MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 15/2000

Arbeitsgemeinschaft Dessins d'enfants

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Die Tagung fand unter der Leitung von Pierre Lochak (Ecole Normale Supérieure, Paris) und Jürgen Wolfart (Johann Wolfgang Goethe–Universität, Frankfurt am Main) statt. Im Mittelpunkt der Vorträge standen Fragen aus dem Bereich der Belyĭ Flächen und der Grothendieck–Teichmüller Gruppe $\widehat{GT}$. Dabei wurden unter anderem folgende Schwerpunkte gesetzt:

Galois–Invarianten von Dessins, die Bedeutung der Ecken eines Dessins, explizite Uniformisierung, Moduli–Körper und Definitionskörper, Teichmüller– und Modulraum, Definition von $\widehat{GT}$ nach Drinfeld und andere Beschreibungen dieser Gruppe, Zusammenhang mit Zopfgruppen, die absolute Galois–Gruppe als Untergruppe von $\widehat{GT}$.

Zwischen den Vorträgen blieb Zeit zu vielen vertiefenden Diskussionen, die allen Teilnehmern zu neuen Einsichten und/oder Denkansätzen verhalfen. In diesem Zusammenhang ist auch ein von Pierre Lochak spontan gehaltener Übersichtsvortrag mit dem Thema „Die Grothendieck–Teichmüller Gruppe: Woher und woin?“zu erwähnen, der einerseits die Motivation zur Betrachtung dieser Gruppe zu erklären versuchte und andererseits Ausblicke in die Zukunft (Vermutungen, ungelöste Probleme) vorstellte.

Vortragsauszüge

Jürgen Wolfart

The arithmetic fundamental group

Some basic concepts were introduced: The topological fundamental group ${\pi_1}^{top}(X-S)$ for a compact Riemann surface X (= smooth algebraic projective curve over some subfield k of $\mathbb{C}$) minus a finite set S of punctures, its profinite completion $\hat{\pi}_1^{top}(X-S)$, and isomorphic to that $\hat{\pi}_1$, the algebraic fundamental group $\pi_1^{alg}(X-S) = \mathrm{Gal}\,(\Omega/\mathbb{C}(X))$, where Ω denotes the maximal subfield $\Omega \subset \overline{\mathbb{C}(t)}$ unramified outside S. In more detail the "Lefschetz principle" was explained, i.e. that for K algebraically closed and $S \subset \mathbb{P}^1(K)$, S finite ($X = \mathbb{P}^1$), we have even $\Gamma_s := \pi_1^{alg}(X-S) \cong \mathrm{Gal}\,(\Omega/K(X))$ the "geometric Galois group", meaning that every finite covering of $\mathbb{P}^1$ unramified outside $S \subset \mathbb{P}^1(K)$ may be defined over K. This implies in particular one direction of Belyĭ's theorem (if $\beta : Y \mapsto \mathbb{P}^1$ is ramified above $0, 1, \infty$ only, then Y can be defined over $\overline{\mathbb{Q}}$). Finally the short exact sequence

$$1 \mapsto \Gamma_s \mapsto \mathrm{Gal}\,(\Omega/k(t)) \mapsto G_k \mapsto 1$$

was explained, where $G_k = \mathrm{Gal}\left(\overline{\mathbb{Q}}/k\right)$ and Ω is again maximal unramified outside a finite set $S \subset \mathbb{P}^1(k)$, $\Omega \subset \overline{k(t)}$, k some number field.

Jörg Zipperer

Belyĭ's theorem and its variants

A meromorphic function on a compact, connected Riemann surface with only three critical values (which can be assumed to be $0, 1, \infty$) is called a Belyĭ function. Belyĭ 's theorem was proved which states that on every smooth complete curve defined over $\overline{\mathbb{Q}}$ there exists a Belyĭ function. Together with Weil's descent theorem this gives the following characterization: a complete smooth complex curve X may be defined over $\overline{\mathbb{Q}}$ iff there exists a Belyĭ function on X.

