MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Random Systems

29.8. - 4.9.1999

The Mathematics Research Institute at Oberwolfach (Germany) hosted the meeting entitled "Random Systems" which was organized by L. Arnold (Bremen), I. Goldsheid (London) and Y. Kifer (Jerusalem). The meeting was attended by 41 scientists from France, Germany, Israel, Italy, Russia, Switzerland, the United Kingdom and the United States.

This was an interdisciplinary meeting whose main purpose was to bring together two cultures: probabilists (from stochastic dynamics, stochastic particle systems, systems in random environment) with dynamical systems people (from smooth/topological dynamics and ergodic theory with applications), and allow for an exchange of ideas, methods and models. The talks can be grouped according to four main themes:

Stochastic Dynamics: K. Athreya (Ames), V. Baladi (Paris), M. Klünger (Bremen), M. Scheutzow (Berlin) and Ö. Stenflo (Jerusalem). In addition, an informal seminar on **Random Attractors** was organized on Monday night, with talks by H. Crauel (Berlin), P. Imkeller (Berlin) and B. Schmalfuss (Merseburg).

Statistical Mechanics Systems: C. Boldrighini (Roma), A. Bovier (Berlin), J.-D. Deuschel (Berlin), D. Ioffe (Haifa), Y. Kondratiev (Bonn), R. Minlos (Moscow), A. Soshnikov (Caltech), O. Zeitouni (Haifa) and E. Zhizhina (Moscow).

Dynamical Systems and Ergodic Theory: J. Aaronson (Tel Aviv), A. Karlsson (Yale), A. Katok (Penn State), G. Keller (Erlangen), F. Ledrappier (Paris), C. Liverani (Roma), M. Pollicott (Manchester), O. Sarig (Tel Aviv), J. Schmeling (Berlin) and R. Sharp (Manchester).

Random Walks on Groups: Y. Guivarc'h (Rennes) and V. Kaimanovich (Rennes).

The feedback received by the organizers about the success of the meeting was rather positive. New contacts between the two cultures were started, and existing ones were extended and deepened.

Vortragsauszüge

J. Aaronson: Group extensions of Gibbs-Markov maps

(Joint work with M. Denker)

These are essentially random walks driven by weakly dependent base transformations and with properties like random walks with i.i.d. jump distributions, as we attempt to show. Here we present two partial results on exactness.

A *Markov map* is a quintuple $(X, \mathcal{B}, m, T, \alpha)$ where $(X, \mathcal{B}, m, T)$ is an ergodic transformation, and $\alpha \subset \mathcal{B}$ is a countable or finite partition satisfying: (i) $\bigvee_{n\geq 0} T^{-n}\alpha = \mathcal{B}$, (ii) $Ta \in \sigma(\alpha)$ $\forall a \in \alpha$ and (iii) $T : a \to Ta$ is invertible, non-singular $\forall a \in \alpha$. Without loss of generality, $\alpha = \{a_s : s \in S\}$, $X \subseteq S^{\mathbb{N}}$, $T =$ shift and $X = sptm = \{x = (x_1, x_2...) \in S^{\mathbb{N}} : m([x_k, x_{k+1}]) > 0 \forall k\}$. Thus $T : X \to X$ is a continuous map of a Polish space. The Markov map has the *Renyi property* if $\exists c > 1$ such that $V_a'(x) \leq CV_a'(y)$ $\forall a \in \alpha_0^{n-1}$, $m \times m$-a.e. $(x, y) \in T^n a \times T^n a$, where $V_a : T^n a \to a$ is an inverse branch of T^n and $V_a' := \frac{dm \cdot V_a}{dm}$.

We say that the Markov map has the *Gibbs property* if $\exists c > 1, 0 < r < 1$ such that $\left|\frac{V_a'(x)}{V_a'(y)} - 1\right| \leq r^{\tau(x,y)}$ for all $n \geq 1$, $a \in \alpha_0^{n-1}$, $m \times m$- a.e. $(x, y) \in T^n a \times T^n a$ where $\tau(x, y) = \min\{k \geq 1 : x_k \neq$

$y_k\}$. A function $\Phi : X \to \Omega$ has *finite memory* if there exists N such that $\Phi(x) = \Phi(x_1, ...x_N)$.

A *Gibbs Markov* map is a Markov map $(X, \mathcal{B}, m, T, \alpha)$ with the Gibbs property, and such that $\inf_{a\in\alpha} m(Ta) > 0$.

Theorem 1: Let $(X, \mathcal{B}, m, T, a)$ be a mixing Gibbs-Markov map, and let the map $\Phi : X \to \mathbb{R}^d$ be uniformly r-Hölder continuous on each $a \in \alpha$, i.e. $\sup_{a\in\alpha} \sup_{x,y\in a} \frac{\|\Phi(x)-\Phi(y)\|}{r^{\tau(x,y)}} < \infty$, *aperiodic* in the sense that $\gamma(\Phi) = z\overline{g}g \circ T$ for $\gamma \in \widehat{\mathbb{R}^d}$, $z \in S^1$, $g : X \to S^1$ measurable has no solution except $\gamma \equiv 1$, $z = 1$, $g \equiv$ const, and suppose that for all $\lambda > 1$ there exists n_k such that $\frac{\Phi_{n_k}}{\lambda^{n_k}} \to 0$ a.e.. Then T_Φ is exact.

