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Representation Theory

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During the last years it became increasingly apparent that there are many instances in the representation theory of Lie groups where techniques from complex analysis and geometry play a decisive role. On the other hand representation theoretic methods, such as Fourier series expansions for compact groups, are nowadays standard techniques in complex analysis. One area of mathematics, where both of these fields interact in a particularly fruitful way is the harmonic analysis on causal symmetric spaces. Motivated by these interactions both researchers working either in representation theory and researchers working in complex analysis were invited to this conference by the organizers J. Faraut, A. T. Huckleberry and K.–H. Neeb. By bringing these people together we tried to create an atmosphere of scientific interaction resulting in a sharpening of some of the still somewhat vague ideas at the interface of representation theory and complex analysis. In this sense, this meeting with its 29 talks was aimed at a cross fertilization that could not be achieved by meetings devoted to just one of the topics of representation theory or complex analysis alone.

Indication of the subject and specific goals

On a general level one of the basic purposes in the area where representation theory and complex analysis flow together is the analysis of the representation of a group of biholomorphic transformations on a complex manifold on the space of holomorphic functions or more generally on the cohomology of a vector bundle. On the geometric side an important class of such representations consists of induced representations on cohomology spaces of homogeneous vector bundles on orbits of real forms in flag manifolds. For compact real forms one obtains in particular the classical Bott-Borel-Weil Theorem.

On the analytic side the method of analytic extension is a central tool in euclidean analysis, where one typically encounters holomorphic functions on certain tube domains or bounded domains whose boundary values display various types of regularity. The technique of passing from spaces of holomorphic functions to their boundary values also shows up in representation theory, where the boundary value maps are interesting intertwining operators from a complex analytic picture to a “real” picture. A natural question in this spirit is how to construct cohomological versions of Hardy spaces which one could use to realize many important representations in a uniform way and hence obtain a better understanding of decompositions of representations on certain natural L^2-spaces. This approach to representation theory is called the “Gelfand-Gindikin Program.” Even though it has been a guiding idea for several years it has not been completed to a satisfactory stage.

In the same way as on the level of function spaces Fourier transforms relate L^2-spaces and holomorphic functions, there exists a geometric version of this picture. On the one hand side one has causal structures on symmetric spaces described by fields of cones and on the other hand certain complex domains obtained by complexifying real domains in a certain set of "imaginary directions" described by a convex cone. For symmetric spaces this correspondence is established by a certain duality generalizing the duality between a cone and the tube domain over the dual cone. This setup is well understood in the context of Jordan theory where traditionally many different areas of mathematics such as pure algebra, infinite dimensional complex analysis and operator theory flow together. Closely related is the theory of spherical functions on ordered symmetric spaces. Originally motivated by the study of integral operators in scattering theory respecting causality it has developed into a mature theory parallel to Harish-Chandra's treatment of Riemannian symmetric spaces, where in some sense the prominent role of the compactness of the isotropy group is replaced by a globally hyperbolic causal structure.

One main objective is to focus the attention of complex geometers working in the area of group actions on problems in representation theoretic contexts. It is hoped that this will lead to a complex analytic basis for new holomorphic realizations of representations of non-compact semi-simple groups G. There has indeed been a great deal of interesting progress in the settings of symmetric spaces, Ol'shanskiĭ domains, and, e.g., more generally in the Stein or Kählerian contexts, where positive definite structures such as plurisubharmonic functions or Bergman-Kähler forms with certain invariance properties play a role. While it is assumed that the participants will report on these developments, it is hoped that the non-positive-definite side will also receive its due attention.

For example, special attention should be paid to the non-Stein, canonical, $G \times G$-invariant domains in $G^{\mathbb{C}}$, i.e., the cohomological side of the Gelfand-Gindikin program. The study of non-Stein, in particular non-measurable, open G-orbits in flag manifolds $G^{\mathbb{C}}/P$ should be intensified. Properties of their Barlet cycle spaces are of essential importance and require understanding and development from the complex analytic side.

Furthermore, a systematic study of the Levi geometry of higher codimensional G-orbits is at the present time an appropriate goal.

Conversely, such a conference will give the complex analysts the possibility of describing results which lead to representation theoretic problems of independent interest. These involve a wide range of spaces of holomorphic functions, differential forms etc., on complex spaces equipped with, e.g., proper actions of Lie groups of holomorphic transformations.

As typical example one can mention the action of an isometry group of a Riemannian manifold (M, g) on tubular neighborhoods in TM equipped with the adapted complex structure or on the canonical Stein-Kähler complexification of a symplectic G-space. In the former example the special case of $M = G/K$ a Riemannian symmetric space is particularly important for the cycle space considerations mentioned above.

On the analytic side the close interplay between harmonic analysis and complex geometry is quite well visible in the theory of invariant Hilbert spaces of holomorphic functions. For the curved tube domains in complexifications of causal symmetric spaces these Hilbert spaces decompose as direct integrals of highest weight representations. These results seem to scratch the surface of a more general theory which vastly generalizes the Fourier series expansion techniques nowadays common in the study of holomorphic actions of compact groups. On the other hand for many invariant Hilbert spaces of holomorphic functions the corresponding reproducing kernel defines in a natural way a Kähler structure for which the action of the group G is Hamiltonian and which corresponds to an embedding of the manifold into a projective space of a Hilbert space. This technique is quite common in algebraic geometry (the correspondence between ample line bundles and projective embeddings) and is closely related to Bargmann transforms in the sense that it establishes a way back from a "quantum object" (a Hilbert space) to a "classical object" (a Kähler manifold).

Abstracts

Minimal Representations and Jordan Algebras

Dehbia Achab

A minimal representation of a simple real Lie group is a unitary (irreducible) representation which is associated to the minimal nilpotent complex coadjoint orbit. The most famous minimal representation of a simple real and Hermitian Lie group is the Segal-Shale-Weil representation. It is known to admit a Fock model realization in a Hilbert space of holomorphic functions.

In the non Hermitian case, the analogues of these Fock models have been recently constructed, in a uniform manner, by the work of R.Brylinski and B.Kostant. They are realized in spaces of holomorphic sections of a half-form bundle over some variety Y, which is intimately related to the minimal nilpotent orbit. More precisely, let $G_{\mathbb{R}}$ a real non Hermitian and non compact form of a complex simple Lie group G and $\mathfrak{g}_{\mathbb{R}}$ and $\mathfrak{g}$ the corresponding Lie algebras. Let $K_{\mathbb{R}}$ be the compact maximal subgroup of $G_{\mathbb{R}}$, $\mathfrak{k}_{\mathbb{R}}$ its Lie algebra, $\mathfrak{g}_{\mathbb{R}} = \mathfrak{k}_{\mathbb{R}} \oplus \mathfrak{p}_{\mathbb{R}}$ the Cartan decomposition and $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ its complexification. Denote by $\mathcal{O}_{min}$ the minimal nilpotent adjoint orbit in $\mathfrak{g}$. An irreducible unitary representation $(\pi, \mathcal{H})$ of $G_{\mathbb{R}}$ is called minimal if the variety of zeroes of the graded ideal associated to the annihilator of π, is equal to the closure of $\mathcal{O}_{min}$. If there exists a minimal representation of $G_{\mathbb{R}}$, then the variety $Y := \mathcal{O}_{min} \cap \mathfrak{p}$ is nonewpty and it is the conical K-orbit in $\mathfrak{p}$. In this case, the non Hermitian symmetric pair $(\mathfrak{g}, \mathfrak{k})$ is called $\mathcal{O}_{min}$-split. These pairs have been classified by R.Brylinski and B.Kostant, and they found a bijective correspondance between the non Hermitian $\mathcal{O}_{min}$-split symmetric pairs $(\mathfrak{g}, \mathfrak{k})$ and the pairs (J, P), where J is a semisimple Euclidean Jordan algebra of rank ≤ 4 and different of 3, and P is a L-semiinvariant homogeneous polynomial of degree 4, over J, with L being the structure group of J. The Lie algebra $\mathfrak{k}$ is the conformal Lie algebra (the Kantor-Koecher-Tits) of the complexified Jordan algebra $V = J_{\mathbb{C}}$.

In this work, we use the Jordan algebras characterize the Lie structure of the pairs $(\mathfrak{g}, \mathfrak{k})$ and to describe more explicitely the minimal representation, which is realized in a Hilbert space of holomorphic functions over $\mathbb{C}^* \times V$. Moreover, an integral formula (over $\mathbb{C}^* \times V$) is also obtained for the Hilbert space norm, the densities have been calculated explicitely using Meijer G-functions.

