Mathematisches Forschungsinstitut Oberwolfach

Meeting Report 22/2000

Self-Interacting Random Processes

21. – 27.5 2000

The goal of the meeting was to bring together researchers from mathematics, physics, chemistry and biology to discuss recent progress in the area of self-interacting random processes. This area, which lies at the interface of the above four disciplines, deals with random processes that interact either with themselves or with a random environment. This interaction typically results in complex behavior that is very different from that of the more traditional models studied in the literature.

The program focussed on the following topics: Brownian intersections and conformal invariance, self-avoiding walks and surfaces, polymers, percolation, reinforced random walk, diffusion with memory and noise, random walk in random environment, random walk in restricted geometries, random surfaces, spin glass models, and interacting Brownian motions.

The program highlighted these topics from the perspective of stochastics, combinatorics and numerics. Key techniques came from field theory, large deviation theory, renormalization group theory, and expansion techniques.

There were 49 participants, and 27 lectures were given. The lectures were organized into small blocks, focussing on a particular subtopic, in order to show the various cross links.

Erwin Bolthausen (Zürich, Switzerland)
David Brydges (Virginia, USA)
Frank den Hollander (Nijmegen, The Netherlands)

Lectures:

Gerard Ben Arous

Anomalous Diffusion and Homogenisation on Infinitely Many Scales

We show how homogenisation on infinitely many scales can lead to anomalously slow or anomalously fast diffusion. More specifically we show anomalously slow diffusion for

$$dx_t = dw_t - \nabla U(x_t)dt \qquad \text{in } \mathbb{R}^d, \quad \text{and}$$
$$U(x) = \sum_{k=0}^{\infty} U_k\Big(\frac{x}{R_k}\Big), \qquad R_k = r_0, r_1, \ldots, r_k,$$

where U_k are smooth 1-periodic functions, r_k are integers.

We also show anomalously fast diffusion for

$$dx_t = dw_t - \nabla\Gamma(x_t)dt, \quad \text{where}$$
$$\Gamma(x) = \sum_{k=0}^{\infty} \gamma_k \Gamma^k\Big(\frac{x}{R_k}\Big) \quad \text{is a smooth stream-matrix,}$$

i.e. skew-symmetric and with appropriate hypothesis on the coefficients γ_k.

The results are very precise in dimension 1 for the slow diffusion model and for the shearflow problem in dimension 2, but in higher dimension anomalous diffusion is established rigorously in a weak form.

Marek Biskup

Parabolic Anderson Model with Bounded Potentials

We consider the parabolic Anderson problem $\partial_t u = \kappa\Delta u + \xi u$ on $(0,\infty)\times\mathbb{Z}^d$ with random i.i.d. potential $\xi = (\xi(z))_{z\in\mathbb{Z}^d}$ and the initial condition $u(0,\cdot)\equiv 1$. Our main assumption is that $\operatorname{esssup}\xi(0) = 0$. In dependence of the thickness of the distribution $\operatorname{Prob}(\xi(0)\in\cdot)$ close to its essential supremum, we identify both the asymptotics of the moments of $u(t,0)$ and the almost-sure asymptotics of $u(t,0)$ as $t\to\infty$ in terms of variational problems. As a by-product, we establish Lifshitz tails for the random Schrödinger operator $-\kappa\Delta - \xi$ at the bottom of its spectrum. In our class of ξ-distributions, the Lifshitz exponent ranges from $\frac{d}{2}$ to ∞; the power law is typically accompanied by lower-order corrections.

This is joint work with Wolfgang König.

Mireille Bousquet-Mélou

Counting Paths on the Slit Plane

Let $a_{i,j}(n)$ denote the number of paths of length n on the square lattice that start from $(0,0)$, end at (i,j), and avoid the half-line $L = \{(-k,0) : k \le 0\}$ once they have left their starting point. Let $S(x,y;t)$ be the corresponding generating function:

$$S(x,y;t) = \sum_{i\in\mathbb{Z}} \sum_{j\in\mathbb{Z}} \sum_{n\geq 0} a_{i,j}(n) x^i y^j t^n.$$

We give a closed form expression of $S(x,y;t)$, which turns out to be algebraic of degree 8:

$$S(x,y;t) = \frac{\left[1-2t(1+\bar{x})+\sqrt{1-4t}\right]^{1/2}\left[1+2t(1-\bar{x})+\sqrt{1+4t}\right]^{1/2}}{2[1-t(x+\bar{x}+y+\bar{y})]}.$$

From this exact enumerative result, we derive

- other enumerative results ($a_{0,1}(2n+1) = 4^n(2n)!/n!/(n+1)!$, for instance),
- the distribution of the point where a random walk starting at (i,j) hits L for the first time: this answers a question raised by Rick Kenyon, which was the starting point of this work,
- a limit law for the position of the endpoint of walks of length n.