Theorem 2: Let $(X, \mathcal{B}, m, T, \alpha)$ be a mixing Markov map with the Renyi property and let $\Phi : X \to \mathbb{R}^d$ have finite memory. If T_Φ is topologically mixing then T_Φ is exact.

Notes: (i) $\mathbb{R}^d$ can be replaced by an arbitrary locally compact, Abelian, polish group equipped with a Lipschitz norm.

(ii) For a relevant preprint, see http://www.math.tau.ac.il/~aaro.

K. B. Athreya: Random logistic and other population dynamics maps

Let $\{C_n\}_1^\infty$ be i.i.d. with values in $[0, 4]$. Let $\{X_n\}_0^\infty$ be the Markov chain $X_{n+1} = C_{n+1}X_n(1-X_n)$ for $n = 0, 1, 2, ...$ We establish

Theorem 1: There exists a probability measure π invariant for $\{X_n\}$ satisfying $\pi(0, 1) = 1$ iff $E \log C_1 > 0$ and in this case $E \log C_1 = -\int \log(1 - x)\pi(dx)$.

We extend this to population dynamics maps. Let $\mathcal{F} = \{f : [0, \infty) \to [0, \infty)$, $f'(0) > 0, \overline{\lim}_{n\to\infty} \frac{f(x)}{f'(0)x} < 1\}$. Let $\{f_i\}_1^\infty$ be i.i.d. from $\mathcal{F}$ and generate a Markov chain $\{X_n\}$ by $X_{n+1} = f_{n+1}(X_n)$, $n = 0, 1, 2, ...$

Theorem 2: Let $E(\log C_1)^+ < \infty$, where $C_1 = f_1'(0)$, and $g(x) = \frac{f_1(x)}{f_1'(0)x} < 1$ for all x with probability 1. Then there exists an invariant probability measure π for $\{X_n\}$ satisfying $\pi(0, 1) = 1$ iff $E \log C_1 > 0$ and in this case $E \log C_1 = -\int \log g(x)\pi(dx)$.

V. Baladi: Rates of random mixing for random (i.i.d.) unimodal maps

(Joint work with M. Benedicks and V. Maume-Deschamps)

It has been known for a while (Jakobsen, Collet-Eckmann, Benedicks-Carleson) that the interval map $f_a(x) = a - x^2$ admits a mixing absolutely continuous invariant measure for a positive Lebesgue measure set of parameters a. Keller-Novicki and Young independently proved that its correlation functions decay exponentially, for smooth test functions. We consider random compositions $x \mapsto f_a(x) + \omega$ with ω chosen in $[-\varepsilon, \varepsilon]$ in an i.i.d. way following an absolutely continuous probability distribution γ_ε. Adapting to random systems, on the one hand a notion of hyperbolic times due to Alves and on the other hand a probabilistic coupling method introduced by Young to study rates of mixing, we proved stretched exponential bounds for the random rates of mixing of almost every itinerary and smooth test functions.

C. Boldrighini: Almost-sure results for random walks in fluctuating random environments

Consider a discrete-time random walk (RW) on the lattice $\mathbb{Z}^\nu$, $\nu = 1, 2, ..$ with transition probabilities $P(X_{t+1} = x + u|X_t = x, \xi = \overline{\xi}) = P_0(u) + \varepsilon c(u; \overline{\xi}(t, x))$, depending on a random field $\xi = \{\xi(t, x) : (t, x) \in \mathbb{Z}^{\nu+1}\}$, which is a collection of i.i.d. random variables distributed with some non-degenerate distribution π. Here $\varepsilon > 0$ is a small parameter and the RW P_0 is finite-range and completely irreducible. $c(u; s)$ is also finite range and $< c(u; \cdot) >_\pi = 0$.

We consider the RW for fixed ξ. The main results are:

(i) For all $\nu \geq 1$ and ε small enough the CLT holds for X_t, and all normalized models tend to the limiting gaussian moments.

(ii) The local CLT holds with the gaussian term of the averaged RW P_0 modulated by a factor depending on the field, as seen from the final point.

Similar results can be obtained in dimension $\nu \geq 3$ for "directed polymer" models in which a trajectory bond $(t,x) \to (t+1, x+u)$ is given weight $P_0(u)(1+\varepsilon\xi(t,x))$.

A. Bovier: Stochastic dynamics of random mean-field models

Recent years have seen considerable progress in the mathematically rigorous understanding of the equilibrium statistical mechanics of some disordered mean field spin-systems: the REM, the Hopfield-model, and others. From a physical point of view, in particular in disordered systems, the question of the dynamics and the approach to equilibria are often far more relevant. One of the fundamental issues that is to be resolved concerns the question of how the knowledge of the equilibrium distribution (Gibbs measures) can give information on the long-time behavior of the dynamics.