Averaging operators on homogeneous spaces and characters of simple compact Lie groups

D. Akhiezer

Let M be a compact Riemannian manifold, $Q = \{g_1, \ldots, g_d\}$ a finite set of isometries of M, and T_Q an operator in $L^2(M)$ acting by

$$(T_Q f)(x) = \sum_{j=1}^{d} (f(g_j x) + f(g_j^{-1} x)).$$

Following A.Lubotzky, R.Phillips and P.Sarnak (1987), we define the operator discrepancy of Q by

$$\delta_Q = \sup_{||f||=1} \left|\left| \ \frac{1}{2d}(T_Q f) - \frac{1}{\mathrm{vol}(M)} \int_M f d\omega \ \right|\right|,$$

where $d\omega$ is the Riemannian measure. The operator discrepancy δ_Q is regarded as the measure of equidistribution of the sequence of isometries $g_1, \ldots, g_d, g_1^{-1}, \ldots, g_d^{-1}$. In order to find optimally distributed sequences, one has to make δ_Q as small as possible. For $M = S^2$, A.Lubotzky, R.Phillips and P.Sarnak established a lower bound for δ_Q. We generalize this result in the following way.

Theorem *Let K be a centerless connected compact simple Lie group and let $M = K/L$, where L is any closed subgroup of K. Then*

$$\delta_Q \geq \frac{\sqrt{2d-1}}{d}$$

for any subset $Q = \{g_1, \ldots, g_d\} \subset K$. The equality is possible only if the group Γ generated by $g_1, \ldots, g_d$ is free and $g_1, \ldots, g_d$ are free generators of Γ. If Γ is amenable then $\delta_Q = 1$.
For any connected simple compact Lie group K denote by χ_λ the character of a simple complex K-module with highest weight λ and let d_λ be the dimension of this K-module. The proof of the above theorem is based on the following fact:

$$\frac{\chi_\lambda(g)}{d_\lambda} \to 0 \qquad \text{as} \qquad d_\lambda \to \infty \tag{$*$}$$

for any fixed non-central element $g \in K$.

We sketched an algebraic proof of (*) using Kempf restriction formula for representations. After the conference, it turned out that (*) is found in the literature though the proofs are different (D.Ragozin (1972), D.Rider (1972), K.Hare (1998)). We are grateful to A.Dooley for drawing our attention to the work of last author, which also contains an interesting estimate of the ratio in question.

Small representations and generalized Bessel functions

L. Barchini

Calculating explicit and natural realizations of unitary representations (especially singular ones) has been a very fruitful field of study in representation theory. Detailed knowledge of a representation frequently comes through use of a good realization. We study realizations of small representations and present two examples. The first example comprise the most singular (scalar) representations in the analytic continuation of the discrete series of $SU(n,n)$. The second example is a finite family of unitarizable representations of $SO(2n,2n)$. These second family of unitarizable representations is studied algebraically by Sahi. The first example is based on joint work with Mark Sepanski. If O_p denotes the set of $n \times n$ hermitian matrices of signature $(p,0)$, then we obtained the known result that the space of L^2-functions on O_p is a unitary representation of $SU(n,n)$. Our approach to this result is knew and has the merit that the group action and invariance of the Hilbert structure come very naturally from the construction. Starting with a certain degenerate principal series of $SU(n,n)$, a Szegö map is used to construct an intertwining operator to sections of a line bundle over $SU(n,n)/S(U(n) \times U(n))$. taking boundary values then yields an intertwin ing map, A, to the opposite degenerate principal series. On the other

hand, we produce a second splitting of the intertwining map A through $L^2(O_p)$ by using restrictions of Fourier transforms. We show that all maps in both splittings of A are continuous. As a result the copy of $L^2(O_p)$ in the degenerate principal series coincides with the image of the G-map A and thus it is G-invariant. The Inner product on $L^2(O_P)$ coincides with the inner standard inner product induced by A and thus it is G-invariant.
In the second example we only do part of the program. We compute the Fourier transform in the sense of distribution of the Szegö kernel. The computations have interest on their own (not just in connection with our project). We express the Fourier transform in terms of Shimura's Generalized Bessel Functions. These functions live on space of $2n \times 2n$ skew symmetric matrices of rank $2j$. We denote that space by $\mathcal{O}_{2j}$. We prove that the FT of the Szegö kernel lies in $L^2(\mathcal{O}_{2j})$.

Analysis on tube domains over symmetric cones

D. Békollé

(report on joint work in progress with A. Bonami and G. Garrigos)
Let Ω be a irreducible symmetric cone in a real Euclidean space V, which we regard as a Euclidean Jordan algebra. We denote $n = dim(V), r = rank(\Omega)$ and $\Delta(x) = det(x) \quad (x \in V)$. Let $T_\Omega = V + i\Omega$ be the tube domain over the cone Ω. For ν real and $p \in [0, \infty)$, set

$$L^p_\nu = L^p(T_\Omega, \Delta(y)^{\nu - \frac{2n}{r}} dxdy)$$

and define the weighted Bergman space A^p_ν to be $A^p_\nu = L^p_\nu \bigcap Hol(\Omega)$. If $\nu \leq \frac{2n}{r}$, then $A^p_\nu = \{0\}$; when $\nu > \frac{2n}{r}$, the space A^p_ν is a closed subspace of L^p_ν. In this case, we define the weighted Bergman projection to be the orthogonal projection of the Hilbert space L^2_ν onto its closed subspace A^2_ν. Then:

$$P_\nu f(z) = \int_{T_\Omega} B_\nu(z, u+iv) f(u+iv) \Delta^{\nu - \frac{2n}{r}}(v) dudv \quad (f \in L^2_\nu)$$

where $B_\nu(z, w)$ is the corresponding weighted Bergman kernel of T_Ω. Define also the operator P^+_ν by

$$P^+_\nu f(z) = \int_{T_\Omega} |B_\nu(z, u+iv)| f(w) \Delta^{\nu - \frac{2n}{r}}(v) dudv.$$

Theorem. There are 3 positive numbers $p_1(\nu), p_2(\nu), p(\nu)$ satisfying $2 < p_1(\nu) < p_2(\nu) < p(\nu)$, such that the following properties hold :
(i) P^+_ν is bounded on L^p_ν if and only if $p \in (p'_1(\nu), p_1(\nu))$ (in which case, P_ν also extends to a bounded operator from L^p_ν to A^p_ν) ;
(ii) if P_ν also extends to a bounded operator from L^p_ν to A^p_ν, then $p \in (p'_2(\nu), p_2(\nu))$; (iii) P_ν also extends to a bounded operator from L^p_ν to A^p_ν if $p \in (p'(\nu), p(\nu))$.

As usual, q' denotes the conjugate exponent of $q \in (1, \infty)$.

Assertions (i) and (ii) have been known for some time ([D. Békollé, A. Bonami, 1995] and [D. Békollé, A. Temgoua Kagou, 1995]).
Assertion (iii) was proved in 1999 for tube domains over Lorentz cones
[D. Békollé, A. Bonami, M.M. Peloso, F. Ricci].

The problem is to generalize the results of [D. Békollé, A. Bonami, M.M. Peloso, F. Ricci] to general symmetric cones. Four geometric estimates on the cone Ω are needed for the proof.

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Complex Analysis and representation theory

W. Bertram

In Lie theory one can "integrate" the bilinear Lie bracket from the Lie algebra to a global group structure on a Lie group, and similarly one can integrate the trilinear Lie triple bracket from a Lie triple system to a global product map on a symmetric space (theory of O. Loos). In this talk we present a counterpart of this feature in Jordan theory: the integrated version of the trilinear product of a *Hermitian Jordan triple system* is what we call a *circled space*, that is, a complex manifold generalizing axiomatically the product map $j : D \times D \to D$ on the unit disc D defined by $j(x,y) := j_x(y) := g(i(g^{-1}.x))$ ($x = g.0$, $g \in \mathrm{SU}(1,1)$), and the integrated version of a general Jordan triple system is what we call a *ruled space* which generalizes the family of product maps $\mu_r : M \times M \to M$ (r a real number) defined on the real projective space M by

$$\mu_r([x],[y]) = [(1-r)\langle x,y\rangle x + r\langle x,x\rangle y]. \tag{1}$$

A closer look shows that this map arises from a natural ternary product map

$$\tilde{\mu}_r([x],[\lambda],[y]) = [(1-r)\lambda(y)x + r\lambda(x)y] \tag{2}$$

defined on a Zariski-dense subset of $M \times M' \times M$ (where M' is the dual projective space). This map satisfies certain algebraic identities which define an object we call a "generalized projective geometry" (this, in turn, is the integrated version of a *Jordan pair*). Identifying $[x]$ and $[\lambda]$ via a polarity $M \to M'$, we get (1), and letting $r = -1$ we get the symmetric space structure of M in the sense of Loos. In fact, all classical and many exceptional symmetric spaces are obtained in a similar way from generalized projective geometries – this can be shown by classification; a conceptual explanation seems to be an open problem.

