MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 24/2000

Optimale Steuerungen komplexer dynamischer Strukturen

4.06. - 10.06.2000

Diese Tagung fand unter der Leitung von Karl-Heinz Hoffmann (Bonn, München), Irena Lasiecka (Charlottsville, VA USA), Günter Leugering (Darmstadt), Jürgen Sprekels (Berlin) und Fredi Tröltzsch (Chemnitz) statt.

Das Gebiet der Steuerung partieller Differentialgleichungen hat in jüngster Zeit, bedingt durch die Erfordernisse der Hochtechnologie einerseits und die Möglichkeiten moderner Höchstleistungsrechner andererseits, einen enormen Bedeutungszuwachs erfahren. Material- und Formoptimierung, Optimierung in der Verfahrenstechnik und Prozesssteuerung allgemein stehen dabei im Mittelpunkt des Interesses. Die zugrundeliegenden Prozesse werden dabei durch 'verteilte Systeme' also durch partielle Differentialgleichungen beschrieben, deren Lösungen durch Steuerungen im Gebiet, auf dem Rand, in den Koeffizienten oder durch die Gestaltung des Gebiets vermöge vorgegebener Kostenfunktionen 'optimiert' werden. Die Entwicklungen komplexer neuartiger Materialien (z.B. piezo-keramische Werkstoffe, Formgedächtnis-Materialien etc.) sowie die Notwendigkeit, auf komplexe Art gekoppelte Strukturen (z.B. Flüssigkeits-Festkörper-Wechselwirkungen, flexible Strukturen, komplexe Verfahren des Kristallziehens etc.) mathematisch modellieren, analysieren sowie numerisch simulieren zu müssen, evoziert den Kontext der Analysis, Optimalsteuerung und Numerik nichtlinearer partieller Differentialgleichungen in seiner ganzen spektralen Breite.

Entsprechend wurden die Vorträge so ausgewählt, dass den einzelnen Aspekten in exemplarischer Weise Rechnung getragen wurde. Angefangen von einer neuartigen Modellierung von Schalen über die optimale Steuerung der Luftströmung an Oberflächen von Tragflügeln vermöge aktiver, semi-aktiver oder formgebender Steuerungen bis zur Steuerung quantenchemischer, thermo-mechanischer sowie anderer technischer Prozesse.

Im Zuge der numerischen Umsetzung ergeben sich neben neuartigen Herausforderungen an die Lösungsstrategien auch Fragen der adäquaten Diskretisierung von Optimalsteuerungsproblemen.

Das Tagungsprogramm spannte also den Bogen von den ingenieurwissenschaftlichen Anwendungen über die mathematische Analyse kontinuierlicher und diskreter nichtlinearer Prozesse und deren Optimierung und Simulation bis zur konkreten Implementierung. Die zahlreichen jungen Wissenschaftler aus aller Welt nutzten die offene und diskussionsfreudige Athmosphäre des Instituts, um mit den herausragenden Vertretern der Disziplin ins Gespräch zu kommen und ihre eigenen Resultate vorzustellen. Die Zusammenführung von Ingenieuren, Analytikern und Numerikern sowie die Förderung des wissenschaftlichen Nachwuchses in diesem Kernbereich anwendungsorientierter Mathematik speziell in Deutschland lag den Veranstaltern besonders am Herzen.

H. T. Banks

Reduced Order Based Compensator Control of Thin Film Growth in a CVD Reactor

We report on an interdisciplinary effort (involving applied mathematicians, material scientists and physicists at N.C. State University) to use computational simulations and mathematical analysis in the complete design (geometric), development and implementation of sensing and control methodology, and construction of a series of high pressure (100 Atm.) organometallic chemical vapor deposition thin film reactors. The approach involves design of the reactor so that control and sensing are a basic component of the original design efforts for the reactor. We report here on the successful use of mathematics in a fundamental role in the development of linear and nonlinear feedback control methods for real time implementation on the reactor. This is done in the required context of nonlinear gas dynamics as well as nonlinear surface reaction layers. The problems are optimal tracking problems (for the chemical component fluxes over the substrate) that employ state dependent. Riccati gains nonlinear observations and the resulting dual state dependent Riccati equations for the compensator gains. This methodology is successfully combined with reduced order model methods based on Proper Orthogonal Decomposition (POD) techniques. Computational results to support the efficacy of our approach and methods are presented.

