Mathematisches Forschungsinstitut Oberwolfach

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Nonlinear Equations in Many-Particle Systems

5.12.–11.12.1999

The present conference was the fourth of its kind organized by J. Batt (Munich) and C. Cercignani (Milano). It was attended by 52 participants from Germany, France, USA, Italy, Sweden, Austria, Spain, Russia, Poland, Israel, Canada, Japan. The two main topics of the conference where

(a) the nonlinear systems governing the evolution of plasmas and gravitational matter (star systems) such as the Vlasov-Poisson, the Vlasov-Maxwell, the Schroedinger-Poisson, and the Vlasov-Einstein systems of partial differential equations and their variants, e. g. in the theory of semiconductors,

(b) the Boltzmann equation and its many applications.

The following 7 survey lectures (of one hour each) gave an overview over specific developments in the area of kinetic theory:

1. Stable steady states in stellar dynamics (G. Rein, Munich)
2. Hydrodynamical limit of the Boltzmann equation (R. Esposito, L'Aquila)
3. Steady state solutions of the Boltzmann equation (L. Arkeryd, Goeteborg)
4. Diffusion limits of kinetic models (F. Golse, Paris)
5. Survey of results on the Einstein-Vlasov system (A. Rendall, Golm)
6. Smoothness results for the full Boltzmann equation (L. Desvillettes, Cachan)
7. Long time asymptotics for the Vlasov-Poisson system (J. Dolbeault, Paris).

There were further 30 lectures (of half an hour each). On Thursday afternoon and Friday morning they were organized in parallel sessions to allow enough time for separate scientific work and discussion. The participants enjoyed the traditionally pleasant and friendly atmosphere of the Institute, which contributed to the success of the meeting, and they expressed their hope for a subsequent meeting in about 3 years.

Abstracts

The Einstein-Vlasov system (global existence questions)

Håkan Andreasson

A global existence theorem, with respect to a geometrically defined time, is shown for Gowdy symmetric globally hyperbolic solutions of the Einstein-Vlasov system for arbitrary (in size) initial data. These spacetimes contain both matter and gravitational waves.

On the stationary Boltzmann equation in $\mathbf{R}^n$

Leif Arkeryd

A survey of the scenario up till now concerning existence for the boundary value problem of this type of equations in the non-linear case was given. The stationary problem was related to the asymptotic aspects of the time dependent problem and to boundary layer questions in the hydrodynamic limit. The survey was followed by a presentation of new ideas and results about existence for the n-dimensional nonlinear Boltzmann equation with given indata. The new approach is based on the entropy production control and a separation of scales in the limit for the approximations.

Various asymptotic limits of kinetic equations

Claude Bardos

In this talk was described a series of results jointly obtained with K. Aoki, C. Bardos, F. Golse & D. Levermore.

The basic idea was to apply convenient scaling and moments methods to the kinetic equation to obtain not only the compressible Euler equation but also a large variety of equations, including

i) The incompressible Euler and Navier Stokes equations with for the energy equation the non linear term of heat production.

ii) The Stokes and acoustic equation

iii) The Prandlt equations for the boundary layer.

Emphasis was put on the fact that these such formal derivation can be obtained for any collision operator which satisfies the basic axioms of kinetic theory.

In the present state of the art rigorous proof of these limit are in good agreement with the classical existing results for the macroscopic equation and this was explained. However up to now no result exists concerning the convergence of the Di Perna Lions solution of the Boltzmann equation to the Leray solution of the incompressible Navier Stokes equation. This is mainly due to the absence of proof concerning the local conservation of momentum.

However it was recently shown that such difficulty can be overcome in the case of the acoustic and Stokes limit. One of the main ingredient for this proof was given.

Half-space problem for the Boltzmann equation and discrete velocity models

Alexander Bobylev

The talk is devoted to some rigorous estimates for solutions of the evaporation/condensation problem, i.e. the half-space ($x > 0$) problem for the nonlinear Boltzmann-type equation with given distribution (usually Maxwellian with zero bulk velocity) of incoming particles at the origin $x = 0$. The asymptotic state at infinity is given by some Maxwellian with, generally speaking, unknown parameters M (Mach number), T (temperature) and p (pressure). We derive some a priori estimates for the parameters on the basis of the following idea. The entropy flux at infinity is a known function $F(M, T, p)$. This function can not exceed the entropy flux at the origin. The entropy flux is not a convex functional, however it is a difference of two convex functionals (fluxes for incoming and outgoing particles). The first (incoming) flux is given whereas the second (outgoing) flux is bounded below by a function of three first moments of unknown distribution of outgoing particles. The moments can be easily expressed through parameters M, T, p of the asymptotic Maxwellian. This consideration leads to inequality $0 < D(f) < G(M, T, p)$, where $D(f)$ is the total entropy production. The function $G(M, T, p)$ is implicitly expressed through some elementary functions and the error function. The inequality $G > 0$ defines a domain of admissible values of M, T, p. Numerical calculations performed by A. Heintz and R. Grzhibovskis show that this domain (in case of evaporation) is a closed "tube" in $3d$ space of parameters. Moreover the "central curve" $T = T(M),\ p = p(M)$ of the tube, that corresponds to maximal total entropy production, practically coincides with the evaporation curve obtained by Sone et al. from direct numerical solutions of the Boltzmann and BGK equations.

