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Projectively equivalent metrics and the geodesic flow on ellipsoids

New proofs of two classical and related facts are discussed: the complete integrability of the geodesic flow on ellipsoids and the complete integrability of the billiard map inside an ellipsoid. The proofs are based on the observation that both dynamical systems have two invariant symplectic forms: the first coming from Euclidian geometry in the ambient space, the second from the Klein model of the hyperbolic geometry inside the ellipsoid. The geometry and topology of a class of transverse line fields along hypersurfaces in linear space are discussed too.   Howard Weiss

Structure of the Decomposition of Phase Space into Level Sets of Fundamental Dynamical Invariants

Lyapunov exponents, local entropies, Birkhoff averages, and pointwise dimensions are fundamental invariants in smooth ergodic theory. For equilibrium states for subshifts of finite type, conformal repellers, and Axiom-A surface diffeomorphisms, we effect a refined analysis of the decomposition of the phase space into level sets of these invariants. Typically we show that these invariants attain a (precisely identified) interval of values, each value attained on a dense set of positive Hausdorff dimension. Furthermore, we provide formulas for the precise Hausdorff dimension of these level sets and show that the sets where these invariants are not defined have maximal Hausdorff dimension. We can also achieve parts of this program for certain non-hyperbolic maps.

One interesting family of applications is to Diophantine approximation algorithms. For instance, the fine structure analysis of the level sets for the Lyapunov exponent for the Gauss map provides new insights on the precise exponential speed of convergence of the continued fraction algorithm.   Lai-Sang Young

Statistics of Billiards

The sketch of a proof of the exponential decay of correlations for billiards corresponding to the 2-dimensional Lorentz process is given. This consists of relating the dynamical system in question to countable state Markov chains with certain recurrence properties. These conditions are met because billiard maps are strongly hyperbolic, and their singularities or discontinuities are relatively mild.

Berichterstatter: Felix Schlenk

spectrum bundle $\Sigma_\Phi \to \mathrm{Ham}(M,\omega)$, where Σ_Φ is the set of critical values of the action functional for H, and secondly a refined algebraic analysis of the pair-of-pants ring structure on Floer homology leading to a power map on local Floer homology. Given the canonical isomorphism $\Phi_H\colon H^*(M) \xrightarrow{\cong} HF_*(H)$ and the long exact sequence for Floer homology filtered by the action

$$\cdots \xrightarrow{\partial_*} HF_k^{(-\infty,a]}(H) \xrightarrow{i^a} HF_k(H) \xrightarrow{j} HF_k^{(a,\infty)}(H) \xrightarrow{\partial_*} HF_{k-1}^{(-\infty,a]}(H) \to \cdots$$

we define for $\alpha \in H^*(M) \setminus \{0\}$,

$$c(\alpha, \Phi_H^1) = c(\alpha, H) = \inf\{a \mid \Phi_H(\alpha) \in \mathrm{Im}\, i^a\} \ \in \Sigma_\Phi.$$

For $\alpha = 1 \in H^0(M)$ we have $c(1,\Phi^n) \leq nc(1,\Phi)$ and study $\lim_{n\to\infty} c(1,\Phi^n)/n$. Friedrich Siburg **Minimal action in geometry and dynamics**

We investigate two situations where Mathers's minimal action plays an important role.

I. Elliptic fixed points of symplectic mappings

It is shown that, associated to an elliptic fixed point of general stable type, there is the germ of the minimal action, which is a symplectic invariant. In particular, the following quantities and properties are determined by this invariant:

- the Birkhoff normal form of the map
- the Liouville classes of ("proper") KAM-tori
- the C^0-integrability in the case of 2 dimensions

Applied to the geodesic flow near an elliptic closed geodesic thus yields that all these are length spectrum invariants under continuous deformations.

II. Hofer's geometry

Considering the group $\mathrm{Ham}(B^*S^1) = \mathrm{Ham}(S^1\times[-1,1])$, equipped with Hofer's metric d, we show that $d_\infty(id,\varphi) \geq \mathrm{osc}\,\alpha^*$, where $\varphi \in \mathrm{Ham}(B^*S^1)$ is generated by a convex Hamiltonian and α^* is the convex conjugate to the minimal action; d_∞ denotes the asymptotic distance

$$d_\infty(id,\varphi) = \lim_{N\to\infty} \frac{1}{N} d(id,\varphi^N).$$

We conjecture that one has equality for convex maps. Finally, we mention a joint work with Leonid Polterovich, where d_∞ is explicitly calculated for autonomous Hamiltonians on surfaces of infinite area. Domokos Szász

Correlation decay for multidimensional dispersing billiards

For dispersing billiards with finite horizon and no corner points given on the d-torus $\mathbb{T}^d$, $d \geq 3$, exponential decay of correlations is established. For $d = 2$, the analogous statement was proved by L.S. Young in 1998.