Harmonic, pluriharmonic and Hua-harmonic functions

E. Damek

Let $\mathcal{D}$ be a symmetric Siegel domain. There exists a solvable Lie group S which acts simply transitively as a group of biholomorphisms on $\mathcal{D}$. We study the class of S-invariant real elliptic degenerate second order operators on $\mathcal{D}$ which annihilate holomorphic functions and, consequently, their real and imaginary parts: the pluriharmonic functions. Such operators will be called admissible.

Theorem(E.Damek, A.Hulanicki, D.Müller, M.Peloso) Let $\mathcal{D}$ be a symmetric Siegel domain. Given an admissible elliptic operator L on $\mathcal{D}$ there are two admissible operators Δ and $\mathcal{L}$ such that if a real valued function F satisfies H^2 condition

$$\sup_{s\in S}\int_{N(\Phi)} |F(us)|^2 \, du < \infty.$$

and $LF = \Delta F = \mathcal{L}F = 0$ then F is the real part of a holomorphic H^2-function. For the domain biholomorphically equivalent to the complex ball L and Δ are sufficient.
For tube domains there is a stronger result.
Theorem(D.Buraczewski, E.Damek, A.Hulanicki) Let $\mathcal{D}$ be a symmetric tube domain. Given an admissible elliptic operator L on $\mathcal{D}$ there is an admissible operator Δ such that if a real valued function F is bounded and $LF = \Delta F = 0$ then F is pluriharmonic.
The classical Hua system for symmetric tube domains can be generalized as follows

$$\mathbf{H}F = \sum_{j,k}(\Delta_{j,k}F)\ R(\bar{Z}_j, Z_k)|_{T^{1,0}}, 1$$

where $Z_1, ..., Z_m$ is an orthonormal basis of the holomorphic tangent boundle $T^{1,0}$.
Theorem(A.Bonami, D.Buraczewski, E.Damek, A.Hulanicki, R.Penney, B.Trojan) Let F be a real valued function satisfying H^2 condition on a non-tube irreducible symmetric domain. If $\mathbf{H}F = 0$ then F is the real part of a holomorphic H^2 function.

Asymptotic spectral geometry

A. Deitmar

Let $M \leftarrow M_1 \leftarrow M_2 \leftarrow \dots$ be a tower of finite coverings of a Riemannian manifold converging to the universal covering M_∞ and let D be an elliptic differential operator on M. Let D_j be its lift to M_j.
If M is compact the spectral distribution of D_j converges in a precise sense to the spectral distribution of D_∞ as j tends to ∞. If the M_j are locally symmetric spaces of the form $\Gamma_j\backslash G/K$ then the assertion can be extended to the representation theoretic spectrum of the group G. The corresponding convergence assertion was conjectured by DeGeorge and Wallach and later proven by Clozel and Delorme.
If M is noncompact very little can be said in general. In the case of arithmetical quotients of symmetric spaces Werner Hoffmann and the author succeeded to show an analogous assertion. It turns out that one has as well to take the continuous spectrum into the picture.

Orbital convolutions, wrapping maps and e-functions

A. H. Dooley

Let G be a compact Lie group. Each coadjoint orbit passes through $\mathfrak{t}^{*+}$ in a unique point λ. Let μ_λ be the measure on the orbit normalised to have measure $\Pi_{\alpha\in\Phi^+} < \mu, \alpha >$. Then we can write

$$\mu_\lambda \star \mu_\xi = \int_{\mathfrak{t}^{*+}} N(\lambda, \xi, \beta)\mu_\beta d\beta.$$

Here, $N(\lambda, \xi, \beta)$ is a certain function on $\mathfrak{t}^{*+}$, which can be described combinatorially.

Further, one can link orbital convolutions and convolution of central measures and distributions on G by the wrapping map, introduced by the speaker and N.J.Wildberger [2]. Define Φ from the Ad-invariant distributions of compact support on $\mathfrak{g}$ to central distributions on G as follows. We define $< \Phi(\nu), f >=< \nu, j.f \circ \exp >$, for $f \in C^\infty(G)$. Here, j is a suitable square root of the Jacobian of exp, given by $j(X) = \Pi_{\alpha\in\Phi^+}\sin\alpha(H)/\alpha(H)$. The wrapping formula then says $\Phi(\mu) \star_G \Phi(\nu) = \Phi(\nu \star_{\mathfrak{g}} \mu)$.
Several applications exist of this global formula in particular to the Duflo isomorphism and the Kirillov character formula and their generalisations. Wildberger, Lipsman and I have found an extension of this formula to semi-direct products of vector times compact Lie groups: one can use this formula to deduce the Lipsman character formula for the semi-direct products, describe the hypergroups of adjoint, coadjoint orbits as duals of each other, and relate this to the convolution of conjugacy classes.
This theory goes through for compact symmetric spaces; one can describe K-invariant convolution on G/K in terms of K-invariant convolution on $\mathfrak{p}$, and this generalises Rouvière's Kashiwara-Vergne formula. Specifically, if we define

$$< \mu \star_e \nu, f >= \int_{\mathfrak{s}\times\mathfrak{s}} \mu(X)\nu(Y)e(X,Y)f(X+Y)dXdY,$$

then we have the formula

$$\Phi(\mu) \star_S \Phi(\nu) = \Phi(\mu \star_{e,\mathfrak{s}} \nu).$$

Here, $e(X,Y)$ is as follows. Let X, Y and $X+Y$ be congugate to H_1, H_2 and $H_3 \in \mathfrak{a}$ respectively. Then

$$e(X,Y) = \Pi_{\alpha\in\Phi^+}\Pi_{\omega_0,\omega_1\in\mathcal{W}} \left(\frac{\cos(\alpha(H_1) + \alpha^{\omega_0}(H_2) + \alpha^{\omega_1}(H_3))}{(\alpha(H_1) + \alpha^{\omega_0}(H_2) + \alpha^{\omega_1}(H_3))} \right)^{m_\alpha}.$$

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Geometry and analysis on symmetric spaces of hermitian type

J. Faraut and P. Graczyk

The theory of Hardy spaces on complex semigroups was generalized by Hilgert, 'Olafsson and Ørsted (1991) to a class of symmetric spaces called **symmetric spaces of Hermitian type** (compactly causal in the terminology of 'Olafsson and Ørsted). In our work we present another, geometrical approach to Hardy spaces on such symmetric spaces, using also tools of Jordan algebras and Jordan triple systems. Molchanov's hyperboloids are one of the most important examples of a symmetric space of Hermitian type.
Let D be a bounded symmetric complex domain in a complex vector space $V \simeq \mathbf{C}^n$. Equipped with the Bergman metric, D is a Hermitian symmetric space $D \simeq G/K$.

Let ν be a complex conjugation of V, i.e. an antilinear involution of V. If $\nu(D) = D$ we will say that ν is a complex conjugation of D. We associate to ν an involution σ of G defined by

$$\sigma(g) = \nu \circ g \circ \nu.$$

This involution commutes with the Cartan involution θ of G for which $K = G^\theta$. Let

$$H = G^\sigma = \{g \in G |\ \sigma(g) = g\}.$$

The symmetric space G/H is called **symmetric space of Hermitian type**. This definition is equivalent to that of 'Olafsson and Ørsted. Using our approach one gives a classification of symmetric spaces of Hermitian type using results of Loos and others on classification of complex conjugations of D. Cartan subspaces, root systems and the Harish-Chandra homomorphism are discussed from our geometrical point of view.
In the analysis of Hardy spaces on symmetric spaces of Hermitian type or, equivalently, of the holomorphic discrete series representations of these spaces, we introduce and exploit properties of **conical functions**.
Let $\Xi \subset G/H$ be the domain of Hardy spaces on G/H. A conical function is a holomorphic function on Ξ which is semi-invariant with respect to a solvable subgroup of G. We discuss the relation of conical functions with irreducible representations of K which are $K \cap H$-spherical. We prove that conical functions with the same weight are proportional and that each (non-trivial) invariant and C-negative Hilbert subspace $\mathcal{H}$ of $\mathcal{O}(\Xi)$ contains a non-zero conical function. This allows us, among others, to give a new proof of the spectral theorem on $\mathcal{H}$.