Derivation of the Euler equations from a caricature of Coulomb interaction

Yann Brenier

A caricature of collisionless plasma involving $2N$ particles of opposite charge is introduced. The N first particles are called 'ions' and don't move. The N other particles are called 'electrons'. At each time, there is a one-to-one matching between electrons and ions and each pair is linked by a 'spring' so that each electron oscillates with fixed frequency ϵ^{-1}. The essential point is that the matching between electrons and ions is updated at every discrete time $n\tau$, $n = 0, 1, 2, ...$, so that the total potential energy of the system stays minimal. This leads to a non trivial interaction which turns out to be a caricature of Coulomb interaction. It is proven that, provided the N ions are equally spaced in a bounded domain D and ϵ, τ and N^{-1} tend to zero at appropriate rates, the electrons behave as the fluid parcels of an incompressible inviscid liquid moving inside D according to the Euler equations. Our proof relies on a result of P. Lax on the approximation of volume-preserving transformations by permutations.

Conjecture on "eternal" solutions, for a model of the Boltzmann equation

Henri Cabannes

According to a classical conjecture, the only "eternal" positive solutions of the Boltzmann equation are the Maxwellian solutions [3]. We consider the equation (E):

$$\frac{\partial N(t;\theta)}{\partial t} = \frac{1}{2\pi}\int_0^{2\pi} \{N(t;\phi)\,N(t;\phi+\pi) - N(t;\theta)\,N(t;\theta+\pi)\}\,d\phi$$

which is a model of the two-dimensional, homogeneous Boltzmann equation, obtained from discrete kinetic theory by a limiting process. When the initial data are π-periodic: $N(0,\theta+\pi) = N(0,\theta)$, the study of equation (E) can be reduced to the study of a linear ordinary differential equation, and three years ago at Oberwolfach we have then obtained, on a parametric form, the general solution [2].

Using this general solution we prove now, for the π-periodic solutions of equation (E), the conjecture on "eternal" positive solutions [1].

[1] H. Cabannes, Proof of the conjecture on "eternal" positive solutions for a semi-continuous model of the Boltzmann equation. *C. R. Acad. Sci. Paris* **327**, série I, 217–222 (1998)

[2] H. Cabannes, "Eternal" solutions for an homogeneous two dimensional model of the Boltzmann equation. *Math. Models and Meth. in Appl. Sci.* **9**, 127–137 (1999)

[3] C. Villani, Contribution à l'étude mathématique des équations de Boltzmann et de Landau cinétique des gaz et des plasmas. *Thèse de'lUniversité de Paris IX-Dauphine*, 289–295 (1998)

Stochastic methods in kinetic theory

Eric Carlen

Although kinetic theory is intrinsically probabilistic at the physical level, mathematical results are usually obtained through analytic, rather than stochastic, methods. In this talk, we present three results in which stochastic analysis plays an important role. The first is joint work with M. C. Carvalho and E. Gabetta, in which we prove an L^1 bound on the error made in truncating the wild expansion for the Boltzmann equation with Maxwellian molecules. The key estimate concerns the behavior a certain random walk on graphs introduced by McKean. This work has recently appeared in *Commun. Pure and Applied Math.* The second is joint work with M. C. Carvalho and M. Loss. In this we consider a model for N colliding molecules introduced by Marc Kac that has a model Boltzmann equation as a limit as N tends to infinity. Through a novel method, obtain sharp bounds on the rate of relaxation to equilibrium as a function of N. The third is joint work with Wilfrid Gangbo. In it we use "mass transport" metrics on probability measures to construct solutions to the spatially inhomogeneous kinetic Fokker–Planck equation using a strategy based on earlier work of Luckhaus and Otto.

Boltzmann's kernel and the spatially homogeneous Boltzmann equation

Laurent Desvillettes

We give a survey concerning the properties of the Boltzmann (and Landau) kernel of monoatomic rarefied gases and its evolution equation: the spatially homogeneous Boltzmann (and Landau) equation.

The cross sections taken into account in this survey are the hard spheres, the hard, Maxwellian and soft potentials, and the Coulombian cross section. We also distinguish between the cutoff and the non cutoff (with respect to the angular variable) case.

