MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 30/2000

Cohomology of Finite Groups: Interactions and Applications

23.07. – 29.07.2000

Organizers: Alejandro Adem (Madison)
Jon F. Carlson (Athens)
Hans-Werner Henn (Strasbourg)

As the title of the meeting suggests, the conference centered around a couple of different but closely related topics in connection with the cohomology of finite groups. The meeting emphasized the interaction between algebraic topology and modular representation theory with cohomology of groups being the unifying theme. Studies such as the applications of subgroup complexes have been the source of new developments in both areas and were recurring subjects in several of the lectures.

The first main theme was the structure of the cohomology rings of finite groups itself, where the term "cohomology" could include extraordinary cohomology theories applied to BG. The results presented in this group of talks were, for example, about finiteness properties of cohomology rings of finite p-groups, characterizations of groups having periodic cohomology, subrings of cohomology rings generated by specific representations, or the cohomology length of finite p-groups. The second main topic was the structure of the classifying spaces of finite groups. Here results were communicated related to the concept of endofiniteness of classifying spaces, about finite groups having homotopy equivalent classifying spaces up to completion, or the classifying space associated to a block of a finite group, among others. Finally, a third topic that received particular attention were actions of finite groups on spaces with predefined properties. Besides these three main areas, other related topics were the subjects of lectures of a more topological or representation theoretical nature.

There were 23 talks of varying length (between 30 and 60 minutes). Particularly noteworthy is the fact that many young mathematicians were given the opportunity to present their work at this meeting. Another point worth mentioning was the possibility to present one's work to the other participants in the form of a poster which one could hang up next to the lecture room. Between the talks there was enough time to exchange ideas with other participants, to discuss new developments or to work on some joint projects related to the topic of the conference.

Besides the traditional hike on wednesday afternoon there was also an excellent concert on thursday night, which featured the performances of some of the participants. The very pleasant and stimulating atmosphere at Oberwolfach contributed to make this once again a very successful meeting.

ABSTRACTS OF THE TALKS PRESENTED

Bill Browder, Princeton University, USA:

Constructing group actions

Given a 1-connected CW complex Y of dimension less than or equal to n, how may one construct and classify free actions of a group G on finite dimensional spaces of the homotopy type of Y?

The method of Postnikov towers represents a space by an (infinite) sequence of fibrations with fibres Eilenberg-MacLane spaces $K(\pi_m(\_), m)$. A related problem then is: how large a portion of the Postnikov tower of a space is necessary to describe an n-dimensional space with fundamental group G?

Let Y be a 1-connected n-dimensional CW complex, $f : Y \to Y(n+1)$ the term of the Postnikov tower so that f induces an isomorphism on π_k for $k \leq n+1$ and $\pi_k(Y(n+1)) = 0$ for $k > n+1$.

Theorem: Suppose that the finite group G acts freely on $Y(n+1)$ with quotient $Z(n+1)$ such that the cohomology groups $H^{n+1}(Z(n+1); A) = 0$ for coefficient systems $A = \mathbb{F}_p G/K$ for all primes p and all p-subgroups K of G. Then there exists a Z of dimension less than or equal to n, such that $Z(n+1)$ is the $(n+1)$st term of the Postnikov tower for Z, and such that its universal cover is homotopy equivalent to Y.

This makes possible the construction of many strange group actions on ordinary spaces, extending the ideas of my paper: "Homologically exotic group actions" (to appear in the birthday volume for Jim Milgram).

One may show that the homological dimension of Z (with $\mathbb{F}_p$ coefficients) is the same as that of Y, and if Y satisfies Poincare duality in dimension q, then a free G action on some X homotopy equivalent to Y has finite dimensional quotient if and only if the quotient satisfies Poincare duality in the same dimension.

John Greenlees, University of Sheffield, UK:

Morita invariance of the Gorenstein condition for rings up to homotopy and cohomology rings of finite groups

(Report on joint work with W.G.Dwyer and S.Iyengar.) The aim of the talk was to give a new proof of the local cohomology theorem for the cohomology ring of a p-group G. The Benson-Carlson CM-implies-Gorenstein theorem, and other geometric consequences follow as described by Greenlees and Lyubeznik (JPAA (2000)).

The first ingredient is the Morita equivalence of Dwyer and Greenlees, stating

$$I\text{-torsion-}R\text{-modules} \simeq \text{modules-}\mathcal{E} \simeq I\text{-complete-}R\text{-modules}.$$

Here R is a ring up to homotopy, I is an ideal and (in the case that $R/I = k$ is small over R) $\mathcal{E} = \mathrm{End}_R(R/I)$.

The next ingredient is that a ring up to homotopy is said to be *homotopy Gorenstein* if $\mathrm{Hom}_R(k, R) = k$ as $\mathcal{E}$ modules. A formal argument shows that this condition is invariant under Morita equivalence in the sense that R is homotopy Gorenstein if and only if $\mathcal{E}$ is homotopy Gorenstein.

The relevant example has $R = C^*(BG; k)$ for a p-group and $R/I = k$ is a field of characteristic p. By the Eilenberg-Moore theorem, $\mathcal{E} = C_*(\Omega BG) = kG$. Now it is well known that kG is a Frobenius algebra and hence homotopy Gorenstein. It follows from Morita invariance that $R = C^*(BG)$ is homotopy Gorenstein. For 'commutative' rings, the local cohomology theorem follows from one formulation of the homotopy Gorenstein condition by taking homology.