Bi-invariant domains in complex semisimple Lie groups

G. Fels

Our investigation is inspired by a paper of Gelfand and Gindikin [GG], wherein the authors study certain domains Ω in $G = \mathrm{SL}(2, \mathbb{C})$, invariant under the bi-action of $G^{\mathbb{R}} = \mathrm{SL}(2, \mathbb{R})$:

$$G^{\mathbb{R}} \times G^{\mathbb{R}} \ \times \ G \to G \qquad (g_1, g_2)\ ,\ x \longmapsto g_1 x g_2^{-1}$$

Some of these domains carry a natural Hilbert space structure of Hardy type $H^2(\Omega) \subset \mathcal{O}(\Omega)$ such that the regular representation of $G^{\mathbb{R}}$ on $H^2(\Omega)$ can be decomposed into a direct sum of irreducible unitary representations with finite multiplicities, each of them belonging to the holomorphic discrete series. This result has been generalized by Olshankiĭ [O1], [O2] for $G^{\mathbb{R}}$ being a Hermitian real form of G. Further, there exists an $\mathrm{Ad}(G^{\mathbb{R}})$–invariant closed and pointed cone $C = C_{\mathbb{R}mmax}$ in $\Upsilon^{\mathbb{R}}$, the Lie algebra of $G^{\mathbb{R}}$, such that $\Omega = \Gamma(C) \cong G^{\mathbb{R}} \times \exp(iC^\circ)$ and $\overline{\Omega} \cong G^{\mathbb{R}} \times iC$ has a semigroup structure. We refer to these domains as Olshanskiĭ domains. There is a hope that other bi-invariant domains in G can also be related to some series of representations of $G^{\mathbb{R}}$, see [GG]. Note that the Olshanskiĭ domains are Stein ([N]). Our contribution to this subject is the following. Let G be complex semisimple and $G^{\mathbb{R}}$ an arbitrary non-compact real form. For simplicity we assume that $(G, G^{\mathbb{R}})$ is irreducible as a symmetric pair. We show that if $D \subset G$ is a Stein bi-invariant domain then either D is equal to G or, in the case when $G^{\mathbb{R}}$ is Hermitian, there are also proper bi-invariant Stein domains which are contained in an Olshanskiĭ domain $\Gamma(C_\pm)$ or in an appropriate translate $\Gamma(C_\pm)n$ with $n \in N_G(\mathfrak{t})$ and $n\sigma(n)^{-1} \in Z(G)$. Here, $\sigma : G \to G$ denote the conjugation with respect to $G^{\mathbb{R}}$ and $\mathfrak{t} \subset \mathfrak{g}$ is a Cartan subalgebra, such that $\pitchfork^{\mathbb{R}} = \mathfrak{t}^\sigma \subset \Upsilon^{\mathbb{R}}$ is compactly embedded. In order to prove this fact we first give a description of the natural $G^{\mathbb{R}} \times G^{\mathbb{R}}$–equivariant stratification of G ([BF1]). Then a quite explicit analysis of the CR-geometry of the principal orbits ([FG]) and some non-principal $G^{\mathbb{R}} \times G^{\mathbb{R}}$–strata ([BF2]), i.e., computing the corresponding Levi cones, yields the result. It shows in particular that in attempting to produce natural representations of $G^{\mathbb{R}}$ from bi-invariant domains not contained in the Olshanskiĭ domains, one has to consider subspaces of higher cohomology groups rather than $\mathcal{O}(\Omega)$.

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Invariant domains in the commplexification of a non-compact Riemannian symmetric space

L. Geatti

Let G/K be an irreducible non-compact Riemannian symmetric space. The group G acts by left translations on the Stein manifold $G^{\mathbb{C}}/K^{\mathbb{C}}$. This action determines a finite number of invariant regions, whose union is dense in $G^{\mathbb{C}}/K^{\mathbb{C}}$ and which roughly correspond to the different types of closed G-orbits of maximal dimension (*generic orbits*). By studying the invariant CR-structure that generic orbits inherit from the complex manifold $G^{\mathbb{C}}/K^{\mathbb{C}}$, we determine which generic orbits can lie in the boundary of an invariant Stein domain in $G^{\mathbb{C}}/K^{\mathbb{C}}$ or in a level set of an invariant plurisubharmonic function. As a result, only some of the above regions may contain invariant Stein subdomains and admit non-constant invariant plurisubharmonic functions.
One of them is the region $\mathbb{X}_0$, introduced in [AG], which consists of all G-orbits intersecting the compact dual symmetric space $U/K \cong U \cdot \overline{e} \subset G^{\mathbb{C}}/K^{\mathbb{C}}$. In general, $\mathbb{X}_0$ contains several copies of the symmetric space G/K, and each of them comes with a distinguished invariant neighbourhood. These domains, say $D_0, \ldots, D_m$, indeed contain Stein invariant subdomains and carry non-constant invariant plurisuharmonic functions. They are conjectured to be Stein [AG] and to be related to the parameter space of linear cycles in flag domains [WZ]. They also carry a canonical G-invariant Kaehler structure compatible with the Riemannian structure of G/K (see [LS][Sz][GS]).

When the group G is of Hermitian type and $G^{\mathbb{C}}/K^{\mathbb{C}}$ contains compactly causal symmetric spaces G/H as minimal orbits, there are other regions in $G^{\mathbb{C}}/K^{\mathbb{C}}$ containing invariant Stein subdomains. They are of the form $S_W := G \exp iW$, where W is a maximal Ad_H-stable regular elliptic cone in the tangent space $T(G/H)_p$, $p \in G/H$. The domains $S_{\pm W}$ were showed to be Stein in [Ne]. Moreover, their invariant plurisubharmonic functions and Stein subdomains were completely characterized. Our results on the CR-structure of generic orbits imply that, with few possible exceptions, all proper G-invariant Stein domains in $G^{\mathbb{C}}/K^{\mathbb{C}}$ are either contained in one of the domains $D_0, \dots, D_m$ or in one of the domains $S_{\pm W_1}, \dots, S_{\pm W_s}$. The same holds for domains admitting non-constant invariant plurisubharmonic functions. The possible exceptions are domains whose boundary entirely consists of non-generic orbits, to which our techniques do not apply. The domains $D_0, \dots, D_m$ are among them.

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Positive-definite functions on cones and tuves in infinite-dimensional spaces

H. Glöckner

1) Let Ω be a convex set in a real vector space V, and $\phi\colon \Omega \to \mathbb{C}$ be a function. Then ϕ is the Laplace transform of a positive measure μ on the algebraic dual space V^*, equipped with the initial σ-algebra $\sigma(\mathrm{ev}_x : x \in V)$, if and only if ϕ is positive-definite and ϕ is continuous on line segments.

2) Let S be a convex cone with non-empty interior in a real topological vector space V, let $\alpha\colon S \to [0, \infty[$ be an absolute value on S (*i.e.*, $\alpha(s+t) \le \alpha(s)\alpha(t)$ for all $s, t \in S$) which is locally bounded, and $\phi\colon S \to \mathbb{C}$ be a function. Then ϕ is the Laplace transform of a Radon measure on $C_\alpha := \{\lambda \in V' \colon \exp \circ \lambda|_S \le \alpha\}$, equipped with the weak-$*$-topology, if and only if ϕ is an α-bounded positive-definite function on S which is continuous on line segments. Here ϕ is called α-bounded if $\phi(s+t) \le \alpha(t)\phi(s)$ for all $s, t \in S$. If S is open, the continuity assumption can be omitted.

Analysis of probability measures on Lie groups and Gelfand pairs

P. Graczyk

Different fundamental properties of probability measures, which are classical and well known in the commutative case, are still unknown on Lie groups and Riemannian symmetric spaces. This last case is an example of a Gelfand pair. It turns out that this very general setting of a Gelfand pair is very natural to ask and answer such important questions as:

- Do the two factorization theorems of Khinchin hold?

(Khinchin's first theorem says that any probability measure on $\mathbb{R}$ can be written as a countable product of indecomposable measures (possibly infinite) and a probability measure without indecomposable factors (called anti-indecomposable).
Khinchin's second theorem says that any anti-indecomposable measure on $\mathbb{R}$ is infinitely divisible.)

- Is the central limit theorem true? (the most general one, belonging to Khinchin in the Euclidean case)

- Do the Gaussian measures have only Gaussian factors? (a celebrated Cramer's theorem gives the positive answer on the real line)

These questions were studied in a joint work with C.R. Raja(Chennai). We prove the Khinchin's Theorems for the following Gelfand pairs (G, K) satisfying a condition (*): (a) G is connected; (b) G is almost connected and Ad (G/M) is almost algebraic for some compact normal subgroup M; (c) G admits a compact open normal subgroup; (d) (G, K) is symmetric and G is 2-root compact; (e) G is a Zariski-connected p-adic algebraic group; (f) compact extension of unipotent algebraic groups; (g) compact extension of connected nilpotent groups. The condition (*):

$$\text{for every compact subgroup } M \text{ of } G \text{ containing } K\,,\ N(M) = N(K)M\,.$$

is always verified when K is a maximal compact subgroup of G or when G is compact.
The main tools of our work are harmonic analysis and what is called "algebraic probability theory", developed by Ruzsa and Szekely in a recent book.
We also prove that Cramer's theorem does not hold for Gaussian measures on compact Gelfand pairs.