Existence and uniqueness are studied, as well as the behaviour in large time or large velocities. The smoothness is investigated, and also the existence of lower bounds.

Whenever it is possible, the properties of the equation are shown to derive from an estimate on the kernel.

On long time asymptotics of the (nonlinear) Vlasov-Poisson system

Jean Dolbeault

In the periodic case, scattering results (nonlinear Landau damping) can be proved, which are unstability results in weak norms, while for problems with injection boundary conditions (and eventually collision terms) convergence results hold in a well chosen relative entropy framework. In the whole space case, time-dependent rescalings provide a convenient tool for studying the dispersion, to look for intermediate asymptotics and to present a unified framework for several related problems.

Hydrodynamical Limit of the Boltzmann Equation

Raffaele Esposito

The derivation of the macroscopic equations for a fluid from microscopic deterministic models is a difficult problem whose solution is beyond the present mathematical knowledge. However, if the Boltzmann equation is assumed as a good approximation of the microscopic dynamics, for rarefied gases, a few rigorous results can be proved.

We present a quick survey of such results, starting with earlier works on the proof of the convergence of the hyperbolic scaling limit to Euler Equations. Then we introduce the problem of the dissipative effects and the Navier-Stokes correction. In this framework we present some situations where dissipative effects become relevant. They are

- Diffusive scaling limit, where, in the low Mach number regime the convergence to the incompressible Navier-Stokes equations has been proved at the end of the eighties;
- Stationary boundary value problems, in the presence of special symmetries that allow to treat weakly compressible situations;
- Phenomena of phase segregation for binary mixture.

Kinetic theory of granular materials

Irene Gamba

I have presented in this lecture recent work in collaboration with Alexander B. Bobylev, Jose A. Carrillo and Carlo Cercignani. We have investigated a Boltzmann equation for inelastic scattering in which the relative velocity in the collision frequency is approximated by the thermal speed. The inelasticity is given by a velocity variable restitution coefficient. This equation is the analogous to the Boltzmann classical equation for Maxwellian molecules.

We study the homogeneous regime using Fourier analysis methods developed by Bobylev in the case of elastic collisions. We analyze the existence and uniqueness questions, linearized operator around Dirac delta function, self-similar solutions and moment equations.

We clarify the conditions under which self-similar solutions describe the asymptotic behavior of the homogeneous equation. We obtain formally a hydrodynamic description for near elastic particles under the assumption of constant and variable restitution coefficient. We describe the linear long-wave stability/instability for homogeneous cooling states.

Finally, we present the corresponding inelastic collision model in a fluidized bath with fixed temperature θ_B. The corresponding transport equation is a pseudo-Maxwellian model with a Fokker-Plank operator added to the collision term. There, we have performed an expansion in energy dissipation as small inelasticity perturbation from the elastic regime. We compute the corresponding stationary solution which are shown to be Maxwellians with variance θ_B times a factor given by a quartic polynomial.

Semiclassical propagation through energy level crossings

Patrick Gérard

The semiclassical limit of the following system

$$ih\frac{\partial\Psi}{\partial t}\frac{h}{i}\begin{pmatrix} \partial_{x_1} & \partial_{x_2} \\ \partial_{x_2} & -\partial_{x_1} \end{pmatrix}\Psi + V(x)\Psi$$

$(x_1, x_2) \in \mathbf{R}^2$, $\Psi(t) \in L^2(\mathbf{R}^2, C^2))$ is discussed, with some emphasis on the critical set $\{\xi = 0\}$ in the phase space where the energy level are crossing. Assuming $\nabla V(x) \neq 0$ at the crossing points, we derive evolution equations for the rescaled Wigner transforms; the obtained formulae involve transition probabilities of Landau-Zener type, namely $e^{-\pi\eta^2/|\nabla V(x)|}$, where η denotes the rescaled distance to the critical set $\xi \wedge \frac{\nabla V(x)}{|\nabla V(x)|} = 0$.

Diffusion limits of kinetic models

Francois Golse

Prof. Golse did not provide an abstract of his talk (The editor).

Some recent results on fluid as well as kinetic models for a collisionless plasma

Yan Guo

(1) In the study of the 'two-fluid' model for a plasma, Tahvildar-Zadeh and I [GT] have proven the formation of singularities for 'large' initial data in the 3D Euler-Poisson system. It was shown that small smooth irrotational flows persist for all time for the same system [G]. In the periodic case, S. Cordier, E. Grenier and I [CGG] have recently verified the well-known 'two-stream' instability in a nonlinear, dynamical setting. Linearized 'two-stream' instability was well studied by physicists long before.