Our results are joint with P. Bálint, N. Chernov and I. P. Tóth. Serge Tabachnikov

Invariant Tori in Hamiltonian Systems near a Singular Point

We consider a Hamiltonian system of the form

$$\dot{u}_\alpha = H_{u_{N+\alpha}}, \quad \dot{u}_{N+\alpha} = -H_{u_\alpha}, \qquad \alpha = 1, \dots N,$$
$$H = H(u) = \sum_{\beta_1+\dots+\beta_{2N}\geq 2} H_{\beta_1\cdots\beta_{2N}} u_1^{\beta_1} \cdots u_{2N}^{\beta_{2N}},$$

where the power series is convergent near $u = (u_1, \dots, u_{2N}) = 0 \in \mathbb{R}^{2N}$ and has real coefficients. Under some non-resonance conditions for the eigenvalues of the linearized system and rather weak non-degeneracy conditions we can prove the existence of invariant tori in the above system in any neighbourhood of $u = 0$. David Sauzin

A new method for measuring the splitting of invariant manifolds

We study the so-called Generalized Arnold Model (a weakly hyperbolic, near-integrable, analytic Hamiltonian system), with $d+1$ degrees of freedom ($d \geq 2$), in the case where the perturbative term does not affect a fixed invariant d-dimensional torus. This torus is thus independent of the two perturbation parameters which are denoted ε ($\varepsilon > 0$) and μ.

We describe its stable and unstable manifolds by solutions of the Hamilton-Jacobi equation for which we obtain a large enough domain of analyticity. The splitting of the manifolds is measured by the partial derivatives of the difference ΔS of the solutions, for which we obtain upper bounds which are exponentially small with respect to ε.

A crucial tool of the method is a *characteristic vector field*, which is defined on a part of the configuration space, which acts by zero on the function ΔS and which has constant coefficients in well-chosen coordinates.

It is in the case where $|\mu|$ is bounded by some positive power of ε that the most precise results are obtained. In a particular case with three degrees of freedom, the method leads also to lower bounds for the splitting. Felix Schlenk

Rigidity and flexibility of symplectic embeddings

We study the rigidity and flexibility of symplectic embeddings of simple shapes. It is first proved that under the condition $r_n^2 \leq 2r_1^2$ the symplectic ellipsoid $E(r_1, \dots, r_n)$ with radii $r_1 \leq \dots \leq r_n$ does not embed in a ball of radius strictly smaller than r_n. We then use symplectic folding to see that this condition is sharp and to construct some nearly optimal embeddings of ellipsoids and polydiscs into balls and cubes. It is finally shown that any connected symplectic manifold of finite volume may be asymptotically filled with skinny ellipsoids or polydiscs. Matthias Schwarz

Continuous Sections in the Action Spectrum Bundle

Let (M^{2n}, ω) be a closed symplectic manifold with $\omega_{|\pi_2} = 0$ (and $c_1(TM)_{|\pi_2} = 0$). We study periodic points of Hamiltonian automorphisms. It was conjectured by C. Conley that under the above assumptions $\Phi = \Phi_H^1 \in \mathrm{Ham}(M, \omega)$ has infinitely many periodic points corresponding to contractible periodic solutions of the Hamilton equation. This conjecture is proven provided that M^{2n} admits a nontrivial finite covering, and so in particular for T^{2n}. The methods comprise firstly the study of homologically visible sections in the action a non-quasiconformal theory with compactness properties. New rectification theorems for degenerate conformal structures are obtained in this way. They can be used to obtain a new theory of renormalization (in the applications $|\{|\mu| > 1-\epsilon\}| \sim \epsilon^{\alpha}$, so David's theorem cannot be used). We obtain in this way an analytic justification of Douady's algorithm for extremal rays of the Mandelbrot set. The same analysis can be carried on (with considerable effort) to parameter space. This is the main ingredient for the proof of local connectivity of the Mandelbrot set. To the knowledge of the author this is the first time that smooth quotients of manifolds are constructed from an equivalence relation without fundamental domain. Paul Rabinowitz

Connecting orbits for a reversible Hamiltonian system

Consider the Hamiltonian system

(HS) $\quad \ddot{q} + V_q(t,q) = 0$

where V satisfies

(V_1) $\quad V \in C^2(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$ and V is 1-periodic in t and $q_1, \ldots, q_n$

(V_2) $\quad V$ is time reversible: $V(-t,q) = V(t,q)$.

Let $L(q) = \frac{1}{2}|\dot{q}|^2 - V(t,q)$ be the Lagrangian for (HS). Minimizing $I_1(q) = \int_0^1 L(q)\,dt$ over the class of 1-periodic functions produces 1-periodic solutions of (HS). If $\mathcal{M}$ is the class of such minimizers, by (V_1), $q \in \mathcal{M}$ implies $q + j \in \mathcal{M}$ for all $j \in \mathbb{Z}^n$. Assume

$(\mathcal{M})$ $\quad \mathcal{M}$ consists of isolated points.