Gang Bao

Modeling and Design of Diffractive Optical Structures

Micro diffractive optics is an emerging technology with many practical applications. Over the past two decades, significant technology developments have been made particularly in two areas. First, high precision micromachining techniques have permitted the creation of gratings (periodic structures) and other diffractive structures of extremely small scales. Second, rapid developments of laser technology and nonlinear optical materials have made important progress possible in nonlinear diffractive optics. The practical application of diffractive optics technology has driven the need for mathematical models and numerical algorithms.

In this talk, recent developments on direct, inverse, and optimal design problems in the mathematical modeling of diffractive optics will be reported. Particular attention will be paid to a variaional approach. For the direct problem, the speaker will present a new variational formulation and results on the well-posedness and convergence analysis of the PDE model. Computationally, an interface least-squares finite element method will be discussed. For the inverse problem, recent results on uniqueness, stability, and numerical reconstruction will be presented. Existence of optimal design is established by a homogenization approach.

Finally, the speaker will highlight ongoing research in the above areas as well as in mathematical modeling of near-field optics, electromagnetic cavities, as well as magnetoencephalagraphy.

N. D. Botkin und K.-H. Hoffmann

Homogenization of a fully coupled model for a nonlinear thin plate controlled by piezoelectric transducers

A model of a nonlinear von Kármán plate controlled by piezoelectric transducers is considered. The complete coupling between elastic deformations and electric fields is assumed. This means that the generation of electric fields via elastic deformations is being taken into consideration. Therefore, the model contains additional variables, the potential functions, that describe electric fields arising both via the voltage applied to piezoelectric transducers and due to elastic deformations. This paper is a continuation of the works [1], [2] where the interface between fields and deformations is rather simple: electric fields generate elastic deformations but not vice versa.

We discuss the derivation of partial differential equations describing the phenomena, the proof of their solvability, and the homogenization procedure when the number of transducers goes to infinity whereas their dimension tends to zero. The main result can be summarized as follows. We introduce a parameter ε that describes the density of transducers. Roughly speaking, the number N of transducers is proportional to $1/\varepsilon$ whereas their dimension is proportional to ε. Then, we derive a limit system corresponding to $\varepsilon = 0$ i.e. $N = \infty$. The limit system is uniquely solvable, though the original system may have many solutions for each value of ε. We prove that all of them converge to the unique solution of the limit system as $\varepsilon \to 0$ i.e. $N \to \infty$. Therefore, the optimal control can be computed for the limit system where the control forces are distributed continuously so that the limit system is controllable. Then, the computed optimal control can be applied to the original system, if the number of transducers is sufficiently large. Computer simulations show that this approach works well for $N > 36/cm^2$.

References

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John A. Burns

Computational Methods for Shape Optimization and Control

In this presentation, we discuss several theoretical and computational issues that arise in the development of numerical methods for sensitivity analysis. We motivate the work by describing a problem concerning the design of an ion beam reactor. A simple 1D-interface problem is used to illustrate the basic ideas and to provide a mathematical framework for developing very weak sensitivity equations as generalized boundary value problems. We present a Hybrid Sensitivity Equation Method that allows for the approximation of the sensitivity equations without requiring mesh sensitivities. Finally, we close with several applications of the algorithm to problems involving high speed chemically reaction flows.