Anton Bovier

Aging in the Random Energy Model

The concept of "aging" is a rather new paradigm for the description of the asymptotics of dynamical behaviour in complex systems, and in particular large random or "glassy" systems that is heavily studied in the physics literature. In general terms, it is characteristic of systems that, on the time scale on which they are observed, do not converge to equilibrium exponentially fast and whose "age" can therefore be deduced from the observation of the system at a given time t. A general indicator for aging is e.g. a autocorrelation function $C(t,t+s)$ that is a function of s/t only.

One of the simplest models that is expected to show aging in its dynamics is the random energy model (REM) under Glauber dynamics. In spite of its simplicity, even in the physics literature, no real results on the dynamics of this model are available; however, Bouchaud introduced several years ago that is known as the "REM-like trap model" which permits exact computations of the autocorrelation functions and which indeed shows very nicely the aging phenomenon. In this talk I explain these ideas in some detail, and present a rigorous derivation of the aging dynamics in the full REM for a special choice of the Glauber dynamics. It is found that one may define a properly rescaled autocorrelation function representing the probability that the process moves from one of the most low-lying states to another one between times tC_N and $(t+s)C_N$ has exactly the same asymptotic behaviour as the corresponding autocorrelation function in the REM like trap model.

This is joint work with G. Ben Arous and V. Gayrard.

Maria Serena Causo

Interacting Self-Avoiding Walks:
A Simple Model for the DNA Denaturation Transition

We study pairs of interacting self-avoiding walks $\{\boldsymbol{\omega}^1, \boldsymbol{\omega}^2\}$ on the 3d simple cubic lattice. They have a common origin $\boldsymbol{\omega}_0^1 = \boldsymbol{\omega}_0^2$, and are allowed to overlap only at the same monomer position along the chain: $\boldsymbol{\omega}_i^1 \neq \boldsymbol{\omega}_j^2$ for $i \neq j$, while $\boldsymbol{\omega}_i^1 = \boldsymbol{\omega}_i^2$ is allowed. The latter overlaps are indeed favored by an energetic gain ϵ.

This is inspired by a model introduced long ago by Poland and Sheraga [J. Chem. Phys. **45**, 1464 (1966)] for the denaturation transition in DNA where, however, self avoidance was not fully taken into account. For both models, there exists a temperature T_m above which the entropic advantage to open up overcomes the energy gained by forming tightly bound two-stranded structures.

Numerical simulations of our model indicate that the transition is of first order (the energy density is discontinuous), but the analog of the surface tension vanishes and the scaling laws near the transition point are exactly those of a second order transition with crossover exponent $\phi = 1$. Numerical and exact analytic results show that the transition is second order in modified models where the self-avoidance is partially or completely neglected.

François Dunlop

A Convergent Expansion around the Gaussian Interface Model

I present the phase space cluster expansion for the random interface model with Hamiltonian $H(\Lambda) = \int_\Lambda d^2x \left[|\nabla h|^2 + V(\nabla h) - u \cdot \nabla h \right]$. The perturbation $V(\nabla h)$ obeys for some $n \geq 2$, K, $K' > 0$ and $\lambda > 0$ sufficiently small

$$\nabla^{(2)} V > \lambda K \left(1 + |\nabla h|^{n-2}\right) \quad \text{as quadratic forms} \qquad \forall\, \nabla h,$$
$$|\nabla^{(m)} V| < \lambda K' \, |m|! \left[1 + (\,1 + |\nabla h|\,)^{n-|m|} \right] \qquad \forall\, \nabla h \quad \forall\, m.$$

The thermodynamic limit is achieved with $\Lambda = \Lambda_L = L\Lambda_1$ where Λ_1 is a connected open domain with $\mathcal{C}^2$ boundary. The slope chemical potential u is a smooth *function* similarly scaled, $u(x) = u_L(x) = u_1(x/L)$. A notable feature coming with the expansion is the renormalisation group flow of this function. Results may be applied to yield properties of surface tension as function of slope, and macroscopic Wulff shape under a volume constraint.