Moreover the notions of (pre-clean, clean) dessin, meaning a "nice" embedding of a 1–complex in a Riemann surface were introduced and it was proved that the isomorphism classes of these combinatorial objects are paramatrized by the transitive permutation representations of the oriented cartographic group ${C_2}^+$. Since ${C_2}^+ \cong \pi_1(\mathbb{P}^1(\overline{\mathbb{Q}})/({l_1}^2)$ (l_1 being the loop around 1 on $\mathbb{P}^1(\overline{\mathbb{Q}})$), isormophism classes of clean dessins are in bijection with those of the clean Belyĭ functions, thus giving a combinatorial description of those complex curves which are defined over $\overline{\mathbb{Q}}$.

Finally the following equivalences were proved (which are all consequences of Belyĭ 's theorem):

(i) a smooth complete complex curve X is defined over $\overline{\mathbb{Q}}$

$\Leftrightarrow$ (ii) $X \cong \mathcal{U}/\Gamma$, for $\mathcal{U} = \mathbb{H}$ (the upper half–plane), $\mathbb{C}$ or S^2 and Γ a subgroup of finite index in cocompact triangle group.

$\Leftrightarrow$ (iii) X is biholomorphic equivalent to a Riemann surface with an equilateral complex structure.

Hilmar Hauer

Examples. Galois actions

The absolute Galois group $G_{\mathbb{Q}} = \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ acts on the set of dessins via the action on the defining coefficients of the corresponding Belyĭ pairs. We saw examples of dessins in genus 0 and their Galois orbits. In addition we proved that $G_{\mathbb{Q}}$ acts faithfully on the set of dessins in genus 0 resp. 1. We introduced the monodromy group and the cartographic group and showed that they are invariant under this action (up to conjugacy). In particular, this implies the invariance of the number of vertices and edges and the genus of a dessin.

Bernhard Köck
Moduli fields and fields of definition
The moduli field of a curve X defined over $\overline{\mathbb{Q}}$ is the smallest subfield K of $\overline{\mathbb{Q}}$ such that X is isomorphic to X^σ for all $\sigma \in \mathrm{Aut}\left(\overline{\mathbb{Q}}/K\right)$. Using Galois descent we proved that X is defined over its moduli field, if X has no automorphisms (which holds for "almost all" X) or if X has many automorphisms (i.e. if the canonical projection $X \mapsto X/\mathrm{Aut}(X)$ is a Belyĭ function). Furthermore we showed that certain "generic" hyperelliptic curves of even genus are not defined over their moduli field.

Manfred Streit
Uniformization, equations and Galois actions made explicit
A Belyĭ function β is called regular if it determines a Galois cover and uniform if ramification orders do not vary on the fiber $\beta^{-1}(x)$, $x \in \mathbb{P}^1$. For regular and uniform Belyĭ pairs the universal covering transformation group can be constructed as a torsion–free subgroup of a cocompact triangle group. In the regular case of a Belyĭ pair the canonical model of the (non–hyperelliptic, non–trigonal, $g > 4$) curve is cut out by quadrics, even the canonical ideal is generated by them, and so the action of the automorpism–group can be used to effectively calculate the canonical model in small genera. Also it can be used to calculate $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$–orbits of special curves, like the Macbeath–Hurwitz curves.

Walter Gubler
Vertices of dessins and CM-points
Let $\beta : C \mapsto \mathbb{P}^1$ be a Belyĭ function. Choose p, q resp. r as a multiple of the ramification orders over $0, 1$ resp. ∞ with $1/p + 1/q + 1/r < 1$. Let Δ be the triangle group $\Delta(p, q, r)$ and look at the following diagram:

$$\begin{array}{ccc} \mathbb{H} & \overset{\exists!\, h}{\cdots\cdots\longrightarrow} & C \\ {\scriptstyle j}\downarrow & \swarrow{\scriptstyle \beta} & \\ \mathbb{P}^1 = \Delta\backslash\mathbb{H} & & \end{array} \Rightarrow C \cong \Delta_0\backslash\mathbb{H},\ \Delta \supseteq \Delta_0 = \text{covering group of } h$$

