MATHEMATISCHES FORSCHUNGSINSTITUT OBEFRWOLFACH

T a g u n g s b e r i c h t 36/1999

Risk Theory

5. 9. – 11. 9. 1999

This meeting was organised jointly by Søren Asmussen (Lund), Hans Bühlmann (Zürich) and Christian Hipp (Karlsruhe). In 32 talks and informal discussions, it covered a wide range of problems dealing with risk, mainly in insurance but also in finance. Key topics included:

- Ruin probabilities,
- Applications of stochastic control in risk theory,
- Risk measures and risk orderings,
- Extreme risks,
- Loss distributions,
- Pricing of risks and risk management.

The meeting had 39 participants.

Abstracts

Philippe Artzner

Allocation of capital via internal trading

A firm with initial capital K has two divisions. Each has access to a f.d. vector space $\mathcal{X}$ or $\mathcal{X}'$ of date 1 random net worth with date prices given by π. The regulators have a set $\mathcal{R}$ of scenarios (probabilities) and the shareholders a set $\mathcal{A}$ and a family G_S, $S \in \mathcal{A}$ of goals. The firms problem is:

$$\max_{X\in\mathcal{X},X'\in\mathcal{X}'} \mathbb{E}[X+X'] \text{ with} \qquad (1)$$
$$\pi[X+X'] = K,\ \forall R\ \mathbb{E}_R[X+X'] \geq 0 \text{ and } \forall S\ \mathbb{E}_S[X+X'] \geq G_S$$

A manager assigns to the divisions capital k, k' $(k+k'=K)$, regulators limits l_R, l'_R $(l_R+l'_R=0)$, shareholders goals g_S, g'_S $(g_S+g'_S=0)$. Divisions are allowed to internally trade capital limits and goals at prices c, l_R, m_S. They decide upon net worth AND quantities internally traded:

$$\text{(p)} \quad \max_{X\in\mathcal{X}} \left(\mathbb{E}[X] - ch - \sum_R \delta_R l_R - \sum_S \epsilon_S m_S\right)$$
$$\pi(X) = k+h$$
$$\forall R\ \mathbb{E}_R[X] + \delta_R \geq 0, \forall S\ \mathbb{E}_S[X] + \epsilon_S \geq g_S$$

and similarly (p'). Choosing the terms of trade equal to an optimal solution (c^*, l_R^*, m_S^*) of the dual of (1), this solution is also optimal for the dual of (p),(p') and adding the $X^*, X^{*\prime}$ parts of the optimal solution of (p),(p') gives an optimal solution of (1): trading at (c^*, l_R^*, m_S^*) has decentralized (1). Dantzig–Wolfs algorithm should be studied for actually implementing the decomposition. (Joint work with F. Delbaen, J.-M. Eben and D. Heath to apply the idea of coherent measures to risk management: choice of portfolio, capital allocation, performance measurement)

Søren Asmussen

On the ruin problem for some adapted premium rules

We consider risk processes where the premium rate $p(t)$ at time t is calculated according to past claims statistics, e.g. $p(t) = (1+\eta)A_{t-}/t$ or

$p(t) = (1+\eta)(A_{t-} - A_{t-s})/s$ where η is the safety loading and A_t the total compound Poisson claims in $[0,t]$. We perform a comparison of the ruin probabilities with those of the Cramér–Lundberg model, and characterize the claims behaviour leading to ruin. With heavy tails, the controlled risk process has typically at least as large a ruin probability as the Cramér–Lundberg model. With light tails, the adjustment coefficient is typically larger so that the ruin probability is smaller. A key tool in the arguments is the Gärtner–Ellis theorem from large deviations.