This is joint work with Jacques Magnen.

Bertrand Duplantier

Conformally Invariant Boundaries, Brownian Paths Intersections and Multifractal Potential Distribution

The multifractal (MF) distribution of the electrostatic potential near any conformally invariant fractal boundary, like a critical $O(N)$ loop or a critical Q -state Potts cluster, is solved in two dimensions [1].

Consider a single (conformally invariant) critical random cluster, generically called $\mathcal{C}$. Let $H(z)$ be the potential at exterior point $z \in \mathbf{C}$, with Dirichlet boundary conditions $H(w \in \partial\mathcal{C}) = 0$ on the outer (simply connected) boundary $\partial\mathcal{C}$ of $\mathcal{C}$, and $H(w) = 1$ on a circle "at ∞", i.e., of a large radius scaling like the average size R of $\mathcal{C}$. $H(z)$ is identical to the *harmonic measure*, i.e, the probability that a Brownian motion started at z, escapes to ∞ without having hit $\mathcal{C}$. The multifractal formalism characterizes subsets $\partial\mathcal{C}_\alpha$ of boundary sites w by a Hölder exponent α, and a Hausdorff dimension $f(\alpha) = \dim(\partial\mathcal{C}_\alpha)$, such that the potential, or harmonic measure content of a ball $B(w,r)$ of radius r, centered at w, locally scales as

$$H(B(w,r)|w \in \partial\mathcal{C}_\alpha) \approx (r/R)^\alpha, r \to 0.$$

By this equation a Hölder exponent α thus defines a local equivalent *harmonic angle* $\theta = \pi/\alpha$, and the MF dimension $\hat{f}(\theta)$ of the boundary subset with such angle θ is found to be

$$\hat{f}(\theta) = f(\alpha = \pi/\theta) = \frac{\pi}{\theta} - \frac{25-c}{12}\frac{(\pi-\theta)^2}{\theta(2\pi-\theta)},$$

with c the central charge of the conformal field theory describing the critical model. Values of c are, for instance, $c = 1/2$ for an Ising cluster ; $c = 0$ for the frontier of a Brownian motion [2], for a self-avoiding walk [3], as well as for a critical percolation cluster [4]. One thus finds for $c = 0$ that these three boundaries all have the statistics of a self-avoiding walk, with a unique external perimeter dimension $D_{\mathrm{EP}} = sup_\theta \hat{f}(\theta) = 4/3$, which is Mandelbrot's conjecture for the frontier of a Brownian motion. It is identical to the external perimeter dimension for percolation, directly derived in [5].

For any value of c, the Hausdorff dimension of the frontier $D_{\mathrm{EP}} = sup_\theta \hat{f}(\theta) = \hat{f}(\hat{\theta})$, and the *typical harmonic angle* $\hat{\theta}$ satisfy $\hat{\theta} = \pi(3 - 2D_{\mathrm{EP}})$. For a critical Potts cluster, the dimensions D_{EP} of the external perimeter (which is a simple curve) and D_{H} of the cluster's hull (which possesses double points) obey the *duality* equation [1]

$$(D_{\mathrm{EP}} - 1)(D_{\mathrm{H}} - 1) = \frac{1}{4},$$

independently of the model.

A related covariant MF spectrum is obtained for any critical system near the cluster boundary.

Related results, obtained via stochastic Loewner equations, are discussed in the Oberwolfach talks by G. Lawler and W. Werner (with O. Schramm).

References:

[1] B. Duplantier. Conformally Invariant Fractals and Potential Theory. *Physical Review Letters*, Vol. 84, (2000), pp. 1363–1367.

[2] B. Duplantier. Random Walks and Quantum Gravity in Two Dimensions. *Physical Review Letters* Vol. 81, (1998), pp. 5489–5492.

[3] B. Duplantier. Two-Dimensional Copolymers and Exact Conformal Multifractality. *Physical Review Letters* Vol. 82, (1999), pp. 880–883.

[4] B. Duplantier. Harmonic Measure Exponents for 2D Percolation. *Physical Review Letters* Vol. 82, (1999), pp. 3940–3943.