I report on some first results in this direction in certain types of meanfield dynamics, i.e. Markov chains on discrete state space $\Gamma_N \subset \varepsilon\mathbb{Z}^d$ that are reversible with respect to the measure Q_N of the form $Q_N(x) \sim \exp(-NF_N(x))$, where F_N converges to some deterministic function F_0 and $F_N - F_0$ is random and after scaling with some N^α it converges to some stochastic field.

H. Crauel: Random set and point attractors

After recalling the notion of random dynamical systems (RDS) we introduce the notion of random point and set attractors, which are characterized as random sets attracting all deterministic points and bounded sets, respectively, of the state space. Basic properties are:

(i) a necessary and sufficient condition for their existence is the existence of a not necessarily invariant random compact set attracting all points or bounded sets, respectively,

(ii) the set attractor is ($\mathbb{P}$-a.s.) unique, connected, supporting all invariant measures and containing all unstable sets of its subsets,

(iii) there exists a minimal point attractor,

(iv) set and minimal point attractor are measurable with respect to the past.

However, (and this is a major difference when comparing with deterministic dynamical systems), the *point* attractor does *not* support all invariant measures. For *white noise* systems (the most prominent examples of which are products of i.i.d. maps and systems induced by stochastic differential equations) the point attractor still does support all invariant *Markov* measures. A simple example, which is a particular realization of Brownian motion on the circle S^1, shows that there may exist anticipating invariant measures not supported by the point attractor.

J.-D. Deuschel: Random interfaces: pinning, entropic repulsion and wetting transition

(Joint work with E. Bolthausen, O. Zeitouni and H. Giacomi)

We consider the anharmonic crystal, or lattice massless field, with zero-boundary conditions outside $D_N = [-N,N]^d \cap \mathbb{Z}^d$ and N a large natural number, that is the finite volume Gibbs measure $\mathbb{P}_N$ with Hamiltonian $\sum_{<x,y>} V(S_x - S_y)$, V a strictly convex even function. We investigate two types of interaction with the wall $\{S_x = 0, x \in D_N\}$: the repulsion $\Omega_N^+ = \{S_x \geq 0, x \in D_N\}$ and a weak δ-pinning of strength e^J. Under the repulsion condition Ω_N^+, the influence is repelled to a height of $O(\sqrt{\log N})$ if $d \geq 3$ and $O(\log N)$ if d=2. On the other hand, any weak pinning produces a localization and generates a mass, i.e. an exponential decay of the covariances in $d \geq 2$. When the two effects, repulsion and pinning compete, M. Fisher has shown a wetting transition for the (simple) $1-d$ model, that is delocalization for weak pinning and localization for strong pinning parameter J. We prove that for the harmonic (gaussian) case, pinning always wins in $d \geq 3$. The delicate 2-dimensional case remains open.

Y. Guivarc'h: Products of random matrices with dynamical counterparts

(Joint work with J.P. Conze)

Let $G = SL(d,\mathbb{R})$, $N =$ group of upper triangular matrices, A the group of diagonal matrices, Γ a discrete subgroup of G with $\mathrm{Zcl}(\Gamma) = \mathrm{G}$. Let μ be a probability measure on G such that $\int \log \|g\| d\mu(g) < \infty$, Γ_μ the group generated by the support of μ. One supposes $\mathrm{Zcl}(\Gamma_\mu) = \mathrm{G}$. One uses the properties of the random walk on G/N in order to get properties of the action of N or A on $\Gamma \setminus G$. One shows that there is an attractor $\mathcal{A} \subset \Gamma \setminus G$ for A such that A has a dense orbit in $\mathcal{A}$, a lot of recurrent points. Also there is a closed invariant set $\mathcal{N} \subset \Gamma \setminus G$ for N such that many points have dense orbits. If Γ is of ping-pong type, then every point of $\mathcal{N}$ has a dense N-orbit. Also in that case every point of $\mathcal{A}$ is recurrent.

P. Imkeller: Random attractors, cohomology of flows of stochastic and random differential equations

We consider random dynamical systems generated by smooth stochastic differential equations. To construct random attractors, we use a comparison argument based on Lyapunov functions. This comparison argument is not well compatible with Ito's stochastic calculus. For this reason we pass, via a random coordinate change, to a random differential equation the dynamics of which is generated by an appropriate random stationary fluctuating vector field. This coordinate change (conjugation) faithfully maps random attractors, but is interacting in its own right, being a Lyapunov cohomology. We show that conjugations exist provided the diffusion vector fields generate solvable Lie algebras. This way we are able to construct random attractors for many interesting examples including randomly perturbed Duffing-van der Pol oscillators.

D. Ioffe: A rigorous approach to the Ornstein-Zernike theory of fluctuations

We show that for every $p < p_c$ and any dimension $d \geq 0$ the two point function $g_p(x) \widehat{=} \mathbb{P}_p(0 \leftrightarrow x)$ of the Bernoulli bond percolation satisfies the Ornstein-Zernike asymptotic relation

$$g_p(x) = \frac{\Psi_p(\vec{n}(x))}{\|x\|^{\frac{d-1}{2}}} e^{-\xi_p(x)}(1 + o(1)),$$

where ξ_p is the (direction dependent) inverse correlation length, and $\Psi_p : S^{d-1} \to \mathbb{R}$ is a positive analytic function acting on unit directions $\vec{n}(x) \widehat{=} x/\|x\|$.