Jan Beirlant

Regression models in extreme value statistics with risk applications

Applications of extreme value theory in the field of actuarial and financial risk analysis concern for instance the calculation of tail indices and extreme quantiles. In the analysis of non-life insurance portfolios the estimation of the Pareto index is a classical and ongoing research topic. In the financial world, extreme value theory receives a growing interest as an analytic tool: financial data show fat tails, and estimation of the Value-at-Risk (*Var*) measure can be placed in the extreme value context as it boils down to the estimation of an extreme quantile.
In trying to provide methodological support towards such applications, statistical extreme value methods still face serious problems and they should be treated with extreme care. Here we can refer to bias reduction and choice of threshold beyond which the fit of parametric tail model is appropriate. Also basic extensions to the analysis of conditional and multivariate tails are still very much under development. In this contribution, regression models are constructed which can help to overcome some of the problems cited above. More specifically, in case of Pareto-type tails given by a regularly varying survival function $1 - F(x) = x^{-1/\gamma}\ell(x)$, it can be argued that a good approximation to log-spacings of the extreme order statistics $X_{n,n} \geq X_{n-1,n} \geq \ldots \geq X_{n-k,n}$ $(1 < k < n)$ from an i.i.d. sample of size n, is given by

$$j\left(\log X_{n-j+1,n} - \log X_{n-j,n}\right) = \left(\gamma + b(\frac{j}{k+1})^{\rho}\right) f_j, \quad j = 1, \ldots, k$$

where $f_1, ..., f_k$ is an i.i.d. sample of size k, and where b and ρ are nuisance parameters arising from a flexible parametrization of the slowly varying function ℓ. Maximum likelihood estimation of this exponential regression model leads to a new estimator for the Pareto tail index with a seriously bias.

The basic asymptotic properties of this maximum likelihood estimator are developed. The small sample properties of the estimator are studied through a simulation study. A simulation study for ARCH processes suggests a promising behaviour of this new estimator under time series models traditionally fitted to financial time series. We also discuss consequences of the above decomposition of log-spacings to extreme quantile estimation and to the adaptive estimation of the optimal number of order statistics to be used when applying popular estimators such as the Hill estimator.

The approach described above can be extended to extreme value regression models incorporating covariate information $\mathbf{z}$, modelling the positive tail index γ through an exponential link function as $\gamma(\mathbf{z}) = \exp(\beta'\mathbf{z})$, where β represents the vector of regression coefficients. A typical application from non-life insurance can be found in the analysis of time trends in Pareto tails of claim data, which leads to a serious gain in estimation efficiency compared to a year-by-year analysis.

Finally extensions to the case of a real valued extreme value index γ are feasible. Such a program is carried out replacing ordinary order statistics by the ordered values from the sequence

$$X_{n-j,n} \;\; \frac{1}{j} \sum_{m=1}^{j} (\log X_{n-l+1} - \log X_{n-j}) \quad (j = 1, \ldots, n-1)$$

in the scheme discussed above. (This contribution is based on joint work with G. Dierckx, Y. Goegebeur, G. Matthys, and C. Stărică)

Patrick Brockett

Data envelopment analysis evaluation of the efficiency of the US property and liability insurance industry

The efficiency effects of different forms of organisatorial structure (stock versus mutual ownership) and different types of marketing distribution systems (direct company marketing versus independent agency) are studied. A new form of data envelopment analysis (DEA) is presented and used together with rank order statistical methods to produce results which differ from those in the literature. A method for adjusting for managerial inefficiency is presented and used in the analysis.

Nicole Bäuerle

The effect of dependencies in risk models

In the first part we consider multivariate risk portfolios, where the risks are dependent. Using the supermodular and the symmetric supermodular ordering we are able to compare portfolios w.r.t. their dependency. For example, it often occurs that the risks can be grouped such that there is a strong dependence between members of one group but much less dependence between members of different groups. It can be shown now that whenever the group structure majorizes another, the portfolio is greater w.r.t. the symmetric supermodular ordering and thus contains more dependency. In the second part we investigate a Markov-modulated ruin model. We vary the dependency with which claims arrive by multiplying the generator of the environment process by a constant. For the finite and also for the infinite horizon ruin probability it can be shown that higher dependence leads to greater ruin functions w.r.t. the stop-loss ordering.

Andrew Cairns

Dynamic stochastic control of pension plans

Here we show how dynamic stochastic control can be applied to pension funding problems. We are concerned primarily with management of uncertainty arising from a funds investments but also concern ourselves with salary and demographic risk.