Then it is known that for every $v \in \mathcal{M}$ there exists $w \in \mathcal{M} \setminus \{v\}$ and a solution Q of (HS) such that Q is heteroclinic from v to w. It was also shown by T. Maxwell that for any distinct pair $v, w \in \mathcal{M}$ there exists a minimal heteroclinic chain $Q_1, \ldots, Q_l$ of solutions of (HS) joining v and w. Suppose Q_i is heteroclinic from z_{i-1} to z_i, $1 \le i \le l$, so $z_0 = v$ and $z_l = w$. Using elementary minimization arguments, we obtain here further results:

Theorem A If the set of heteroclinics joining z_{i-1} to z_i is appropriately nondegenerate, there exist infinitely many actual heteroclinics joining v to w distinguished by the amount of time they spend near the intermediate states $z_1, \ldots, z_{l-1}$.

To state the second result, note that (V_2) implies that $Q_i(-t)$ is heteroclinic from z_i to z_{i-1}. Consider any admissible heteroclinic chain $u_1, \ldots, u_p$ constructed from $Q_1(t), \ldots, Q_l(t), Q_1(-t), \ldots, Q_l(-$ with e.g. u_1 asymptotic to the periodic $v(t)$ as $t \to -\infty$ and u_p asymptotic to the periodic $w(t)$ as $t \to \infty$.

Theorem B Under the hypothesis of Theorem A there exist infinitely many actual heteroclinics joining v and w distinguished by the amount of time they spend near the intermediate periodics. Helmut Rüssmann

Set $h^n(x,y) = \min\{h(x_0,x_1)+\cdots+h(x_{n-1},x_n),\ x_0 = x, x_n = y\}$, $h^\infty(x,y) = \lim_{n\to\infty} h^n(x,y)$ and $d(x,y) = h^\infty(x,y) + h^\infty(y,x)$. Then d is non-negative, symmetric, and satisfies the triangle inequality. Let

$$\mathcal{M} = \{x \in M \mid d(x,x) = 0\}.$$

Let $\mathcal{R}$ be the equivalence relation defined by $(x,y) \in \mathcal{R} \Leftrightarrow d(x,y) = 0$. Thus d is a metric on $\mathcal{M}/\mathcal{R}$. We say that elements X and Y in $\mathcal{M}/\mathcal{R}$ are separated if there exists $Z \in \mathcal{M}/\mathcal{R}$ such that $d(X,Z) + d(Z,Y) = d(X,Y)$. If X, Y are not separated then there is an action minimizing orbit $\mathcal{O}$ with $\alpha(\mathcal{O}) \subset \widetilde{X}$ and $\omega(\mathcal{O}) \subset \widetilde{Y}$, where $\widetilde{X}$ and $\widetilde{Y}$ are subsets of $TM \times S^1$ such that e.g. $\widetilde{X} \cap (TM \times 0)$ projects to X. This result, which is closely related to results in our article in Annales de l'Institut Fourier (Grenoble), 1993, gives a generalization simultaneously of results of Rabinowitz (discussed in this conference), Morse (class A geodesics), Aubry-LeDaeron and Bangert (for twist diffeomorphisms). The purpose of this talk was to describe this general theory, in which all these results can be stated and proved. Evgeni Neduv

Prescribed Minimal Period Solutions for Convex Hamiltonian Systems via Hofer-Zehnder capacity

We generalize the formula $\frac{dA(e)}{de} = T$, where $A(e)$ is the area bounded by the energy surface $H = e$ in $\mathbb{R}^2$ and T is the period of the periodic solution which lies on this energy surface. In $\mathbb{R}^{2n}$ the analogue of this area is given by the Hofer-Zehnder capacity. If we consider a family of energy surfaces $H = e$, where H is a convex Hamiltonian function on $\mathbb{R}^{2n}$, then on each energy surface the capacity is given by the action of some distinguished periodic solution. This action is known to be minimal. Using these properties one is able to show that the capacity as a function of the energy, $c(e)$, is continuous and differentiable on the left and on the right. Moreover, the left derivative $c'_-(e)$ is always bigger or equal than the right derivative $c'_+(e)$. In addition, $c'_-(e) = T_-$ and $c'_+(e) = T_+$, where T_-, T_+ are the periods of different periodic solutions on the energy surface $H = e$. These properties of $c(e)$ can be used efficiently for answering the question of existence of periodic solutions with given period for convex Hamiltonian systems. Some examples considered are convex asymptotically quadratic Hamiltonians, subquadratic Hamiltonians an so called classical Hamiltonians. Ricardo Pérez-Marco

Degenerate conformal structures and renormalization

We first presented a solution to a question of A. Katok (asked to the author in a precedent Oberwolfach meeting): there exists an analytic circle diffeomorphism that is C^∞ but not analytically linearizable. Then we presented some of the ideas and the analytic tool in a new construction of the renormalization for polynomials with focuss on the quadratic family. The main tool is the extension of the quasi-conformal theory to a theory that deals with degenerate conformal structures (that is $\|\mu\|_{L^\infty} = 1$ where μ is the Beltrami coefficient). Previous attempts (Lehto, David) impose too strong restrictions on μ that are incompatible with the application to renormalization. The novel idea is to combine the classical quasiconformal theory and the classical potential theory in order to obtain