[5] A. Aizenman, B. Duplantier, A. Aharony. Path Crossing Exponents and the External Perimeter in 2D Percolation. *Physical Review Letters* Vol. 83, (1999), pp. 1359–1362.

Tony Guttmann

Self-Avoiding Walks and Polygons:
Exact New Results from Numerical Data

Since the pioneering work of Hammersley and collaborators nearly half a century ago, the problems of self-avoiding walks (SAW) and polygons (SAP) have resisted almost all attacks. In two- and three dimensions the existence of a critical exponent has not even been proved.

Based on algorithms which are exponentially faster than pre-existing ones, we have obtained very long series expansions for the SAW and SAP generating functions. Based on certain properties of these expansions we are able to make plausible conjectures about the generating functions which exclude certain classes of functions. Many exactly solved models are in these excluded classes.

For two-dimensional self-avoiding polygons these conjectures can be proved. Analogous conjectures for the susceptibility of the two dimensional Ising model have also recently been confirmed. These results make clear the intractability of these problems.

Takashi Hara

Decay of Correlations for High Dimensional
Self-Avoiding Walk and Percolation

We consider self-avoiding walk and percolation on a d-dimensional hypercubic lattice $\mathbb{Z}^d$. We concentrate on their two-point functions at their criticality, and ask about their decay properties with distance. Our interest is mainly in the high dimensional case, where we expect the same decay as that of simple random walk two-point functions. Our main theorem can be stated as follows:

Theorem 1. *Let $G(x)$ denote two-point functions of self-avoiding walk or percolation on $\mathbb{Z}^d$ at their criticality. For sufficiently large d, $G(x)$ obeys the asymptotic form*

$$G(x) \sim \frac{\text{const.}}{|x|^{d-2}} \qquad \text{as} \quad |x| \to \infty.$$

In the above, the "const" can be expressed in terms of quantities appearing in the lace expansion, and a_d defined below.

During the course of the proof, we also prove the following, which gives a sufficient condition under which a given gaussian propagator obeys the above asymptotic form. Here we denote the Fourier transform of $J(x)$ as $\hat{J}(k) \equiv \sum_x J(x) e^{ikx}$.

Lemma 2. *Let $J(x)$ be a function on $\mathbb{Z}^d$, which satisfies*

$$\hat{J}(0) = 1, \quad 1 - \hat{J}(k) \geq K_1 |k|^2, \quad |J(x)| \leq \frac{K_2}{|x|^{d+2}}, \quad \sum_x |x|^2 |J(x)| < \infty$$

with positive constants K_1, K_2. Then, the inverse of $(1 - J)$, defined by

$$G(x) \equiv \int_{[-\pi,\pi]^d} \frac{d^d k}{(2\pi)^d} \frac{e^{-ikx}}{1 - \hat{J}(k)}$$

obeys the asymptic form

$$G(x) \sim \frac{a_d}{K_3} \frac{1}{|x|^{d-2}} \qquad \text{as} \quad |x| \to \infty$$

where

$$a_d \equiv \frac{d}{2} \frac{\Gamma(\frac{d}{2} - 1)}{\pi^{d/2}}, \qquad K_3 \equiv \sum_x |x|^2 J(x).$$

Theorem 1 is proved by employing the existing results by lace expansion, which essentially states that the two-point functions of self-avoiding walk and percolation satisfies the assumptions of Lemma 2.

Remco van der Hofstad

Critical Oriented Percolation above $4+1$ Dimensions

We consider oriented bond percolation on $\mathbb{Z}^d \times \mathbb{Z}_+$, at the critical occupation density p_c, for $d > 4$. The model is a "spread-out" model having long but finite range. We condition the cluster of the origin to reach time n, and scale space by $n^{1/2}$. We prove that the moment measures converge, as $n \to \infty$, to those of super-Brownian motion. This extends a previous result of Nguyen and Yang, who used the lace expansion to prove Gaussian behaviour for the critical two-point function, to all r-point functions ($r \geq 2$).

Our method is based upon the expansion for unoriented percolation by Hara and Slade that is used to prove that unoriented percolation converges to integrated super-Brownian excursion (ISE). This inclusion/exclusion expansion derives the same equation for the two-point function as the lace expansion, but can be generalized to include a magnetization. We prove convergence for the two-point function using induction.

The recurrence equation with non-zero magnetization allows us to derive a recurrence equation for the higher point functions by differentiating with respect to the (site dependent) magnetization parameter. This allows us to prove that all higher point functions converge to the ones of super-Brownian motion.