V. Kaimanovich: Markov chains on equivalence relations

Usually probabilists prefer to study Markov chains whose transition probabilities are absolutely continuous with respect to the stationary measure of the chain. However, there are numerous examples of Markov chains with a finite stationary measure whose transition probabilities are concentrated on countable (or just finite) sets. Such sets arise in ergodic theory (e.g. random walks on the preimages of an endomorphism), but also in a very natural way in probability (random walks in random environment). The typical situation is that, although there is a global finite stationary measure, the "local" behavior of sample paths (provided the starting point is fixed) may be rather complicated. This is analogous to what happens with Brownian motion on fiber bundles (ergodic "global" behavior vs. complicated leafwise behavior).

The talk consists of two parts. The first part is of more methodological nature and explains the relationship between Markov chains with discrete transition probabilities and equivalence relations. Namely, any such chain determines an equivalence relation generated by pairs of points $(x, y) : p(x - y) > 0$. In a simple way one can express stationarity of the measure on the state space in terms of the Radon-Nikodym derivative (with respect to the equivalence relation) of this measure. Now the properties of the equivalence relation are connected with the leafwise behavior of the Markov chain. For example, if the equivalence relation is non-amenable, then the leafwise Poisson boundaries have to be non-trivial.

In the second part we give an entropy criterion of triviality of the leafwise Poisson boundaries (it is a far reaching generalization of the analogous criterion for random walks on groups obtained by Vershik and myself and and by Derriennic). This criterion also readily applies to the problem of identification of leafwise Poisson boundaries. One just has to replace the original Markov chain with its "Poisson extension" whose state space is optimal by "adding" the leafwise Poisson boundaries to the points of the original state space: the new state space $\overline{X}$ projects onto the original one X with the fibers (E_x, ε_x) which are the Poisson boundaries of the leafwise chains started from the points $x \in X$. The transition probabilities of the new chain are the conditional transition probabilities with respect to E_x.

A. Karlsson: A multiplicative ergodic theorem and non-positively curved spaces

In a joint work with G.A. Margulis, we study integrable cocycles $u(n,x)$ over an ergodic measure preserving transformation that take values in a semigroup of non-expanding maps of a non-positively curved space Y, e.g. a Cartan-Hadamard space or a uniformly convex Banach space. It is proved that for any $y \in Y$ and almost all x, there exists $A \geq 0$ and a unique geodesic ray $\gamma(t,x)$ in Y starting at y such that

$$\lim_{n\to\infty} \frac{1}{n} d\left(\gamma(An,x), u(n,x)y\right) = 0.$$

In the case where Y is the symmetric space $GL_N(\mathbb{R})/O_N(\mathbb{R})$ and the cocycles take values in $GL_N(\mathbb{R})$ this is equivalent to the multiplicative ergodic theorem of Oseledec. Two further applications, concerning Hilbert-Schmidt operators and Poisson boundaries, are also described.

A. Katok: Slow entropy

Invariants describing the speed of sub-exponential growth in dynamical systems with zero entropy are introduced both in topological and measure-theoretic setting. In the topological situation, i.e. for a continuous map $f: X \to X$ of a compact metric space with the distance function d let us consider the integral metric between the orbit segments: $\partial_n^f(x,y) = \frac{1}{n}\sum_{i=0}^{n-1} d \circ f^i$. Let $\tilde{\mathcal{N}}_d(f,\varepsilon,n)$ be the minimal of an (n,ε)-spanning set in the metric $\partial_n^f(x,y)$. The asymptotic growth of these quantities is described using scale functions. A function $a: (0,\infty)\times(0,\infty) \to (0,\infty)$ is called a scale function if $a(\cdot,t)$ is increasing for all t and $\lim_{t\to\infty} a(s,t) = \infty$ for all s. The power scale $a(s,t) = t^s$ is a typical example. The quantity

$$\lim_{\varepsilon\to\infty}\left(\sup\left\{s : \overline{\lim}_{t\to\infty}\tilde{\mathcal{N}}_d(f,\varepsilon,n)/a(s,t) > 0\right\}\right)$$

is called the upper a-entropy; the lower a-entropy is defined with $\underline{\lim}$.

In the measurable setting one considers a measure preserving transformation $T: (X,\mu) \to (X,\mu)$ and a finite measurable partition ξ. Let $\Phi_{T,\xi}: X \to \Omega$ be the corresponding covering map and $X_n = (\Phi^n_{T,\xi})_*\mu$ the marginal of the pushforward measure $(\Phi^n_{T,\xi})_*\mu$ corresponding to the first n coordinates. The Hamming metric in the sense of finite codes is defined by

$$d_n^H(\omega,\omega') := \frac{1}{n}\sum_{i=0}^{n-1}(1-\delta_{\omega_i\omega'_i}).$$

Let $N_\xi^H(T,\varepsilon,n,\delta)$ be the minimal number of balls in d_n^H metric whose union have λ_n measure at least $1-\delta$. The upper (lower) measure-theoretic a-entropy for the scale function a is defined as

$$\sup_\xi \lim_{\delta\to 0}\lim_{\varepsilon\to 0}\left\{s : \overline{\lim_{n\to\infty}} N_\xi^H(T,\varepsilon,n,\delta)/a(s,n) > 0\right\}$$

(and correspondingly with $\underline{\lim}$). For the a-entropy a generator theorem holds (sup is achieved at the generating partition). Variational inequality between topological and measure-theoretic a-entropies holds, but not necessarily the variational principle.