(2) In the study of the Vlasov theory for a collisionless plasma, W. Strauss and I [GS] recently have obtained sharp conditions to determine nonlinear stability and instability of an anisotropic homogeneous equilibria in the presence of magnetic perturbations.

[CGG] S. Cordier, E. Grenier, Y. Guo, Two-stream instabilities in plasmas, submitted to JMAA, special volume in honor of C. S. Morawetz.

[G] Y. Guo, Smooth irrotational flows in the large to the Euler-Poisson system in $\mathbf{R}^{3+1}$, Commun. Math. Phys., 195, 249-265 (1998).

[GT] Y. Guo, A. Shadi Tahvildar-Zadeh, Formation of singularities in relativistic fluid dynamics and in spherically symmetric plasma dynamics, Contemp. Math., 238, 151-161, (1999).

[GS] Y. Guo, W. Strauss, Magnetically created instabilities in a plasma, preprint 1999.

Renormalization process for the Vlasov-Poisson system

Pierre-Emmanuel Jabin

We consider solutions to the Vlasov-Poisson system with infinite mass and energy. The solution to the Poisson equation cannot be defined directly because the macroscopic density has no decay at infinity. To solve this problem, we decompose the solution to the kinetic equation in an homogeneous function and a perturbation. We are then able to prove a global existence result of weak and strong solutions to the equation even though there are no a priori estimates for the solution by lack of positivity.

Nonlinear problems in quantum semiconductor modeling

Ansgar Jüngel

A hierarchy of macroscopic equations for quantum semiconductor devices is presented, namely the hydrodynamic, energy-transport and drift-diffusion equations including quantum correction terms. The quantum hydrodynamic model (QHD) can be derived from the mixed-state Schrödinger equation (or the quantum Boltzmann equation), whereas the quantum energy-transport (QET) and quantum drift-diffusion (QDD) equations are zero-relaxation-time limits of the QHD.

An existence result for the stationary solutions of the QHD is proven in the case where the thermal energy is much larger than the electric energy. For the transient QDD, a positivity preserving numerical scheme and convergence results are presented, based on a semi-discretization of time. Finally, numerical results for a one-dimensional resonant tunneling diode are given.

Phase space transport in noisy Hamiltonian systems

Henry E. Kandrup

This talk was comprised of two parts, namely: (1) a description of several difficult questions about the gravitational Vlasov-Poisson system, which the galactic physicist would like to see answered; followed by (2) the results of an investigation of how phase space transport through cantori or along Arnold webs in complex two- and three-dimensional Hamiltonian systems is accelerated by low amplitude irregularities modeled as periodic driving and/or noise.

Recent observations suggest that real galaxies are genuinely three-dimensional, neither spherical nor axisymmetric, and that they often contain a near-singular cusp with the density increasing without bound down to very short scales. Does Vlasov-Poisson admit cuspy, triaxial equilibria, or must these galaxies be viewed as out of equilibrium?

Low amplitude noise, like periodic driving, can dramatically accelerate phase space transport in a complex phase space. This accelerated transport is driven by a resonant coupling between the frequencies for which the power spectrum of the noise has substantial support and the natural frequencies of the orbit, which wiggles the orbit and thus helps it to find phase space holes. The efficacy of the perturbation scales logarithmically in amplitude. For coloured noise, this efficacy decrease logarithmically with t_a for autocorrelation times t_a long compared with t_{cr}, the natural time scale for the orbit, but seems largely independent of other details.

For galaxies, (near-)white noise can mimic discreteness effects, periodic driving can mimic individual companion galaxies, and coloured noise can mimic a complex, near-random environment. Environmental perturbations often have fractional amplitudes as large as $\sim 10^{-2}$; discreteness effects can be as large as $\sim 10^{-6} - 10^{-5}$. The timescale over which such perturbations can be important is, in many cases, short compared with the age of the Universe.

(joint work with Ilya V. Pogorelov and Ioannis V. Sideris)

Diffusion approximation and kinetic boundary layers in plasmas

Brigitte Lucquin-Desreux

We consider a magnetized plasma composed of electrons of low mean density n_0^e and of one species of ions (of density n_0^i) with a high ionization degree Z. More precisely, we suppose that:

$$Z^2\varepsilon = 1, \quad n_0^e = \varepsilon n_0^i, \quad \text{with } \varepsilon = \sqrt{m^e/m^i},$$

where m^e (resp. m^i) is the electron mass (resp. ion mass).

Starting with the Vlasov-Fokker-Planck equations for both particles, we derive, in the limit $\varepsilon \to 0$, a diffusion model for the electrons, with explicit transport coefficients which are related to the ions. We simultaneously obtain a standard system of hydrodynamic equations for the ions; this system is, up to the first order in ε, totally independent of the electrons, which confirms physical results [1].