Eduardo Casas

Finite Element Approximations for Some State-Constrained Control Problems

In this talk we study the convergence of the numerical discretizations, by using finite elements, of some optimal control problems with pointwise state constraints. The presence of these constraints introduces some difficulties in the analysis of the convergence. Some stability of the optimal cost functional with respect to small perturbations of the set of feasible states is necessary to obtain the convergence. On the other side, instead of integral constraints on the state, we are considering pointwise constraints. This obliges us to study the convergence in the L^∞-norm of the solutions of the discrete states to the continuous states, solutions of the partial differential equation. More precisely, uniform convergence of the finite element method is investigated, assuming minimal regularity of the state. We can distinguish the cases of distributed controls and boundary controls. In the first case H^2-regularity of the state is used. However only $W^{1,p}$-regularity can be assumed for the case of boundary controls. Collecting all these results and assuming some convexity of the cost functional with respect to the control, we can prove a convergence result.

Michel Delfour and M. Bernadou

Intrinsic Models of Piezoelectric Shells

Completely intrinsic methods have been recently developed and used to obtain linear models of thin and asymptotic shells without local coordinates or Christoffel symbols. They make use of polynomial approximations with respect to the variable normal to the midsurface. It has been shown that the $P(2,1)$ model is specially well suited to obtain "Naghdi's type" model without the usual assumption on the stress tensor. Moreover that model converges to an asymptotic model made up of two coupled variational equations. By reduction of the number of variables, the usual membrane shell equation and a generalized bending equation are obtained. They coincide with the classical equations for the plate and the bending dominated case.

The object of this conference is to use those intrinsic methods in the study of linear piezoelectric shells. Starting form a three-dimensional model of piezoelectric material, we obtain models of the Naghdi and Koiter types and a coupled set of asymptotic membrane shell and bending equations. The shell approximation is achieved via an extended version of the $P(2,1)$ model and a scalar $P(2,1)$-approximation of the electrical potential. Effective mechanical and electrical constitutive laws are obtained from arbitrary three dimensional constitutive mechanical and electrical constitutive laws.

Max Gunzburger

Optimization-based methods for multidisciplinary simulation and optimization

We discuss algorithms for multidisciplinary simulation and optimization that efficiently couple existings single-disciplinic codes.The algorithms are based on a strategy in which unknown data at the interfaces are determined through an opzimization process. The strategy allows the user to select the data type at the interfaces for each discipline, so that the methods can be tailored to existig codes. We focus on the fluid-structure interaction problem for which we describe the optimization-based methods.

Bill Hager

Discrete Approximations in Optimal Control

Convergence rate estimates for the Euler approximation to control problems with state/ control constraints will be presented. For union strained control problems, we gave the order conditions for schemes of orders up to 4. These conditions were more stringent than the corresponding order conditions for differential equations.

Matthias Heinkenschloss

Optimal Control of Compressible Navier-Stokes Equations

We consider the minimization of final-time kinetic energy of a two dimensional compressible viscous flow through suction and blowing on part of the boundary. The spatial domain is $\{(x, y)|\ y > 0\}$ and control is exercised on part of the botton wall $\{(x, y)|\ y = 0\}$. The no-control flow represents two counter rotating vortices, which move towards the bottom wall, hit it, and bounce back. For the numerical solution, the computational domain is restricted to $\Omega = (a, b) \times (0, c)$ and far field boundary conditions are imposed on $\partial\Omega\backslash(a, b) \times \{0\}$. Existence of optimal controls, even of solutions of the compressible Navier-Stokes equations are yet unknown in the general case. Our approach assumes existence of sufficiently smooth states. We derive the optimality system. Theoretical and practical issues related to the adjoint equations are discussed. Numerical results are presented.

Michael Hinze

Second order methods for control of the instationary Navier-Stokes equations

Es werden lokale Konvergenzresultate für die gängigen Verfahren zweiter Ordnung bei Anwendung auf die Kontrolle der instationären Navier-Stokes Gleichungen bewiesen (Newton, SQP, reduced SQP, BFGS...). Weiterhin werden implementierungstechnische Details präsentiert, wobei insbesondere auf die Komplexität bei zu lösenden SQP- bzw. Newtonverfahren eingegangen wird. Abschliessend werden numerische Resultate für die Kavität präsentiert und dabei der Black-Box Zugang anhand des Newtonverfahrens mit dem All-at-once Zugang anhand des SQP-Verfahren verglichen.