Δ_0 may not be arithmetic, but we have a "modular embedding theorem" (Cohen, Wolfart):
a) There exists an arithmetic group Γ acting on $\mathbb{H}^r$, a Shimura variety $V = \Gamma\backslash\mathbb{H}^r$ such that Δ_0 is a subgroup of Γ.
b) There exists an analytic embedding $F : \mathbb{H} \hookrightarrow \mathbb{H}^r$ which is $\Delta_0 - \Gamma$–equivariant.
c) The quotient morphism $\overline{F} : C \mapsto V$ is defined over $\overline{\mathbb{Q}}$.
d) $\overline{F}\left(\beta^{-1}(\{0, 1, \infty\})\right) \subset \{$ special points of the Shimura variety $V\}$.
The goal of the talk was to construct V as a moduli space of abelian varieties with certain generalized complex multiplication.

Georg Hein
Weierstraß points on regular maps
We reported on the following result of Singerman and Watson:

Let $\beta : X \mapsto \mathbb{P}^1$ be a Belyĭ map which is Galois, then we have:

a) For genus(X) $\in \{2, 3, \}$ all Weierstraß points of X are ramified.

b) for genus(X) $\in \{4, 5, \}$ the following holds:

(i) all Weierstraß points are ramified, or

(ii) all Weierstraß points are ramified with respect to a second Belyĭ function, or

(iii) X is of genus 5 and the ramification indices of β are $2, 6$ and 15.

Martin Möller

Further Galois invariants

We introduced a new Galois invariant for a curve X with many automorphisms induced by the trace of the action of $\mathrm{Aut}(X)$ on $H^0(X, \Omega_{X/\overline{\mathbb{Q}}})$. The Eichler trace formula enables us to compute this invariant when the fixed–point–behaviour of $\mathrm{Aut}(X)$ is known. As an example, we studied a class of curves with $\mathrm{Aut}(X) = \mathbb{Z}_p \rtimes \mathbb{Z}_q =: G$ defined over $\mathbb{Q}(\zeta_q)$, where this invariant is able to distinguish the different Galois orbits while the cartographic group is always $(G \times G) \rtimes S_2$.

Niko Naumann

Rigidity and the inverse Galois problem

We prove the most basic rigidity theorem over $\mathbb{Q}$ mainly following Serre's Topics in Galois theory. As an example we derive the theorem, due to Hilbert, that S_n and A_n have regular G-realizations over $\mathbb{Q}$. Finally, by way of example, we explain variants and generalization.

Razvan–Dinu Litcanu

Belyĭ functions, degrees and heights

The results of this talk try to give an answer to a question of Bogomolov and Szpiro, who had the intuition that height functions on the moduli space of curves could be obtained using the degree of Belyĭ functions. We define the Belyĭ degree of a curve X defined over a number field as the minimal degree of a Belyĭ function $\beta : X \mapsto \mathbb{P}^1$, and the Belyĭ degree of a rational point $x \in X$ as the minimal degree of a Belyĭ function $\beta : X \mapsto \mathbb{P}^1$ such that $x \in \beta^{-1}(\{0, 1, \infty\})$. We prove finiteness results for these invariants, results which show that they have some properties of height functions, but they are not "arithmetic enough". We also give an upper bound for the Belyĭ degree of a rational point on the projective line in terms of its algebraic degree and height. The proofs use, on the one hand, the Grothendieck correspondence between Belyĭ pairs and "dessin d'enfants" and Belyĭ's algorithm on the other hand.

Dan Fulea

$\mathbb{P}^1$minus three points and polylogarithms

There is a tentative, close but speculative parallelism between the "world of the Grothendieck–Teichmüller–group actions" ("anabelian geometry") and the motivic (and/or K–theoretical) world ("abelian geometry"). The talk tried to present "common" objects, ideas and conjectures. The starting point is the Knizhnik–Zamolodchikov equation in the (special) form $\nabla_{KZ} G = 0$, where $\nabla := \frac{d}{dz} - \left(\frac{A}{z} + \frac{B}{z-1}\right) dz$, A, B non–commuting symbols generating the group ring of the free group with 2 generators F_2. Its solution leads to the construction of o n e associator, and after having it and simultaneously the transitive action of the Grothendieck–Teichmüller group on the set of all associators (Drinfeld 1990) we obtain a huge set of associators. One instance of this general solution is given by specializing A, B to the (nilpotent) matrices $e_0, e_1 \in gl_{n+1}(\mathbb{Q})$:

$$e_0 := \left[\begin{array}{c|cccc} & & & & \\ \hline & 0 & & & \\ & 1 & \ddots & & \\ & & \ddots & \ddots & \\ & & & 1 & 0 \end{array}\right], \qquad e_1 := \left[\begin{array}{c|cccc} 0 & & & & \\ \hline 1 & 0 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & 0 \end{array}\right]$$