Although for $n \geq 3$ there is a Cantor set of scattering orbits with given incoming and outgoing directions $\hat{\theta}^-, \hat{\theta}^+ \in S^2$, the Rutherford cross section (for the Kepler problem with $W = 0$) is a good approximation to the differential cross section of the n-centre problem:

$$\frac{d\sigma}{d\hat{\theta}^+}(E, \hat{\theta}^-, \hat{\theta}^+) = \frac{d\sigma}{d\hat{\theta}^+}(E, \hat{\theta}^-, \hat{\theta}^+)_{Ru} \left(1 + O\left(\frac{1}{E}\right)\right).$$

Finally we show that most results become wrong if the NC condition is violated. Sergei Kuksin

Randomly perturbed Hamiltonian PDE's

We discuss qualitative properties of dynamical systems which PDE's of the form

$$\langle \text{Hamiltonian equation} \rangle + \langle \delta\text{-small dissipation} \rangle = \langle \text{random forcing} \rangle$$

define in function spaces of periodic functions of the space variable x. We study quantities E_s, equal to time- and ensemble-average of the squared Fourier coefficients $|\hat{u}_s(t)|^2$. For them we prove a result which can be regarded as a weak form of the Kolmogorov-Obukhov law from the theory of turbulence. Patrice Le Calvez

A new proof of Bell's Theorem

Bell's Theorem asserts that if K is a non separating continuum invariant by an orientation reversing homeomorphism of the plane, then there is a fixed point in K. We give a simpler proof of this theorem.

Mark Levi

Feynman's racket, curvature and averaging

We describe an observation that behind the standard averaging procedure there lies some previously unobserved geometry. In particular, the effective forces in systems of the form, e.g. $\ddot{x} = a(t)f(x)$ (in $\mathbb{R}^n$) with rapid periodic t-dependence in a, are, as it turns out, the constraint forces (Lagrange multipliers) for an associated constrained system. As an example, the curvature of the pursuit curve (the tractrix) enters the averaged equations of the pendulum with vibrating suspension.

This observation gives a new physical and geometrical insight into the mechanism of the Paul trap, into the behaviour of particles in oscillating media, etc. John Mather

Variational construction of orbits

We consider a time periodic Lagrangian $L\colon TM \times S^1$ satisfying the Legendre condition, fiberwise superlinear growth, and completeness of the Euler-Lagrange flow. The Mañé critical value α_0 is $-\min \int L\, d\mu$ where μ varies over Euler-Lagrange invariant probabilities. Let

$$h(x, y) = \min \left\{ \int_0^1 (L + \alpha_0)(\gamma(t), \dot{\gamma}(t), t)\, dt,\ \gamma(0) = x, \gamma(1) = y \right\}.$$

translations one is able to produce conjugating maps which are entire with their inverses but have very large derivative and hence provide enough flexibility to produce a uniform distribution with increasing degree of accuracy.

Svetlana Katok

Frame flows and automorphic forms on complex hyperbolic spaces

Let G be a connected real semisimple Lie group of non-compact type, K its maximal compact subgroup, and Γ a lattice in G. We study automorphic forms on G in the case when its real rank is equal to one using a dynamical approach based on properties of the homogeneous flow on $\Gamma \setminus G$ and a Livshitz type theorem that we prove for such a flow. In the Hermitian case $G = SU(n,1)$, for one-dimensional representations of K we construct relative Poincaré series associated to closed geodesics on $\Gamma \setminus G/K$ and prove that they span the corresponding spaces of cusp forms.

This work is joint with Tatyana Foth. Andreas Knauf

The n-Centre Problem for Large Energies

The motion of a fast comet in the gravitational field of n fixed celestial bodies is considered. Their positions are $s_1, \ldots, s_n \in \mathbb{R}^3$, and their masses $z_1, \ldots, z_n \neq 0$. (Because of applications in molecular scattering repelling forces are considered, too.) The Hamiltonian $\widehat{H}\colon T^*\widehat{M} \to \mathbb{R}$, where $\widehat{M} = \mathbb{R}^3 \setminus \{s_1, \ldots, s_n\}$, is of the form

$$\widehat{H}(p,q) = \frac{1}{2}p^2 + V(q), \quad V(q) = -\sum_{l=1}^{n} \frac{z_l}{|q - s_l|} + W(q)$$

with $W \in C^\infty(\mathbb{R}^3, \mathbb{R})$ decaying at infinity. We first uniquely complete the Hamiltonian System $(T^*\widehat{M}, \sum dq_i \wedge dp_i, \widehat{H})$, obtaining (P, ω, H), where P is a smooth 6 dimensional manifold, ω a smooth symplectic form and $H\colon P \to \mathbb{R}$ smooth. Then we study the complete flow $\Phi^t\colon P \to P$. For large energies E we get a complete symbolic dynamics on $H^{-1}(E)$, assuming that no three centres s_l are on one line (NC condition). Whereas for $n = 1$ (Kepler problem) there is no bounded orbit and for $n = 2$ (Jakobi problem) there is just one, for $n \geq 3$ there is a Cantor set b_E of bounded orbits in $H^{-1}(E)$. The Hausdorff and box counting dimension of b_E are both of the order

$$\dim(b_E) = 1 + d(E)\left(1 + O\left(\frac{1}{E \log E}\right)\right),$$

where $d(E)$ is explicit. The topological entropy of $\Phi_E^t := \Phi^t_{|H^{-1}(E)}$ is given by

$$h_{top}(\Phi_E^1) = h_{top}(\Phi_E^1|_{b_E}) = c_1\sqrt{2E}\left(1 + c_2\frac{\log E}{E} + O\left(\frac{1}{E}\right)\right)$$

with explicit constants c_1, c_2.