Dietmar Hömberg

Optimal shape design of inductor coils

We consider a shape optimization problem related to the design of induction hardening facilities. The mathematical model consists of a vector potential formulation for Maxwell's equations coupled with the energy balance and an ODE to describe the solid phase transition in steel during heating. Depending on the shape of the coil we control the volume fraction of the high temperature phase.

The coil is modeled as a tube and is defined by a unit-speed curve. We formulate the shape optimization problem over the set of admissible curves and prove existence of an optimal control. To obtain the form of the shape gradient of the cost functional, we apply the material derivative method. Finally, the first order necessary optimality conditions are established for an optimal tube.

(Joint work with Jan Sokolowski, Nancy)

Ronald H. W. Hoppe

Topology Optimization of High Power Electronic Devices

We consider the structural optimization of electric drives for high power electromotors. Such drives consist of specific semiconductor devices (IGBTs and/or GTOs) that are linked to each other as well as with the power source and the motor to be driven by conducting bus bars. Due to fast switching times of the IGBTs and GTOs and to high voltages, eddy currents are generated leading to parasitic inductivities that cause a substantial power loss in the transmission. Therefore, the idea is to design the bus bars in such a way that the total inductivity is minimized. This can be formulated as a topology optimization problem with inequality constraints for the design parameter, which is the conductivity, and equality constraints for the state variables in terms of the quasistationary limit of Maxwell's equations in its potential formulation. After discretization by non-conforming finite elements for the scalar electric potential and edge elements for the magnetic vector potential, the discretized optimization problem is solved by a primal-dual Newton interior point method combined with a watchdog strategy relying on a hierarchy of two merit functions. The method features transforming null space iterations applied to the algebraic system for the condensed primal-dual Hessian involving multigrid iterative solvers for the discretized field equations.

Mary Ann Horn

Asymptotic Behaviour of a Cylindrical Shell Model with Boundary Stabilizing Feedback

Uniform stabilization of a cylindrical shell is established through the use of boundary feedback control. At issue is the question of whether the stability result is robust in the limit as the thickness of the shell tends to zero. To establish an affirmative answer to this question, the effects of the thickness must be tracked throughout the estimates required to prove uniform stabilization. Our approach is based on multiplier methods, however, the desire to track the thickness requires the use of sharp trace regularity estimates as well as unique continuation for two related cylindrical shell models. Proof of sharp trace regularity is achieved by using microlocal analysis to analyze the behaviour of the symbol of the differential operators corresponding to the system. Unique continuation properties are derived via reduction to a stationary problem and the use of Carleman estimates.

Nobuyuki Kenmochi

Variational Evolution Problem associated with Total Variation Functional with Constraint

We consider a variational evolution inclusion of the form

$$w'(t) + k\partial V(w(t)) \ni w(t) \ , \ \ t > 0 \ , \text{ in } \ L^2(0, L) \ ,$$

where k is a given (small) positive constant, $0 < L < +\infty, V(w)$ denotes the total variation of $w \in L^2(0, L)$ with constraint $|w| \leq 1$ a.e. on $(0, L)$ and $\partial V(\cdot)$ is its subdifferential in $L^2(0, L)$. This type of evolution inclusions was discussed by A. Visintin as the kinetic equation for the order parameter in a solid-liquid system.

In this talk, we are interested in the stability of the stationary solutions. We give some results related to it, using the order-preserving property of the semigroup associated with our variational evolution inclusion.