The solution $\nabla_{KZ} L = 0$ gives the "polylogarithmic extension" L. Polylogarithms appear naturally and essentially in K–theory as regulators to the Deligne–Cohomology. A striking parallelism at conjectural level appears between:

- the (Deligne)–Beilinson–Soule conjecture about the graded K–theory pieces $K_n^{[j]}(F)$, $j \leq n$, that (should) "jump only in odd degrees" $K_1^{[1]}(F)$, $K_3^{[2]}(F)$, $K_5^{[3]}$, ...
- and a conjecture in Drinfeld (1990) about the Lie algebra $\mathfrak{grt}_1(F)$ that also should "only jump in odd degrees".

Thilo Kuessner
Braids
The braid group $B_n(X)$ of a space X is defined as π_1 of the configuration space of n–tuples. We proved the classical presentation of Artin's braid group $B_n(\mathbb{R}^2)$. Furthermore we showed that the mapping class group $M(0,n)$ of the n–punctured sphere can be obtained from Artin's braid group by quotienting out the sphere relation and the center relation. The moduli space $M_{0,n}$ of complex structures with n marked points on the sphere is the configuration space of $(n-3)$–tuples on $\mathbb{P}^1(\mathbb{C})\backslash\{0,1,\infty\}$ (for $n = 4$ this correspondence is just the cross–ratio). $\pi_1(M_{0,n})$ equals $M(0,n)$. Finally we discussed various aspects of the stable compactification of $M_{0,5}$ and how it may be described in a combinatorial way.

Alexis Marin
An "elementary proof" of the Uniformization theorem
A Riemann surface is simple if each of its components is either isomorphic to $\widehat{\mathbb{C}}$ or elementary ($\pm$ isomorphic to D).

Lemma A: Let U and V be two elementary open sets in a compact Riemann surface $S = U \cup V$ then $S \cong \widehat{\mathbb{C}}$.

An open h–polygon is an open set U such that $U = Int(\overline{U})$ and such that the boundary of U is a piecewise analytic curve ∂U. A closed h–polygon is the closure of an open one. A cell is a compact h–polygon E with $Int(E)$ elementary. A cell E is peripherical in a closed h–polygon F if $E \subset F$ and $\partial E \cap \partial F$ is an arc.

Lemma A': If a compact h–polygon $K = Int_K(E_1) \cup Int_K(E_2)$ is the union of the relative interior of two peripherical cells then K is a cell.

Lemma B: If Ω is an open h–polygon in a planar Riemann surface P such that $\overline{\Omega}$ is compact and $\overline{\Omega} \subset U_1 \cup U_2$ where the U_i are are simple and $P\backslash\Omega$ is connected then Ω is simple.

Theorem 1: A compact planar Riemann surface is simple.

Lemma C: A compact component X of the boundary of a Riemann surface with boundary has a neighbourhood isomorphic to a neighbourhood of S^1 in $\overline{D}$.

Lemma D: If K is a compact set in a Riemann surface T then there is a compact Riemann surface S containing an h–polygon V isomorphic to a neighbourhood U of K in T and such that $S\backslash U$ is a collection of cells. In particular S is planar if T is.

Theorem 2: A connected planar Riemann surface is isomorphic to an open set of $\widehat{\mathbb{C}}$.

The implications are: $A \Rightarrow A' \Rightarrow \left\{ \begin{array}{ll} B & \Rightarrow 1 \\ C & \Rightarrow D \end{array} \right\} \Rightarrow 2.$

$A \Rightarrow A'$ and $C \Rightarrow D$ use the construction on the double of an open h–polygon U along C, a curve of its boundary, of a holomorphic structure making $U \cup C$ a Riemann surface with boundary. $A, A' \Rightarrow C$ and $D \Rightarrow 2$ uses the Montel criterion: if a connected set U in a Riemann surface is a union of charts C_i such that for all i, j there is a k with $C_k \supset (C_i \cup C_j)$ then U is a chart.