Another group of questions concerns properties of Gaussian measures on Lie groups and symmetric spaces. They are motivated by an absence of a non-analytical characterization of Gaussian measures on these spaces (the only known definition of Gaussian measures is via Laplace-Beltrami operator as the generator). In particular the following problems have been and still are studied:

- Do the Gaussian measures are characterized by a Bernstein type property? (independence of XY and XY^{-1}). This is a joint work with J.J. Loeb (Angers).

- Are the K-invariant Gaussian measures on Riemannian symmetric spaces stable? (the negative answer is based on recent estimates of the heat kernel on symmetric spaces by Anker and Ji)

• Are the K-invariant Gaussian measures on Riemannian symmetric spaces the only anti-indecomposable ones? This is known in rank one case and may be obtained for any rank if one knows a product formula for spherical functions:

$$\phi_\lambda(X)\phi_\lambda(Y) = \int_{\frown} \phi_\lambda(Z)F_{X,Y}(Z)dm(Z)$$

with some information on the kernel $F_{X,Y}$. In a joint work with P. Sawyer (Sudbury) we have obtained such a formula in the complex case.

Hua and Ahlfors operators associated with generalized conformal structures

W. Bertram, J. Hilgert, B. Orsted, A. Pasquale

Let $M_{p,q}$ be the bounded symmetric domain of complex $p \times q$ matrices Z for which $ZZ^* - \mathbb{1}_p$ is positive definite. 1958 Hua introduced a system of second order differential operators on $M_{p,q}$ having as kernel precisely the Poisson integrals over the Shilov boundary. Analogous constructions for general bounded symmetric domains have been studied e.g. by Johnson-Koranyi, Berline-Vergne, Lassalle, and others. Bounded symmetric domains admit a generalized conformal structure which can be described in terms of a Jordan triple system. We describe the Hua operators for bounded symmetric domains in terms of the Jordan triple system and show how this generalizes to arbitrary symmetric spaces with generalized conformal structure. This in particular gives a way to define Hua operators for real bounded symmetric domains.

In the same framework we introduce a general Ahlfors operator which, in contrast to the Hua system, is conformally invariant and characterizes conformal vector fields. It turns out that Hua systems as well as Ahlfors operators are closely related to (complementary) generalized gradients.

K-invariant differential operators for a multiplicity-free-action

R.M. Howe and G. Ratcliff

Let V be a complex vector space of dimension m and let K be a compact subgroup of $U(V)$, the group of unitary operators on V. If $\mathcal{P}(V)$ is the algebra of polynomial functions on V, then the action of K on V induces an action on $\mathcal{P}(V)$. Let $\mathcal{PD}(V)$ denote the polynomial coefficient differential operators on V, and denote by $\mathcal{PD}(V)^K$ those operators in $\mathcal{PD}(V)$ that commute with the action of K on $\mathcal{P}(V)$. Via the usual identification of $\mathcal{P}D(V)^K$ with $\mathcal{P}(V) \otimes \mathcal{P}(V^*)$ we can identify $\mathcal{P}D(V)^K$ with the K-invariant tensors in $\mathcal{P}(V) \otimes \mathcal{P}(V^*)$. Since K is compact, the space $\mathcal{P}(V)$ decomposes into an algebraic direct sum of finite dimensional irreducible subspaces,

$$\mathcal{P}(V) = \sum_{\alpha \in \Lambda} \mathcal{P}_\alpha$$

where Λ is a countably infinite index set that parameterizes the representation, and where the index α is usually the highest weight of an irreducible representation. We are interested in the case where the above decomposition is multiplicity-free.

Via this identification we have

$$\mathcal{P}D(V) = \mathcal{P}(V) \otimes \mathcal{P}(V^*) = \sum \sum \mathcal{P}_\alpha \otimes \mathcal{P}_\beta^*,$$

and it is a classical result that the identity representation appears in $\mathcal{P}_\alpha \otimes \mathcal{P}_\beta^*$ with multiplicity one if and only if $(\mathcal{P}_\alpha)^* = \mathcal{P}_\beta^*$. Thus, for each α there is a unique (up to a scalar) K-invariant tensor $S_\alpha \in \mathcal{P}_\alpha \otimes \mathcal{P}_\alpha^*$. The collection $\{S_\alpha\}$ is a vector space basis for the space of K-invariants $\mathcal{P}D(V)^K$. For multiplicity free actions, $\mathcal{P}D(V)^K$ is a commutative algebra by Schur's lemma.
The following questions are natural:
1) What is the K-decomposition of $\mathcal{P}(V)$?
2) What are the K-invariant differential operators $\mathcal{P}D(V)^K$? Equivalently, what are the K-invariant tensors in $\mathcal{P}(V) \otimes \mathcal{P}(V^*)$?
3) What are the eigenvalues?
In particular, we seek explicit formulas for the canonical invariants S_α and their eigenvalues. Complete results have been obtained for certain cases.

Branching laws of unitary highest weight modules with respect to semisimmple symmetric paris

T. Kobayashi

Let $G \supset H$ be reductive Lie groups, and $\pi \in \widehat{G}$, an irreducible unitary representation of G. The restriction $\pi|_H$ decomposes uniquely into irreducibles:

$$\pi|_H \simeq \int_{\widehat{H}}^{\oplus} n_\pi(\sigma)\sigma d\mu(\sigma) \qquad \text{(branching law)}.$$

An interesting setting is the case $n_\pi(\sigma) \leq 1$ (multiplicity free). However, for a general $\pi \in \widehat{G}$, the multiplicity $n_\pi(\sigma)$ can be infinite even though (G, H) is a symmetric pair (cf. $[K-2]$). We give a sufficient condition on π and (G, H) such that $n_\pi(\sigma) \leq 1$ for any $\sigma \in \widehat{H}$.
Theorem A Let G be a non-compact Hermitian Lie group, (G, H) a symmetric pair, and $\pi \in \widehat{G}$ a scalar highest weight module. Then $\pi|_H$ decomposes with multiplicity free. Analogous results also hold for $\otimes$-product, and for finite dimensional representations. Theorem A gives a uniform explanation of multiplicity free results in classical cases, such as $GL_m \times GL_n$-duality, the Clebsch-Gordan formula, the Plancherel formula for line bundles over Hermitian symmetric spaces, the Kostant-Schmid formula, and so on, together with new multiplicity free formulae. Among other cases, we give an explicit formula, when $\pi|_H$ splits discretely: We say (G, H) is *holomorphic type* if H is defined by $\tau \in \mathrm{Aut}(G)$ acting holomorphically on G/K. Take $\mathfrak{t}^\tau \subset \mathfrak{k}^\tau$ and extend $\mathfrak{t} \subset \mathfrak{k}$. Let $k = \mathbb{R}\text{-rank}G/H$ and take a maximal set of strongly orthogonal roots $\{\nu_1, \ldots, \nu_k\}$ in $\Delta(\mathfrak{p}_+^{-\tau}, \mathfrak{t}^\tau)$. Here is a generalization of the Kostant-Schmid formula to non-compact H:
Theorem B: If $L^G(\mu)$ is a holomorphic discrete series of scalar type, and (G, H) is a symmetric pair of holomorphic type, then

$$L^G(\mu)|_H \simeq\subset a_1 \geq \cdots \geq a_k \geq 0 \to \sum^{\oplus} L^H(\mu|_{\mathfrak{t}^\tau} - \sum a_j \nu_j).$$

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Jacques Faraut, man and mathematician

A. Koranyi

On the occasion of J. Faraut's sixtieth birthday this was a brief account of his biography and of his mathematical accomplishments. His work starts in potential theory and goes on to cover a wide range of subjects. His most important results belong to the large field of analysis on semi-simple Lie groups: he is a pioneer of the study of ordered symmetric spaces and he did important work on special functions and Jordan algebras. He is the author of several monographs and he is known for the large number of his former students who have beome eminent mathematicians in their own right.

Analytic continuation of holomorphic forms

B. Krötz

This is a report on joint work with Dehbia Achab and Frank Betten.
Let G be a hermitian linear Lie group and $\Gamma < G$ an arbitrary discrete subgroup. We write $(\pi_\lambda, \mathcal{H}_\lambda)$ for a unitary highest weight representation of G with highest weight $\lambda \in it^*$ and $v_\lambda \in \mathcal{H}_\lambda$ for a highest weight vector.