Ralf Kornhuber and Rolf Krause

On Monotone Multigrid Methods for the Signorini Problem with Coulomb friction

We present a new class of multigrid methods for the Signorini problem with Coulomb friction in linear elasticity. The methods are based on fixed point iteration on the normal stresses, constrained Newton linearization and inexact subspace correction with respect to d-dimensional subspaces associated with the nodes of the underlying triangulation. $d = 2, 3$ is the dimension of the reference configuration. The subspaces are intended to represent the high and low frequency contributions of the error. Monotonically decreasing energy provides global convergence of the algorithms. Special care has to be taken in case of varying normal directions, because corresponding slip boundary conditions then are no longer resolved by the coarse grid. Efficiency and reliability is illustrated by numerical experiments in $d = 2, 3$ space dimensions.

Werner Krabs

Controllability of Rotating Beams

We consider a slowly rotating Timoshenko beam in a horizontal plane whose movement is controlled by the angular acceleration of the disk of a driving motor into which the beam is clamped. The problem to be solved is to transfer the beam from a position of rest into a prescribed position under a given angle within a given time. This problem is transferred into a trigonometric moment problem and is solved, if the time of rotation which is prescribed is large enough. The infimum of times for which the problem of controllability is solvable depends on Young's modulus, the shear modulus and the cross section area of the beam which are all assumed to be constant.

K. Kunisch

Optimal Control of Fluids

We give a survey of recent contributions for solving optimal control problems related equations describing viscous and non-viscous fluids. The role of the choice of cost-functionals is discussed and the optimality conditions are specified. The size of these systems suggests the use of suboptimal strategies: we discuss the receding horizon approach with instantaneous control as a special case, and proper orthogonal decomposition (POD) based techniques. Both these techniques can be used in open as well as in closed loop control. In the context of real time control they will ultimatively also have a significant bearing on "smaller" size problems. - We also address second order methods for solving optimal control problems related to the Navier-Stokes equation.

(The talk draws upon joint work with K. Afanasiev, M. Hinze, K. Ito, X. Marduel and S. Volkwein).

J. Lagnese

A nonoverlapping domain decomposition for optimal control of the Maxwell system

A nonoverlapping domain decomposition for optimal control of the Maxwell system of dynamic electromagnetism, in which the control enters through an impedance condition in the boundary of the electromagnetic region, and a cost functional that penalizes the deviation of the final state from a target state. A nonoverlapping, iterative domain decomposition with interface condition of impedance type is introduced that reduces the global optimality system to a sequence of uncoupled systems on the subdomains. Convergence of the local iteration to the solution of the global optimality system is then established with some restriction on the impedance parameters that appear in the global problem and in the local iterations, and provided the local time is sufficiently large.

Michel Lenczer

Two Scale Control of Wave Equation

A two-scale model for the wave equation with periodic coefficients will be presented. It has been derived using an asymptotic method based on a two-scale convergence and on the Bloch waves decomposition. The homogenized wave equation and the Bloch waves expansion are particular cases of this new formulation. An application of this two-scale model for an optimal control problem will be discussed.

Ingo Müller

Adaptive Air Foil with Shape Memory Wires

Shape memory wires contract upon heating and expand when cooled. Since the heating may be performed electrically, the wires may act as sensitive muscles for the realization of quick shape changes in structural elements to which the areas are attached.

Thus the profile of an air foil may be adapted to changing flight conditions, i.e. angle of incidence and speed of the incoming air. If the optimal shapes are known and registered in a data bank, and if sensors are monitoring the actual shape, the wires may be heated so as to minimize the difference between the actual shape and the set point shape. This adjustment may occur by feedback control or - more ambitiously - by optimal control, if the expenditure of energy for the control and the control time must be minimized.

Models are shown for the feedback control of a slice of an air foil and for a winglet. The latter reduces drag while the lift force is constant.

Peter Philip

Transient Numerical Simulation of Sublimative Growth of Sic Single Crystals

A transient model for the Modified Lely Method for the sublimative growth of Sic single crystals is presented. The model is based on a mixture theory for the gas phase and consists of all conservation laws including reaction diffusion equations. Moreover a model for the heat transport via radiation will be discussed. The heat transport in the whole growth apparatus is simulated numerically and includes the heat transport by radiation. Some aspects of the numerical method will be treated and some results of simulations will be shown.