In a second part, we consider the case of a bounded domain in space variables. In order to derive sufficiently accurate boundary conditions for the above diffusion model, we study the kinetic boundary layers. For that purpose, we are led to analyze a Milne problem and to introduce a generalized extrapolation length.

The present work is detailed in [2].

[1] L. Landau, E. Lifchitz, *Physique Théorique, tome 10, Cinétique Physique*, Editions Mir, Moscou (1990).

[2] B. Lucquin-Desreux, *Diffusion of electrons by multicharged ions*, Internal Report R 99007, Paris 6 (article accepted for publication in M3AS).

A kinetic model describing electron flow in semiconductors: analytical and numerical results. Mobility and diffusivity in silicon devices according to the Chapman-Enskog expansion

Armando Majorana

In this paper the Boltzmann equation describing the carrier transport in a semiconductor is considered. A modified Chapman-Enskog method is used, in order to find approximate solutions in the weakly non-homogeneous case. These solutions allow to calculate the mobility and diffusion coefficients as function of the electric field. The integral-differential equations derived by the above method are numerically solved by means of a combination of spherical harmonics functions and finite-difference operators. The Kane model for the electron band structure is assumed; the parabolic band approximation is obtained as a particular case. The numerical values for mobility and diffusivity in a silicon device are compared with the experimental data. The Einstein relation is also shown.

High density limits to the Thomas-Fermi-von Weizsäcker-Dirac model via deformations of plane waves

Norbert Mauser

We deal with local density approximations for the kinetic and exchange energy terms of a periodic Coulomb model. For the kinetic energy, we give a rigorous derivation of the usual combination of the von-Weizsäcker term and the Thomas-Fermi term in the "high density" limit. Furthermore, we justify the inclusion of the Dirac term for the exchange energy and the Slater term for the local exchange potential. Our method is based on deformations (local scaling transformations) of plane waves in a periodic box.

An integral transform for the continuous spectrum in fluid and plasma dynamics

Philip Morrison

Fluid and plasma dynamics obtained by expansion about equilibria (steady states) generally possess a continuous spectrum. A general integral transform, which is a generalization of the Hilbert transform, will be described and shown to make the solution of such a linear problem trivial. The transform will also be shown to amount to a coordinate change to action-angle variables in the infinite-dimensional Hamiltonian description of fluid and plasma dynamics. Examples include: Vlasov-Poisson dynamics, shear flow, and possibly Maxwell-Vlasov.

To appear in *Transport Theory and Statistical Physics.*

Initial value problem for Einstein-Boltzmann system

Piotr Mucha

I have presented a model of a relativistic collision gas described by the Einstein-Boltzmann system

$$\begin{aligned} p^\alpha f_{,\alpha} - \Gamma^i_{\mu\nu} p^\mu p^\nu f_{,p^i} &= Q(f,f), \\ G^{\mu\nu} &= T^{\mu\nu}, \qquad\qquad (EB) \\ T^{\mu\nu} &= \int f p^\mu p^\nu \frac{|g|^{\frac{1}{2}}}{p^0} d\bar{p}. \end{aligned}$$

First, I showed a local in time result on existence of solutions of (EB) in the harmonic coordinates.

Next, in the cosmological case, in the flat Robertson-Walker spacetime

$$ds^2 = dt^2 - R^2(t) \left((dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right)$$

I proved global in time existence of mild solutions for (EB) in such cosmological case.

Kinetic formulation of elastodynamic equations

Benoit Perthame

The kinetic approach to conservation laws concerns various aspects of the theory of hyperbolic systems. For those with a rich enough family of entropies, it allows to give a kinetic formulation which is a way to represent by a single equation the whole family of entropies. The example of elastodynamics is particularly interesting since it is not possible to find explicit formulas for the entropy kernel and it relies on a mathematical construction. For scalar conservation laws it turns out to be a powerful tool to understand their mathematical structure and unify in a simple formalism most of the theory. As an example remains the proof of Sobolev regularizing effects for nondegenerate fluxes in multidimensional scalar conservation laws. Many open theoretic questions are still open which will be mentioned during the talk.

More general, and useful for applications, is the derivation of stable numerical schemes for systems of gas dynamics type. As an example, we will explain how the kinetic schemes furnish a natural answer to the question of deriving numerical methods which preserve equilibriums for the Saint-Venant model of shallow water.

Stable steady states in stellar dynamics

Gerhard Rein

In the astrophysics literature steady states of the stellar dynamic Vlasov-Poisson system have a long history as models for galaxies. Besides the existence question for such steady states the question of their non-linear stability is both physically important and mathematically nontrivial. We present recent results on a variational approach to this problem which was developed in joint work with Yan Guo (Brown University). We give the main ideas of this approach, compare it with other results in this area, and state open problems.