Under a defined benefit plan, pensions are taken as given with no randomness or with demographic risk only. Optimal controls are derived using the Bellman equation for the contribution rate and the asset-allocation strategy. For two different loss functions (quadratic and power) problems or undesirable characteristics of the solution are discussed.

Second we consider a defined contribution plan where investment risk is borne entirely by an individual. With a regular contribution rate we consider the dynamics as the individual fund rolls up over time and is then converted at the date of retirement into an annuity. Characteristics of the optimal asset allocation strategy are derived and a particular solution is given for the power utility function.

Claudia Czado

Multivariate regression analysis for binary response variables

In many studies binary outcome measures together with time stationary and time varying explanatory variables are collected over time on the same individual. Therefore, a regression analysis for this type of data must allow for the correlation among the outcomes of an individual. The multivariate probit model of Ashford and Sowden (Biometrics, 1970) was the first regression model for multivariate binary responses. However, a likelihood analysis of the multivariate probit model with general correlation structure for higher dimensions is intractable due to the maximization over high dimensional integrals thus severely restricting its applicability so far. A Markov Chain Monte Carlo (MCMC) algorithm is proposed to overcome this difficulty. An application of this algorithm to unemployment data from the Panel Study of Income Dynamics involving 11 waves of the panel study is presented. In addition Bayesian model checking techniques based on the posterior predictive distribution (see for example the book by Gelman, Carlin and Rubin, 1995) for the multivariate probit model are adapted. These help to identify mean and correlation specification which fit the data well.

Griselda Deelstra

Some issues common to finance and insurance: Incomplete markets, long-term investment and stochastic interest rates

Together with Grasselli and Koehl, we are looking at different aspects of pension funds. In a first paper, we consider the case that financial markets and insurance markets are incomplete. We define the concept of conditional dominance and use it for the obtention of bounds on the hedging prices of random variables. These bounds depend only on the characteristics of the financial market and the random variables to hedge. Moreover, they are coherent with the equilibrium and tighter than the ones obtained by the classical super-replication approach, significantly in some cases. This approach can be applied in static as well as dynamic frameworks. In a second paper, we stress the problem that in pension funds, the fund manager should hedge the promised pension during a long period. We study an optimal investment problem in a continuous-time framework where the interest rates follow the Cox-Ingersoll-Ross dynamics. Closed form formulae for the optimal investment strategy are obtained by assuming the completeness of financial markets and CRRA utility function. As an application, we consider the case of a defined contributions pension fund in the presence of a minimum guarantee. In particular, we study the behaviour of the solution when time approaches to the terminal date.

F. Etienne De Vylder

Optimization problems on spaces of mixtures with a polynomial structure

Let $F_\theta(a \leq \theta \leq b)$ be a family of probability distribution on $\mathbb{R}$. We say that it has a *polynomial structure of order* n if the F_θ up to order n are polynomial in θ:

$$\int x^i dF_\theta(x) = \sum_{0 \leq j \leq i} a_{ij}\theta^j \ (0 \leq i \leq n, a \leq \theta \leq b)$$

with $a_{ii} \neq 0$. Let $\mathbf{Space} = \mathbf{Mixt}[F_\theta(a \leq \theta \leq b)]$ be the space of mixtures of the distributions $F_\theta(a \leq \theta \leq b)$. We denote by

$$\sup_{F \in \mathbf{Space}} \{T(F) \| \mu_1, \ldots, \mu_n\} \tag{2}$$

the upper bound of $T(F)$ when F ranges over **Space**, subject to the *moment constraint* $\int x^i dF = \mu_i (1 \leq i \leq n)$ for fixed $\mu_1, \ldots, \mu_n$ and where T is some functional on **Space**. We show that the problem of determination of (2) can be reduced to the problem of determination of

$$\sup_{\mathbf{Prob}[a,b]} \{T(F_U) \| \mu'_1, \ldots, \mu'_n\}, \tag{3}$$

where **Prob**$[a, b]$ is the space of probability distributions concentrated on $[a, b]$, $\mu'_1, \ldots, \mu'_n$ are new moments *associated* to the old moments $\mu_1, \ldots, \mu_n$, F_U is the mixture of the distributions F_θ with respect to the mixing distribution U.