René Pinnau

An Optimization Strategy for the Drift Diffusion Model for Semiconduct Devices

A typical design question in modern semiconducter device modelling is to fit a given current voltage characteristic. Usually, some geometrical properties are fixed and so the problem is reduced to a finite dimensional problem. We consider the infinite dimensional setting and try to gain an amplified current only by a slight change of the doping profile. This yields a constrained optimal control problem for the drift diffusion equations for semiconductors. Existence of an optimal control is proven and the first order optimality system is analyzed, yielding estimates on the sensitivity of the problem. Numerical tests for a u-p diode are presented.

Jean-Pierre Puel

Bilinear optimal control for Schrödinger equations via an action on a potential

We are interested in controlling a system occuring in quantum chemistry, describing, the Hartree-Fock approximation, the state of a molecule on which we may have an external action via an electric potential (typically produced by a laser beam). A particular case where the external electric field is uniform in space has been treated by E. Concas and C. Le Bris for the existence and regularity part. They also give (with M. Pilot) an existence result for an optimal control for the full nonlinear system (but they don't derive an optimality system).

We consider here, in a first step, a simple model of the form

$$\left.\begin{array}{rcl} i\,\frac{\partial y}{\partial t} & + & \Delta y + \frac{2}{|x-\tilde{x}(t)|}\,y + (1+|x|^2)^{\frac{1}{2}}\,w_1(x,t)\,y = 0 \text{ in } I\!R^3 \times (0,T) \\ y(0) & = & y_0 \end{array}\right\} \qquad (1)$$

where w_1 is the (reduced) external electric potential which will also be our control variable (w_1 is not assumed to be uniform in x). We prove existence, uniqueness and regularity for a solution of (1) in spaces like $H^2 \cap H_2$ where H^2 is the usual Sobolev space and

$$H_2 = \{z \in L^2,\ |x|^2\, z \in L^2\}\ .$$

The difficulty appears in the study of regularity. We then consider a functional

$$I(w_1) = \frac{1}{2} \int_{I\!R^3} |y(T) - y_1|^2 dx + \frac{h}{2}\, \|w_1\|^2_W$$

where y_1 is the given "target state", $r > 0$ and W is, for simplicity, a Hilbert space such that $W \subset W^{1,1}(0,T; L^\infty(I\!R^3))$ (e.g. $W = W^{1,2}(0,T; H^2(I\!R^3))$.)

We (easily) show existence of an optimal control and then we give necessary conditions of first order which can be written, using a suitable adjoint state, as an optimality system which turns out to be interesting, specially for computational purposes.

J. P. Raymond, J. M. Buchot, P. Villedieu

Control and stabilization in a boundary layer

For a stationary longitudinal incoming flow on a flat plate, the transition location from laminar to turbulent flow is stationnary. When the velocity of the incoming flow varies the transition location also varies. We are interested in stabilizing it by a suction device through the plate. The transition location is a nonlinear function of the incoming flow velocity and on the laminar flow velocity. The laminar flow is described by the Prandtl's equations. Using the so-called Crocco transformation, the Prandtl's system is transformed into a parabolic degenerate equation (the Crocco equation). Linearizing this equation and the transition location around the stationnary solution, we formulate our stabilization problem as a linear quadratic regulation problem. We prove the existence of a unique solution to the corresponding algebraic Riccati equation, which ist not completely standard since the generator of the linearized state equation is a degenerate elliptic operator. We next study a robust control problem.

Peter Rentrop, Sven-Olaf Stoll

Sensitivity Calculations for Profile Optimization of Turbine Blades

In coal fired power plants the last blade row of a turbine is crucial for efficiency considerations. The profile of the turbine blade is defined by Bezier polynomials. Their coefficients are used as variables in a nonlinear optimization procedure. The talk deals with the question of sensitivity calculations. The steam flow is modelled by the $2D$ Euler Gas equations. Their sensitivity due to geometry parameters (Bezier coefficients) and initial or boundary conditions is discussed. Numerical results and simulations are based on CLAWPACK by LeVeque.