Let $\lambda_0 = \frac{1}{2}[q \cdot dp - p \cdot dq]_{|S^3}$ be the standard contact form on $S^3 \subset \mathbb{C}^2$, where $q + ip \in \mathbb{C}^2$. For any smooth function $f\colon S^3 \to (0,\infty)$ the 1-form $\lambda = f\lambda_0$ is a contact form, i.e., $\lambda \wedge d\lambda$ is a volume form. Associated to λ we have the so-called Reeb vector field X_λ defined by $d\lambda(X_\lambda, \cdot) = 0$, $\lambda(X_\lambda) = 1$. The aim is to describe the dynamics of X_λ. The main tool is a pseudoholomorphic curve theory. Define $\xi = \ker(\lambda) = \ker(\lambda_0) \subset TS^3$ (the associated contact structure). Fix a compatible complex multiplication $J\colon \xi \to \xi$ and extend it to an $\mathbb{R}$-invariant almost complex structure $\tilde{J}$ on $\mathbb{R} \times S^3$ by putting $X \mapsto 1 \in \mathbb{R} \mapsto -X$. Let (S, j) be a closed Riemann surface and Γ be a finite set in S. A finite energy surface is a map $\tilde{u}\colon S \setminus \Gamma \to \mathbb{R} \times S^3$ which is proper such that

$$T\tilde{u} \circ j = \tilde{J} \circ T\tilde{u} \quad \text{on } S \setminus \Gamma \qquad \text{and} \qquad \int_{S\setminus\Gamma} u^* d\lambda < \infty. \tag{$*$}$$

We call Γ the set of punctures. If the $\mathbb{R}$-component near a puncture goes to $+\infty$ we call it positive and negative otherwise. A finite energy foliation $\mathcal{F}$ of $(\mathbb{R} \times S^3, \tilde{J})$ is a foliation of $\mathbb{R} \times S^3$ by the images of solutions of $(*)$ (assumed to be embedded) so that with $F \in \mathcal{F}$ also $a + F \in \mathcal{F}$ (observe that there is a natural $\mathbb{R}$-action on $\mathbb{R} \times S^3$. We show that such a foliation with additional properties exists and deduce strong dynamical information for the Reeb flow. For example, one can conclude that a Reeb vector field always has at least two geometrically distinct periodic orbits.

This work is joint with Krys Wysocky and Eduard Zehnder. Thomas Kappeler

Action-angle coordinates for KdV and applications

Theorem. On the Sobolev spaces H_0^N, $N \geq 0$, of 1-periodic functions with average 0 the Korteweg-deVries equation (KdV) admits global Birkhoff coordinates $(x_n, y_n)_{n\geq 1}$, where $I_n = (x_n^2 + y_n^2)/2$ are the action variables found by Flaschka and McLaughlin and $\theta_n = \arctan \frac{y_n}{x_n}$ are the angles prompted by the Abel map. Anatole Katok

New examples of uniquely ergodic real-analytic diffeomorphisms

Theorem. Let G be a compact connected Lie group with Haar measure χ, and let $L_{g_0}\colon g \mapsto g_0 g$ be the left translation. For any $g_0 \in G$ and any neighbourhood U of L_{g_0} in a real analytic topology there exists a diffeomorphism $S\colon G \to G$ in U such that χ is the only invariant Borel probability measure for S.

This is a progress in an area which started with the 1969 paper by D.V. Anosov and the author where a conjugation-approximation construction was developed which allowed to construct ergodic C^∞ perturbations of diffeomorphisms arising from actions of S^1 (e.g. rotations of the disc D^2). The version allowing to control all (as opposed to almost all) orbits were developed in the seventies by the author and by A. Fathi and M. Herman. The method is remarkably flexible and powerful as in producing examples with various ergodic and topological properties, but until now has not been made to work in the real analytic category.

The principal new ingredient is the use of right translations on the left homogenous space of G factored by a one-parameter group. By using twists and products of such where m_1 again denotes Lebesgue measure on $\mathbb{R}$.

Consequence. Let

$$A_\lambda(x) = \begin{pmatrix} \lambda a & \lambda b \\ \lambda^{-q} c & \lambda^{-q} d \end{pmatrix} \in M_{n,n}(\mathbb{R}),\ \lambda \in \mathbb{R}^*,\ q > 0,$$
$$p \in \mathbb{N}^*, a \in M_{p,p}(\mathbb{R}),\ \log^+ \|A_\lambda\| \in L^1 \text{ and } \log|\det a| \in L^1(\mu).$$

We denote by $\Lambda^p A_\lambda$ the p'th exterior product of A_λ acting on $\Lambda^p \mathbb{R}^n$. Then, when $|\lambda| \to \infty$, for λ except in a set of small Lebesgue measure at ∞,

$$\lambda_+(f, \mu, \Lambda^p A_\lambda) = p \log|\lambda| + \int_X \log|\det a(x)|\, d\mu(x) + \epsilon(\lambda),$$

where $|\epsilon(\lambda)| < \epsilon$ for any fixed $\epsilon > 0$ that we must fix if we want estimates of the measure of the bad set.