JESPER GRODAL, M.I.T., USA:

Higher limits, group cohomology, and subgroup complexes

A homology decomposition of the classifying space BG of a finite group G can be thought of as a "topological induction theorem" telling that BG can be recovered from BP for various subgroups P together with "gluing" or "fusion" information. Topological induction theorems are related to algebraic induction theorems via certain higher limits. I will give a very small model for these higher limits in terms of subgroup complexes. Topologically, this model is very useful for calculating maps between classifying spaces or addressing rigidification issues. Algebraically, it yields refinements of classical algebraic induction theorems, for instance in the case of group cohomology.

BERNHARD HANKE, UNIVERSITÄT MÜNCHEN, GERMANY:

Witt classes of inner products and actions of finite p-groups

Let a cyclic group of odd prime order p act on a finite dimensional $\mathbb{Z}_{(p)}$-Poincaré duality space X. Extending work of Alexander and Hamrick, we prove a relation between the Witt classes associated to the $\mathbb{F}_p$-cohomology rings of the fixed point set of this action and of X. This is applied to show a similar result for actions of finite p-groups on $\mathbb{Z}_{(p)}$-homology manifolds.

Furthermore, we construct connecting homomorphisms on spectral sequences associated to a short exact sequence of filtered complexes. Specializing to the case of the Leray-Serre spectral sequence of the Borel fibration we compare the Witt classes of the fixed point set and the total space of $\mathbb{Z}/p$-actions on finite dimensional $\mathbb{F}_p$-Poincaré duality spaces on which the Bockstein operator acts as the zero map.

BRUNO KAHN, UNIVERSITÉ PARIS VII, FRANCE:

Orthogonal and symplectic analogues of a theorem of Rector

For a finite group G, Atiyah's work gives an isomorphism between the completion of its complex representation ring $R(G)$ with respect to the augmentation ideal and the representable K-theory of BG. This theorem was generalised by Atiyah and Segal to real representations, and also by Rector to representations over a finite field k. In a joint work with Hinda Hamraoui, we prove an analogue of Rector's theorem for orthogonal and symplectic representations of G over k, provided k is of characteristic $\neq 2$ and the order of G is prime to the characteristic of k. Our result is an isomorphism

$$ {}_{\varepsilon}\hat{L}_i(k[G]) \xrightarrow{\sim} [\Sigma^i BG, {}_{\varepsilon}L_0(k) \times B_{\varepsilon}O(k)^+] $$

for $\varepsilon = \pm 1$ and all $i \geq 0$, between the Karoubi L-groups of $k[G]$ completed with respect to the augmentation ideal of $R_k(G)$ and the representable ${}_\varepsilon L(k)$-theory of BG. The method is to reduce to the Rector case via a "homotopy limit theorem" whose archetype is the following well-known result: the natural map $KO \to KU^{h\mathbb{Z}/2}$ is a weak equivalence of spectra, where $\mathbb{Z}/2$ acts via complex conjugation. In practice, the proof involves various technical steps, among which comparison between topologies, reduction to characteristic 0 and also a rectification process in the case of symplectic K-theory.

Henning Krause, Universität Bielefeld, Germany:

Endofiniteness in stable homotopy theory

A CW-complex X is said to be *endofinite* if all stable homotopy groups $\pi_n^s(X)$ have finite length as module over the ring $\{X, X\}$ of stable self maps. This concept was introduced in joint work with Ulrike Reichenbach. In my talk I discussed some of the basic properties of endofinite complexes. For example, every endofinite complex has a stable wedge decomposition into indecomposables which is essentially unique. Moreover, a complete classification of all indecomposable endofinite complexes in terms of certain ideals in the category of finite spectra is possible. An example of particular interest is the classifying space BG of a finite group G. This follows from a simple criterion for endofiniteness in terms of singular homology.

Originally, endofiniteness is a concept from representation theory of finite dimensional algebras, which Crawley-Boevey introduced about 10 years ago: a module is *endofinite* if it is of finite length as a module over its endomorphism ring. At the end of my talk I discussed a few interesting examples of endofinite modules which arise in representation theory of finite groups. In fact, some of Rickard's idempotent modules are endofinite.

Ian J. Leary, University of Southampton, UK:

The subring of group cohomology generated by permutation representations

(Joint work with David Green and Björn Schuster.) For G a finite group and X a G-set of cardinality n there is an induced homomorphism $\rho_X^* : H^*(\Sigma_n) \to H^*(G)$. (Throughout, H^* will denote mod-p cohomology, and Σ_n the symmetric group.) We study the subring of $H^*(G)$ generated by the $\mathrm{Im}(\rho_X^*)$, where X ranges over all finite G-sets. In the case when $G = GL(n,p)$ we also consider the subrings obtained by allowing X to range over just the partial flags or partial frames in $\mathbb{F}_p^n$.

We study these subrings by comparing their varieties of ring homomorphisms to algebraically closed fields of characteristic p, using the technique introduced by two of us whilst studying the subring of $H^*(G)$ generated by Chern classes of complex representations. To each of the subrings considered, we associate a category with objects the elementary abelian p-subgroups of G, and morphisms a subset of the injective group homomorphisms. The variety for the subring is described as a colimit over this category, by analogy with Quillen's description of the variety for $H^*(G)$ itself.

For the subring of $H^*(G)$ generated by the $\mathrm{Im}(\rho_X^*)$ for X in some collection $\mathcal{F}$ the category is as follows. An injective group homomorphism $f : E \to F$ from one elementary abelian subgroup of G to another is a morphism in the category if and only if, for each X in $\mathcal{F}$, the E-sets X and $f^!X$ are isomorphic.