The authors are indebted to Dr. U. Wever from the SIEMENS AG for stimulating discussions and for a Ph. D. grant for S.O. Stoll.

Arnd Rösch

Fast and Stable Methods for the Identification of Nonlinear Heat Transfer Laws

We will investigate the following optimal control problem.

$$
\begin{aligned}
\text{Minimize} \quad \phi(\alpha) &= \int_0^T \int_\Gamma (\vartheta(t,x) - q(t,x))^2 \, dx dt \\
\text{where} \quad \frac{\partial \vartheta}{\partial t} &= \Delta_x \vartheta \\
\vartheta(0,x) &= \vartheta^0(x) \\
\frac{\partial \vartheta}{\partial n} &= \alpha(\vartheta)(u - \vartheta) \; .
\end{aligned}
$$

The control α is a function of the unknown state ϑ. Assuming that α belongs to a set of uniformly Lipschitz continuous functions, it is possible to derive several theoretical results in this general case. In a special situation we will show that the standard assumptions, which are needed for the convergence analysis of higher order numerical methods, are not fulfilled. These properties cause a very slow convergence of gradient type methods. For that reason we will propose different Gauss-Newton type methods to overcome the slow convergence. Our numerical experiences show several advantages of such algorithms. In addition to a significant faster convergence we get also more precise results, even for heavily perturbed data. The derived stability results can be used as a-posteriori estimates of the control.

Keywords: Identification, Optimality Conditions, Stability, Gauss-Newton Algorithm, Heat Equation, Optimization, a-posteriori estimates.

Jan Sokolowski

Topological derivatives of shape functionals for elasticity systems

The form of topological derivative in $3D$ elasticity is derived for an arbitrary shape functional. Topological derivative is used for numerical solution of shape optimization problems in structural mechanics e.g., in the framework of the so-called buble-method as well as for numerical solution of shape inverse problems.

References

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S. Stojanovic

Control Methods in finance

Two kinds of control methods with applications in mathematical finance were considered:

- optimal control of parabolic obstacle problems, as a method for computing of implied volatility for American options; and
- stochastic control via numerical solutions of Hamilton-Jacobi-Bellman, and in particular Mongle-Ampere type pde's, as a method for computing optimal diversification strategies for portfolios of stocks and options.

Dan Tiba

Shape optimization problems in mechanics

It is our aim to give a new treatment for some classical models of arches and plates and for their optimization. In particular, our approach allows to study nonsmooth arches, while the standard assumptions from the literature require $W^{3,\infty}$-regularity for the parametric representation. Moreover, by a duality-type argument, the deformation of the arches may be explicitly expressed by integral formulas.

As examples for the shape optimization problems under study, we mention the design of the middle curve of a clamped arch or of the thickness of a clamped plate such that, under a prescribed load, the obtained deflection satisfies certain desired properties. In all cases, no smoothness is required for the design parameters.

Roberto Triggiani

Thermo-Elastic Plate Systems

This talk presents two main results for thermo-elastic plate systems:

1) A backward uniqueness result for thermo-elastic plate systems, in the fully space variable coefficient case, and for all canonical boundary conditions, including the most challenging case of "free boundary conditions".

2) Simultaneous exact/approximate controllability for thermo-elastic plates under boundary controls in the hinged or clamped boundary conditions. The solution of problem 1) is critical for the the solution of problem 2).

Marius Tucsnak

Global weak solutions for a fluid-rigid body problem

We consider the motion of several rigid bodies immersed in an incompressible viscous fluid. The main novelty of this work is that we show *global* existence of solutions. More precisely, unlike in previous literature, we don't need any assumption concerning the lack of collisions between several rigid bodies or between a rigid body and the boundary.