F. Etienne De Vylder

Can we trust in asymptotic ruin probabilities?

The Cramér asymptotic expression $\psi_{CR}(u)$ for the ruin probability $\psi(u)$ as $u \to \infty$ (u is the initial reserve) in the classical actuarial risk model, is safe for large classes of claim size distributions if the security loading is not excessive.
On the contrary, in case of subexponential claim size distributions, the Anthrea-Ney asymptotic expression (also known as the Embrechts–Veraverbeke expression) is a rather academic result with very little practical applicability.

Jan Dhaene

Stochastic Upper Bounds For Present Value Functions

In most practical cases, it is impossible to find an explicit expression for the distribution function of the present value of a sequence of cash flows that are discounted using a stochastic return process. In this paper, we present an easy computable approximation for this distribution function. The approximation is a distribution function which is, in the sense of convex order, an upper bound for the original distribution function. Explicit examples are given for pricing stochastic annuities with stochastic return process, more general stochastic cash flows as well as pricing Asian options. Numerical results seem to indicate that the approximation will often be rather close to the original distribution function. (Joint work with M. Goovaerts and A. Shepper)

David C M Dickson

Ruin problems for phase-type(2) risk processes

We consider a Sparre Andersen risk process in which the distribution of inter-arrival times is phase-type(2). We show how techniques for the classical risk model can be used to find ruin and related probabilities. In particular, we find the compound geometric representation of the distribution of the maximum aggregate loss, and the distributions of the surplus immediately prior to ruin and the deficit at ruin. We also consider an approach to finding the moments of the time to ruin.

Hansjörg Furrer

How to deal with a deficit in a pooling arrangement

Multinational pooling is a system under which insurance coverages in various countries are brought together into one pool: The risks of high liability claims are pooled with the group portfolio. When the pooling result is negative, various solutions can be adopted to write off the loss. We extend the traditional stop-loss and m-year loss-carry-forward systems by intermediate solutions where the write-off is only a fraction of the total annual loss. We also consider reductions in the pay-off which allow for a minimal guaranteed dividend payment.

Jan Grandell

Simple approximations of ruin probabilities

A "simple approximation" of a ruin probability is an approximation using only some moments of the claim distribution and not the detailed tail behaviour of that distribution. Such approximations may be based on limit theorems or on more or less *ad hoc* arguments. The most successful simple approximation is certainly the De Vylder approximation, which is based on the idea to replace the risk process with a risk process with exponentially distributed claims such that the three first moments coincide. That approximation is known to work extremely well for "kind" claim distributions. The main purpose of this paper is to analyse the De Vylder approximation and other simple approximations from a more mathematical point of view and to give a possible explanation why the De Vylder approximation is so good.

Bjarne Højgaard

Optimal risk control for large insurance corporations

We consider the problem of finding optimal reinsurance policies, which maximizes the expected discounted pay-out of dividend until the time of eventual ruin of the company.
The model considered is diffusion model and in this setting the problem results in finding a solution of a Hamilton-Jacobi-Bellman equation. the latter is a non-linear differential equation to whom only seldomly solutions can be found. We present some earlier results from Højgaard and Taksar (Mathematical Finance,1999) and Asmussen *et al.* (Finance & Stochastics, 1999, to appear) where the problem is solved for different reinsurance forms. We also present recent results of Højgaard and Taksar where stochastic interest and optimal investment strategies are incorporated in the model.