We deduce that for n large and $G = GL(2n,p)$, the varieties for $H^*(G)$, the subring generated by all G-sets, and the subring generated by just the actions of G on partial frames and partial flags are distinct.

Markus Linckelmann, Université Paris VII, France:

Cohomology in functor categories

The present talk is based on common work with Peter Webb; its content is motivated by the question as to what should be the classifying space $B(G,b)$ of a block b of a finite group G over a field k of prime characteristic p. Such a space, if it exists, should certainly have the following properties:

- if the block is the principal block, then $B(G,b)$ should be the $p-$localisation of the classifying space of G;

- in general, the cohomology of $B(G,b)$ should be the block cohomology algebra $H^*(G,b)$ (defined in terms of stable elements in the cohomology ring $H^*(P,k)$ of a defect group P of b with respect to the local structure of b).

Using recent work of Broto, Grodal, Jackowski, Levi, McClure, Oliver we describe how a possible solution for this problem comes essentially down to being able to compute higher limits of functors defined on certain categories of $p-$groups. We show that the source algebra of b gives rise to an idempotent in the stable endomorphism ring of $BP_p^{\hat{}}$, splitting off a stable summand whose cohomology is indeed $H^*(G,b)$. Thus we have a classifying spectrum of b; whether there is an actual space behind this, is still open at this moment.

Dagmar M. Meyer, Université Paris 13, France:

An equivariant version of the Kuhn–Schwartz non-realizability theorem

Lionel Schwartz has recently proved the following surprising theorem which goes back to a conjecture of Nick Kuhn: if the mod p cohomology of a space is finitely generated as a module over the Steenrod algebra $\mathcal{A}$ then it is finite (i.e. a finite dimensional $\mathbb{F}_p$-vector space). We generalize this result to the category of spaces over a fixed base space B. The mod p cohomology of a space X in this category is in a natural way an object in $H^*B-\mathcal{U}$, i.e. an unstable H^*B-$\mathcal{A}$-module. We prove that under certain conditions on B we have the following implication: if H^*X is finitely generated as an object in $H^*B-\mathcal{U}$ then it is finitely generated over H^*B. In particular, under certain conditions on the group G we can apply this to the case where X is the Borel construction on a G-space and B is the classifying space BG. In this way we obtain an equivariant analogue of Schwartz's non-realizability result. For compact Lie groups this equivariant version has also been derived more directly as a consequence of Schwartz's theorem by Dorra Bourguiba (Tunis) and myself.

Daniel K. Nakano, Utah State University, USA:

Cohomology for finite Chevalley groups via algebraic groups and Frobenius kernels

In this talk I will present recent results with C. Pillen and C. Bendel on relating the cohomology of finite Chevalley groups with the cohomology for their corresponding algebraic groups and Frobenius kernels. Our approach uses work of Donkin, and CPS (Cline-Parshall-Scott) to construct a spectral sequence which relates the cohomology of the Chevalley group to cohomology for the algebraic group. This construction has many applications. We will show how one can derive a nice Ext^1-formula for extensions between two simple modules for the finite Chevalley group. This formula can be used to show that self-extensions vanish for simple modules when the underlying field has more than p^2 elements. Other results on self-extensions will be given, thus answering several questions raised by S. Smith and J. Humphreys in the mid 1980s.

Bob Oliver, Université Paris 13, France:

Homotopy equivalences of p-completed classifying spaces of finite groups

This talk summarized joint work with Carles Broto and Ran Levi. To each finite group G and each prime p, we associate a finite category $\bar{\mathcal{L}}_p^c(G)$, defined as follows. The objects of $\bar{\mathcal{L}}_p^c(G)$ are the p-centric subgroups of G: those p-subgroups $P \leq G$ such that $C_G(P) = Z(P) \times C'_G(P)$ for some subgroup $C'_G(P)$ of order prime to p. And for each P and Q, $\mathrm{Mor}_{\bar{\mathcal{L}}_p^c(G)}(P,Q) = N_G(P,Q)/C'_G(P)$, where $N_G(P,Q) = \{x \in G \,|\, xPx^{-1} \leq Q\}$. The following is one of our main theorems:

Theorem A: For any prime p and any finite groups G and G', $BG^\wedge_p \simeq BG'^\wedge_p$ if and only if the categories $\bar{\mathcal{L}}_p^c(G)$ and $\bar{\mathcal{L}}_p^c(G')$ are equivalent.

Theorem A follows immediately from two propositions. The first says that for all p and G, $|\bar{\mathcal{L}}_p^c(G)|$ has the mod p homotopy type of BG; i.e., that $BG^\wedge_p \simeq |\bar{\mathcal{L}}_p^c(G)|^\wedge_p$. The second says that there are categories $\mathcal{L}_p^c(X)$, defined for all spaces X, such that $\mathcal{L}_p^c(BG) \simeq \mathcal{L}_p^c(G)$ (equivalent as categories), and $\mathcal{L}_p^c(BG^\wedge_p) \simeq \bar{\mathcal{L}}_p^c(G)$.

Theorem A is closely related to the following conjecture, due to Martino and Priddy. If $S \leq G$ and $S' \leq G'$ are Sylow p-subgroups, then an isomorphism $f : S \to S'$ is called *fusion preserving* if for any isomorphism $P \xrightarrow[\cong]{\varphi} Q$ between subgroups of S, φ is induced by conjugation in G if and only if the corresponding isomorphism $f(P) \xrightarrow[\cong]{\varphi'} f(Q)$ is induced by conjugation in G'.