Leonardo Zapponi

Galois invariants and Strebel differentials

There are two central and related questions concerning dessins d'enfants: how far is a valency class (set of dessins having the same ramification data) from being a Galois orbit? Is it possible to give a combinatorial description of the $G_{\mathbb{Q}}$–action? These two questions are really difficult, and in order to have some new results, the general strategy is to reduce to special cases (families) of dessins having some common combinatorial and topological properties. In this talk, we are concerned with the so–called diameter four trees, a class of dessin d'enfants on the Riemann sphere that are really simply described from a combinatorial point of view. The aim of the talk is to prove a conjecture stated by Kotchetkov in 1997, relating the galois action to some arithmetical properties of the ramification indices. The central tool we use is a correspondence between dessins d'enfants and ribbon graphs. These last objects can be considered as a generalization (continuous) of the concept of dessins d'enfants. They arise from the theory of Strebel differentials and were used in order to perform a cellular decomposition of the moduli space of curves.

Gregory Ginot

Braided categories and the Grothendieck–Teichmüller group

The main goal of the talk was to give Drinfeld's description of $\widehat{GT}$, the Grothendieck–Teichmüller–group. The exposé split into two parts: In the first one, the definition of braided categories was given: roughly speaking one considers a category with a tensor product which is neither commutative nor associative, but endowed with natural families of isomorphisms satisfying some relations (triangle, pentagon, hexagon) as substitute for the lack of associativity and commutativitiy. Then we described the category of braid (and profinite braid) which is braided and in fact has some universal properties which we briefly mentioned. In the second part $\widehat{GT}$ was defined following Drinfeld as a "subgroup" of $\widehat{\mathbb{Z}} \times \hat{F}_2$ (with a different product!). The idea of Drinfeld, that is to make pure braid groups K_n "act" on a braided category, was explained. In fact we sketched the proof that the 3 equations describing $\widehat{GT}$ come from the fact that one wants to find an action which carries the structure of a braided category into another structure of braided category. Finally we used this categorical construction to explain the action of $\widehat{GT}$ on braid groups.

David J. Green

The Grothendieck–Teichmüller group again

A second description of the profinite Grothendieck–Teichmüller group $\widehat{GT}$ was given, this time as a group of automorphisms of the profinite completion of the free group on two generators. The embedding of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ in $\widehat{GT}$ was constructed explicitly and shown to be an injective group homomorphism. In particular, it was stressed that automorphisms coming from $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ satisfy the pentagon relation. The two–level principle of Grothendieck was mentioned briefly.

Bernhard Hanke, Dieter Kotschik

Moduli space of curves

Let S be a closed smooth real surface of genus g. Denote by $\mathcal{C}$ the set of complex structures on S compatible with a given orientation. The diffeomorphism group $G = Diff_+(S)$ acts on $\mathcal{C}$

via pull–back. The moduli space is $\mathcal{M}_g = \mathcal{C}/G$. Let $G_0 \subset G$ denote the path component of the identity. The Teichmüller space is $\mathcal{T}_g = \mathcal{C}/G_0$. The mapping class group $T_g := G/G_0$ acts properly discontinously on $\mathcal{T}_g$ with finite isotropy groups. Hence $\mathcal{M}_g = \mathcal{T}_g/T_g$ is a rational $K(T_g, 1)$ since Teichmüllers theorem states $\mathcal{T}_g \cong \mathbb{R}^{6g-6}$. This is shown introducing Fenchel–Nielsen coordinates depending on a pants decomposition of S and the fact that $\mathcal{C} \cong Met_{-1}$, where Met_{-1} is the space of hyperbolic metrics on S. The moduli space has functorial properties with respect to families of curves, making it a coarse moduli space. Simple examples show that it cannot be a fine moduli space due to the presence of non–trivial automorphisms of curves. It can be constructed algebraically via geometric invariant theory as a variety defined over $\overline{\mathbb{Q}}$. Let $\mathcal{H}_{g,N,d}$ be the Hilbert scheme of curves of degree $d = 2n(g-1)$ and genus g in $\mathbb{P}^N$, where $N + 1 = \dim H^0(C, nK)$ for $n \geq 3$. Then for some suitable subvariety $\mathcal{K} \subset \mathcal{H}_{g,N,d}$ we have $\mathcal{M}_g = \mathcal{K}/PGL_{n+1}$. This moduli space can be compactified by allowing curves with nodal singularities and finitely many automorphisms. This compactification is closely related to the Fenchel–Nielsen coordinates, and is a projective variety and a moduli space for families of stable curves. One can think of the moduli space as a stack, i.e. a fibered category in the sense of Grothendieck.