Survey of results on the Einstein-Vlasov system

Alan Rendall

Known results on global existence and qualitative behaviour of solutions of the Einstein-Vlasov system are surveyed. Due to the difficulty of the general problem, most of these results concern solutions with symmetry or other restrictions. A summary of the different relevant symmetry types is given. There follows a tour through many of these, proceeding from greater to lesser symmetry. Similarities and differences in comparison to the Vlasov-Poisson and Vlasov-Einstein systems are pointed out whenever possible. Finally, some promising directions for future progress are indicated.

Time relaxed Monte Carlo methods for the Boltzmann equation

Giovanni Russo

A new family of Monte Carlo schemes is derived for the numerical approximation of the Boltzmann Equation of rarefied gas dynamics. The objective of the research is the development of a robust Monte Carlo method for the Boltzmann equation which can be effectively used over a large range of Knudsen number.

A common procedure to solve the Boltzmann equation is to use splitting. Space is divided into cells. The solution is computed as a sequence of two steps: a convection step, and a collision step. In the latter, the space homogeneous equation is solved in each cell for one time step Δt. We consider the space homogeneous equation first.

The schemes are based on a variant of the Wild sum expansion in time of the solution of the Boltzmann equation for Maxwell molecules. The Boltzmann equation in this case can be written as

$$\frac{\partial f}{\partial t} = P(f,f) - \mu f,$$

where μ is constant. m-th order scheme is written as

$$f^{n+1} = \sum_{k=0}^{m} A_k f_k^n + A_{m+1} M,$$

where the coefficients $A_k > 0$ are chosen in such a way to guarantee accuracy in time, conservation and asymptotic preservation. The terms f_k are obtained by a recursive formula.

These schemes can be generalized to non Maxwellian molecules, by computing at each time step a bound on the collision frequency $\mu \geq \nu(v)$. In this case $P = Q_+ + \mu - \nu(v)$. Because A_k and f_k are positive, Monte Carlo scheme can be derived from the probabilistic interpretation of the above equation.

The schemes can be made conservative if pairs of particles are sampled, and if the particles sampled from the Maxwellian have prescribed average velocity and temperature.

Numerical tests on the space homogeneous equation shows that it is possible to achieve the same accuracy of Nanbu-Babowsky scheme using a smaller time step.

Tests on computation of the shock structure show that good results are obtained for very high Mach number shocks, even if an underresolved grid is used.

Hole Equilibria in Vlasov-Poisson Systems— A challenge for plasma wave theories?

Hans Schamel

Electron holes [1] (as well as ion holes and double layers [2]) are 1D stationary solutions of the Vlasov-Poisson system and can in this sense be considered as BGK-type solutions [3]. Being characterized by a notch in the thermal part of the distribution function at resonant velocities, they show up in real space as localized density depressions and associated field spikes. Improved diagnostics have enabled physicists recently to detect them experimentally in space and laboratory plasmas, including beam plasmas in particle accelerators [5].

Mathematically, hole equilibria (or phase space vortices) are best described by the so-called potential method introduced in [4]. This method differs from the original BGK-method [3] by the fact that all distributions can be prescribed as functions of the constants of motion allowing the incorporation of distribution functions of arbitrary degree of regularity [6]. Of special interest are solutions which are continuous across the separatrix with a jump of its first derivatives. This "physical" subclass of BGK-type solutions is peculiar in so far as in the small amplitude limit a linear counterpart can not be found, the solution would be associated with. Neither a Landau nor a van Kampen analysis are capable of extracting the specific features of small amplitude hole solutions, namely their phase velocity and spectral content. In fact, hole equilibria appear to be heavily Landau damped if a linear Landau treatment of the initial value problem would be applicable. Furthermore, the marginal modes of the van Kampen continuum (or generalizations [7]) can generally not be used either as a complete basis to represent by superposition undamped hole equilibria (except perhaps modes in the harmonic limit). It is therefore claimed that any linearization procedure will fail to describe a hole solution no matter how small the wave amplitude is. Even in the infinitesimal wave limit, the solution will keep its nonlinear character due to the presence of resonant particles. Hole solutions, hence, represent a challenge for standard wave theories, as there is no threshold value for the wave amplitude below which a linear analysis can be justified.