Conjecture B (Martino-Priddy): For any prime p and any finite groups G and G', $BG^\wedge_p \simeq BG'^\wedge_p$ if and only if there is a fusion preserving isomorphism between Sylow p-subgroups of G and G'.

Conjecture B is known to hold in many cases; for example whenever $\mathrm{rk}_p(G) < p^2$.

Jonathan Pakianathan, University of Wisconsin-Madison, USA:

On commuting and non-commuting complexes

I will discuss joint work with Ergün Yalçın where we study various simplicial complexes associated to the commutative structure of a finite group G. We define $NC(G)$ (resp.

$C(G)$) as the complex associated to the poset of pairwise non-commuting (resp. commuting) sets of nontrivial elements in G.

We observe that $NC(G)$ has only one positive dimensional connected component, which we call $BNC(G)$, and we prove that $BNC(G)$ is simply connected.

Our main result is a simplicial decomposition formula for $BNC(G)$ which follows from a result of A. Björner, M. Wachs and V. Welker on inflated simplicial complexes. As a corollary we obtain that if G has a nontrivial center or if G has odd order, then the homology group $H_{n-1}(BNC(G))$ is nontrivial for every n such that G has a maximal noncommuting set of order n.

We discuss the duality between $NC(G)$ and $C(G)$, and between their p-local versions $NC_p(G)$ and $C_p(G)$. We observe that $C_p(G)$ is homotopy equivalent to the Quillen complexes $A_p(G)$, and obtain some interesting results for $NC_p(G)$ using this duality.

Finally, we study the family of groups where the commutative relation is transitive, and show that in this case, $BNC(G)$ is shellable. As a consequence we derive some group theoretical formulas for the orders of maximal non-commuting sets.

Geoff Robinson, University of Birmingham, UK

A cancellation theorem related to conjectures of Alperin and Dade

In a 1990 Astérisque paper, R. Staszewski and I showed that the Knörr-Robinson formulation of Alperin's weight conjecture was particularly amenable to Clifford-theoretic reductions, and proved a number of results which could be viewed as simplifying certain alternating sum calculations in the presence of normal subgroups, among other things.

We discuss some analogues of these results for Dade's projective conjecture (DPC). The main result is :

Theorem: Suppose that the formula appearing in Dade's projective conjecture fails to hold (for some defect d and some linear character, λ, of the central subgroup $Z = O_p(G)$) for the block B of RG, and that first $[G : Z(G)]$, then $|G|$, have been minimized subject to such a failure occurring. Then whenever $N \geq Z$ is a non-central normal subgroup of G, the block B covers blocks of N with defect groups strictly containing Z, and we have

$$\sum_{\sigma \in \mathcal{N}(G,Z)/G} (-1)^{|\sigma|} k_d(B_\sigma, \lambda) = \sum_{\sigma \in \mathcal{N}(N,Z)/G} (-1)^{|\sigma|} k_d(B_\sigma, \lambda)$$

for each linear character, λ, of Z, and each positive defect d.

Björn Schuster, Universität-GHS Wuppertal, Germany:

Transfers of Chern classes in BP-cohomology and Chow rings

(Joint work with Nobuaki Yagita.) Let G be a finite group and BG its classifying space. For a complex oriented cohomology theory h one can define Chern classes of complex representations of G in $h^*(BG)$. We are interested in studying the Mackey closure $\overline{Ch}_h(G)$ of the ring of Chern classes in $h^*(BG)$, the subring generated by transfers of Chern classes. For ordinary mod p cohomology, Green-Leary showed that the inclusion map $\overline{Ch}_{H\mathbf{Z}/p} \hookrightarrow H^*(BG; \mathbf{Z}/p)$ is an F-isomorphism, i.e., the induced map of varieties is a homeomorphism. Next consider BP or $K(n)$, the n-th Morava K-theory, at a fixed prime p. Call a group G "h-good" if $h^*(BG)$ is generated (as an h^*-module) by transferred Euler classes of representations of subgroups of G. If the Sylow p-subgroup of G is good, then so is G and one has an isomorphism $h^*(BG) \cong \overline{Ch}_h(G)$. Furthermore, it follows from work of Ravenel, Wilson and Yagita that G is good for BP if it is good for $K(n)$ for all n. Examples for groups that are $K(n)$-good for all n are the finite symmetric groups. Another typical case are p-groups of p-rank at most 2 and $p \geq 5$, where one has an isomorphism $BP^*(BG) \otimes_{BP^*} \mathbf{Z}_{(p)} \cong H^{even}(BG)$. However, Igor Kriz found a p-group G with $K(n)^{odd}(BG) \neq 0$.

Our calculations for the BP-cohomology of extraspecial 2-groups produce examples where $BP^*(BG)$ is not generated by transfers of Chern classes.

A second calculation gives torsion elements in the kernel of the cycle map from the Chow ring of BG to 2-local homology, using Totaro's factorisation of the cycle map through BP-theory.

Jeffrey H. Smith, Purdue University, USA:

A new E_∞-operad

This is joint work with Jim McClure.

Let $C^* : \mathcal{S} \to \mathcal{C}$ be the cochain functor from the category of spaces to the category of cochain complexes. For each $n \geq 0$, let D_n be the chain complex of natural transformations from the n-fold tensor of functors $C^* \otimes \cdots \otimes C^*$ to the functor C^*. Each of the chain complexes is contractible and together they form an operad which acts naturally on C^*X for $X \in \mathcal{S}$ showing that C^*X is an E_∞-algebra. However the operad D has some bad properties: it is too big as the complexes are uncountably generated in each degree, it is not an E_∞-operad and it is not clear that the tensor product of two D-algebras is again a D-algebra.