Ralf Korn

Introduction to stochastic control with applications to finance and risk theory

An introduction to the tasks and main tools of continuous-time stochastic control such as the Bellman principle and the Hamilton-Bellman-Jacobi equation is given. Together with some variants of verification theorems some examples highlighting the principle differences in applications in finance and risk theory are presented

Anders Martin-Löf

The Petersburg game with interest

In the celebrated Petersburg game let M be the number of losses until heads comes up for the first time. We have $P(M = m) = 2^{-m}, m = 1, 2, \ldots$. Let $X = 2^M$ denote the payment received. The cost of playing is $1 + 2 + 4 + \cdots + 2^{M-1} = X - 1$, so there is a gain of one unit in each game. If many independent games are played one can hence get very rich if one can afford the cost. We analyse the situation where the player has to borrow the successive costs in a bank and pay interest for them. Then

the total present value of the costs of interest minus the gain is $V = (1/r-1)\sum_k (X_k-1)r^k - \sum_k r^k = V_+ - V_-$. This is finite if $r<1$.
The distribution of V is studied when r is close to 1 ($r = e^{-\rho 2^{-n}}$, $n \to \infty$). It is shown that the value of the costs has a limit distribution, which has a Levý representation of an interesting form, that allows a simple asymptotic formula for the tail of the distribution to be found:

$$P(V_+ - n > 2^m) \simeq c2^{-m}.$$

When $m = n$ it says that $P(V > 0) \simeq c(1 - r^{-1})$ when n is large so the probability of a negative value is quite small.

Alex McNeil

Correlation and dependence in risk management

Modern risk management calls for an understanding of stochastic dependence going beyond simple linear correlation. We deal with the static (non-time-dependent) case and emphasize the copula representation of dependence for a random vector. Linear correlation is a natural dependence measure for multivariate normally and, more generally, elliptically distributed risks but other dependence concepts like comonotonicity, rank correlation and tail dependence should also be understood by the risk management practitioner. Using counterexamples the falsity of some commonly held views on correlation is demonstrated; in general, these fallacies arise from the naive assumption that dependence properties of the elliptical world also hold in the non-elliptical world. In particular, the problem of finding multivariate models that are consistent with prespecified marginal distributions and correlations is addressed. Pitfalls are highlighted and simulation algorithms avoiding these problems are mentioned.

Thomas Mikosch

Ruin probabilities for random walks with heavy-tailed dependent step sizes

Let $X_1, X_2, \ldots$ be a stationary ergodic sequence with common distribution F, $S_0 = 0$ and $S_n = X_1 + \cdots + X_n$. We will refer to (S_n) as the random walk generated by (X_n). Moreover, we assume the tail balance condition

$$\bar{F}(x) \sim L(x)px^{-\alpha}\,, F(-x) \sim L(x)qx^{-\alpha}\,, x \to \infty\,,$$

where L is slowly varying, $\alpha > 1$ and $p+q = 1$, $p > 0$. The classical result by Embrechts and Veraverbeke (1982) states that the ruin probability

$$\psi(u) = \mathbb{P}\left(\sup_{n\geq 0}(S_n - n\mu) > u\right) \sim \frac{1}{\mu}\int_u^\infty \bar{F}(y)dy \sim \frac{1}{\mu(\alpha-1)}u\bar{F}(u)\,, \quad (4)$$

as $u \to \infty$, provided the X_ns are iid. For dependent X_ns the behaviour of $\psi(u)$ can deviate from (4) significantly. In Mikosch and Samorodnitsky (1998) it is shown for linear processes (X_n), including the class of ARMA processes, that

$$\psi(u) \sim c\,\frac{1}{\mu(\alpha-1)}u\bar{F}(u)\,,$$

where the constant c depends on all coefficients of the linear process and on α. Mikosch and Samorodnitsky (1999) study $\psi(u)$ for symmetric α-stable stationary ergodic X_ns with $\alpha \in (1,2)$. In this case, $\mathbb{E}X^2 = \infty$ but $\mathbb{E}|X| < \infty$. Such processes have a stochastic integral representation $X_n = \int_E f_n(x)M(dx)$ wrt a stable random measure M. The dependence is flexibly modelled via the deterministic functions f_n on the space E, describing the flow on E. We show that, depending on how strong the dependence in (X_n) is, one can get asymptotics of the form $\psi(u) \sim \tilde{L}(u)u^{\gamma(1-\alpha)}$ for any slowly varying function $\tilde{L}$ and $\gamma \in [0,1]$. The mentioned papers are available via www.math.rug.nl/∼mikosch .