[1] H. Schamel, *Physica Scripta* **20** (1979) 336

[2] H. Schamel, *Phys. Reports* **140** (1986) 161

[3] I. B. Bernstein, J. M. Greene, M. D. Kruskal, *Phys. Rev.* **108** (1957) 546

[4] H. Schamel, *Plasma Phys.* **14** (1972) 905

[5] H. Schamel, *Phys. Rev. Lett.* **79** (1997) 2811

[6] J. Korn, H. Schamel, *J. Plasma Phys.* **56** (1996) 207, 339

[7] P. J. Morrison, D. Pfirsch, *Phys. Fluids* **B4** (1992) 3038

Flat solutions of Vlasov-Poisson system

Svetlana Shevchenko

One of the interesting mathematical problems arising in astrophysics is the modeling of "flat" galaxies, which means that these galaxies have the shape of a very flat disc. Mathematically, such galaxies can be described approximately by the Vlasov-Poisson system, where the distribution function has a δ-singularity in the third component. In this case the Poisson equation becomes more singular than in the usual 3-dimensional case. But by choosing appropriate function spaces, which are preserved under the singular solution operator of the Poisson equation, we prove the existence of local classical and global weak solutions.

Positive solutions for nonlinear singular boundary value problem of magnetic insulation

Alexander Sinitsyn

On the basis of generalization of upper and lower solution method to the singular two-point boundary value problems, the existence theorem of solutions for the system

$$\frac{d^2\varphi}{dx^2} = j_x \frac{1+\varphi(x)}{\sqrt{(1+\varphi(x))^2 - 1 - a(x)^2}}; \quad \varphi(0) = 0, \quad \varphi(1) = \varphi_L,$$

$$\frac{d^2a}{dx^2} = j_x \frac{a(x)}{\sqrt{(1+\varphi(x))^2 - 1 - a(x)^2}}; \quad a(0) = 0, \quad a(1) = a_L,$$

which models a process of magnetic insulation in plasma is proved. We develop this technique for the free-boundary problem. This work was done collaborate with P. Degond.

A variational approach to the Schrödinger-Poisson system

Juan Soler

In this communication a variational formulation of the three-dimensional Schrödinger-Poisson system is proposed with the aim of solving the open problem of the asymptotic behaviour in time of the solutions in the case of attractive Coulomb forces. A dispersive equation relating density and linear moment dispersions is found. Optimal bounds for the kinetic energy are obtained which leads to study the asymptotic behaviour in time for the solutions in the attractive case with positive energy. The description of the asymptotic behaviour properties of the solutions such as a the existence of *breathing* mode solution, i.e. a changing size oscillatory wave function, in the case of attractive potential with negative kinetic energy are also given. On the other hand, a long time global description of the 3–D Schrödinger–Poisson system is derived in terms of the Wigner formalism through the choice of the mass–preserving scale group $\psi_\epsilon(x,t) = \psi(x, \epsilon^{-1}t)$. This group of scale transformations leads to high frequency (time oscillatory) wave functions, which are Wignerized via the introduction of a rescaled space–time Wigner transform. This 'extended' Wigner transform produces an attenuating effect on the temporal oscillations as time grows up, which allows to overcome the lack of compactness in time. In a certain sense, it transforms high frequency asymptotics into a long time, low oscillating limit ($\epsilon \to 0$) which allows to recover some of the macroscopic properties of the solutions of the original Schrödinger–Poisson problem.

Slow motion of charges interacting through the Maxwell field

Herbert Spohn

I report on joint work with Markus Kunze, FB Mathematik, University of Cologne. We study the Abraham model for N charges interacting with the Maxwell field. On the scale of the charge diameter, R_φ, the charges are a distance $\epsilon^{-1}R_\varphi$ apart and have a velocity $\sqrt{\epsilon}c$ with ϵ a small dimensionless parameter. We follow the motion of the charges over times of the order $\epsilon^{-3/2}R_\varphi/c$ and prove that on this time scale their motion is well approximated by the Darwin Lagrangian. The mass is renormalized. The interaction is dominated by the instantaneous Coulomb forces, which are of the order ϵ^2. The magnetic fields and first order retardation generate the Darwin correction of the order ϵ^3. Radiation damping would be of the order $\epsilon^{7/2}$.

The asymptotic behaviour of kinetic equations

Giuseppe Toscani

Consider the flow of gas in an N-dimensional porous medium with initial density $v_0(x) \geq 0$. The density $v(x,t)$ then satisfies the nonlinear degenerate parabolic equation $v_t = \Delta v^m$ where $m > 1$ is a physical constant. Assuming that $\int(1+|x|^2)v_0(x)dx < \infty$, we prove that $v(x,t)$ behaves asymptotically, as $t \to \infty$, like the Barenblatt-Pattle solution $V(|x|,t)$. We prove that the L^1-distance decays at a rate $t^{1/((N+2)m-N)}$ which is sharp. Moreover, if $N = 1$, we obtain an explicit time decay for the L^∞-distance at a suboptimal rate. The method we use is based on recent results obtained recently for the Fokker-Planck equation, namely the so-called entropy-entropy method. The results have been obtained jointly with J. Carrillo of the University of Granada (Spain).