McClure and Smith have constructed a suboperad of D, the minimal sequence operad MS, which resolves these problems. The minimal sequence operad has the following properties: the chain complexes MS_n are of finite type and free as Σ_n-modules. There is a map of operads $MS \to MS \otimes MS$ and so the tensor product of two MS-algebras is an MS-algebra. There is a filtration of MS by suboperads F_nMS for which F_nMS is quasi-isomorphic to the operad obtained by taking the chain complex of the little n-cubes operad. An F_1MS-algebra is the same as a DGA. In an earlier paper we prove that F_2MS acts naturally on the Hochschild cohomology complex of a DGA.

Stephen D. Smith, University of Illinois at Chicago, USA:

Interactions of simple-group geometries with homology approximations of group cohomology

Work of Brown and Quillen as further developed by Webb produced an alternating sum formula for the p-part of group cohomology, summed over a suitable simplicial complex, typically of p-subgroups.

Ryba, Smith and Yoshiara verified the condition for the p-local geometries of many sporadic groups, and Smith and Yoshiara extended that work with general methods for homotopy equivalences of local geometries with standard collections of p-subgroups.

In the later 90s, homology approximations were further developed by Maginnis and Webb; and particularly by Dwyer, with more recent work by Grodal. Dwyer's demonstration that the collection of p-radical p-centric subgroups is ample allows the methods of Smith-Yoshiara to show that the remaining local geometries essentially provide the minimal complex on which the group cohomology can be thus computed. More recent work of Smith, and especially of Masato Sawabe, provides further methods for demonstrating some of the homotopy equivalences in a standardized way, and hence further explaining the minimal properties of the 2-local geometries.

PETER SYMONDS, UNIVERSITY OF MANCHESTER, UK:

The cohomology of permutation modules and the Segal conjecture

We show that the mod-p cohomology of a finite group, considered as a global Mackey functor, contains every simple cohomological Mackey functor as a composition factor. Our proof uses methods from stable homotopy theory in an essential way. (In particular we use the (proved) Segal Conjecture.)

One consequence is an easy proof of Mislin's theorem on group homomorphisms inducing an isomorphism on mod-p cohomology.

For another, recall that the permutation projective (=trivial source) modules for a group G over an algebraically closed field of characteristic p are parametrised by a pair (P, V), up to conjugacy, where P is a p-group and V is a simple module for $N_G(P)$. Denote this module by $\mathcal{P}_{P,V}$.

Theorem: $H^*(G, \mathcal{P}_{P,V}) = 0$ if and only if $C_G(P)$ acts non- trivially on V.

OLYMPIA TALELLI, UNIVERSITY OF ATHENS, GREECE:

On complete resolutions

A group G is said to have periodic cohomology with period q after k-steps if the functors $H^i(G, -)$ and $H^{i+q}(G, -)$ are naturally equivalent for all $i > k$.

It was conjectured by G. Mislin and O. Talelli that periodicity after k-steps is the algebraic characterization of those groups G which admit a finite dimensional free G-CW-complex X homotopy equivalent to a sphere, and it was proved for all groups in HF, the class of hierarchically decomposable groups introduced by P. Kropholler, for which there is a bound on the order of the finite subgroups.

A. Adem and J. Smith proved that if the periodicity isomorphisms for a group G are induced by cup product with an element in $H^q(G, \mathbb{Z})$ then G admits a finite dimensional free G-CW-complex X homotopy equivalent to a sphere.

Here we show that for a group G with period q after k-steps to have the periodicity isomorphisms given by cup product with an element in $H^q(G, \mathbb{Z})$ is equivalent to the projective dimension of the coinduced module $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}G, \mathbb{Z})$ being finite. We then show that for a group G in HF the coinduced module $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}G, \mathbb{Z})$ has finite projective dimension if and only if the group G admits a complete resolution.

We know however that if a group G has periodic cohomology after some steps then G admits a complete resolution. It now follows that if a group G in HF has period q

after k-steps then the periodicity isomorphisms are given by cup product with an elment in $H^q(G, \mathbb{Z})$.

JAQUES THÉVENAZ, UNIVERSITÉ DE LAUSANNE, SWITZERLAND:

Endo-trivial modules

(Joint work with Jon F. Carlson.) We prove that the group $T(G)$ of endo-trivial modules for a non-cyclic finite p-group G is detected on restriction to the family of subgroups which are either elementary abelian of rank 2 or (almost) extraspecial. This result is closely related to the problem of finding the torsion subgroup of $T(G)$. When p is odd, we can eliminate extraspecial groups of exponent p^2 and almost extraspecial groups from the detecting family. The question of eliminating the other extraspecial groups remains an open question. We also give the complete structure of $T(G)$ when G is dihedral, semi-dihedral, or quaternion. Finally we deduce from our main result a detection theorem for the Dade group of all endo-permutation modules for G.