Theorem A. *There exists a parameter* λ *(in fact almost all subject to the condition of being sufficientlty far away from the walls) such that the Poincaré-series*

$$P(v_\lambda) = \sum_{\gamma \in \Gamma} \pi_\lambda(\gamma).v_\lambda$$

converges in the module of hyperfunction vectors $\mathcal{H}_\lambda^{-\omega}$ *to a non-zero element in* $(\mathcal{H}_\lambda^{-\omega})^\Gamma$.■

If $\eta \in (\mathcal{H}_\lambda^{-\omega})^\Gamma$ and $v \in \mathcal{H}_\lambda^\omega$, then we can form the matrix coefficient

$$\theta_{v,\eta} \colon \Gamma \backslash G \to \mathbb{C}, \quad \Gamma \mapsto \langle \pi_\lambda(g).v, \eta \rangle.$$

If v is K-finite, then $\theta_{\eta,v}$ is called a *holomorphic automorphic form.* The functions $\theta_{v,\eta}$ have the remarkable property that they extend to holomorphic functions on a very interesting G-biinvariant open Stein domain $S \subseteq G_\mathbb{C}$, namely S is the open compression semigroup of the bounded symmetric domain $G/K \subseteq G_\mathbb{C}/P_{\max}$. One calls S a *complex Ol'shanskiĭ semigroup.* Note that Γ acts on S properly discontinuously so that we can form the quotient $\Gamma \backslash S$ in the category of complex manifolds.

Theorem B. **1.** *The quotient $\Gamma\backslash S$ is Stein, provided the analytically continued automorphic forms vanish at infinity,*

$$(VAI) \qquad \lim_{\substack{s\to\infty (in\ \overline{S}) \\ s\in S}} \theta_{v,\eta}(s) = 0.$$

2. *If $\Gamma < G$ is a uniform lattice, then (VAI) holds true.* ■

Jordan Compression Semigroups and Triple Decompositions

J. Lawson

G. I. Ol'shanski introduced a remarkable class of subsemigroups of Lie groups which have come to be called Ol'shanski semigroups. A typical example of such a semigroup arises in the complexification $G_{\mathbb{C}}$ of a semisimple hermitian Lie group G by taking an $\mathrm{Ad}G$-invariant convex cone $\mathbb{C}$ in the Lie algebra $\mathfrak{g}$ of G and forming the semigroup $S = G\exp(i\mathbb{C})$ in $G_{\mathbb{C}}$.

The existence of these (infinitesimally generated) semigroups at the group level manifests itself in the existence of causal structures and causal partial orders at the homogeneous space level. In the harmonic analysis carried out at the homogeneous space level (e.g. in the analysis of kernels in Volterra algebras [1]), it is frequently crucial to know that the partial order is "globally hyperbolic," i.e., that the order intervals are compact. The property of being globally hyperbolic has also played an important role in other contexts, e.g. in the study of partial differential equations and in the causal orders that arise in Lorentzian geometry. Mittenhuber and Neeb have exploited this condition in their study of the exponential function on ordered manifolds with affine connections [6].

We use recent results of B. Krötz and K.-H. Neeb [2] on hyperbolic cones to prove that the homogeneous causal order arising from an Ol'shanski semigroup is always globally hyperbolic; this general result extends earlier work of J. Faraut [1], followed by J. Hilgert and G. Ólafsson [3], who proved it for special cases.

Let G be a Lie group equipped with an involution τ. Then τ induces an involution on the Lie algebra $\mathfrak{g}$ (making it a symmetric Lie algebra), and $\mathfrak{g}$ is the direct sum of the $+1$-eigenspace $\mathfrak{h}$ and the -1-eigenspace $\mathfrak{q}$. Let H be a τ-fixed subgroup with Lie algebra $\mathfrak{h}$. If $\mathfrak{q}$ contains an $\mathrm{Ad}H$-invariant hyperbolic cone $\mathbb{C}$, then $H(\exp\mathbb{C})$ is an Ol'shanski semigroup. One extremely useful structural property of such semigroups is the existence and uniqueness of the "Ol'shanski polar decomposition": each element s factors uniquely as $s = h\exp(X)$, $h \in H$, $X \in \mathbb{C}$. We consider the important special case that the symmetric algebra $\mathfrak{g}$ is of Cayley type (this means that $\mathfrak{q}$ can be written as AdH-invariant summands $\mathfrak{q}^+ + \mathfrak{q}^-$, each of which is an abelian subalgebra). In this case we establish that the Ol'shanski semigroup has a unique triple decomposition $S = \exp(\mathbb{C}^-)H\exp(\mathbb{C}^+)$, which may be viewed as a semigroup variant of the Harish-Chandra decomposition. In [5] necessary and sufficient conditions are given for the existence of the Ol'shanski semigroup, given an $\mathrm{Ad}(H)$-invariant hyperbolic cone in $\mathfrak{q}$. A very pleasant feature of the theory established in this paper is that under the mild restriction that the cone is pointed, then for cones of Cayley type the triple decomposition obtains whenever the Ol'shanski semigroup exists.

Semigroups for which the triple decomposition holds include sympletic semigroups, or more generally the conformal compression semigroup of a symmetric cone in an Euclidean Jordan algebra. Such semigroups have been studied in detail by K. Koufany in [4]. Relying heavily on Jordan algebra theoretic methods, he established the triple decomposition for this class of semigroups. We revisit this class of examples and show how these semigroups fit within our framework and how the triple decomposition follows from our general results. We also develop order-theoretic aspects of the structure of these semigroups, which we call Jordan compression semigroups. In particular, we show that there is a unique closed partial order in the compactification of a symmetric cone in the conformal compactification of the real Jordan algebra in which it sits that extends the natural order of the cone. With respect to this order the Jordan compression semigroup acts in an order preserving way. Furthermore, there is a natural Finsler structure that can be defined from the order so the members of the Jordan compression semigroup are actually contractions with respect to the Finsler metric and members of the interior of the semigroup are strict contractions. The preceding work represents joint work with Yongdo Lim.

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Coherent state representations and highest weight representations

W. Lisiecki

A coherent state (CS) representation of a connected Lie group G is an irreducible unitary representation for which there is a complex G-orbit on the projective space of all rays in the representation space. The group G is called a CS group if it admits a CS representation with discrete kernel. It is well known that any irreducible representation of a compact Lie group is a CS representation (the complex orbit being the orbit through a highest weight line). Some time ago I showed that noncompact reductive CS groups are precisely the Hermitian groups and that their CS representations coincide with highest weight (HW) representations. This was generalized by K.-H. Neeb who extended the theory of highest weight representations to the class of connected Lie groups with admissible Lie algebra and showed that any CS representation of such a group is a HW representation. An admissible Lie algebra is necessarily unimodular. Here I present a complete classification of unimodular CS groups. My approach is based on the structure theory of homogeneous Kähler manifolds due to Vinberg and Gindikin and Dorfmeister and Nakajima. On the Lie algebra level, the classification theorem asserts that a unimodular Lie algebra is a CS Lie algebra (i.e. is the Lie algebra of a CS Lie group) iff both its radical and Levi part are CS Lie algebras. Moreover, unimodular solvable CS Lie algebras can also be classified. It turns out that they need not be admissible. Thus even for unimodular groups the class of CS representations is larger than that of HW representations.

Invariant complex structures on the punctured cotangent bundle of compact symmetric spaces