Periodic solutions of the Boltzmann equation

Seiji Ukai

We discuss the Boltzmann equation with time–periodic boundary data and/or time-periodic external source terms, and show that a time-periodic solution exists near an absolute Maxwellian if the data are small and if the semi-group of the linearized Boltzmann operator at the Maxwellian has nice decay properties.

In the case of a domain with boundary, the nonhomogeneous boundary condition can be reduced to a homogeneous one (which we assume time-independent) by introducing a suitable extension of boundary data over the domain, which gives rises to an additional linear part and source term. Thus, the evolution equation we shall solve has the form

$$\frac{du}{dt} = Bu + L(t)u + \Gamma[u] + h(t),$$

where B is the linearized Boltzmann operator (with the time-independent homogeneous boundary condition), $L(t)$ the linear operator coming from boundary data which vanishes in the case of the whole space $\mathbf{R}^n$, Γ the nonlinear remainder and $h(t)$ a given periodic source term including the term coming from boundary data.

Our aim is to seek a time periodic solution $u = u(t)$ with the same period as $h(t)$. We can show that u is such a solution if and only if it solves the integral equation

$$u(t) = \int_{-\infty}^{t} e^{(t-s)B}\{L(s)u(s) + \Gamma[u(s)] + h(s)\}ds,$$

if the integral on the right hand side converges. The convergence is guaranteed if the semi-group e^{tB} has an enough decay as $t \to \infty$ and then the contraction argument applies if h is small.

Conservation laws for kinetic equations

Victor Vedenyapin

Generalized discrete velocity models are considered that include triple and more collisions and chemical reactions. Simple algebraic constructions give their conservation laws. We consider

1. Connection with Hamiltonian dynamics [1, 2],
2. hyperbolicity of macroscopic equations,
3. discrete velocity models for mixtures [3, 4].

[1] O. Mingalev, Yu. Orlov, V. Vedenyapin, *Physics Letters A* (1994)

[2] V. Vedenyapin, Yu. Orlov, *Theor. Math. Physics* (1999)

[3] V. Vedenyapin, S. Amasov, L. Toscano *Russian Math. Surveys* (1999)

[4] V. Vedenyapin, *Transport Theory and Stat. Phys.* (1999)

A mathematical theory of grazing collisions

Cedric Villani

We present some new results and techniques for the study of the (spatially inhomogeneous) Boltzmann equation without cutoff. Grad's assumption of angular cutoff (integrability of the cross-section) is a huge mathematical simplification, but is unnaturally restrictive from the physical point of view, since it is never satisfied for long-range interactions, due to the abundance of *grazing collisions.*

Recently, a lot of progress was made in the understanding of the non-cutoff case. Here we present an almost complete analog of the DiPerna-Lions theory for non-cutoff cross-sections, under fully realistic assumptions. The main results, based on several new tools, are

1) existence of solutions to the Cauchy problem, in a suitable class of weak solutions (renormalized solutions with a defect measure);

2) immediate damping of oscillations (passing from weak to strong convergence) as time becomes positive. This regularizing effect of grazing collisions was conjectured by Lions in 1994;

3) a rigorous proof of the Landau approximation in plasma physics: roughly speaking, this means the replacement of the Boltzmann collision operator by the Landau operator for Coulomb interaction.

[1] R. Alexandre, L. Desvillettes, C. Villani, B. Wennberg, Entropy dissipation and long-range interactions. *Arch. Rat. Mech. Anal.*, to appear

[2] R. Alexandre, C. Villani, On the Boltzmann equation for long-range interactions and the Landau approximation in plasma physics. Preprint ENS Paris, DMA. (To be split for publication)

Static solutions of the Vlasov-Einstein system

Gershon Wolansky

Static solutions of the SO(3)-symmetric Vlasov-Einstein system are studied via a variational approach. These solutions are given as minimal solutions of the Lagrangian

$$\int_0^\infty \sqrt{\frac{r}{r-2m}}Q(m^{'},r)dr$$

for the mass function $m(r) = 4\pi \int_0^r s^2\rho(s)ds$, where ρ is the spatial distribution. For the constitutive relation of the Emden-Fowler type $\phi(E,F) = E^\sigma F^k$ we prove the existence of such solutions of sufficiently small mass-energy provided $0 < \sigma < k + 3/2$. These solutions are local minimizers of the energy-Casimir functional, subjected to a variational barrier.

On a problem with two-time data for the Vlasov equation

P. Zhidkov

Dr. Zhidkov did not provide an abstract of his talk. He gave a report on his paper P. E. Zhidkov, On a problem with two-time data for the Vlasov equation, *Nonlinear Anal., Theory, Methods & Appl.* **31**, 537–547 (1998) (The editor)

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