PETER WEBB, UNIVERSITY OF MINNESOTA, USA:

Computing resolutions without computing kernels

I describe a way to produce resolutions of the trivial module over group rings which seems to be quite effective computationally, and which also conveys with some clarity the structural properties of group cohomology (cup products, restriction, corestriction). The resolutions produced have polynomial growth and avoid the computation of the kernels of boundary homomorphisms. The difficult part of the calculation is lifting maps along resolutions, which comes down to solving equations over group rings. This procedure has been implemented in GAP in the case of group rings of p-groups over the field of p elements. If time permits I will describe:

1. The easiest form of the construction which works for 2-groups over a field of characteristic 2.
2. The slightly more complicated construction which works for p-groups with p odd (and indeed in greater generality).
3. The special properties of the resolution with regard to restriction and corestriction maps, and cup products.
4. How to solve equations over group rings of p-groups in characteristic p. I use a method which exploits the special properties of Jennings' basis for the group ring. There is available a fast algorithm for base change between the standard and Jennings' bases. It is analogous to the fast Fourier transform algorithm.
5. How to strip off contractible summands to obtain a minimal resolution.
6. Computational performance.

CLARENCE WILKERSON, PURDUE UNIVERSITY, USA:

Applications of generalized quaternionic tori

A *generalized quaternionic torus* H is a 2-compact group isomorphic to $(SU(2)^k)/E_H$, for some $E_H \subset Center(SU(2)^k) = (Z/2Z)^k$.

Theorem I. a) If X is a connected 2-compact group of rank k and X contains a generalized quaternionic torus of rank k, then $W(X)$ has a nontrivial center, represented on its action on $\pi_1(T_X)$ as multiplication by -1.

b) Conversely, if $W(X)$ contains -1, then there is at least one choice of a generalized quaternionic torus $H \subset X$ of rank = rank X.

Applications: Recall that a Lie group or 2-compact group X is *almost-simple* if and only if $\pi_1(T_X) \otimes Q$ contains no proper $Q_2(W)$-submodules.

1) Let X be a connected almost-simple 2-compact group such that X contains a subgroup $H = SU(2)^k$ for k =rank(X), then $H^*(BX) \approx_{A_2} H^*(BSp(k))$.

2) If X is a connected almost-simple 2-compact group with $W(X)$ isomorphic to $W(G)$, for G one of the exceptional Lie groups G_2, $DI(4)$, F_4, $Ad(E_7)$, or E_8, then $N_X(T_X) \approx N_G(T_G)$.

Items in this abstract are joint work with Antonio Viruel and Bill Dwyer.

Ergün Yalçin, McMaster University, Canada:

Set covering and Serre's theorem on the cohomology algebra of a p-group

Let G be a p-group which is not elementary abelian. The *cohomology length* of G, denoted by $chl(G)$, is defined as the minimum number of one dimensional classes $x_1, \dots, x_n \in H^*(G, \mathbb{Z}/p)$ such that $x_1 x_2 \cdots x_n = 0$ when $p = 2$, and $\beta(x_1)\beta(x_2)\dots\beta(x_n) = 0$ when $p > 2$. In this work, we study $chl(G)$ by defining a new group theoretical invariant, denoted by $s(G)$, which is relatively easier to compute and closely related to $chl(G)$.

A set $S \subseteq G$ is called a *representing set* if it includes at least one non-central element from each maximal elementary abelian subgroup of G. We define $s(G)$ as the minimum cardinality of a representing set for G (we assume G is not p-central.) When G is an extra-special p-group, we prove that $chl(G) \leq s(G)$, where the equality holds when G has self-centralizing maximal elementary abelian subgroups. Studying $s(G)$, we prove the following:

Theorem: If G is a p-group and $k = \dim H^1(G, \mathbb{Z}/p)$, then $chl(G) \leq p+1$ if $k \leq 3$ and

$$chl(G) \leq (p^2 + p - 1)p^{[\frac{k}{2}]-2} \quad \text{if} \quad k \geq 4.$$

Theorem: Let G_n be an extra-special 2-group isomorphic to an n-fold central product of D_8's. Then,

$$chl(G_n) = \begin{cases} 2^{n-1} + 1 & \text{if} \quad n \leq 4, \\ 2^{n-1} + 2^{n-4} & \text{if} \quad n \geq 5. \end{cases}$$

Alexander Zimmermann, Université de Picardie, France:

Auto-equivalences of derived categories acting on group cohomology

Let S, S_1, S_2 be commutative rings, let R be a complete discrete valuation ring of characteristic 0 with residue field k of characteristic p and let G be a finite group. In joint work with Raphaël Rouquier I defined and studied the group of autoequivalences of standard type $TrPic_S(A)$ of an S-algebra A which is finitely generated projective over S.

The stabilizer $HD_S(G)$ of the trivial module S in $TrPic_S(SG)$ acts on $H^*(G,S)$. A ring homomorphism $S_1 \to S_2$ induces a group homomorphism $HD_{S_1}(G) \to HD_{S_2}(G)$ and this induces an $S_1(HD_{S_1}(G))$-module homomorphism $H^*(G,S_1) \to H^*(G,S_2)$. Let Q be a p-subgroup of G and let $HSplen_k(G)$ be the subgroup of $HD_k(G)$ formed by splendid equivalences. Then the Brauer construction defines a group homomorphism $HSplen_k(G) \to TrPic_k(C_G(Q))$. The action of those elements in $HSplen_k(G)$ whose image still fixes the trivial module commutes this way with restriction to and transfer from $C_G(Q)$. If the Sylow p subgroups of G are abelian, lifting to R, a theorem of Roggenkamp and Scott applies to characterize the action of splendid equivalences on cohomology.