Thomas Møller

On indifference pricing for insurance contracts

We give an introduction to the financial variance and standard deviation principles of Schweizer (1997) and apply these principles for the valuation of (re-)insurance contracts which depend on "pure insurance risk" in addition to some "purely financial risk". Examples considered are: Unit-linked life insurance contracts, double-trigger stop-loss contracts and financial stop-loss contracts. We derive optimal trading strategies for the two financial principles and investigate how these strategies and the fair premiums depend on the amount of information available to the seller of the contracts. By considering different filtrations for the pure insurance risk, we arrive at simple upper and lower bounds for the fair premiums. The upper bound is obtained when the seller receives no information concerning the insurance risk; the lower bound corresponds to the situation where all information is revealed immediately after the selling of the contract. These bounds seem particular relevant for the valuation of reinsurance contracts, since reinsurers often receive only scarce information after the selling of the contracts.

Ragnar Norberg

Bounds for ruin probabilities under Levy driven interest

The principal aim of this talk is to show that, typically, the probability of ruin decreases slowly (not faster than a power function) if the surplus is currently deposited in an investment portfolio that may earn negative interest. More specifically, consider an insurance business commencing at time 0, with initial reserve x (> 0). Let C_t be the total net income (premiums less claims) by time t and let R_t be the log accumulation factor in the time interval $[0, t]$. Then the surplus X_t at time t develops in accordance with

$$dX_t = dC_t + X_{t-}dR_t \,.$$

If the processes C and R obey classical rules of calculus, then the solution is

$$X_t = e^{R_t - R_s} X_s + \int_s^t e^{R_t - R_u} dC_u \,, \tag{5}$$

which is easy to interpret. We are interested in the probability of ultimate ruin,

$$\psi(x) = \mathbb{P}[\inf_{t \geq 0} X_t \,|\, X_0 = x] \,.$$

Some structure must be added to obtain results. We assume that C and R are Levy (independent and stationary increment) processes, thus covering the case with classical risk process and e.g. Poisson driven accumulation factor. Clearly, ruin can occur only due to negative increments (claims) of C. Having the classical case in mind for the time being, let $\tau_1 < \tau_2 < \cdots$

be the times of successive claims occurrences, put $\tau_0 = 0$, and consider the skeleton process $S_i = X_{\tau_i}$, $i = 0, 1, 2, \ldots$ Then

$$\psi(x) = \mathbb{P}[\inf_i S_i < 0 \mid S_0 = x] .$$

Due to (5), the embedded process S satisfies a recurrence relation of the type

$$S_{i+1} = \xi_{i+1} S_i + \eta_{i+1},$$

where the (ξ_i, η_i) are i.i.d. pairs and $\xi_i > 0$ a.s. (The process S_i is recognized as an "ARCH(1) process with positive drift", thus non-stationary). Other choices of stopping times τ_i may be considered, e.g. fixed equispaced times, which is particularly relevant if C is a Brownian motion with drift. The basic result is: If there exist constants $\xi^* < 1$ and $q^* > 0$ such that $\mathbb{P}[\xi \leq \xi^*] \geq q^*$ and if $\mathbb{P}[\eta < -u] > 0$ for all $u > 0$, then there exist constants $a > 0$ and $b > 0$ such that

$$\psi(x) \geq \frac{a}{x^b} .$$

Thus, the probability of ruin cannot decay faster than a power function. The result sheds light on some previous results by the second author, which will be commented upon. Under certain conditions we obtain two-sided bounds of similar type. (Joint work with V. Kalashnikov)

Harri Nyrhinen

Large deviations and ruin problems

Ruin probabilities in various models have been of constant interest in risk theory. Typically, we are interested in the case where the initial capital of the company is large. Then the probabilities in question are small and there is a possibility to make use of large deviations techniques to deal with the problem. The resulting estimates will then be crude in the sense that the method only gives the rate of convergence associated with the probabilities. An advance is that it allows us to deal with general models for the risk process. Large deviations estimates may also be used to obtain shape results concerning the process (given that ruin occurs) and to analyse efficiency questions in simulation.