I. V. Mykytyuk

Let G be a compact connected Lie group and $K \subset G$ it's closed subgroup. The natural action of G on G/K extends to the (left) action of G on $T^*(G/K)$. This G-action on $T^*(G/K)$ preserves the canonical symplectic 2-form Ω. We denote by $\mathfrak{g}$ the Lie algebra of G and by Φ a negative definite bilinear form on $\mathfrak{g}$ associated with a faithful representation of $\mathfrak{g}$. This form defines the G-invariant Riemannian metric g on G/K. Using g we can identify the cotangent bundle $T^*(G/K)$ and the tangent bundle $T(G/K)$. The Hamiltonian function H which is associated with the given metric g on G/K defines the geodesic flow on $T^*(G/K)$: $H(gK,\xi) = g_{gK}(\xi,\xi), \xi \in T^*_{gK}(G/K) \simeq T_{gK}(G/K)$.
Different kind of geometric constructions which comes from geometric quantization naturally lead to G-invariant complex structures defined on the punctured cotangent bundle $T^*_0(G/K) = T^*(G/K) - \{\text{zero section}\}$. Such structure J_S for the spheres were found by Souriau [So]. Later it was observed by Rawnsley [Ra1], that the length function $\sqrt{H}$ is strictly plurisubharmonic with respect to the above complex structure J_S and thus defines a Kähler metric on $T^*_0S^n$ with the Kähler form Ω. He also observed that J_S is invariant with respect to the Hamiltonian flow of the length function $\sqrt{H}$ (the normalized geodesic flow) and used the Kähler structure J_S to quantize the geodesic flow on the spheres [Ra2]. Subsequently Furutani and Tanaka [FT] defined a Kähler structure J_S with the analogous properties on the punctured cotangent bundle of complex and quaternionic projective spaces $\mathbb{C}P^n$, $\mathbb{H}P^n$ and used it for quantization.
In [Sz] Szoke explored the relationship of J_S and so-called adapted complex structure J_A on the respective cotangent bundle $T^*(G/K)$ (associated with Riemannian metric g). He showed that for all compact, rank-1 symmetric spaces (also for Cayley projective plane $\mathbb{C}aP^2$) the family of complex structures obtained by pushing forward the adapted complex structure with respect to an appropriate family of diffeomorphisms has a limit and this limit complex structure coincides with J_S.
Let $\mathfrak{m}$ be the orthogonal complement to Lie algebra $\mathfrak{k}$ of K in $\mathfrak{g}$ relative Φ and $\mathfrak{m}^*_0 = \mathfrak{m}^* \setminus \{0\}$. We have the natural Ad^*-action of K on the dual space $\mathfrak{m}^*$.
Theorem. Let G/K be a symmetric space and J a Kähler structure on $T^*_0(G/K)$ with the Kähler form Ω. Suppose that J is G-invariant and invariant with respect to the normalized geodesic flow $X_{\sqrt{H}}$. Then rank of the symmetric space G/K is equal 1. For every symmetric space $G/K \subset \{S^n, \mathbb{C}P^n, \mathbb{H}P^n (n \geq 2), \mathbb{C}aP^2\}$ there is one-to-one correspondence between the space of G-invariant and invariant with respect to the normalized geodesic flow Kähler structure on $T^*_0(G/K)$ with the Kähler form Ω and the space of Ad^*K-invariant smooth function $\lambda : \mathfrak{m}^*_0 \to (\mathfrak{m}^*_0)^{\mathbb{C}}$ with positive real part. In particular, $J_S = J_\lambda$, where $\lambda(\xi) = \alpha\sqrt{-\Phi(\xi,\xi)}$, $\xi \in \mathfrak{m}^* \simeq \mathfrak{m}$, and $\alpha \in \mathbb{R}^+$ is a constant.
It is well known that the symmetric spaces G/K $\mathbb{C}P^n = U(n+1)/U(1)\times U(n)$ and $\mathbb{H}P^m = Sp(m+1)/Sp(1) \times Sp(m)$ are the quotient of the spheres S^k, $k = 2n+1, 4m+1$ with respect to the action of the subgroup $K_1 \subset K$ isomorphic to $U(1)$ and $Sp(1)$ respectively. This action of K_1 on S^k defines natural Hamiltonian action of K_1 on $T^*_0S^k$ and the moment mapping $P : T^*_0S^k \to \mathfrak{k}^*_1$. The reduced space $P^{-1}(0)/K_1$ is isomorphic to the punctured cotangent bundle $T^*_0(\mathbb{C}P^n)$ and $T^*_0(\mathbb{H}P^m)$ respectively. By ([GS],Th.3.5) for the Kähler structure J_S on $T^*_0S^k$ there is canonically associated reduced Kähler structure J^r_S on the quotient space $P^{-1}(0)/K_1$.
Theorem. The reduced Kähler structure J^r_S on the punctured cotangent bundle complex and quaternionic projective spaces $\mathbb{C}P^n$, $\mathbb{H}P^m$ coincides with Kähler structure J_S.

This theorem allow us to quantize the normalized geodesic flow on $\mathbb{C}P^n$, $\mathbb{H}P^m$ using the simple quantization procedure on S^k and general reduction theorems for Kähler and vertical polarizations [GS,Go] .

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Plancherel formula for Berezin deformation of L^2 on Riemannian symmetric space

Yu. A. Neretin

For instance, we consider Riemannian noncompact symmetric space $B_{p,q} = O(p,q)/O(p)\times O(q)$ (we assume $p \le q$). We realize this space as the space of real $p\times q$ matrices with norm less than 1. Let $\alpha = 0,1,\dots,p-1$ or $p > 1$. We consider the Hilbert space H_α defined by the positive definite kernel $K_\alpha(z,u) = \det(1-zu^*)^\alpha$ on $B_{p,q}$. We also consider the natural representation of the group $O(p,q)$ in the space H_α .

1. There obtained the complete Plancherel formula for this representation.
For $\alpha > (p+q)/2-1$ the Plancherel measure is supported on principal nondegenerate series and its density is

$$C\cdot 2^{p\alpha}\frac{1}{\prod_{j=1}^p \Gamma(\alpha-j+1)}\times \prod_{k=1}^p \left\{\Gamma(\tfrac12(\alpha-(p+q)/2+1+s_k))\Gamma(\tfrac12(\alpha-(p+q)/2+1-s_k))\right\}\times$$
$$\prod_{k=1}^p \frac{\Gamma((q-p)/2+s_k)\Gamma((q-p)/2-s_k)}{\Gamma(s_k)\Gamma(-s_k)}\times \prod_{1\le k<l\le p}\frac{\Gamma(\frac12(1+s_l+s_k))\Gamma(\frac12(1+s_l-s_k))\Gamma(\frac12(1-s_l+s_k))\Gamma(\frac12(1-s_l-s_k))}{\Gamma(\frac12(s_l+s_k))\Gamma(\frac12(s_l-s_k))\Gamma(\frac12(-s_l+s_k))\Gamma(\frac12(-s_l-s_k))}.$$

For $\alpha < (p+q)/2-1$ support of the Plancherel measure contain many components and the density on each component is a long explicit product of Γ-functions.

2. A natural limit of the spaces H_α as $\alpha\to\infty$ is L^2 on the Riemannian noncompact symmetric space $O(p,q)/O(p)\times O(q)$. Hilbert spaces H_α are also well-defined for negative integer α . A natural limit of the spaces H_n as $n\to\infty$ is the space L^2 on the Riemannian compact symmetric space $O(p+q)/O(p)\times O(q)$.

Reference.

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Harmonic analysis on big groups and determinantal point processes

G. Olshanski

The aim of the talk is to describe new phenomena arising in harmonic analysis on groups with infinite-dimensional dual space. For certain such groups, irredicible representations are naturally parametrized by infinite point confugurations on the line, and then spectral measures on the dual space give rise to random point configurations (or point processes), which can be described in terms of the correlation functions. I would like to give an understandable introduction to the subject, with emphasis on main ideas and new promising connections.

Representations associated infinite-dimensional cones

B. Orsted

This is a report on joint work with Karl-Hermann Neeb. Consider the automorphism group of a classical infinite-dimensional tube domain corresponding to a Jordan algebra U; an example would be the unit ball of Hilbert-Schmidt operators on a complex Hilbert space. One would like to give L^2 - realizations of the unitary highest weight representations of these groups, including the so-called vector-valued case, where the corresponding reproducing kernel is operator-valued rather than just scalar-valued. For this we use the "Bochner principle", which under suitable conditions gives the existence of an operator-valued measure μ on a cone in the algebraic dual U^* of the Jordan algebra U with Laplace transform equal to the reproducing kernel. For a natural Hilbert-Schmidt completion L_2 of the structure group L of U, and a similar completion U_2 of U, we construct an infinite family of irreducible unitary representations of a natural extention of the semi-direct product $U_2 \times L_2$ on $L^2(U_2^*, \mu_2)$, where μ_2 is a positive operator-valued measure with Laplace transform equal to the reproducing kernel. This kernel has been renormalized in order to allow taking determinants of operators of the form $I + X$, X a Hilbert-Schmidt operator.

On the meromorphic extension of spherical functions on NCC symmetric spaces

A. Pasquale

Joint research with G. Ólafsson (Louisiana State University)

Let G/H be a noncompactly causal (NCC) symmetric space. We determine the meromorphic extension in the λ-parameter of the spherical functions $\varphi_\lambda(x)$ on G/H as an application of Bernstein's Theorem on the complex powers of polynomials. It is known that for all λ is a certain subset $\mathcal{E}$ of the complexification $\mathfrak{a}_{\mathbf{C}}^*$ of a Cartan subspace $\mathfrak{a}$ of the Lie algebra of G, the spherical functions are given by Poisson integrals. For a suitably fixed translation parameter $\delta \in \mathfrak{a}^*$ and all $m \in \mathbf{N}$, the meromorphic extension to $\mathfrak{a}_{\mathbf{C}}^*$ satisfies on $\mathcal{E} + m\delta$ the functional equation $b_m(\lambda)\varphi_\lambda(a) = I_m(\lambda, a)$, $a \in S^0 \cap A$. Here S^0 is the interior of the maximal semigroup for G/H, the function $I_m(\lambda, a)$ is given in the form of an integral, and $b_m(\lambda)$ is a product of m δ-translates of the Bernstein polynomial. The regularity properties of φ_λ are deduced. In particular, it follows that the possible poles of φ_λ are contained in the δ-translates of the zero set of the Bernstein polynomial. Application of the functional equations are the asymptotic estimates for $\varphi_\lambda(a)$ as $a \to \infty$. The expression of the Bernstein polynomial is conjectured. The relation between the proposed conjecture and the product formula of the function c_Ω is analyzed.