These invariants are useful in the study of elliptic and parabolic phenomena in dynamics.

There is a natural extension of these constructions to action of amenable groups. One application is the construction of a $\mathbb{Z}^k$ actions, $k \geq 2$, by a zero entropy transformation which cannot be realized as action by a diffeomorphism on a compact differentiable manifold with respect to a Borel measure.

G. Keller: Coupled maps - between dynamics and statistical mechanics

We study coupled systems of mixing, piecewise $\mathcal{C}^2$ and piecewise expanding interval maps $\tau : [0,1] \to [0,1]$. Let I be a finite or countable index set and denote the state space of the coupled system by $\Omega_I := [0,1]^I$. The dynamics of the system is described by the map $T_\varepsilon : \Omega_I \to \Omega_I$, $T_\varepsilon := \Phi_\varepsilon \circ T$, where $T : \Omega_I \to \Omega_I$, $(T\omega_i) = \tau(\omega_i)$, and $\Phi_\varepsilon : \Omega_I \to \Omega_I$ is $\mathcal{C}^2$-close to the identity on Ω_I in the following sense: $\|\Phi_\varepsilon(x) - x\|_\infty \leq C \cdot \varepsilon$, $\|D\Phi_{\varepsilon|x} - 1\|_\infty \leq C \cdot \varepsilon$ and $\|DD_i\Phi_{\varepsilon|x} - 1\|_\infty \leq C \cdot \varepsilon$ for all $x \in \Omega_I$ and $i \in I$. Under the (unavoidable) additional assumption that there is some $k > 0$ such that $|(\tau^k)'| > 2$ and no critical point of τ is of period less than or equal to k we have for finite I:

Theorem 1: (Keller-Känzle) $\exists \varepsilon_0 = \varepsilon_0(\tau, C) > 0$ such that $\forall \varepsilon \in [0, \varepsilon]$ and $\forall I$ finite: $T_\varepsilon : \Omega_I \to \Omega_I$ has a unique absolutely continuous invariant probability measure $\mu_{\varepsilon,I}$. $(T_\varepsilon, \mu_{\varepsilon,I})$ is exponentially mixing, and the density $h_{\varepsilon,I}$ of $\mu_{\varepsilon,I}$ satisfies $var(h_{\varepsilon,I}) \leq \text{card } I \cdot const(\tau, C)$ (bounded variation!).

For infinite systems (card $I = \infty$) one can deduce from this the *existence* of a T_ε-invariant probability measure $\mu_{\varepsilon,I}$ all of whose finite dimensional marginals are absolutely continuous (for short: $\mu_{\varepsilon,I} \in AC_1(\Omega_I)$). Even more is known: If $\Lambda \subseteq I$ is finite, then the conditional measures $\mu_{\varepsilon,I}(\cdot|\mathcal{F}_{I\setminus\Lambda})(\omega)$ on Ω_Λ have densities of bounded variation for $\mu_{\varepsilon,I}$-almost all ω. For one-sided couplings on $I = \mathbb{N}$ we can prove the following

Theorem 2: (Keller-Zweimüller) If $\mu_{\varepsilon,I}, \hat{\mu}_{\varepsilon,I} \in AC_1(\Omega_I)$ are T_ε-invariant and if $\mu_{\varepsilon,I}|\mathcal{F}_{\text{tail}} = \hat{\mu}_{\varepsilon,I}|\mathcal{F}_{\text{tail}}$, then $\mu_{\varepsilon,I} = \hat{\mu}_{\varepsilon,I}$, where $\mathcal{F}_{\text{tail}}$ denotes the spatial tail field of Ω.

In order to deduce uniqueness of $\mu_{\varepsilon,I}$ we need one more assumption: Suppose that $\left|\frac{\partial \Phi_{\varepsilon,i(\omega)}}{\partial \omega_j}\right| \leq const \cdot ((1-\varepsilon)\delta_{ij} + \varepsilon p_{j-i})$ for a probability vector $(p_k)_{k\in\mathbb{Z}}$ with $p_k = 0$ for $k < 0$ and $\sum_k p_k e^{tk} < \infty$ for all $t > 0$. Then

Theorem 3: (Keller-Zweimüller) $T_\varepsilon : \Omega_I \to \Omega_I$ has a unique invariant measure $\mu_{\varepsilon,I} \in AC_1(\Omega_I)$. Correlations of Lipschitz observables decay exponentially in time and space under $\mu_{\varepsilon,I}$.