Jostein Paulsen

Importance Sampling for a class of semimartingales

Let Y be a semimartingale. We are interested in the probability of eventual ruin, i.e. the probability that Y ever becomes negative. One natural way of doing this is of course to use crude Monte Carlo simulation, i.e. simulate Y repeatedly and count the number of times it goes below zero before it hits an upper bound $\bar{y}$. If $\bar{y}$ is chosen sufficiently high, this will lead to an approximate unbiased estimator of the probability of eventual ruin. A more efficient, but also more dangerous method is to simulate Y under another probability measure. Using the generalized Girsanov theorem, we discuss how this can be done. Based on analytical results for some simple models, we propose a simple class of measure changes for classical risk processes compounded by independent Levy processes with a finite number of jumps on finite time intervals. It is shown by Monte Carlo simulations that this method works well, at least when jumps are of exponential type. The case with subexponential jumps is unfortunately more delicate.

Holger Rootzén

Catastrophic risk and dependent extremes

What distinguishes catastrophic (economic) risks from ordinary risks, and what are the consequences? From a risk management point of view, the defining property may be “a catastrophe can happen at most once”. If a risk is catastrophic isn't determined by the “physical” situation alone. E.g., a major earthquake may be both a human catastrophe and an economic catastrophe for an insurance company but is only an ordinary risk for an owner of a cat bond related to the event. It is not possible to adjust and improve procedures as one gains more and more experience from occurrences of the catastrophe – the risk have to be treated as one-shot gambles. What can be done is (i) to take measures to reduce risk, and (ii) to determine the odds of the gamble. In this presentation, both options were discussed. In addition, for (ii) some stochastic issues which arise from clustering of extreme events caused by dependence were discussed.

This included dependent compound Poisson limits with explicit rates of convergence and estimation of the extremal index by non-parametric and Markov chain methods.

Sabina Schlegel

Asymptotics of ruin probabilities in random walks with m-dependent heavy-tailed increments

We consider a random walk $\{S_n, n \in \mathbb{N}\}$ with m-dependent heavy-tailed increments as $S_n = \sum_{i=1}^{n} \xi_i$, where $\xi_i = \sum_{k=i-m}^{i} c_{i-k}\eta_k - a$ with i.i.d. zero mean random variables $\{\eta_n, n \in \mathbb{N}\}$ and some constants $a > 0$ and $c_0, \ldots, c_m \in R$. Conditions are given under which the asymptotic tail behaviour of $\sup_n S_n$ can be easily related to the asymptotic behaviour of the left and right integrated tails of the distribution function F of η_n. In order to get an asymptotic lower bound for $P(\sup_n S_n > x)$ we assume that F belongs to the class of long-tailed distributions. For an upper bound we additionally assume that the left and right tails are the tails of subexponential distributions. Finally lower and upper bounds are shown to coincide.

Hanspeter Schmidli

On the distribution of the surplus prior and at ruin

Consider a classical compound Poisson model $\{X_t\}$. The safety loading can be positive, negative or zero. Denote the ruin time by τ. Define the function

$$u \mapsto f(u; x, y) = \mathbb{P}[\tau < \infty, X_{\tau-} > y, -X_\tau > x] \,.$$

This is an expression for the distribution of the surplus prior and at ruin. We find the integro-differential equation for $f(\cdot; x, y)$, and determine its Laplace transform. This leads to an explicit expression for $f(0; x, y)$. Inversion of the Laplace transform yields expressions for the distributions of the surplus prior and at ruin in terms of the ruin probability. Moreover, the asymptotic behaviour of these distributions as the initial capital tends to infinity are obtained. In particular, for positive safety loading the Cramér case, the case of subexponential distributions and some intermediate cases are discussed.