Example.

$$A_\lambda(x) = \begin{pmatrix} E + \lambda\varphi(x) & -1 \\ 1 & 0 \end{pmatrix} \in \mathrm{SL}(2, \mathbb{R}),\ E \in \mathbb{R},\ \log|\varphi| \in L^1.$$

Then

$$\lambda_+(f, \mu, A_\lambda) = \log|\lambda| + \int \log|\varphi(x)|\, d\mu(x) + \epsilon(\lambda)$$

when $|\lambda| \to \infty$ except for λ in a set of small Lebesgue measure at $+\infty$. To prove this consider $\frac{1}{\lambda} A_\lambda$ and consider $z = \frac{1}{\lambda^{q+1}}$ at $z = 0$ along the real axis.

Perturbation Lemma.

We suppose that X is a compact metric space, $f_j : X \to X$ are homeomorphisms, $f_j \mu_j = \mu_j$ are invariant probability measures, and $A_j \in C^0(X, \mathrm{SL}(2, \mathbb{R}))$. Moreover, when $j \to +\infty$, then $f_j \to f$ in the C^0-topology, $\mu_j \rightharpoonup \mu$ weakly, and $A_j \to A$ in the C^0-topology.

By upper semicontinuity, $\limsup_{j \to +\infty} \lambda_+(f_j, \mu_j, A_j) \le \lambda_+(f, \mu, A)$, and many natural examples show that equality in general does not hold.

Proposition. There exist sequences $j_k \to \infty$, $\lambda_{j_k} \to 0 \mod 1$ such that

$$\lambda_+(f_{j_k}, \mu_{j_k}, r_{\lambda_{j_k}} A_{j_k}) = \lambda_+(f, \mu, A).$$

Here, $r_\lambda = \begin{pmatrix} \cos 2\pi\lambda & -\sin 2\pi\lambda \\ \sin 2\pi\lambda & \cos 2\pi\lambda \end{pmatrix}$. This proposition is a consequence of the Brelot Cartan theorem in potential theory. We discussed various applications to 2-dimensional dynamics and generalizations to symplectic diffeomorphisms. Helmut Hofer

Global systems of surfaces of section

Remarks on Liapunoff exponents

Let $(X, \mathcal{B}, \mu) \cong ([0,1], \mathcal{B}, dx)$ be a Lebesgue measure space and $f\colon (X, \mathcal{B}, \mu) \to (X, \mathcal{B}, \mu)$ an automorphism. Let $x \mapsto A(x) \in M_{n\times n}(\mathbb{R}) \subset M_{n\times n}(\mathbb{C})$ be a μ-measurable mapping, where $M_{n\times n}(\mathbb{R})$ denotes the $n \times n$ matrices over $\mathbb{R}$. On $M_{n\times n}(\mathbb{R})$ we put the operator norm induced by the hermitian metric $\sum_{j=1}^{n} |z_j|^2$ on $\mathbb{C}^n$. We suppose that

$$\log^+ \|A(x)\| \in L^1(\mu)$$

and define the integrated maximal Liapunoff exponent

$$\lambda_+(f, \mu, A) = \lim_{p\to\infty} \frac{1}{p} \int_X \log \|A^p(x)\| \, d\mu(x), \quad p \in \mathbb{N}^*$$

where

$$A^{(p)}(x) = A(f^{p-1}(x)) \circ A(f^{p-2}(x)) \circ \cdots \circ A(x).$$

If $A(x) \in \mathrm{SL}(n, \mathbb{R})$ for all x, then $\lambda_+(f, \mu, A) \geq 0$. We suppose that A_λ depends C^ω on a parameter $\lambda \in [a, b]$ and furthermore that there exists an open subset $U \subset \mathbb{C}$ containing $[a, b]$ such that the family A_λ extends to a family A_λ with $\lambda \in U$ in such a way that for μ almost every x the mapping $U \to M_{n\times n}(\mathbb{C})$, $\lambda \mapsto A_\lambda(x)$ is holomorphic and (the problem is local) such that there exists $\psi \in L^1(\mu)$ such that

$$\sup_{\lambda\in U} \log \|A_\lambda(x)\| \leq \psi(x) \quad \text{for } \mu \text{ almost every } x.$$

Proposition. The function $U \to \mathbb{R}$, $\lambda \mapsto \lambda_+(f, \mu, A_\lambda) =: \varphi(\lambda)$ is a subharmonic function.