M. Klünger: Random logistic maps

We study the random dynamical system φ over an ergodic dynamical system $(\Omega, \mathcal{F}, \mathbb{P}, \vartheta)$ that is generated by

$$\psi(\omega) := c(\omega)x(1-x),$$

where $c : \Omega \to [0,4]$. This is a random perturbation of the logistic family. We construct random attractors and give necessary and sufficient conditions for the existence of invariant measures that are not concentrated at zero. In the case of existence, we give conditions for the uniqueness of these measures.

Y. Kondratiev: Stochastic dynamics for continuous systems

We propose a new approach to the construction of Markov semigroups corresponding to infinite particle diffusion generators for continuous point system. This approach is based on a version of harmonic analysis on configuration spaces. More precisely, using a "proper" version of the Fourier transform we transport the generator from functions on configuration spaces into an operator on functions defined on finite configurations. The transformed generator has a special form which gives a possibility to construct a semigroup recursively. Then the inverse Fourier transform produces a Markov semigroup on observables of our system.

Note, that the same approach works also in the case of deterministic and Hamiltonian dynamics. In the latter case it gives the rigorous deriving of the BBGKY-hierarchy for the time evolution of the correlation functions.

F. Ledrappier: Local characteristics for dynamical systems.

We are in the setting of smooth ergodic theory: $f : M \to M$ is a $\mathcal{C}^{1+\alpha}$-diffeomorphism of the compact Riemannian manifold M, μ is an ergodic invariant measure without exponent zero. We consider "Takens-Thieullen dynamical α-balls" defined as follows:
for $\alpha \geq 0$ $B_\alpha(n,x,\varepsilon) = \{y | d(f^ix, f^iy) \leq e^{-i\alpha}, i = 0,1,...,n-1\}$
for $\alpha \leq 0$ $B_\alpha(n,x,\varepsilon) = \{y | d(f^ix, f^iy) \leq e^{(n-i)\alpha}, i = 0,1,...,n-1\}$.
One result says that there exists a piecewise affine function $\varphi(\alpha)$ such that for μ-a.e. x, all α, we have

$$\varphi(\alpha) = \lim_{\varepsilon \to 0} \left\{ \begin{array}{c} \overline{\lim}_{n\to\infty} \\ \underline{\lim}_{n\to\infty} \end{array} \right\} - \frac{1}{n} \log \mu(B_\alpha(n,x,\varepsilon)).$$

(Compare with Brin-Katok's entropy).
Actually $\varphi(\alpha) = \sum(\lambda_i + \alpha)^+ \gamma_i$ for $\alpha \geq 0$ and $\varphi(\alpha) = \sum(\lambda_i + \alpha)^- \gamma_i$ for $\alpha \leq 0$, where λ_i are the exponents and γ_i are "partial dimensions in the direction i" as introduced by Young and Ledrappier.

C. Liverani: Random perturbations of dynamical systems

I discuss random perturbations of a given dynamical system (X,T) defined by transition probabilities

$$P_\varepsilon(x \in A | y) = \int_A q_\varepsilon(Ty, \xi) d\xi$$

where q_ε is smooth and $q_\varepsilon(x,y) = 0$ if $d(x,y) > \varepsilon$. The focus is on the stability of the statistical properties of the deterministic dynamical system (invariant measure, speed of convergence to the equilibrium, etc.).

The method is based on the study of the spectrum of the family of transfer operators $\mathcal{L}_\varepsilon$ associated to the random perturbations ($\mathcal{L}_0$ being the Perron-Frobenius operator associated to the dynamical system). The essential tool consists in a rather general result, obtained in collaboration with G. Keller, concerning the stability of the spectrum for a family of operators satisfying a uniform Lasota-Yorke type inequality. The above setting applies immediately to the case of piecewise smooth expanding maps but can also be applied to certain Anosov diffeomorphisms. This is achieved by introducing appropriate functions spaces on which the operator $\mathcal{L}_0$ turns out to be quasi-compact. These last results are work in progress together with G. Keller and M. Blank.

R. Minlos: The spectral analysis of the disorder stochastic 1-D Ising model

We consider the generator of the Glauber dynamics for a 1-D Ising model with random bounded potential at any temperature. We prove that for any realization of potential the spectrum of the generator is the union of separate branches (so-called k-particle branches, $k = 1,2,...$) and with probability one it is a nonrandom set. We find the location of the spectrum and prove the localization for the one-particle branch of the spectrum. As a consequence we find a lower bound for the spectral gap for any realization of the random potential. It is contained in the paper S. Albeverio, R. Minlos, E. Scucciatelli, E. Zhizhina. Com. Math. Phys. 204, 651–668 (1999).