Martin Schweizer

Financial valuation of actuarial contracts

To value actuarial contracts in a financial environment, we construct a transformation on valuation rules by using a global indifference approach. Given an a priori rule u, we define the associated a posteriori rule v by the requirement that the u-value of optimally investing in the financial market alone should equal the u-value of first selling the contract at its v-value and then choosing an optimal investment strategy inclusive of the contract's payoff. The transforms of the variance and standard deviation principles are constructed explicitly and shown to possess the same type of structure as the corresponding a priori rules.

Uwe Schmock

Allocation of risk capital

Allocation of the risk bearing capital to the business units of a financial institution is an important tool for performance measurement, risk control and steering of the company. Allocation principles for risk capital are also applicable to portfolios of defaultable bonds or credit risks. We shed some light on the advantages and shortcomings of allocation principles, in particular the Euler principle, the covariance and the expected shortfall principle, which all take dependencies into account. In addition, we show how the capital allocation can be calculated in practice, for example for dependent risk processes. (Joint work with Daniel Straumann, RiskLab, ETH Zürich)

David Stanford

Ruin theoretic computations based on recursions at claim instants

An exact method is presented for the Sparre-Andersen model which calculates ruin probabilities embedded at claim instants. Following a review of previously published results for the Poisson arrival case, new results for a limited selection of non-Poisson renewal processes for the inter-claim time were presented. The driving force in the method is the distribution of the

”increment”, which is defined to be the amount of change in the surplus from claim instant to claim instant. The increment equals the difference between the inter-claim revenue (which itself is a linear multiple of the inter-claim time) and the size of the subsequent claim. The method we present uses the increment distribution to develop a recursion between the incomplete densities for the surplus immediately following the (n-1)th and nth claims. (These densities are incomplete because ruin may have occurred already.) Laplace transforms of these densities are then used to determine an algorithm for the computation of the probability of ruin on the nth claim. While the method we discuss has the advantage of being exact, it also has a significant limitation in that the time of ruin and the likelihood of ruin are obviously related. Therefore there is no direct translation from the nth claim ruin probabilities to standard finite-time ruin probabilities. On the plus side, however, the method does provide insight about the period of time when (relatively speaking) ruin is most likely. In related work performed at Oberwolfach, it was shown that the same philosophy of using recursions based at claim instants can be used to determine actual finite-time ruin probabilities. Preliminary results for the case of Poisson claims and exponential claim amounts show that the ruin probability can be expressed as an infinite mixture of Erlang distributions, a useful alternative to the usual solution, which involves Bessel functions. It is hoped that useful results can be developed for the family of phase type distributions.

Bjørn Sundt

Recursions for multivariate distributions

In the talk we generalise some recursions related to univariate distributions to the multivariate case. We first consider compound distributions where each claim event generates a vector of claim amounts. In that situation we first generalise Panjer’s recursion and then the extension presented by Sundt in 1992. Motivated by a special case of the latter recursion we introduce the De Pril transform of a multivariate distribution with a positive mass at zero and study some of its properties. We point out that most of these properties hold for more general functions than distributions. Finally we generalise to the multivariate case some error bounds of approximations to distributions and indicate how these bounds can be applied for trivial multivariate generalisations of the approximations of Kornya, Hipp, and De Pril.

Howard Waters

Multi-period aggregate loss distributions for a life portfolio

In papers in the ASTIN Bulletin in 1986 and 1989, De Pril derived a recursive algorithm to calculate the probability function for the aggregate loss from a portfolio of life insurance policies. However, an interesting feature of a life portfolio is that losses in successive years are likely to be negatively correlated; large losses in one year indicate lower losses in the next, and *vice versa*. In this talk I discussed a multi-variate extension of De Pril's recursion which can be used to calculate the joint distribution of the losses from a life portfolio over several periods. This recursion was illustrated by applying it to a simplified portfolio of 10-year endowment assurances. The (exact) numerical results were used to compare the accuracy of different ways of approximating the joint distribution. This talk was based on joint work with David Dickson to be published in a forthcoming issue of the ASTIN Bulletin.

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