By the properties of subharmonic functions on the real axis this implies

- If φ is not identically equal to $-\infty$, then $E = \varphi^{-1}(-\infty)$ has log capacity $c(E) = 0$, and hence has Hausdorff dimension 0.
- $\varphi|_{[a,b]}$ is $L^1_{loc}(dx)$, where $m_1 = dx$ is the Lebesgue measure on $\mathbb{R}$.
- If $[a_1, b_1] \subset [a, b] \subset U$, then $\limsup_{x\searrow a_1} \varphi(x) = \varphi(a_1)$, i.e., $[a_1, b_1]$ is non thin at a_1.
- This implies that $\varphi|_{\mathbb{R}}$ has the Darboux property.

Let us suppose that $a_1 = 0$ and $\epsilon > 0$. We consider the open subset of $]0, b_1]$

$$U_\epsilon = \{x, \varphi(x) < \varphi(0) - \epsilon\},$$

and, for $r > 0$, $U_{\epsilon,r} = U_\epsilon \cap [0, r]$. Wiener's theorem on the characterization of thin sets (1924) implies

$$\limsup_{r\to 0} \frac{\log \frac{1}{r}}{\log \frac{1}{c(U_{\epsilon,r})}} = 0 \quad \Longrightarrow \quad \lim_{r\to 0} \frac{c(U_{\epsilon,r})}{r^\alpha} \geq \frac{1}{4} \frac{m_1(U_{\epsilon,r})}{r^\alpha} = 0 \ \text{ for all } \alpha >> 1,$$

Action minimizing solutions of the 4-body problem in $\mathbb{R}^3$

We look at minima of the action functional among H^{-1} loops $x(t)$ of a given period T in the configuration space of the n-body problem in $\mathbb{R}^p$ such that $x(t+T/2) = -x(t)$. For p even, these are relative equilibria. For $p = 3$, $n = 4$ and equal masses, the minima do not correspond to homographic motions. Adding a $\mathbb{Z}/4\mathbb{Z}$ (respectively $\mathbb{Z}/3\mathbb{Z}$ symmetry assumption on the configurations, we can exclude collisions: The minimum is then a periodic orbit whose configuration "hesitates" between the regular tetraedron and a planar central configuration which is the square (respectively the equilateral triangle with mass at the center of mass).

This work is joint with A. Venturelli. Giovanni Forni

Deviation of ergodic averages for measured foliations on compact surfaces

A theorem of A. Zorich (Ergodic Theory Dynam. Systems, 1997) states the following: Let $E\colon [0,1] \to [0,1]$ be a generic interval exchange transformation (uniquely ergodic), let I be a subinterval and let χ_I be the characteristic function of I. Then

$$\max_{x\in[0,1]} \limsup_{N\to+\infty} \frac{\log \left| \sum_{i=0}^{N-1} \chi_I(E^i x) - N \operatorname{length}(I) \right|}{\log N} = \lambda_2$$

where $0 \le \lambda_2 < 1$ is the second Lyapunov exponent of a suitable renormalization dynamics. We prove that $\lambda_2 > 0$ and we give a description of the invariant subspaces of the renormalization dynamics in terms of *basic currents* for the horizontal and vertical foliations of quadratic differentials. In addition, we prove the following generalization of Zorich's theorem.

Theorem. Let (M, Φ_X^t, ω) be the suspension of the generic interval exchange transformation E to a Riemann surface M of genus $g \ge 2$. The flow Φ_X^t is almost everywhere defined and preserves the area form ω which is degenerate at a finite set of points. There exists an $H^{-1}(M)$ X-invariant distribution (i.e. a solution of the equation $X\mathcal{D} = 0$, where $\mathcal{D}$ is in the Sobolev space $H^{-1}(M)$), $\mathcal{D}_2^X$, such that for all $f \in H^1(M)$, $\int_M f\,\omega = 0$, $\mathcal{D}_2^X(f) \neq 0$ and

$$\limsup_{T\to+\infty} \frac{\log \left| \int_0^T f(\Phi_X^t(x))\,dt \right|}{\log T} = \lambda_2 > 0$$

for almost all $x \in M$. John Franks

Regions of Instability for Non-Twist Maps

We investigate an analogue of the concept of regions of instability for twist maps introduced by Birkhoff. We consider the case of area preserving annulus diffeomorphisms which satisfy certain generic properties but are not twist maps. Several properties analogous to those of classical regions of instability are proved.

These results represent joint work with Patrice LeCalvez. Michael Herman

Dynamics of codimension 1 laminations and minimal volume of real homology classes

In Riemannian geometry J. Mather's minimal measures occur as "minimizing closed normal 1-currents" from Geometric Measure Theory, cf. [B]. While much is known about these concepts if $\dim M = 2$, in particular due to work by D. Massart, the situation is less well understood if $\dim M > 2$. It was J. Moser's insight that similar phenomena as in the 2-dimensional case should occur if one looks at codimension 1 minimizing objects in n-manifolds.

We pursue this idea and study minimizing closed normal $(n-1)$-currents in compact oriented n-dimensional Riemannian manifolds. These are the natural representatives of minimal $(n-1)$-volume in a given real $(n-1)$-homology class. It turns out [AB] that they can be described as measured laminations by minimal hypersurfaces (possibly with singularities). Now ideas developed by J. Plante, P. Arnoux - G. Levitt and G. Levitt can be used to decompose these laminations (and hence the corresponding currents) into a set of compact leaves (representing finitely many homology classes) and finitely many minimal sets consisting of noncompact leaves. This implies results on the boundary structure of the stable norm ball in $H_{n-1}(M, \mathbb{R})$.