M. Pollicott: The linear action of some discrete subgroups of $SL(2,\mathbb{C})$

The purpose of this talk is to explain the natural generalization of distribution results on orbits of a point in $\mathbb{R}^2 \setminus \{(0,0)\}$ under the linear action of a discrete group $\Gamma \subseteq SL(2,\mathbb{R})$ [due to Ledrappier]. In the case of $SL(2,\mathbb{R})$, an element $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ maps $(\gamma, (z_1, z_2)) \mapsto (az_1 + bz_2, cz_1 + dz_2)$. If the orbits are ordered by norm $\|\gamma\| = \left(|a|^2 + |b|^2 + |c|^2 + |d|^2\right)^{\frac{1}{2}}$ then for $(z_1,z_2) \neq (0,0)$ and $\Gamma \setminus SL(2,\mathbb{R})$ compact (say) the orbits are distributed as $\|(z_1,z_2)\|_2^{-1} dz_1 dz_2$. The proof uses an equivalence with the usual action of $SL(2,\mathbb{R})$ on horospheres in the Poincaré upper half-plane $\mathbb{H}^2$. To generalize to the case of discrete subgroups $\Gamma \subseteq SL(2,\mathbb{C})$, with linear actions on $\mathbb{C}^2$, it is appropriate to consider horospheres in the Poincaré upper half-space $\mathbb{H}^3$, along with a choice of frames (and their translations under transport on horocycles). Using properties on the (uniform) distribution of horospheres (and frames) one recovers the analog of Ledrappier's result for $SL(2,\mathbb{C})$. There are partial generalizations to subgroups $\overline{\Gamma} \subseteq \Gamma \subseteq SL(2,\mathbb{C})$ such that $\Gamma \setminus SL(2,\mathbb{C})$ is compact and $\overline{\Gamma} \setminus \Gamma \cong \mathbb{Z}^d$, say. This is joint work with F. Ledrappier.

O. Sarig: Thermodynamic formalism for countable Markov shifts

A version of the Thermodynamic Formalism is presented in the setting of a topological Markov shift with a countable number of states. The talk is divided into three parts: Defining topological pressure; generalizing Ruelle's Pesin formula theorem; a discussion of phase transition phenomena. The main theorem is that there exist three different types of qualitative behavior, similar to that of positive recurrent, null recurrent and transient Markov chains. The passage of a one-parameter family of potentials $\{\beta\Phi\}_{\beta>0}$ from one mode of recurrence to another yields critical phenomena.

M. Scheutzow: Dispersion of sets under a stochastic flow

(Joint work with David Steinsaltz and Mike Cranston)

It has been conjectured by R. Carmona that under "reasonable" conditions the diameter of the image of a ball (say) under a stochastic flow on $\mathbb{R}^d$, $d \geq 2$, grows linearly almost surely. We show that the conjecture holds for stochastic flows without drift under boundedness, Lipschitz and non-degeneracy assumptions on the quadratic variation of the driving martingale field. This class of flows includes (essentially) all isotropic Brownian flows. The question is motivated by applications in oceanography.

B. Schmalfuss: Stability for random sets and random Lyapunov functions

Over the metric dynamical system $(\Omega, \mathcal{F}, \mathbb{P}, \theta)$ we consider a random dynamical system φ with state space $\mathbb{R}^d$. A random invariant compact set A is called globally asymptotically stable if for any $\varepsilon > 0$ there exist two random compact neighborhoods U_1, U_2 of A such that

$$\mathbb{P}\{\text{distance}(U_2(\cdot), A(\cdot)) > \varepsilon\} < \varepsilon$$
$$\varphi(t,\omega,U_1(\omega)) \subset U_2(\theta_t\omega), \text{ for all } t \geq 0$$

and for any random variable X

$$\mathbb{P}\text{-}\lim_{t\to\infty} \text{distance}(\varphi(t,\omega,X(\omega)), A(\theta_t\omega)) = 0.$$

Theorem: The random compact invariant set A is globally asymptotically stable if and only if there exists a Lyapunov function, i.e. a function Vsuch that:

(i) $V : \Omega \times \mathbb{R} \to \mathbb{R}^+$ is measurable,

(ii) $V(\omega, \cdot)$ is continuous and uniformly unbounded,

(iii) $V(\omega, x) = 0 \Leftrightarrow x \in A(\omega)$,

(iv) $V(\theta_t\omega, \varphi(t,\omega,x)) = e^{-t}V(\omega,x)$, for $t \geq 0$.

J. Schmeling: Dimension theory in dynamical systems

We give an overview over the dimension theory established recently. We emphasize that dimensions are characteristics which connect "orbit complexity" with the geometry of the manifold the system lives on. In particular there was recent progress in understanding this connection in the case of smooth diffeomorphisms. We discuss how this theory can be generalized to the case of piecewise smooth endomorphisms. Here the connection between entropy, Lyapunov exponents and dimensions is more subtle. There is a crucial connection between the dimension being equal to the "expected" value and the invertibility of the system on the attractor. Moreover, the (physically meaningful) SRB-measure can be characterized by a dimension formula in "typical" cases. This allows us to show the stochastic stability of the SRB-measure in a "typical" situation.