This is joint work with Takashi Hara and Gordon Slade.

John Imbrie

Self-Avoiding Walk, Supersymmetry and All That

There is a close connection between the time τ_x spent by a continuous-time Markov chain at the state x and the differential form $\phi_x\overline{\phi}_x + \frac{1}{2\pi i} d\phi_x \wedge d\overline{\phi}_x$. The relationship, which we call the τ isomorphism, has its origins in the connection between self-avoiding walk and the $n \to 0$ limit of $(\overrightarrow{\phi}^2)^2$ field theory. This connection was expressed in terms of anticommuting variables (related to ϕ by a supersymmetry) in 1980 by Parisi and Sourlas, and McKane. In 1983, Luttinger wrote down the τ isomorphism in a form close to the following:

$$\int_0^\infty dT\, \mathrm{E}_a \left[f(\tau) \mathbf{1}_{X(T)=b} \right] = \int_{\mathbb{C}^N} e^{-S_A} \phi_a \overline{\phi}_b f(\tau).$$

On the left-hand side, we have the process generated by A (assumed to have positive real part) and starting at state a; $\tau = \{\tau_x^{[T]}\}$ is the collection of local times spent at the state x up to time T. On the right-hand side, $S_A = \sum_{xy} \left(\phi_x A_{xy} \overline{\phi}_y + \frac{1}{2\pi i} A_{xy} d\phi_x \wedge d\overline{\phi}_y \right)$ and $\tau = \{\tau_x\}$ with $\tau_x = \phi_x\overline{\phi}_x + \frac{1}{2\pi i} d\phi_x \wedge d\overline{\phi}_x$. David Brydges and I have used the τ isomorphism to derive the logarithmic corrections to the large-T behavior of the end-to-end distance of a hierarchical weakly self-avoiding walk in four dimensions where $f(\tau) = \exp(-\sum_x \lambda \tau_x^2)$. Our result is roughly the following:

$$\begin{aligned}\langle |X(T)| \rangle_\lambda &\equiv \frac{\sum_x |x|\, \mathrm{E}_0 \left[e^{-\sum_x \lambda \tau_x^2} \mathbf{1}_{X(T)=x} \right]}{\sum_x \mathrm{E}_0 \left[e^{-\sum_x \lambda \tau_x^2} \mathbf{1}_{X(T)=x} \right]} \\ &= c(\lambda) T^{1/2} (\ln T)^{1/8} \left(1 + d(\lambda) O\left(\frac{\ln \ln T}{\ln T} \right) \right).\end{aligned}$$

Tom Kennedy

Weakly Self-Avoiding Walks

The first part of this talk will introduce a (non-rigorous) real space renormalization group transformation for weakly self avoiding walks. I will consider several different WSAW's and use the transformation to understand some known results and make some new predictions. The second part of the talk will concern the forgetful WSAW in which the penalty for self-intersection decays like $1/t^p$, where t is the time between the two visits to the same site, and p is a parameter. I will sketch the proof that in one dimension this walk is ballistic for $p \leq 1$.

Greg Lawler

Intersection Exponents for Planar Brownian Motion

The intersection exponents for planar Brownian motion measure the rate of decay of various events on nonintersection of Brownian paths. They are important for determining the dimension of exceptional sets on a Brownian path (outer boundary, cut points, pioneer points) and now seem to be closely related to exponents for other models. A universality principle is given which shows that intersection and crossing exponents for conformally invariant processes which satisfy a certain "restriction property" can be given in terms of the Brownian intersection exponents. Percolation and self-avoiding walk are expected to satisfy this property and the ideas are used to derive nonrigorously some well-known conjectures for planar self-avoiding walks. This is joint work with W. Werner. The idea of universality is used in joint work with O. Schramm and W. Werner to calculate the intersection exponents rigorously.

Jean-François Le Gall

Scaling Limits of Branching Particle Systems with Reflection

We consider systems of Brownian particles on the real line which are subject to a critical binary branching mechanism and interact when they meet by reflecting against each other. We consider scaling limits of these systems when the initial number of particles and the branching rate tend to infinity. We are able to prove tightness of the corresponding distributions but leave the problem of uniqueness of the limit open. However, we can show that various path properties hold under any limit distribution. In particular, we prove that in the scaling limit the reflected paths satisfy a Hölder continuity property with exponent $\frac{3}{4} - \varepsilon$, in contrast to the usual behavior for Brownian paths.