Report written by Dagmar M. Meyer

EMAIL ADDRESSES AND INTERNET PAGES OF THE PARTICIPANTS

Alejandro Adem	adem@math.wisc.edu
Jonathan L. Alperin	alperin@math.uchicago.edu
Gregory Arone	arone@maths.abdn.ac.uk
	http://www.maths.abdn.ac.uk/maths/department/
Carlos Broto	broto@mat.uab.es
	http://mat.uab.es/broto/
William Browder	browder@math.princeton.edu
Jon F. Carlson	jfc@sloth.math.uga.edu
	http://www.math.uga.edu/~jfc/
Tammo tom Dieck	tammo@uni-math.gwdg.de
Karin Erdmann	erdmann@maths.ox.ac.uk
Mark Feshbach	feshbach@math.umn.edu
David J. Green	green@math.uni-wuppertal.de
	http://www.math.uni-wuppertal.de/~green/index_en.html
John Greenlees	J.Greenlees@sheffield.ac.uk
	http://www.shef.ac.uk/~pm1jg/
Jesper Grodal	jg@math.mit.edu
	http://www-math.mit.edu/~jg/
Bernhard Hanke	hanke@rz.mathematik.uni-muenchen.de
	http://www.mathematik.uni-muenchen.de/personen/hanke.html
Manfred Hartl	hartl@univ-valenciennes.fr
Hans-Werner Henn	henn@math.u-strasbg.fr
	http://www-irma.u-strasbg.fr/~henn/
Thorsten Holm	thorsten.holm@mathematik.uni-magdeburg.de
	http://fma2.math.uni-magdeburg.de/~holm/
Stefan Jackowski	sjack@mimuw.edu.pl
	http://www.mimuw.edu.pl/~sjack/
Bruno Kahn	kahn@math.jussieu.fr
	http://www.math.jussieu.fr/~kahn/
Steffen König	koenig@mathematik.uni-bielefeld.de
	http://www.mathematik.uni-bielefeld.de/~koenig/
Henning Krause	henning@mathematik.uni-bielefeld.de
Ian Leary	ijl@maths.soton.ac.uk
Ran Levi	ran@maths.abdn.ac.uk
Markus Linckelmann	linckelm@math.jussieu.fr
Dagmar M. Meyer	meyerd@member.ams.org
	http://www-math.univ-paris13.fr/~dagmar
Haynes Miller	hrm@math.mit.edu
Daniel Nakano	nakano@sunfs.math.usu.edu
	http://www.math.usu.edu/~nakano/
Dietrich Notbohm	notbohm@mpim-bonn.mpg.de

Robert Oliver	bob@math.univ-paris13.fr
	http://zeus.math.univ-paris13.fr/~bob
Jonathan Pakianathan	pakianat@math.wisc.edu
	http://www.math.wisc.edu/~pakianat/
Stewart B. Priddy	priddy@math.nwu.edu
Volker Puppe	Volker.Puppe@uni-konstanz.de
Jeremy Rickard	j.rickard@bris.ac.uk
Geoffrey R. Robinson	g.r.robinson@bham.ac.uk
	http://www.mat.bham.ac.uk/grr/
Björn Schuster	schuster@math.uni-wuppertal.de
	http://www.math.uni-wuppertal.de/~schuster/
Jeff Smith	jhs@math.purdue.edu
Stephen D. Smith	smiths@math.uic.edu
	http://www.math.uic.edu/~smiths/
Nicole Snashall	n.snashall@mcs.le.ac.uk
Urs Stammbach	stammb@math.ethz.ch
	http://www.math.ethz.ch/~stammb/
Peter Symonds	peter.symonds@umist.ac.uk
Olympia Talelli	otalelli@nikias.cc.uoa.gr
Jacques Thevenaz	jacques.thevenaz@ima.unil.ch
	http://www.unil.ch/ima/docs/Personnes/jthevenaz.html
Antonio Viruel	viruel@uma.es
Peter J. Webb	webb@math.umn.edu
Thomas Weigel	weigel@maths.ox.ac.uk
Clarence Wilkerson	wilker@math.purdue.edu
Nobuaki Yagita	yagita@mito.ipc.ibaraki.ac.jp
Ergün Yalçın	yalcine@math.mcmaster.ca
	http://www.fen.bilkent.edu.tr/~cvmath
Alexander Zimmermann	alexander.zimmermann@u-picardie.fr
	http://delambre.mathinfo.u-picardie.fr/alex/azim.html

MAIL ADDRESSES OF THE PARTICIPANTS

Prof. Dr. Alejandro Adem
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive
Madison WI, 53706-1388
USA

Prof. Dr. Jonathan L. Alperin
Department of Mathematics
The University of Chicago
5734 South University Avenue
Chicago , IL 60637-1514
USA

Dr. Gregory Arone
Department of Mathematics
University of Aberdeen
GB-Aberdeen AB24 3UE

Prof. Dr. Carlos Broto
Departament de Matematiques
Universitat Autonoma de Barcelona
E-08193 Bellaterra Barcelona

Prof. Dr. William Browder
Department of Mathematics
Princeton University
Fine Hall
Washington Road
Princeton , NJ 08544-1000
USA

Prof. Dr. Jon F. Carlson
Department of Mathematics
University of Georgia
Athens , GA 30602-7403
USA

Prof. Dr. Tammo tom Dieck
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5
37073 Göttingen

Prof. Dr. Karin Erdmann
Mathematical Institute
Oxford University
24 - 29, St. Giles
GB-Oxford OX1 3LB

Prof. Dr. Mark Feshbach
School of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.
Minneapolis , MN 55455
USA