This work is joint with Franz Auer.

[B] V. Bangert: Minimal measures and minimizing closed normal one-currents, to appear in GAFA.

[AB] F. Auer, V. Bangert: The structure of minimizing closed normal currents of codimension one, in preparation. Misha Bialy

Around E. Hopf rigidity

We discuss three classes of variational problems where one can expect that E. Hopf rigidity holds. The first class is the twisted geodesic flow, i.e., the motion of a particle on the torus in the presence of a magnetic field. Here one expects that the absence of conjugate points for all extremals implies that the metric is flat and the magnetic field vanishes. This can be shown at least for conformally flat Riemannian metrics.

The second example is the non-linear elliptic PDE

$$\Delta u = -V'(u, x_1, \ldots, x_n).$$

It is interesting that the answer here depends on the dimension n.

Finally, we discuss the case $n = 1$, i.e., we consider the Hamiltonian system on $\mathbb{R}^2$

$$(*) \qquad \begin{cases} \dot{q} &= p \\ \dot{p} &= -V'(q,t). \end{cases}$$

Theorem. If V is 2π-periodic in q and satisfies $\int_0^{2\pi} \left|V_q'(q,t)\right|^2 dq < const$, then E. Hopf rigidity holds in the following sense: For any initial foliation of the cylinder $\mathbb{R}(p) \times S^1(q)$ by graphs over $S^1(q)$ some of the leaves develop shocks under the evolution of the system $(*)$. The only case when this does not happen is the case $V_q' \equiv 0$. Alain Chenciner

Abstracts of the talks

Alberto Abbondandolo

Morse-Floer theory on Hilbert spaces

In this talk we describe a joint research carried on with Prof. Pietro Majer, Università di Parma, Italy. Such a research was motivated by trying to deal with the problem of existence of infinitely many periodic points for a certain class of symplectic diffeomorphisms of $\mathbb{R}^{2N}$ (in some sense a many dimensions generalization of a theorem of Franks). Here we present only the abstract theory.

Consider a functional f on a Hilbert space E of the form

$$f(x) = \frac{1}{2}(Lx, x) + b(x),$$

where L is an invertible self-adjoint operator on E and $b \in C^2(E)$ has a compact gradient. If L is negative on an infinite dimensional subspace, all the critical points of f have an infinite Morse index. However, in view of the special form of the functional, it is possible to define a finite relative Morse index.

Using such an index we develop a Floer homology theory for such functionals, under the following extra assumptions: f satisfies the Palais-Smale condition, f is a Morse-Smale function and f is bounded from below on the set of critical points of relative Morse index q, for every $q \in \mathbb{Z}$. If $C_q(f)$ is the $\mathbb{Z}_2$-module spanned by the critical points of f of relative Morse index q, the flow lines of the negative gradient of f allow to define a homomorphism $\partial : C_q(f) \to C_{q-1}(f)$. The first relevant fact is

Theorem 1 *∂ is a boundary operator, meaning that $\partial \circ \partial = 0$.*

The proof can be achieved by looking at the geometry of the flow, and does not require the intricate gluing construction which is necessary for proving the corresponding statement in Floer theory for pseudo-holomorphic curves. Theorem 1 allows to define the Floer homology $FH_*(f)$ of f as the homology of the complex $(C_*(f), \partial)$.

Then we deal with the problem of computing such a homology. Of course, since the function f is not defined on a compact manifold, $FH_*(f)$ will depend on the behaviour of f at infinity. The main result is the following.

Theorem 2 *Assume that f_0 and f_1 satisfy either*
1. $\|f_0 - f_1\|_\infty$ is bounded,
or
2. the family $\{f_\lambda = \lambda f_1 + (1-\lambda) f_0 \mid \lambda \in [0,1]\}$ satisfies the Palais-Smale condition uniformly and there exists $R > 0$ such that $\nabla f_\lambda(x) \neq 0$ for every $\lambda \in]0,1[$, if $\|x\| \geq R$.
Then $FH_*(f_1) = FH_*(f_0)$.

A two-variables extension of the functionals f_0 and f_1 allows to derive such a result quite easily from the identity $\partial \circ \partial = 0$ of Theorem 1. Victor Bangert

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 29/1999

Dynamical Systems

18.07. bis 24.07.1999

Die Tagung fand unter der Leitung von H. Hofer (New York), J. C. Yoccoz (Orsay) und E. Zehnder (Zürich) statt.

Gegenstand der Tagung waren die neuen Resultate und Entwicklungen im Gebiet der klassischen dynamischen Systeme, insbesondere der endlich und unendlich dimensionalen Hamilton'schen Systeme. Themen waren unter anderem Stabilität und Instabilität von Systemen, Mather Theorie und Verallgemeinerungen, Reebflüsse und Fragen der symplektischen Geometrie.