This is joint work with Chris Burdzy, University of Washington.

Neal Madras

Self-Avoiding Walks: The Isotropic and Anisotropic Cases

The self-avoiding walk has long been used as a lattice model of a long polymer molecule in a good solvent. We begin with a review of the classical self-avoiding walk model, and then present a model in which steps in different directions have different probabilities. This can model a type of polymer that is oriented by an external field. It can also be used in a model of flux lines in superconductors. We shall discuss some rigorous results and their physical significance.

This is joint work with Christian Borgs, Jennifer Chayes, and Christopher King.

Carl Mueller

Hitting Properties of the Random String

We study geometric properties of a model for the random string, introduced by Funaki. The position of the string is given by the function $u(t,x)$, where $x \in \mathbf{R}$. It solves the equation

$$\frac{\partial u}{\partial t} = \Delta u + \dot{W}(t,\cdot). \tag{1}$$

We assume that $u(t,x)$ takes values in $\mathbf{R}^d$. Furthermore, $\dot{W} = \dot{W}(t,x)$ is 2-parameter white noise, also taking values in $\mathbf{R}^d$. This model was introduced about 20 years ago, but almost nothing is known about its qualitative behavior. We answer some natural questions about the hitting behavior of the string, and about its multiple points. In particular, we find the dimensions in which the string hits a preassigned point with positive probability, and for which multiple points exist with positive probability. In some cases, we can construct a collision local time, raising the possibility of a self-avoiding or weakly self-avoiding version.

Note that (**??**) is a nonhomogeneous heat equation, meaning that we are taking the high viscosity or low mass limit (in the physical situation). One could also study the corresponding wave equation, but such questions have already been answered using potential theory for multi-parameter Markov processes and for the Brownian sheet. Our process $u(t,x)$ is only Markov in the first parameter, so other methods are required.

This is joint work with Roger Tribe.

Enzo Orlandini

Self-Averaging in Models of Polymer Collapse

We give a set of conditions under which a system is thermodynamically self-averaging and show that several lattice models of interacting polymers satisfy these conditions. We prove this result for a general potential which is linear in the numbers of the various types of contacts, and show that this includes two potentials which have previously been used in models of random interacting linear polymers. In addition we have discussed to which extent the annealing approximation is a good approximation for the quenched problem. It turns out that in the limiting case of very low temperatures the annealing free energy approximation is arbitrary different from the quenched one.

Olivier Raimond

Self-Interacting Diffusions

A general class of random processes on a compact Riemannian manifold, the self-interacting diffusions, is studied. These are solutions of SDEs of the type : Brownian increment + drift term depending on the present position and of the normalized occupation measure at time t, μ_t. It is proved that the asymptotic behavior of $\{\mu_t\}$ can be described by a deterministic dynamical system on the space of the Borel measure. Some interesting examples are studied in details.

Yvan Saint-Aubin

Partition Functions for the Free Boson on Domains with Boundary

The concept of boundary state in conformal field theory (e.g. the free boson, the Ising model) is introduced as a tool to compute partition functions on domains with boundary. For the free boson an explicit expression for these states is given. It allows for the verification that they transform properly under conformal transformations that preserve the reality of the boson and that stabilize the boundary.

Gordon Slade

Critical Two-Point Functions for Spread-Out Percolation and Related Models in High Dimensions

We consider spread-out models of self-avoiding walk, bond percolation, lattice trees and bond lattice animals on Z^d, above their respective upper critical dimensions $d = 4$, $d = 6$ and $d = 8$. We use a new unified approach to the lace expansion to prove that for sufficiently spread-out models above the upper critical dimension, the two-point function of each model decays, at the critical point, as a multiple of $|x|^{2-d}$.

This is joint work with Takashi Hara and Remco van der Hofstad.

Angela Stevens

Self-Attracting Random Walks and Self-Organization in Microbiology

The reinforced self-attracting random walk which was analyzed by Davis in 1990 is related to models which are used to describe self-organizing bacteria following trails produced by themselves. The formal diffusion approximation of Davis' random walk results in a system of partial differential equations which is a so called chemotaxis system. The solutions of these systems are known to develop singularities for suitable parameters. These are understood to reflect the situation when self-organization might happen.

