes_A \Omega^\bullet_A$ and established a theory of variation of Hodge-Tate structures. Soon after that, N. Tsuzuki established a B_{dR}-version. In the case of B_{crys} it is known that it also has a resolution, B_{crys} $(\overline{A}) \to B_{crys}$ $(\overline{A}) \otimes_A \Omega^\bullet_A$.

Theorem. The resolution is $\pi_1(A_{\overline{K}})$-acyclic.

As a corollary, one can describe the Geometric Galois cohomology of a crystalline representation of $\pi_1(A_K)$ in terms of the associated A_K-module with φ, ∇ and $\mathrm{Fil}^{\cdot}$. These results are based on the theory of almost étale extensions by G. Faltings.

Annette Werner (Münster)

Arakelov intersection numbers on $\mathbb{P}^n$ via buildings and symmetric spaces: Manin's analogies continued

Manin raised the question if one could enrich the somewhat formal picture of Arakelov theory at the infinite places by a suitable "geometry at ∞". In fact, he suggested certain three-dimensional hyperbolic manifolds as canditates for "models at infinity" for curves and interpreted some intersection numbers on them via hyberbolic geometry. He finds these manifolds by analogy with the non-archimedean places. In this talk, we generalize some of the results he showed for $\mathbb{P}^1$ to higher-dimensional projective spaces.

We calculate several non-archimedean local intersection indices of linear cycles in $\mathbb{P}^{n-1}$ via the combinatorial geometry of the Bruhat-Tits building for PGL_n. Besides, we show a formula for Néron's local height pairing (applied to hyperplanes and zero cycles) in the same spirit. This formula has an archimedean analogue using the differential geometry of the symmetric space $Z = SL(n,\mathbb{C})/SU(n)$. This might suggest to regard Z as some kind of "model at infinity" for $\mathbb{P}^{n-1}_{\mathbb{C}}$.

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