R. Sharp: Asymptotic expansions for closed geodesics in homology classes

Let V be a compact Riemannian manifold with negative sectional curvatures. Let γ denote a typical closed geodesic on V, $l(\gamma)$ its length, and $[\gamma] \in H_1(V,\mathbb{Z})$ its homology class. We report on recent work of Anantharaman (1998), Pollicott and Sharp (1998) and Kotani (1999) on the distribution of closed geodesics in $H_1(V,\mathbb{Z})$. We suppose $b = \text{rank } H_1(V,\mathbb{Z}) \geq 1$. For convenience we assume that $H_1(V,\mathbb{Z})$ has no torsion. We introduce the counting function $\pi(t,\alpha) = \sharp\{\gamma : l(\gamma) \leq T, [\gamma] = \alpha\}$.

Theorem: (Anantharaman; Pollicott and Sharp) For any $N \geq 1$,

$$\pi(T,\alpha) = \frac{e^{hT}}{T^{\frac{1}{2}b+1}} \left(C_0 + \frac{C_1(\alpha)}{T} + \frac{C_2(\alpha)}{T^2} + \ldots + \frac{C_N(\alpha)}{T^N} + O(\frac{1}{T^{N+1}}) \right),$$

where h is the topological entropy of the geodesic flow on SV.

Theorem: (Kotani) $C_1(\alpha) = C_1(0) - D\|\alpha\|^2$, where $D > 0$ is a constant and $\|\cdot\|$ is a natural norm on $H_1(V,\mathbb{R})$.

Corollary: If $\|\alpha\| < \|\beta\|$ then $\pi(T,\alpha) > \pi(t,\beta)$ for all sufficiently large T.

A. Soshnikov: Universality in random matrices at the edge of the spectrum

We study large random hermitian (resp. real symmetric) Wigner matrices. In the hermitian case our ensemble can be defined as $A = \|a_{ij}\|_{1\leq i,j\leq n}$, where Re $a_{ij} = \frac{\xi_{ij}}{\sqrt{n}}$, $i \leq j$, and Im $a_{ij} = \frac{\eta_{ij}}{\sqrt{n}}$, $i < j$, are independent random variables such that

(i) all moments of $\{\xi_{ij}, \eta_{ij}\}$ exist,

(ii) all odd moments vanish,

(iii) $E\xi_{ij}^2 = E\eta_{ij}^2 = \frac{1}{8}$ for $i < j$, $E\xi_{ii}^2 \leq$ const,

(iv) higher moments do not grow very fast: $E\xi_{ij}^{2k}, E\eta_{ij}^{2k} \leq (\text{const } k)^k$.

Under these conditions we prove that the first few largest eigenvalues of A after proper rescaling converge in distribution to the Tracy-Widom law established in mid-nineties for the limiting distribution of the largest eigenvalue in the gaussian ensemble (G.U.E.). In particular

$$P\left\{\lambda_{\max} \leq 1 + \frac{x}{2\cdot n^{\frac{2}{3}}}\right\} \longrightarrow_{n\to\infty} F(x) = \exp\left(-\int_x^{+\infty}(y-x)q^2(y)dy\right),$$

where q is the solution of

$$q''(x) = xq(x) + 2q^3(x),\ \ q(x) \sim A_i(x),\ x \to \infty.$$

Ö. Stenflo: Markov chains in random environments and random iterated function systems

We consider random independent iterations of functions where the function to iterate is random according to an, in each step, randomly chosen probability distribution (environment). Ergodic theorems are obtained under average contraction conditions and the limiting probability regime is analysed.

O. Zeitouni: Asymptotics for random walks in random environment

Let α denote an iid law on $[0,1]^{\mathbb{Z}}$. The random walk in random environment s_n is the Markov process with transitions

$$\begin{aligned} P_\omega(X_{n+1} = i+1 | X_n = i) &= \omega_i \\ P_\omega(X_{n+1} = i-1 | X_n = i) &= 1 - \omega_i. \end{aligned}$$

I will describe several asymptotic results for $\frac{X_n}{n}$. In particular, the relation between quenched large deviations

$$\lim \frac{1}{n} \log P_\omega \left(\frac{X_n}{n} \in B \right) \asymp - \inf_{x \in B} I^q(x), \quad \alpha\text{-a.s.}$$

and annealed ones

$$\lim \frac{1}{n} \log \alpha \otimes P_\omega \left(\frac{X_n}{n} \in B \right) \asymp - \inf_{x \in B} I^a(x),$$

is explored. Analogous results for random walks in random environment on a Galton-Watson tree will be described.

E. Zhizhina: The Lifshitz tail and relaxation to equilibrium in the 1-D disordered Ising model

We study spectral properties of the generator of the Glauber dynamics for a 1-D disordered stochastic Ising model with random bounded couplings. By an explicit representation for the upper branch of the spectrum of the generators we get an asymptotic formula for the integrated density of states near the upper edge of the spectrum. This asymptotic behavior appears to have the form of the Lifshitz tail, which is typical for random operators near fluctuation boundaries. As a consequence we find the asymptotics for the average over the disorder of the time-autocorrelation function to be

$$\left\langle \langle \sigma_0^\omega(t) - \sigma_0(0) \rangle_{P(\omega)} \right\rangle_\omega = e^{-gt - kt^{\frac{1}{3}}(1+o(1))}$$

as $t \to \infty$ with constants g, k depending on the distribution of the random couplings.

Berichterstatter: L. Arnold and M. Klünger

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