Hua harmonicity on Non-symmetric Domains

R. Penney

Let X be a Kähler manifold. The Hua-Johnson-Korányi operator (HJK) is the $Hom(T^{0,1}(M))$ valued differential operator defined by

$$\text{HJK } (F) = \sum_{i,j} R(Z_i, \overline{Z}_j)\mathfrak{D}^2(F)(\overline{Z}_i, Z_j)|T^{0,1}$$

where R is the curvature operator and Z_i is a local orthonormal basis for $T^{0,1}(M)$. (It is easily seen that it is independent of the basis.) A function F is Hua-harmonic if HJK $(F) = 0$.

If X is a symmetric tube domain, then results of Johnson-Korányi, which generalize earlier results of Hua, together with a result of Oshima-Sekiguchi, show that a Hua harmonic function of exponential growth is the Poisson integral of a distribution over the Shilov boundary. In this talk we presented generalizations of these results to the case of bounded, homogeneous, but not necessarily symmetric, domains $X = G/K$ in $\mathbf{C}^n$. Our main results are:

a) If F is Hua-harmonic with exponential growth, then F has a Van den Ban-Schlichtkrull type asymptotic expansion where the coefficients are distributions on the unipotent radical of G. This expansion is explicitly computable from its "leading" terms, which are, by definition, the boundary distributions for F. They uniquely determine the solution F.

b) The Poisson transform is explicitly computable.

c) If F satisfies an $\mathcal{H}^2$ like growth condition and X is "sufficiently non-tube like," then F is the sum of a holomorphic and anti-holomorphic function. In the symmetric case "sufficiently non-tube like" means that X is a Siegel II domain.

Property (c) was recently proved in the symmetric case by the author together with Bonami, Buraczewski, Damek, Hulanicki, and Trojan. Our result above generalizes it to the non-symmetric case. It represents a partial solution to a problem proposed in 1980 by Berline and Vergne.

Choquet theory applied to harmonic analysis

E. G. F. Thomas

Choquet theory, following the work of G. Choquet, concerns the representation of elements in a closed convex proper cone *Gamma* by sums or integrals over extremal generators of the cone. One has precise results on the existence and uniqueness of such integral representations.

A particular case is where *Gamma* is a cone of distributions of positive type on a Lie group G. These distributions are reproducing kernels of G-invariant Hilbert spaces of distributions. The theory then predicts the existence of extremal generators which are characters of irreducible representations or spherical distributions as the case may be. This gives rise

to Plancherel formulas for homogeneous spaces in a general framework. The decompositions are unique iff the situation is multiplicity free. This is the case if there exists an antilinear automorphism of the surrounding space leaving invariant the G-invariant Hilbert subspaces.
Another particular case is the one where *Gamma* is a cone of positive definite kernels, reproducing kernels of Hilbert spaces of holomorphic functions on complex manifold, invariant under the action of a complex group. One then similarly obtains Plancherel formulas within the framework of holomorphic analysis. In joint work with J. Faraut (J. Lie theory 9 (1999) 381-400) we obtain a simple criterion implying that the situation is multiplicity free. The method has been extended to the case of holomorphic line bundles by T. Kobayashi.

Non-commutative Hardy spaces and Toeplitz-Berezin quantization

H. Upmeier

The Gelfand-Gindikin program for a semi-simple Lie group S gives a decomposition $L^2(S) = \sum_i^{\oplus} H_i^2(S)$ into "non-commutative" Hardy spaces $H_i^2(S)$ described in terms of complex geometry of certain domains in $S^{\mathbf{C}}$. For $S = SL(2,\mathbb{R})$, the Hardy space $H_+^2(S) = \sum_{n\geq 2}^{\oplus} n\,\langle S\rangle_n \otimes \overline{\langle S\rangle_n}$ for the holomorphic discrete series representations $\langle S\rangle_n$ is associated with the pseudo-convex domain $S_+^{\mathbf{C}} = \left\{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in SL(2,\mathbf{C}) : \; Im \begin{pmatrix} a\bar{b} & b \\ b & d\bar{b} \end{pmatrix} > 0 \right\}$.
In joint work with Alexander Alldridge, University of Marburg, we study the C^*-algebra $\mathcal{T}_+(S)$ generated by all Toeplitz operators
$T_+(f)\,h := P_+(fh)$ on $H_+^2(S)$, where $f \in \mathcal{C}_0(S)$. Here P_+ denotes the orthogonal projection onto $H_+^2(S)$. The main result gives a geometric realization of (all) irreducible representations of $\mathcal{T}_+(S)$ in terms of boundary faces of $S_+^{\mathbf{C}}$. The most interesting ones belong to 1-dimensional upper half-planes $N_+^{\mathbf{C}}$ associated with the nilpotent component $N \approx \mathbf{R}$ of the Iwasawa decomposition $S = KAN$.

Commutative homogeneous spaces

E. B. Vinberg

A Riemannian homogeneous space $X = G/K$ is called **commutative** (or (G,K) is called a **Gelfand pair**) if it satisfies the following equivalent conditions:
(1) the convolution algebra of K-invariant measures with compact support on X is commutative;
(2) the algebra of G-invariant differential operators on X is commutative.
Any **symmetric** and, more generally, any **weakly symmetric** space is commutative.
For reductive G, there are known some extra conditions equivalent to (1) and (2), namely:
(3) the representation $G : \mathbf{C}[X]$ is multiplicity free;
(4) X is weakly symmetric, i.e. there is a diffeomorphism s of X normalizing G such that sG can permute any two points of X;
(5) the Poisson algebra of G-invariant functions on T^*X is commutative.
The classification of such homogeneous spaces is known.
In general, (2) implies (5). We call a homogeneous space satisfying (5) **weakly commutative**. There are two main types of such spaces, namely, homogeneous spaces of reductive groups and homogeneous spaces of the form $(N \mathfrak{h} K)/K$ where N is a nilpotent Lie group.

For both types, weak commutativity implies commutativity, and it is likelyhood that this implication is true in general.
In the case $X = (N \mathfrak{h} K)/K$ the group N must be at most 2-step nilpotent. If it is 2-step nilpotent, we call X a homogeneous space of **Heisenberg type** and set $Z = (N, N)$, $V = N/Z$. We call X **irreducible** if the representation $K : V$ is irreducible. A classification of all irreducible commutative homogeneous spaces of Heisenberg type was presented in the talk.

Holomorphic Convexity for Cycle Spaces of Flag Domains

J. A. Wolf

Let G be a connected complex semisimple Lie group, Q a parabolic subgroup, and $Z = G/Q$ the corresponding complex flag manifold. Fix a real form G_0 of G and an open G_0–orbit $D = G_0(z) \subset Z$. Choose a maximal compact subgroup $K_0 \subset G_0$ such that Y_0 is a complex submanifold of D . For reasons of representation theory of G_0 and algebraic geometry in Z we look at the cycle space

$$M_D = \text{ component of } Y_0 \text{ in } \{gY_0 \mid g \in G \text{ and } gY_0 \subset D\}$$

If D has a G_0–invariant measure then D is known to be Stein. The situation is unsettled if there is no invariant measure.
Here I describe a new method for constructing holomorphic functions on M_D which seems to be independent of that measurability.
The method itself is a method recently developed by Barlet and Koziarz. Let $Z' \subset Z$ be a subvariety that meets every element of M_D and such that $Z'' = Z' \cap D$ is Stein. Let f be a holomorphic function on Z'' then Barlet and Koziarz show that

$$F : Y \mapsto \sum_{y \in Y \cap Z'} f(y)$$

is a well defined holomoprhic function on M_D , and if $Y_1 \in \mathrm{cl}(M_D)$ meets D at a boundary point contained in Z' then f can be chosen so that F blows up at Y_1 . Thus, to prove that M_D is Stein, it suffices to find enough subvarieties Z' so that every boundary point of D is contained in at least one of them. I do that in the case where G_0/K_0 is a bounded symmetric domain, Z is the compact dual hermitian symmetric space, and D is any open G_0–orbit on Z .

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