When comparing the conditions for the random walk to be recurrent, respectively for the particle to localize, with the conditions for the solutions of the continuous approximation to stay bounded, respectively blowup in finite time, they correspond in both models.

Since this formal approximation and analysis of blowup conditions can be done also for other random walk models and their continuous counterparts, the analysis of the PDE-models might help to conjecture the behavior of the respective random walk model.

Buks van Rensburg

Interacting Models of Lattice Surfaces

Consider a collection of plaquettes in the hypercubic lattice which is an embedded surface homeomorphic to (say) a disk or a sphere. Let $s_n(m)$ be the number of such plaquette surfaces, modulo translation, of total area (number of plaquettes) n, counted with respect to a property which occurs m times (for example, enclosed volume, number of edges with incident plaquettes at right angles, and so on). The partition function of this model is $Z_n(z) = \sum_m s_n(m) z^m$, and its generating function is $G(x, z) = \sum_n Z_n(z) x^n$. I discuss the existence of limiting free energies and critical behaviour in these models within the framework of tricriticality.

Yvan Velenik

Depinning of the Massless Free Field and Wiener Sausage Asymptotics

We consider the two-dimensional massless free field, with very general quadratic interactions, localized by a δ-pinning of strength ε. We derive the asymptotics of the variance of the field and of the decay-rate of its covariances as ε goes to zero. This provides a precise description of the critical behaviour of the transverse and longitudinal correlation lengths of the interface at the depinning transition, in a non-mean-field regime. This is done by obtaining precise estimates on the distribution of pinned sites, which allows a reduction of the problem to that of computing the asymptotics of the Green function of a random-walk in a Bernoulli random environment of killing obstacles when the density of the latter goes to zero.

This is joint work with Erwin Bolthausen.

Stanislav Volkov

Vertex Reinforced Random Processes with Discrete and Continuous Time

A vertex reinforced random process is a random walk on graph G which is more likely to visit the vertices of G it visited before. The processes of this type can exhibit different behaviors: they can be recurrent, transient or may get stuck on a finite subset of vertices. I will talk about the results obtained recently in this area, partly by myself and partly as a result of joint work with Robin Pemantle and also with Burgess Davis.

Wendelin Werner

Schramm's Process and the Value of Brownian Intersection Exponents

We describe a new conformally invariant stochastic process (Stochastic Loewner evolution process, or Schramm's process). We show that it is the unique possible conformally invariant candidate for the scaling limit of $2d$ critical percolation cluster interfaces. Using some of its properties, together with the universality ideas presented in Greg Lawler's talk, we derive the value of all intersection exponents between planar Brownian motions and random walks. In particular (these result had been predicted by Duplantier and Mandelbrot):

1) If S and S' denote two independent simple random walks in Z^2, then $P(S(0,n] \cap S'(0,n] = \emptyset)$ decays like $n^{-5/8}$ when $n \to \infty$.

2) If B denotes a planar Brownian motion and F the outer frontier of $B[0,1]$, then the Hausdorff dimension of F is almost surely $4/3$.

These results give justifications of the value of critical exponents for planar self-avoiding walks and critical percolation cluster crossings (that had been predicted by Nienhuis, Cardy, Duplantier, Saleur...) modulo the asumption that these objects converge in the scaling limit to an object with conformal invariance properties.

This talk is based on joint work with Greg Lawler and Oded Schramm.

Stu Whittington

Random Copolymers at an Interface:
A Coloured Self-Avoiding Walk Model

Copolymers can be modelled as self-avoiding walks in which the vertices are of two colours to represent two different kinds of comonomers. The colours can be assigned at random, or in some periodic manner. These models can be used to model polymer adsorption at an impenetrable surface, or localization at an interface between two immiscible liquids. We shall present some rigorous results on these models for the case when the vertices of the walk are coloured independently but where the colouring sequence is then fixed. The system can be shown to be self-averaging (ie in the limit of long walks almost all colourings have the same properties) and the free energy is non-analytic, corresponding to a phase transition in the system.

Ofer Zeitouni

Random Walks on Galton-Watson Trees

We consider biased random walks on Galton Watson trees. We study the large deviations for these walks and prove that the quenched large deviation is the same as the annealed one, in sharp contrast with the one dimensional case. The analysis involves some coupling arguments.

This is joint work with A. Dembo, N. Gantert, Y. Peres.

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