Ivan Kausz

The Teichmüller modular group and geometry at infinity

Teichmüller space is a fine moduli space for marked n–pointed Riemann surfaces of genus g over complex analytic spaces. The mapping class group of a n–pointed 2–dimensional compact manifold of genus g acts properly discontinously on Teichmüller space. We considered three possible interpretations of the quotient with respect to this action: In the category of topological spaces, of orbifolds and of analytic stacks. For each of these interpretations there exists a concept of fundamental group. In the orbifold– and stack–interpretation this fundamental group coincides with the mapping class group. This is not true for general g, n in the topological–space–interpretation. We also showed that Dehn–twists correspond to loops around divisors at infinity.

Oliver Bültel

On the Teichmüller tower and complexes of cut systems

The main goal of this talk was to introduce into ideas of Hatcher, Lochak and Schneps to define a $\widehat{GT}$–like group that acts on all Teichmüller mapping class groups $\Gamma^m_{g,n}$ simultaneously. This is achieved by appealing to methods of Hatcher/Thurston who constructed and analysed a 2–dimensional cell complex of curve decompositions. Firstly one shows that this cell complex is simply connected, secondly one shows that the structure of two cells put further conditions on the profinite words in $\widehat{GT}$ forcing one to consider a subgroup. This subgroup is than shown to act on all $\hat{\Gamma}^m_{g,n}$ – the main result of the Lochak/Hatcher/Schneps paper.

Volker Braungardt

Elements of finite order in $\Gamma_{g,n}$, special loci

Fixed point sets of torsion mapping classes in $T_{g,n}$ correspond to loci of symmetric curves in $M_{g,n}$. These loci admit finite–to–finite correspondences to other moduli spaces $M_{\gamma,\nu}$ and play a prominent role in Grothendieck's "Lego–Teichmüller–game".

Carl–Friedrich Bödigheimer

Cell decomposition of moduli and Teichmüller space

The talk started with an overiew of various uniformization methods: almost all of them give a specific graph (and thus a dessin d'enfant) on the surface to be uniformized. The isomorphism type of this graph determines a cell in the moduli or Teichmüller space consisting of all surfaces whose graph has this isomorphism type. Very often, the center of a cell is an "arithmetic" surface.

This general feature of most uniformization methods was then examplified following Bowditch, Epstein and Penner and their method of "ideal triangulations" and "decorated Teichmüller space". The background is the uniformization by Fuchsian groups $\Gamma \subset PSL_2(\mathbb{R})$. The decoration consists of a horocycle in the universal covering (Poincaré disc) $\mathbb{D} \mapsto F = \mathbb{D}/\Gamma$ of the surface F around each puncture $P_1, \ldots, P_n$ of F. A horocycle is determined by a point w in the light-cone $\mathbb{L}$ above $\mathbb{D}$ and tangent to the hyperboloid. The points $w_1, \ldots, w_n$ in Minkowski space give the extra continuous parameters in the decorated version of the Teichmüller space. Each conjugacy class of a Fuchsian group Γ gives an ideal triangulation induced by geodesics between punctures $P_1, \ldots, P_n$.

In this cell decomposition of the Teichmüller space the barycenters are arithmetic, i.e. the Fuchsian group Γ is conjugate to a finite–index–subgroup in $PSL_2(\mathbb{Z})$.

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