Dr. David J. Green
Fachbereich 7: Mathematik
U-GHS Wuppertal
42097 Wuppertal

Prof. Dr. John Greenlees
Dept. of Mathematics
University of Sheffield
Hicks Building
GB-Sheffield S3 7RH

Jesper Grodal
School of Mathematics
Institute for Advanced Study
Einstein Drive
Princeton, NJ 08540
USA

Dr. Bernhard Hanke
Mathematisches Institut der LMU
Theresienstr. 39
80333 München

Prof. Dr. Manfred Hartl
ISTV
Universite de Valenciennes
F-59304 Valenciennes

Prof. Dr. Hans-Werner Henn
U.F.R. de Mathematique et
d'Informatique
Universite Louis Pasteur
7, rue Rene Descartes
F-67084 Strasbourg Cedex

Dr. Thorsten Holm
Fakultät für Mathematik
Otto-von-Guericke-Universität
Magdeburg
Postfach 4120
39016 Magdeburg

Prof. Dr. Stefan Jackowski
Instytut Matematyki
Uniwersytet Warszawski
ul. Banacha 2
02-097 Warszawa
POLAND

Prof. Dr. Bruno Kahn
U. F. R. de Mathematiques
Case 7012
Universite de Paris VII
2, Place Jussieu
F-75251 Paris Cedex 05

Prof. Dr. Steffen König
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
33501 Bielefeld

Dr. Henning Krause
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
33501 Bielefeld

Dr. Ian Leary
Faculty of Mathematical Studies
University of Southampton
GB-Southampton , SO17 1BJ

Dr. Ran Levi
Dept. of Mathematical Sciences
University of Aberdeen
Meston Building 339
GB-Aberdeen AB24 3UE

Dr. Markus Linckelmann
U.F.R. de Mathematiques
Case 7012
Universite de Paris VII
2, Place Jussieu
F-75251 Paris Cedex 05

Dr. Dagmar M. Meyer
Departement de Mathematiques
Universite Paris 13
Inst. Galilee
Avenue Jean-Baptiste Clement
F-93430 Villetaneuse

Prof. Dr. Haynes Miller
Department of Mathematics
2-237 MIT
Cambridge , MA 02139
USA

Dr. Daniel Nakano
Dept. of Mathematics and Statistics
Utah State University
Logan , UT 84322-3900
USA

Prof. Dr. Jeremy Rickard
School of Mathematics
University of Bristol
University Walk
GB-Bristol BS8 1TW

Dr. Dietrich Notbohm
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5
37073 Göttingen

Prof. Dr. Geoffrey R. Robinson
School of Maths and Statistics
The University of Birmingham
Edgbaston
GB-Birmingham B15 2TT

Prof. Dr. Robert Oliver
Departement de Mathematiques
Institut Galilee
Universite Paris XIII
Av. J.-B. Clement
F-93430 Villetaneuse

Dr. Björn Schuster
Fachbereich 7: Mathematik
U-GHS Wuppertal
Gaußstr. 20
42119 Wuppertal

Dr. Jonathan Pakianathan
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive
Madison WI, 53706-1388
USA

Prof. Dr. Jeff Smith
Dept. of Mathematics
Purdue University
West Lafayette , IN 47907-1395
USA

Prof. Dr. Stewart B. Priddy
Dept. of Mathematics
Lunt Hall
Northwestern University
2033 Sheridan Road
Evanston , IL 60208-2730
USA

Prof. Dr. Stephen D. Smith
Dept. of Mathematics, Statistics
and Computer Science, M/C 249
University of Illinois at Chicago
851 St. Morgan
Chicago , IL 60607-7045
USA

Prof. Dr. Volker Puppe
Fakultät für Mathematik
Universität Konstanz
D 201
Postfach 5560
78434 Konstanz

Dr. Nicole Snashall
Dept. of Mathematics
University of Leicester
University Road
GB-Leicester , LE1 7RH

Prof. Dr. Urs Stammbach
Mathematik Departement
ETH Zürich
ETH-Zentrum
Rämistr. 101
CH-8092 Zürich

Dr. Peter Symonds
Dept. of Mathematics
UMIST (University of Manchester
Institute of Science a. Technology)
P. O. Box 88
GB-Manchester , M60 1QD

Prof. Dr. Olympia Talelli
Dept. of Mathematics
University of Athens
Panepistemiopolis
157 84 Athens
GREECE

Prof. Dr. Jacques Thevenaz
Institut de Mathematiques
Universite de Lausanne
CH-1015 Lausanne Dorigny

Dr. Antonio Viruel
Depart. de Algebra, Geometria y
Topologia, Facultad de Ciencias
Universidad de Malaga
E-29071 Malaga

Prof. Dr. Peter J. Webb
Department of Mathematics
University of Minnesota
206 Church Street S.E.
127 Vincent Hall
Minneapolis , MN 55455
USA

Dr. Thomas Weigel
Mathematical Institute
Oxford University
24 - 29, St. Giles
GB-Oxford OX1 3LB

Prof. Dr. Clarence Wilkerson
Dept. of Mathematics
Purdue University
West Lafayette , IN 47907-1395
USA

Prof. Dr. Nobuaki Yagita
Dept. of Mathematics
Ibaraki University
I-I Bunkyo 2-chome
Mito-Shi
Ibaraki Ken 310
JAPAN

Dr. Ergün Yalçın
Department of Mathematics
Mc Master University
1280 Main Street West
Hamilton , Ont. L8S 4K1
CANADA

Dr. Alexander Zimmermann
UER de Mathematiques
Universite de Picardie
33 rue Saint Leu
F-80039 Amiens Cedex