Varvara L. Turova and V. S. Patsko

Numerical solution of optimal time control problems for differential games with the "homicidal chauffeur" dynamics

The construction of value functions is of great interest for the theory of differential games and applications. The value functions have, as a rule, very complicated structure even for differential games with relatively simple dynamics and, therefore, they can not be found analytically. Nowadays, various grid methods based on solving appropriate Hamilton-Jacobi equations are developed for the computation of value functions of differential games. The authors use another approach: the algorithm proposed for construction of level sets of the value functions for time optimal game problems of the second order is based on the direct computation of a propagating front which is the set of all points in the plane with the same optimal guaranteed time of attaining the terminal set. The algorithm provides very accurate construction of level sets and allows us to find out fine peculiarities of solutions.

In this work, the algorithm is applied to three differential games with the dynamics of the "homicidal chauffeur" type. The first problem is the classical Isaacs' homicidal chauffeur differential game. In this game, a pursuer P minimizes the capture time of an evader E. The objective of the evader is to avoid the capture or to maximize the capture time. The magnitude of the velocity is constant for the pursuer, and his maneuverability is bounded through a minimum turn radius. The pursuers control is the rate of turn. The maneuverability of the evader is not bounded. The evader can choose his velocity vector from a circle of a radius. The main difference of the second problem from the classical Isaacs' problem is that the size of the constraint on the control parameter of the evader depends on the position of the game. The third problem is the conic surveillance-evasion game where the dynamics is the same as in the Isaacs' problem but the aims of the players differ from those in the classic formulation: an evader E minimizes the time of escaping from a detection set which is a two-dimensional semi-infinite cone. The detection set is attached to the velocity vector of a pursuer P whose objective is to keep the evader within the detection set for maximum time. The statements considered are based as well on the Isaacs' book as on works of Breakwell, Merz, Lewin, Olsder, Bernhard, Cardaliaguet, Quincampoix and Saint-Pierre.

Many numerically computed examples that demonstrate very complicated structure of level sets of the value functions are given. The problem of arising holes in the victory domains of a pursuer is discussed.

Stefan Ubrich

A sensitivty and adjoint calculus for the optimal control of flows with shocks

We present a sensitivity calculus for discontinuous solutions of conservation laws that is based on shift-variations and implies the differentiability of trocking type functionals. To obtain an adjoint gradient representation, we derive an adjoint formula for the shock sensitivity. The appropriate adjoint state has to be carefully chosen as reversible solution of the adjoint equation. By a duality approach we are able to characterize measure solutions of the sensitivity equation. Finally we discuss, which numerical schemes lead to convergent sensitivity and adjoint schemes and present numerical results.

Axel Voigt

Optimal control of Industrial Silicon single crystal growth

Our aim is to reduce the point defect concentration in silicon single crystals during the Czochvalski growth process. By relating the lattice structure of the growing crystal to macroscopic growth conditions the concentration of self-interstitials and vacancies can be controlled by controlling the ratio $v\backslash g$, where v is the growth velocity and c is the temperature gradient in the growing crystal close to the phase boundary. We solve an inverse nonlinear heat equation in order to calculate the power to the heaters in the Czochvalski furnance which gives us the wanted temperature profile in the crystal.

Stefan Volkwein

Numerical Treatment of Optimal Control Problems for Partial Differential Equations Based on Proper Orthogonal Decomposition

Practically relevant problems often involve complicated systems of partial differential equations for which numerical optimal control methods were close to being out of reach. Hence, the need of for developing novel techniques emerges. Suboptimal control strategies based on proper orthogonal decomposition (POD) guarantee reasonable performance of the controlled plant while being computationally tractable. In this talk error estimates for Galerkin-POD methods for linear and certain nonlinear parablic systems are presented. Moreover, POD is utilized to solve open and closed loop optimal control problems for the Burgers equation. The relative simplicity of the equation allows comparison of POD-based algorithms with numerical results obtained from finite element discretization of the optimality system. For closed loop control suboptimal state feedback strategies are presented.

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